Strength of Materials
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Lecture – 19

This is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department IIT Roorkee. I am going to deliver my lecture 19 on the course of Strength of Materials and this course is developed under the national programs on technology enhanced learning NPTEL. Pride to start this lecture, I just want to refresh you see in whatever we discussed in the lectures in previous lectures as such.

In the previous lectures we discussed about that why we have you know thin cylinder then what the types of stresses which are inducing, when it is subjected by the internal pressure P. And we found that they were two dominant you know like stresses were there and corresponding strains were there like that, like the longitudinal stress and longitudinal strain and the other one is the hoop stress or we can say rather it is a circumference in the stress and circumferential strains.

And, then what the interactions are there, so if you want to compute simultaneous effect, then, you know like we consider the Poisson ratio and then you see know like we also discussed there actually. When these two types of stresses or the strains are there in an thin cylinder, Then how you know like we can compute the total volumetric stress and volumetric strain particular. And then we found that in the volumetric strain, there were contributions from both of the strain component like the longitudinal plus two types of hoop or circumferential strain, if you want to compute the volumetric strain.

And, then you see we just we were trying to you know like see that if we have a thin cylinder without there is means without the two extreme corners of any spherical or any kind of shape or if we have the thin cylinder with the spherical shape. Then what the differences are there if the spherical shape has different thickness and the thin cylinder wall is the is having a different thickness.

So, this kind of relations, which we developed and then we found that actually, if this thin cylinder or a shaft it is just low you know like subjected by a twisting moment or a torque or couple. Then, we derived that you know like these only where whenever shaft is rotating or it if it is not rotating and only this there is a kind of twisting moment is applied.

Then, there is pure state of stresses acting in this cases, because in the previous case we found that whatever the longitudinal and these hoop stresses are there they are mutually perpendicular to each other. But, they are only the normal stress component, but whenever there is a torque applied, then only we you see the kind of shear stresses are acting and the corresponding shear strain is there.

And also, we just we have trying to develop the relationship in between those stresses that actually what will happen, if you know like these kind of stresses are there that what is the shearing stress. And, since we are talking about the shearing stress and corresponding shearing strain, then we found that there is one shear modulus of rigidity is there to compute that part.

And, let then also we found that, the if we are talking about shear strain, but if we have a spring component, then it also exhibit the similar kind of trained as we found in a previous case. Like in a this, solid bar or any kind of rod or shaft is there and if it is under the action of any twisting movement or couple.

So, even we have a spring then similar kind of you know like that twisting movements are occurring and it depends on that what the number of coils are there and you see how much torque is applied. And, what will be due to the under the application of these torque or due to application of these torque, what will be the maximum shearing strains shearing stresses are inducing.

And, then you see what that shape or the sizes are there of the different at the different types of coils. So, all these are if you want to design either you see the circular shaft or either the bar or even about the spring, then these are the key parameters, which always you see. One has to consider a just you know like for the safety measures or you see if you want to strengthen your either the bar or the springs kind of that.

So, you see this kind of discussion, which we have already made in the previous lectures, and we also found that you see that whenever is there is a close this spherical shapes are there in you know like in the thin cylinder. Then, there is a chance of the distortion or there is a chance of the maximum deformation can be occur, wherever the joined points are there or wherever you see these you know like the two different sections are clubbing together.

So, to remove that actually there was a standard you know like the thickness ratio there and if you are keeping the thickness of this cylinder wall is you know like 2.4 times the thickness of the spherical walls. Then we found that there is very, very less chance to be having a kind of distortion at the junction point.

So, this you see you know like these are all the real key points and also you see you know like just I just want to you know pride to start the next lecture. I just want to refresh your formulas also that you see, if you are you know like measuring the longitudinal strain then again you see the key parameters are this P times of Pd divided by 4 times of e into T into 1 minus 2 mu. So, because you see you know like these are the key formulas, so while solving the numerical problems one has to remember.

And, then you see you know like if you are talking about the hoop stresses, then it is epsilon to which we are you know like denoting in the previous lecture in that is nothing but, equals to Pd by 4 T e into 2 minus epsilon. And, if we are talking about the volumetric strain and as I told you recently that actually volumetric strain is nothing but, equals to this longitudinal stress plus 2 times of hoop stresses.

So, again you see we can simply compute that Pd by 4 T e into this 5 minus 4 times of this Poisson ratio, so these three are the key you know like formula and then you see if you are talking about the thickness ratio. Then, again depends on absolutely as I told you that we have to be very much careful by taking you know like that what the spherical the you know like the ends are there and what exactly the thin cylinder is there the walls are there.

So, even you see the T 1 by T 2, the T 1 if the wall thickness of the cylinder and T 2 is the spherical walls are there at the extreme end, if T 1 by T 2 if you want to compute then it is nothing but, equals to 1 minus mu divided by 2 minus mu. So, you see by keeping the mu, mu is nothing but the property of the material, and if you are talking about the mild steel then we were using 0.3 or if you see the different kind of you know like the material is there then Poisson ratio. So, is different but again you see there is a limiting part is there for the Poisson ratio, because Poisson ratio is simply computing the lateral strain by the longitudinal strain.

So, again you see we have to you know like concern that which material we are using and then corresponding what the thickness ratios are there just to avoid the kind of distortion at the junction point. So, you see these are the key features, which we discussed in the previous cases; but, now you see again we are goanna discuss that actually whenever a shaft is there and when it is twisting.

And definitely you see as we discussed that actually you know like that twisting moments are there and you see, because of the twisting moment or the couple or the torque. You know like the shearing stresses are there and there is a maximum shearing stresses there, which you need to consider, and then the two other parameters are the size and the shape of a material or we can say an object.

So, again you see if you are talking about all these parameter, then again we found that the shear modulus of rigidity is again a key parameter. And to compute that since the Young's modulus of a elasticity in you know like in the normal stress and strain component is the basic property of materials.

So, similarly you see here if you are talking about the shear stress and shear strain, the shear modulus of rigidity is again a key feature and it is a property of materials. So, which material you are using corresponding values are coming for the value of G, and G is nothing but, equals to the shearing stress by shearing strain or we can say tau by gamma.

So, again we are computing here, you know like the shear stresses, which are coming due to the torque and we are also measuring the corresponding distortion in the material due to the application of the shear forces. So, you see since we are computing and we are saying that you see you know like the shear modulus of rigidity as well as the Poisson ratio is absolutely occurring within the phenomena.

So, we are saying that this Hooke's law is valid for those kind of deformation or we can say that the elastic deformation is there. So, we are not going beyond the region only our discussion is well defined the period of the elastic region. So, that is why you see you know all either the elastic modulus of this rigidity or this Young's modulus of elasticity or we can say that the bulk modulus of elasticity or the Poisson ratio. All four, you know like the parameters which are very much valid for the elastic deformation is applicable, because we are only we are only applying the load for the Hooke's law.

So, now you see here whatever we discussed in the previous lecture, now we just want to derive in the forms of the mathematical formulation. So, again we are going for the Torsion formula.

