

Strength of Materials
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Lecture – 18

Hi, this is Dr. S. P Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to deliver my lecture 18 on the subject of the Strength of Materials and this course is basically developed under the National Program on Technological Enhanced Learning. As you see you know like in the previous lecture, we are discussed about, that if we have cylinder then and in the cylinder if we have the steam or any kind of pressure is exerted on the walls of the cylinder along any direction, that means along longitudinal or radial or the circumferential direction, then what the stress component are there, which stress components were dominating in that like you see you like as we discussed that the two main stress components are there, there are always dominating in the longitudinal as well as the hoop or we can say the circumferential part.

So, generally what are the stress components which are inducing, due to the application of this internal pressure are always considering in two mutual perpendicular direction. One is along the length, that is the longitudinal stress component that is you know like the σ_l , another one is which is along the circumference that stress component is known as the this circumference or the hoop stress.

So, that was we discussed and also we found that, if you compare the equilibrium portions of under the application of this pressure, then this is longitudinal stress component is nothing but equals to $p d$ by $4 t$ or this hoop stress or circumference stress component will be you know like it is $p d$ by $4 t$. So, this we discussed and then we found that you know like, if we are going for you know like to measuring, just to measure the deformation that, actually how the deformation is there along the longitudinal as well as the circumferential direction.

Then we found that we have the longitudinal strain component and we have the hoop strain component, and if you want to compute the longitudinal strain component then it is nothing but equals to σ_h by σ_l by e , because this is the longitudinal 1. So, σ_l by e minus the Poisson ratio into σ_h by e , so if we compute both the thing in

the terms of the dimensional parameter, then we have a longitudinal strain component which is nothing but equals to $\frac{p d}{4 t e} (1 - 2 \mu)$ of Poisson ratio.

So, that we computed and similar we applied the same theory along the circumference of circumferential portion of the thin cylinder, and we found that we have a circumferential or the hoop strain component, that is the ϵ_2 . Also we computed that it is nothing but equals to $\frac{\sigma_h}{e} - \mu \frac{\sigma_l}{e}$ where σ_h and σ_l are hoop stress as well as the longitudinal stress component.

And if we compute, these two terms on the basis of their dimensional parameter then we found that the ϵ_2 which is a circumferential or hoop strain component is nothing but equals to $\frac{p d}{4 t e} \frac{1}{E} (2 - \mu)$. That means the Poisson ratio, so we computed both the component that what exactly the corresponding change is there or corresponding deformation is there, in terms of the strain component. And then accordingly we computed that what the change of length is there that the ϵ_1 that is the longitudinal strain into original length.

And if you want to compute the change in diameter is nothing but equals to ϵ_2 into real the original diameter ϵ_2 is nothing but the circumferential strain. So, corresponding you see you know like we can simply calculate that what the deformation is there or what the change is there in terms of length as well as in terms of diameter. And then we discussed about the volumetric strain that you see we are finding that, the change in length is there change in diameter is there then definitely you see, if we are considering in the volumetric domain and then we have a volumetric strain.

And we found that the volumetric strain is computed on the basis of longitudinal strain, and hoop strain and there is an exact relationship which we set up in the previous lecture; that this volumetric strain is nothing but equals to the this longitudinal strain plus 2 times