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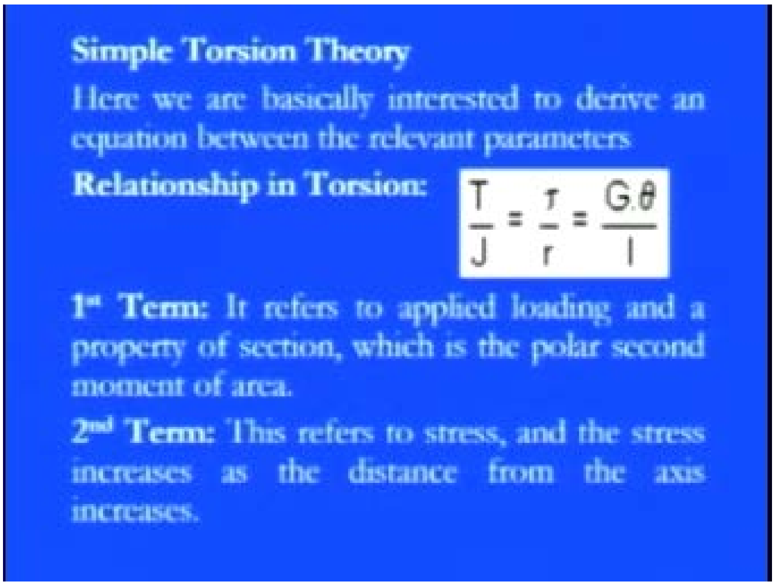

So, here you see you know like we are starting with the Simple Torsion Theory, so basically you know like we are interested to derive an equation between all the relevant parameters, which we discussed. We discussed that actually we have a circular bar and we are applying the torque. So, due to the torque the shear stresses are there and due to the shear stress, there is due to the shear stress is which are occurring on the circumferential part of you know this bar.

So, due to that there is an angular twist is there, and due to the angular twist we have a resultant shear strains are there. So, if you just go by 1 by 1, then you will find that we have a torque, which is applied on and as the this circular bar and due to this torque, we have a shear stresses which are inducing due to the application of this torque.

So, tau is there so starting from T to go to the tau and, then when we have a shear stresses definitely the corresponding shear strains are there, because due to the this shearing stresses we have a angular twist is there. And, due to the angular twist what we have we have a distortion and we want to measure the distortion, so it is nothing but, we can only measure by the shearing strain. So, once you have shearing strain once you have the shearing strain, so shearing stress and shearing strain can be computed with the using of shear modulus of rigidity, so G is there.

So, now you see with all those perfect parameters, we just want to relate these passive parameters. So, the relationship in the torsion will be coming as T by J is equals to tau by r is equals to G theta by l. So, now you see you will find that there are three different terms and these three terms are exactly equal.

So, what these terms signifies, so these terms signifies that we have the first term which is T by J. So, the first term refers to the applied loading that actually what exactly the torque is there how they are applying in which direction it is to be a applied, and when the torque is applied you see what the corresponding property of the sections means what is the affected area is.

So, you see you know like if I am saying that T is the torque applied and J is the polar section or moment of inertia is there. So, what is the area moment means actually here, what we are taking; we are not going for the mass moment of inertia, we are going for the area moment of inertia and since it is a polar section is there.

So, what we are doing here? We are simply dividing by it this T by J, so whatever the property which is getting, it is exactly equal to the second part of the second term which is coming as tau by rho. So, you see the tau is nothing but the shearing stresses which is coming due to the application of this torque.

So, you see the shearing stresses are there and if you are saying that the shearing stresses is there which is increasing as we increase the radius. So, you see here this is a quite to significant part is there that if we are starting to apply the torque always as I told you that it is increasing from you know like central point and it is at maximum at the circumference part is there.

So, this relation is well very much you know like valid that actually as we move further means as we are starting from the centrifugal part the origin as we are moving towards the outer circumference. Then, we found that actually it is simply increasing and it is the maximum at the outer periphery of this cylinder or we can say the rotating shaft.

So, this as you move as you move you know like or as you increase the value of r, the shearing stress is also increasing. So, these two sections T by J which is exactly equals to tau by r, and then the third term gives the relationship between the shear modulus of rigidity into the angular twist divided by the l.

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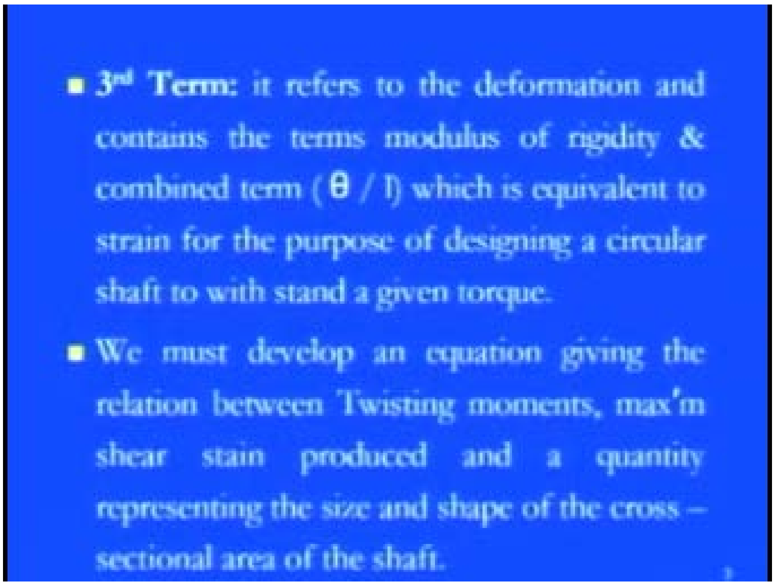

So, third term refers to the deformation and that contains the term of modulus of rigidity that G and the combined term of the angular displacement that is theta by l. So, which is nothing but equivalent to the strain for a purpose of designing a circular shaft to withstand a given torque. That means you see whatever the torque applied is there then how much the relative displacement is there, in terms of you know the circumference with the corresponding to the length of that bar.

So, you see this will give you a clear cut feeling that actually, since if we have you know like more and more this length is there. If you are increasing the length of you know like starting from 0 to 1 2 and if you have the 10 millimeter of the length of this bar is there, so at the extreme corner you will find that the more angular twist.

So, generally you see you know like if you remember the previous case, then we found that if AB was the length is there you see in the you know like in the circular bar. In which you see whenever there is no loading condition is there, so all those layers were all those layers are well setup within the, this circular bar. But as you as we applied the torque you see the B was shifted to B to B dash, but that B 2 B dash shifting was at the extreme corner. But whatever the a was there you see it was exactly at the same point even after the loading.

So, the meaning is pretty simple that as you know like move this l to as you are moving to the l right from the origin to the extreme end then you will find that the theta is also increasing, and since you see know like the distortion is more. So, you can simply capture these kind of distortion or the deformation in terms of the strain and this strain is known as the shear strain.

So, the meaning is pretty simple that all those terms are well signified that if we have T by J, which is exactly equals to this tau by r which is exactly equals to G theta by l. So, T by J is also giving you the similar kind of feeling that as you know like when you apply the torque, the effective area is always maximum, whenever you are at the extreme end. So, whatever you see the polar moment of inertia is there, it all as you increase those things you see you know like the torque is also the effect of torque is also more. So, all three terms are giving you the similar kind of feeling and that is why we can say that for any kind of rotational shaft, this formula is very well valid.

So, now you see we must develop an equation which is giving the relationship between the twisting moment, maximum shear strain you know produced, because of the shear stresses are there and a quantity which represents the size and shape of the cross sectional area of the shaft.

Because, you see if you know the cross sectional area of the shaft based on what the size and shapes are there, then you can also find out that. Actually, how the distortion can be taken place or we can say the, how these microstructures are to be disturbed, whenever the torque is to be applied on the circular shaft. So, that is why you see, we can simply signify that actually all these three terms are very much valid whenever you know like the torque application is there on in a circular shaft. So, now you see you know like we just wanted to visualize those concepts, which we discussed recently that we have a circular shaft as it you know can see on the figure on the screen.

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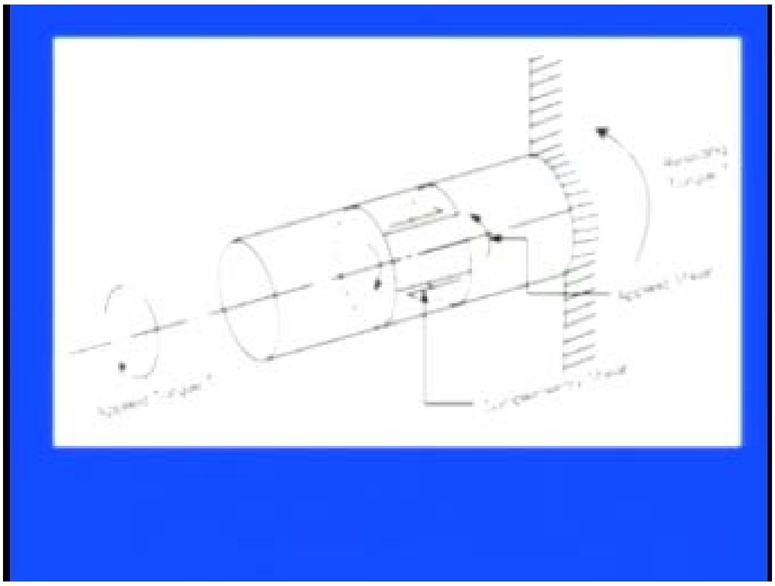

And, the circular shaft is fixedly rigidly fixed at one corner and this at another corner it is free. So, now what we have doing here we are simply applying the torque in this a clockwise direction, so you can see this on this particular extreme corner that there is load or the torque application is there on this free end. So, due to the load application you see this torque will always try to tend the shaft in any respective direction.

So, you see you know like our shaft is having you know like the kind of rotation it has a tendency to move towards this direction wherever the torque application is there. But you see if you are saying that under the torque application if our shaft is in equilibrium position, so definitely you see we have a resisting torque exactly equal and opposite to the applied torque.

So, then we can see you know like when you are applying the torque, there is the internal mechanism is there, because of the microstructure of this uniform bar, through which we can simply apply or we can say through which the microstructure valve micro structures are always trying to put you know opposition force to just balance whatever the disturbances are there.

So, what we have you see, if you are simply cutting a portion which you can see here, we have the a simple segment. So, if you want to analyze the segment then we found that due to the applied torque. Now, you see here this is the tendency to moving of kind of section, so whatever the microstructures or we can say the layers are there of this particular shaft they will just tend to move here. So, what we are doing here we are simply taking the small section of this particular bar and we found that after you know like observing that particular phenomena.

We found that all those you know like right from the elements which are lying along the central line of this particular see here. They have the less tendency, but as we you know moving from center to outward direction; that means, on the circumference we found that this particular tendency to move in an a particular direction; that means. Where the torque is applying they have more tendency to move in that; that means, you see the effect of the torque is more on the circumferential side rather than it is on the this axis part.

So, that is what you see you know like the shear stresses, whatever the shear stresses are generating, they are always not along the axis they are always along the circumference or we can say they are always along the plane part. So, here due to that you see due to the rotation, now you can simply see that we have a shear stress which is applying in this direction.

So, this you see you know like this is the shear stress which is inducing because of this torque, so but if you are saying that the shaft is in well balanced it is in the equilibrium position. So, definitely there is a counter this or we can say the counter torque or the resisting torque is there. So, this is you see the resistant torque is there, and this is resistant torque which is coming and due to that you see what we have? We have a complimentary shear stresses are there. So, what exactly things are due to the torque we have a tendency in an object the two put the shear forces.

So, you see here, these are the applied shear force they applied shear stresses are coming due to that part due to this particular you know like applied torque and the corresponding you know like due this these resisting torque. We have you know like these complimentary shear stresses are there. So, this is the applied shear stress, this is the complementary shear stress.

And if they are you know like and if you see the notations then you will find that this has a tendency to move the object in a clockwise direction and this has a tendency to move in an just repulsive direction. So, if you are shear stress and complementary shear stress if they are exactly opposite and equal.

Then, all we can say that the total you know like the this whatever under this particular our bar, it is exactly equilibrium or we can say just they can maintain the equilibrium under the application of these combination. These four combination you see like you see you know like if you see this part we will find that they are acting in this particular opposite direction and they are opposite in this particular direction.

So, under this application of these two forces or we can say these two shear forces or these two shearing stresses this is well perfectly balance and this diagram is clearly. So, in this kind of phenomena that we have a applied torque we have a resisting either this resisting torque we have a applied shear stress. We have a complimentary shear stress both are acting on and same time both are exactly equal and opposite.

So, whatever you see the torque application is there or whatever the shear stress application is there, we can say that this, whatever you know like our this uniform bar is well balanced under the application of these torque or these shear stresses. So, this is the exact phenomena is there, which you know like we want to discuss.

So, this is what actually you know like we discussed you can again you know like gone through from the this return material that you know like when a uniform circular shaft is subjected to a torque. It can be shown at every section that, shaft is subjected a state of pure shear that you see the applied shear part was there, and the moment of resistance which has to be developed by the shear stresses being everywhere equal a in the magnitude and opposite in the sense of the direction. So, you see if it is there that is what we are saying that the resisting torque is there when the applied torque is there and when applied shear is there the complementary shear stress is there just to balance those things.

And for the purpose of the deriving a simple theory we need to describe you know like the behavior shaft which is subjected to a torque it is necessary to make certain assumptions. Because, you see you know like we cannot say if you go for the realistic situation then you always find that whatever the distortions are there.

And, we which, we are you know like measuring that the distortion it is just uniform starting from zero to maximum at the extreme corner, but certainly you see it is not in practically it is not there. Because whatever the displacements are there of the microstructure from one part to another part it is not uniform, because even you know like the torque is applying, but this torque is you know like one has to be assume that this torque is uniformly the distributed all across the cross section of this particular circler shaft.

And, also with that whatever the tendency of the material is there it has to shown a clear you know like the similar kind of whenever the you know like the torque application of the shear force applications are there. And, then immediately if you are saying that we are going up to only the elastic deformation, then whatever the torque due to the torque whatever the shear stresses are inducing.

One has to you know like exhibit the complementary shear stresses exactly in a equal amount in magnitude and in the opposite direction. So, there are you know like lots of apes and birds are there to apply this kind of theory, so one has to make certain assumptions pride to derive those relationship which we have you know like shown in the first slide. So, here of some of the assumptions are like that.

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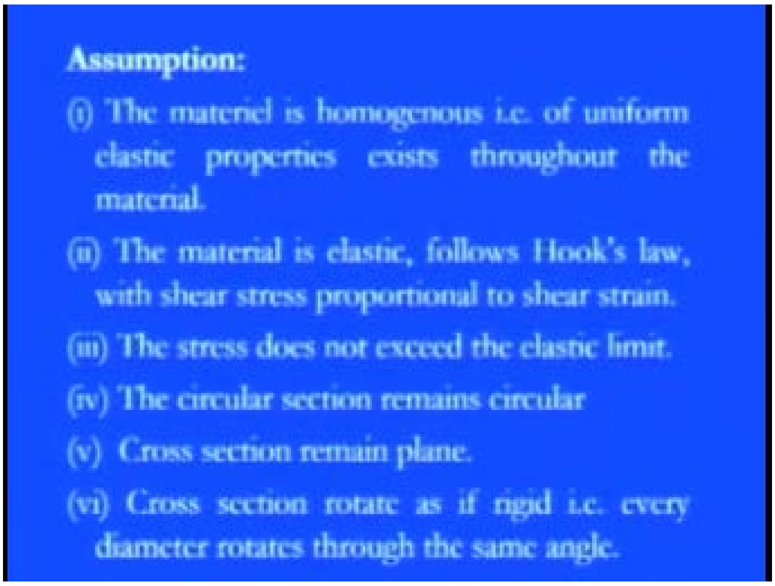

First of all the basic thing for every you know like kind of theory in which we are assuming that you know like an even actually when we say that the generalized Hooke's law is valid. One has to you know like consider that whatever the material is there it is to be homogenous. And it is to be you know like isotropic material with that you see you know like whatever the shear a this a shearing directions are there or whatever the components are there of the this stresses normal and shear stresses they are suppose to be equal in all the directions.

So, coming to that point to for the generalized Hooke's law, here also we are applying the similar kind of theories here. So, that is what we can say that first the material is suppose to be homogenous means it has to be it is must that more material is homogenous; that means, the uniform elastic properties are to the existing throughout the material, that means there is no a kind of like non-linearity is there in between the stress and strain or any kind of you know like the elastic properties of those that particular material

So, first the material is supposed to be homogenous, second the material is elastic follows the Hooke's law. So, when the shear stress is proportional shear stress then all you see we can say that G which is the shear modulus of rigidity is valid. Because, if you see you can if you are saying that your material is not elastic or it is not. So, in exactly the ductile material property then you cannot exhibit you know like the proportional limit or up to you cannot go up to the yield limit.

And, then you can now say that you see all the Young's modulus of elasticity or shear modulus of rigidity or the shear stress strain you cannot make you know like the relationship, because if the stress is not proportional to strain or we can say if the they are not. So, in the linear relationship then you see all those formulas which we are deriving is absolutely incorrect.

So, both are either the material is homogenous or material is suppose to be within the elastic deformation these two are the biggest assumptions are there. Then certain more assumptions are there like the stress does not exist the you know like exceed the elastic limit. Because once it is going in the plastic deformation there is a permanent set of deformation is existing within the elastic within the material.

Then, you see even if you release the load body whatever the object is there or the circular shaft is there, there is a permanent deformation or the distortion is there. Then, even if you apply the load it will exhibit more and more kind of you know like the distortion.

So, you see we need to put our load up to the elastic region only, and then you see we have across the circular cross section remains circular. Because, you see once you cross the elastic limit definitely the distortion is there and due to there is distortion the circulars cross section of the shaft will be either in elliptical or any irregular shape is there.

And, then we can say that you see you know like whatever these theory which we are assuming with the uniform cross section with the uniform stress distribution, so which is not at all valid. So, this is one and then you see the fifth assumption is the cross section remain plane also. Because, if you know like if it is not planar then definitely whatever the stress distribution is there within this total cross sectional area of the circular shaft which not will not be remain same.

And, you see the stress concentration or any other you know like the phenomena or any other properties are there and they are different you know like all together, and you see in the first even we are saying that this is the material is homogenous. So, it has to be exhibit you know like all the elastic properties through the common or we can say the uniform all across the material.

And, the last assumption is the cross section you know like the rotate as if it is rigid; that means, every diameter rotates throughout the you know like this particular this rotation at the same angle Because, if you see there is a different cross sectional areas are there or even you see if you have a same cross sectional area, but if it is rotating at a different angular twisting.

Then, definitely you see whatever the angle of twist there the theta which will not uniformly deform and you see since our whole measurement is absolutely based on the theta because even in the formula you will find that the G theta by l is there. So, the theta is very, very important parameter because whatever the distortion is there and if it is not uniform all across the body. Then, whatever the shear strain because what we are doing here we are simply checking with the angle of twist only, so whatever distortion is coming it is coming due to the angle of twist theta.

So, if any irregularities there in that particular rotation, so obviously, the shear strain is not coming exactly which we want to measure and once you do not have the proper shear strain then you cannot say that this shear stress and shear strain relations are proper or whatever you see you know like the formulas which are coming under the Hooke's law they are valid.

So, one has to be very clear that actually the first of all your circular shaft is uniform throughout the cross section and it is to be rigid, there is no you know hollowness is there and there is no weaker sections are there that the is the shaft is weaker at the central position and rigid at the outer part or vice versa. Then, definitely you see this shaft will rotated at different angles and definitely whatever the angular twist is coming or the shear strain is coming at a particular section they are absolutely different all together.

So, these assumptions are very very valid, if you want to you know like apply that kind of theory where we are saying that the T by J is equals to this tau by r is equals to G theta by l. And, once you know like you want to apply this theory one has to first satisfy these assumptions, and then you can apply this theory. So, you see here come to the main point as we know you know like discuss that if we you know like if we are applying the torque on a circular shaft.

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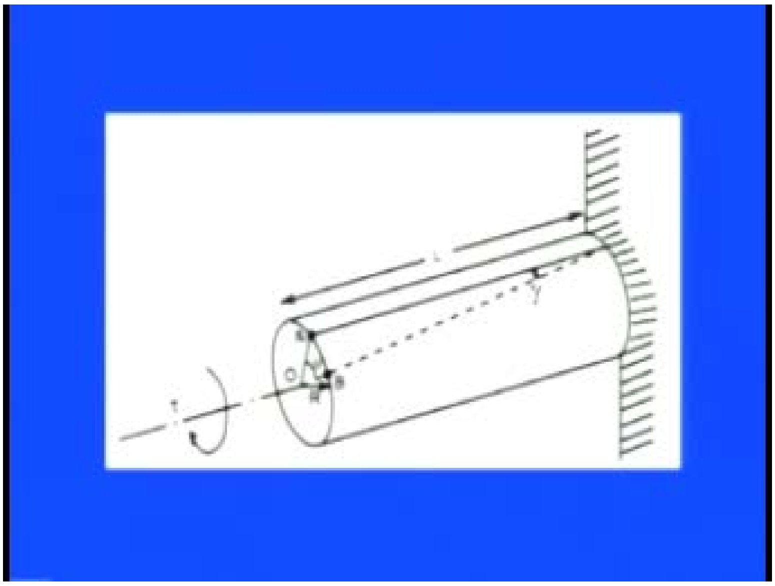

Then, what will happen since as we discussed, that actually you know like this is the apply torque and here the resisting applied torque is you know like generating here. And, if you are saying that this is the length of this bar is there, so obviously you know like it is starting from these angle of twist and it is maximum at that.

So, you see if you want to measure that how must distortion is there always we are measuring from this point to this point. So, now you see earlier it this point was A and now you see A to B this these layers are shifted from this point to this point. So, we are measuring in terms of angle of twist, and if I am saying that this point O is the origin is here from where thus the central axis of the shaft is passing then we can simply say that this is my radius capital R and this is my angle of twist that is theta from A to B. So, this arc is there.

And, if I want to measure in terms of the distortion obvious; obviously, the shear strain is there and shear strain is nothing but equals to gamma. So, this figure clearly signifies that whenever the angular twist is there a always you see there is a kind of distortion and which is maximum at the end starting from the zero, where the rigidness is there and which we can simply measure with the angle of twist. So, this theta is there.

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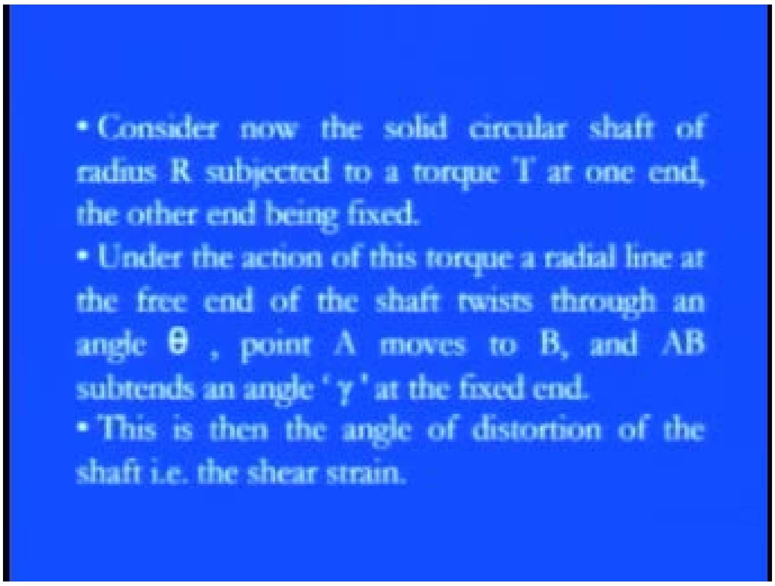

So, as we seen the previous diagram that there was there is a solid circular shaft is there, which has radius of R and you know like the torque applied at the free end.So, the fix end was you know like the free end as I told you that you know like when the applied torque is there at the free end there is a resisting torque is coming.

So, under the action of this particular torque you know like we found that there is you know like the radial line is there at the free end which is just so in the twisting is there at the extreme end and the twist which we are we are measuring as the this angle of twist that is theta at point you know like A to if you are moving to B. So, A to B is a circular point is there and we are saying that the arc AB is there and we just try to set up the relationship between in this arc AB, because you know like when it starting from O to A this will move from A to O to A to O to B. So, this is a kind of measurement, this is the kind of angular twist is there and which we are saying that this is the theta and you know like O to A is the circular radius is there, that is the capital R.

And you know like if you are taking the substantial amount of this twisting we are always measuring a kind of distortion as far as the total length is concerned l as, so this is coming as the gamma that is the you know like the total distortion. So, this we can say the angle of distortion of the shaft all along the line is always the shear strain is there. So, now we would like to you know like setup the relationship in between all those you know like the kind of a this applied torque and all those things.

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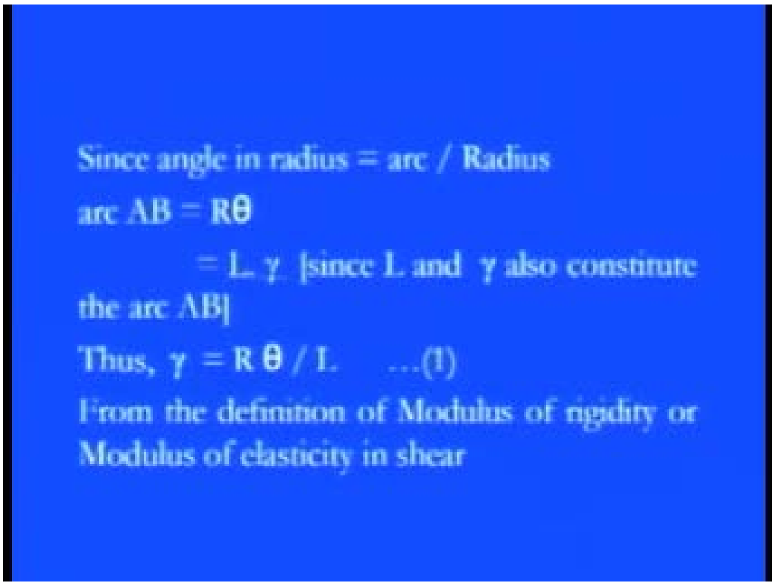

So, here you just go to the basic part that you see at the extreme free end corner what we had there is a twisting is there right from O to A to O to B. So, AB the at this arc is there, so angle in radians is nothing but equals to arc divided by whatever the radius is coming. So, arc AB is nothing but equals to the radius that was theta and the angle of twist is there that the radius is R and the angle of twist is there that is the theta in terms of the radians, so this is just for the outer periphery of that.

But, if you are going for the all along that part where you see you know like this total length L is there, and the shear strain or we can say the total distortion is coming in this particular shaft where the all there is a shifting is there of the layers right from this free end free end to the resident corner. So, we found that this arc AB is exactly equals to R into theta where the frontal portion is there and you see if you are going for the circumferential then it is exactly equals to the length L into the shear strain gamma.

So, you see if you are equating, since the arc AB is exactly equal to both the things, so if we equate both the this angular twisting, then you will find that R theta is equals to L into gamma, where R is the radius of this circular shaft theta is the angle of twist and L is the total length of the circular shaft and gamma is the shear strain which consisting the total distortion of this particular shaft. Then you will find that we have the shear strain gamma is equals to R theta by L.

So, this is you see the you know like from the definition of a modulus of rigidity, now we can simply say that you have you know like the shear strain you have the shear stress. So, you can make you know like the main relation between these two terms the shear stress and shear strain with the using of shear modulus of rigidity. So, under the you know like the load application and of this Hooke's law or we can say under the this elastic limit we can say that the stress shear stress is proportional to shear strain or shear stress is equals to G times of shear strain or G is equals to shear stress by strain.

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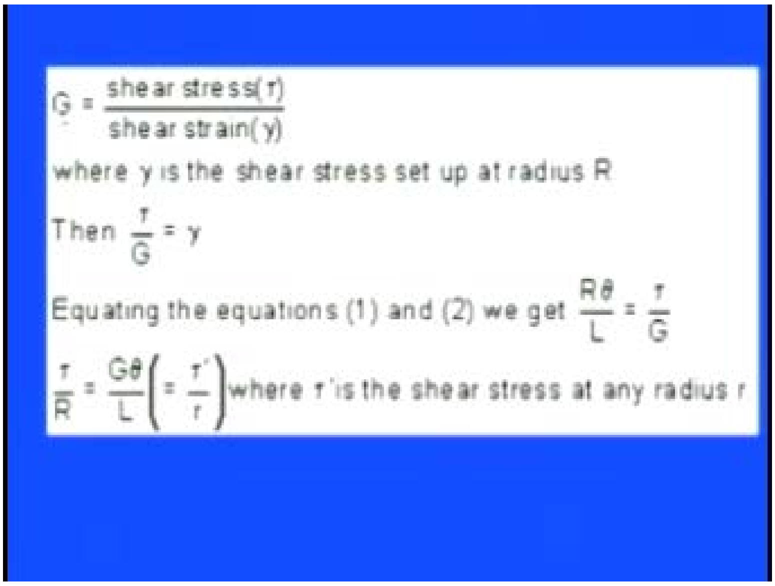

So, you see you know like this a relation is there, shear stress is equals to you know like this modulus of rigidity is equals to shear stress tau divided by shear strain as I told you recently where these gamma is the shear stress which is to be set up whatever you see at the radius of R. So, just by equating those things what we have we have the shear strain gamma is equals to the shear stress divided by G. So, you see from here this gamma is nothing but equals to this G will come in the denominator side and on top of side we have this tau. So, tau by G is gamma.

But, you see you know like if you go to the previous section then you will find that this gamma is nothing but equals to R theta by L by equating R theta and gamma L, what we get? We get the shear strain is equals to R theta by L. So, now you see if we are equating both the term here, this equation which is tau by G.

And, the previous equation that is R theta by L, What we have? We have a relationship in between these two term that in terms of the shear strain is R theta by L is equals to tau by G or you see you know like in that term you see we can say that R this tau by R this tau if you take it by R is equals to G theta by L. So, this relation is there is valid for the outer radius of that.

But, if you are talking about the inner radius or any radius which is that r dash we can say that tau by small r or this tau dash by small r, if I am saying that the tau dash is the shear stress any radius small r is equals to tau by capital R which is recently we have shown that if we have a solid shaft and it has only one radius that is R, the shear stresses tau. So, tau by R is equals to G theta by L or it is also equals to tau dash by r.

So, you see if you just remind that actually on the first slide you know like I have shown you that we have a relationship between you know like all three components. So, the second and third terms will be you see tau by you know like R is equals to the third term which is G theta by L, so you see this we have proven here. So, now as we move further because we just want to you know a set up all three equations.

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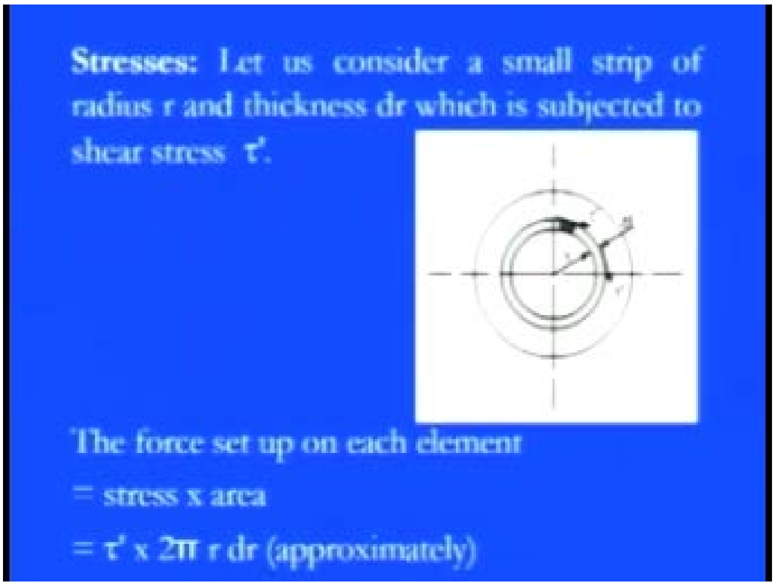

So, for that, now we are considering the circular section of that particular shaft, so here just by considering the small strip of the radius on that circular section. And, also you are just cutting the portion of the small segment and it has a thickness of delta are you can see this particular figure which is subjected to a radius which subjected to a shear stress tau dash.

So, again you see what we have we have a circular section and only we are because you know like we are simply assuming that this whatever the distortion or whatever the shear force is are shear stresses are coming under the section of shear forces. They are uniformly distributed all across the cross section of the area.

So, here you see what we can do here, we are simply taking this is origin and this is the circular portion and it has you see you know like the radius is small r and if I am considering a small portion like that at the radius small r, what I have? I have you know like the shear stresses, due to the torque application is tau dash. So, this is the tau dash directions are there and this segment which we are considering for our this study purpose the segment thickness is delta r.

So, the you see now if we want to set up the force on this particular element, then the force set up on this particular element is nothing but equals to the stress developed under the action of this applied torque into the area concern. So, you see since stress is nothing but tau dash which we are already taken here and the area because it is the circular section. So, the area is 2 pi r this is the 2 pi circumference is there 2 pi r into the thickness the dr. So, now my area is 2 pi r dr or generally we are taking T here, but we have taken here as a small element and dr is the thickness, so 2 pi r dr is the area and the tau dash is there. So, force is nothing but equals to shear stress into area of tau dash into 2 pi r dr.

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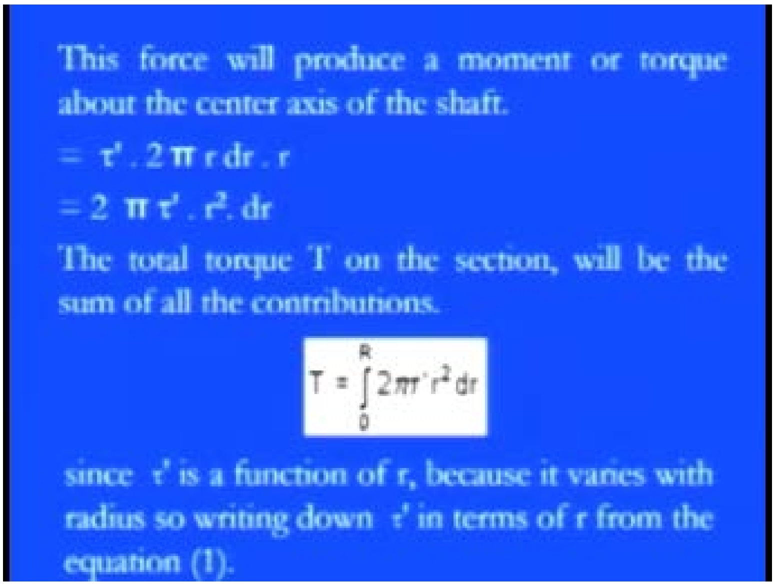

So, once you have the force, then things this force will produce a moment, because r torque about the central axis to resist that part because you see you know like when you apply the torque does these shear stresses are forming. So, now you see due to the internal reactions always they are suppose to set up the kind of this torque or we can say the moment exactly apply to this applied torque and that is why we are saying that this is the resisting torque or we can say the twisting torque again that applied torque. So, this is nothing but equals to tau dash into 2 pi r dr.

So, tau dash is the shear stress 2 pi r dr is the this area, so this tau dash into area is the force and force into this radius is the moment. So, tau dash 2 pi r dr into r will be the just opposite torque is there or if you computing the moment or torque, then it is exactly equals to 2 pi this tau dash into r square dr.

Now, this torque is you know like simply applying on this particular section, but you see as we are saying that whatever the distribution is there irrespective of the shear stresses or it is the irrespective whatever the forces are there all across the layers or the microstructure of this particular shaft is uniform.

So, you see you can simply compute this torque for the whole section because this whatever this moment which we are gaining it is just for the small this section which has the thickness of dr. So, now if I am saying that this is very well valid for all across the section. So, now, the total torque on that particular you know like the circular shaft have is nothing but the sum of the contributions for contributions from these small section of dr.

So, T is nothing but equals to integration of 0 to R for you know like starting from central axis to the extreme corner R the integration of 0 to R 2 pi, you know 2 pi into this tau dash r square dr. So, 2 pi tau dash r square dr will be coming and now we are computing the this is the for the small section and we are computing for all across this where you see the tau dash is a function of r; obviously, you know like this tau dash as we have clearly shown in the previous figure tau dash by r.

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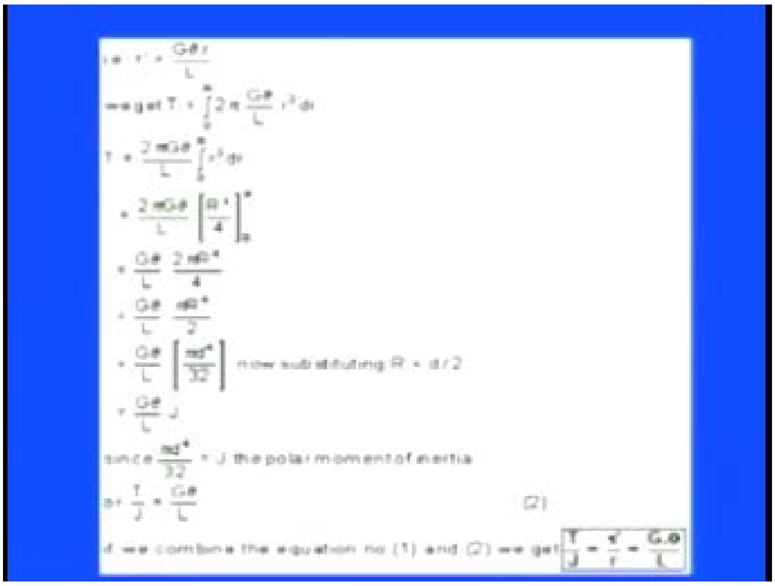

So, tau dash is a function of r and it varies with the radius, so you know like we can simply write the tau dash in terms of this particular first equations. So, it will come exactly as tau dash by r is nothing but equals to G theta by L, L as we have shown in the previous equations by equating tau dash by r and that.

So, tau dash is nothing but equals to G theta, r by L and now you see if we are you know like keeping in the previous equation what we have the total torque T is the nothing but equals to 0 to R 2 pi instead of tau dash. Now, I am simply writing because tau dash is a function of r. So, we need to described this tau dash in terms of r, so you see here we are simply replacing this tau dash in terms of r G theta r by L. So, and then you see r dr was already there.

So now, what we have? We have 2 pi G theta by L r square dr or what we can r cube while dr or what we can do here? We can simply take it out all those component at outside. So we have, now 2 pi G theta by L these are the constant terms. So, if you look at these figures then you will find that the G is the shear modulus of rigidity which is a material property, theta is the angular twist. So, once measure the angle twist angle of twist then it is constant, and then L is the length which is you know like the constant for all across the you know like this particular body and even under the application of load also.

So, now you see what we have the torque is the variable function of r cube dr under the integration of 0 to R. So, now you see you know like after integrating that term, what we have? We have T equals to 2 pi G theta by L into R four by R after integration r cube dr is nothing but R four by R and 0 to R which we need to apply the definite integral.

So, after keeping these you know like values what we have we have G theta by L into 2 pi R four by R or we can say that after cancelling two, what we have? We have at the end G theta by L pi R to the power 4 divided by 4 of divided by 2. So, now you see here if I am replacing these R by the total diameter of the circular shaft, then R is nothing but equals to d by 2, since we have R to the power 4. So, what we have at the end? We have the torque is equals to G theta by L pi d 4 by 32.

So, now you see you know like as we were discussing about we have a this polar moment of inertia polar, moment of inertia is nothing but equals to what will be you know like whatever the effective area is there under the application of the torque. So, J is nothing but equals to pi d 4 by 32. So, whatever the component which we have right, now pi d 4 by 32 can be easily replaced by J and J is the polar moment of inertia.

Because, it is coming under the concern area or the defected area, so you see here what we have now, we have the torque which is equals to G theta by L into J or we can also equate T by J the applied torque divided by this polar moment of inertia is equals to G theta by L.

So, now you see you know like this relation is well set up that in the previous equation, what we had? We had tau dash by r or even tau by capital R was equal to G theta by L and here we have T by J equals to G theta by L. So, G theta by L is equals to this tau by capital R or we can say tau dash by small r means whatever the shear of stresses are there at a particular section r which is equals to T by J.

So, here you see what we have we have the final relations which we you know like started from the beginning that if we have you know like the solid circular shaft which has a uniform cross section having this homogenous material property and which is under the application of elastic loading condition or elastic deformation. And, it has you know like the uniform cross section, so that whatever the angle of twist is coming or the shear strain is coming it has to be uniform.

And, then you see you know like it does not have the different cross section throughout the particular in this length. So, under all these assumptions we can simply derive that when so solid circular shaft is is angular under the angular twist is there this. We have a this shear stresses at a particular radius tau and due to that you see if the angular this angular twist is coming as a theta. So, these all you know like this technical terms are summing up with the kind of relation as T by J is equals to tau dash by r is equals to G theta by L. So, this is a very well relationship you see and we can simply derive for this kind of shearing stresses.

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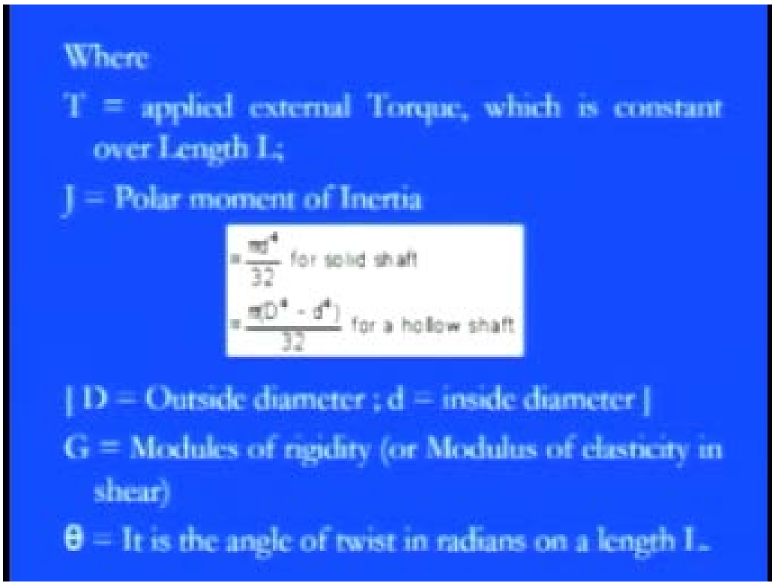

So, now you see in this particular condition what we have we have the torque which is the applied external torque which is constant over the length L. So, you see here, this was the biggest assumptions which we were assuming that whatever the torque application is there it has to be uniform and it does not vary as with the time.

So, it this is kind of so called the static torque is there and J is the polar moment of inertia and if we are using the solid shaft then as we have discussed already that it is based on the area. So, it is pi d 4 by 32 for solid shaft and if we have a hollow shaft; that means, you see if you are simply removing a portion from this solid circular shaft. So, we have a some hollow circular section.

So, for that you see we have a two different diameter, so if we have a D is the outer diameter and this small d is the inside diameter. So, we can derive the J the polar moment of inertia for hollow shaft also which is nothing but equals to pi capital D to the power 4 minus small d to the power 4 divided by 32.

And, you see in that also we found that the G was there that is the modulus of rigidity or we can say the modulus of elasticity in the shearing action. And, since you see we are only assuming the elastic deformation, so this G is very well valid in which the shear tress is proportional to shear strain and the value of G is coming and it is also the property of material. So, which material you are using generally we are using the this or this ductile material in which the percent elongation is always measuring the ductility property.

So, this kind of relations which we can easily set up for you know like the circular solids shaft and you see you know like the solid or we can say the shaft, but it has to be having a uniform cross section. And, the last is the theta which is nothing but equals to the angle of twist in terms of radius on a length L.

So, you see and it has to be uniform as we have already assume, so with under all those applied conditions we can say that the T by J is nothing but equals to tau dash by r is nothing but equals to G theta by L. And, you see now if you are talking about the this new term that was the this G that the G J by L or T by theta we can define the new term as the tensional stiffness.

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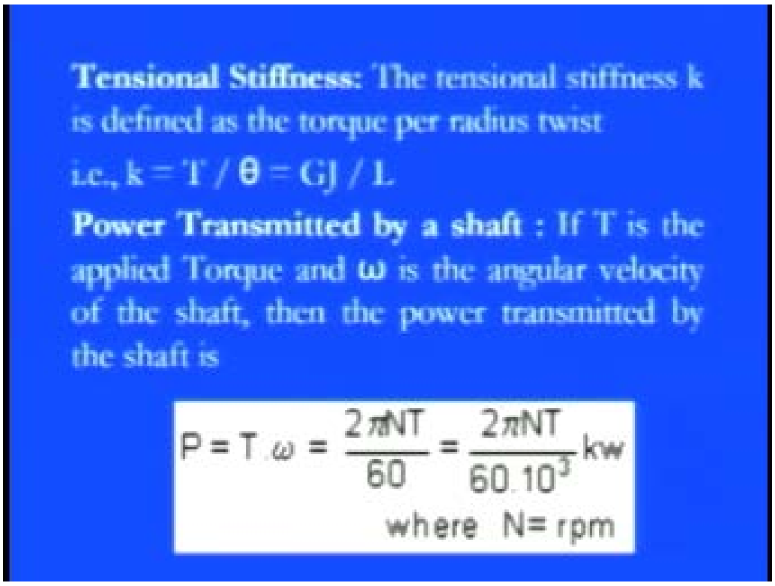

Stiffness as you discussed, you know like since we are talking about the spring is there and whenever it is under the application of torque, it is showing somewhat you know like the twisting part is there. And, you know like based on what the maximum shearing stresses are coming or whatever the size and shapes are there, how these moments can be computed.

So, for all those kind of things here, we are defining that new term that is the tensional stiffness because whenever under the you know like the torque is there a kind of deformation is there. And, we just want to measure the deformation always the stiffness term is coming. So, tensional stiffness the k is nothing but equals to applied torque divided by this radius the twisting part means how much distortion is there.

So, we can say that this whatever the angular twist is there or we can say the this radial twisting is there. So, k is nothing but equals to applied torque T divided by the angular twisting theta or once you put those things here, then you will find that it is nothing but equals to since T by J was nothing but equals to G theta by L. So, T by theta is nothing but equals to G J by L.

So, you see you know like we have either you can compute the tensional stiffness which is nothing but simply measuring how much deformation is there or how much you can deform irrespective whether it is a spring component or whether it is a the circular bar section it can be simply computed based on what material is there, because G is a property of material. So, G into this, what is the effective area is there; that means, what is the this some polar moment of this twist this twisting is there or inertia is there.

So, either polar moment of inertia or we can say the shear modulus of rigidity is there, and then what the total length is there if the total length is more definitely you see you have there is a kind of different stiffness is there. So, this is you see the tensional stiffness is there it is just to be defined for the torque application.

And, then we have the last term that is the power transmitted by shaft, because this is the basic application of any shaft that if the T is the applied torque and omega is the angular velocity of the shaft then power transmitted is nothing but equals to P equals to T times of tau or we can say you know like the tau since it is the angular velocity.

So, we can again describe in terms of what is the rotational speed is there, so tau is nothing but equals to this 2 pi N divided by 60. So, once you multiply 2 pi N by 60 into tau then you have the power transmission which is 2 pi NT by 60 in terms of what or if you are calculating in terms of kilowatt. Then we have 2 pi NT by 60 into this 1000 and which is you know like the kilowatt is there, where N is there this revolution of the shaft per minute.

So, if you are saying that you know like if you know the rpm of any shaft, then so you can simply computed you know like the power if know the torque or if you know the power that you need to you know transfer this much power. And, if you know that this one speed of shaft then the corresponding the torque can be easily calculated.

So, you see here in this chapter, we discussed about you know like that if we have a different kind of you know like the loading conditions or a I should say that actually, if we have a shaft and if I am saying that is the under of simple torque or couple or any kind of you know like the twisting moment is there. And, if it is having a uniform cross sectional and also it has the material which is homogenous in the nature.

Then, you see the kind of relations which we set up is very much valid if the elastic deformation is a deformation is there under the application of the torque. And, you see a under that you see the equation is very well known equation and this is the basic equation for analysis of any rotating shaft, that is the T by J is equals to tau dash by r is equals to G theta by L.

And, once you know those things then the a probably you can also find it out that what is the stiffness is there of the circular shaft of the or of the spring. So, you see it is nothing but equals to T by theta or we can say G J by L, so that means, you see it is depending on G as well as the J also. And, once you know you know like the rotational speed and once you know the either the power or the torque you can simply find it out the remaining parameters.

So, in this lecture you see we have we were simply discussing about the rotating shaft and in the next lecture you see we have goanna discuss about that if we have a spring. And, if you are applying this kind of you know like the torque and all those things. Then what exact the exactly the relations are there in there between the shearing stress and what will be the impact is there of the what are the material of the spring as well as the number coils.

Thank you.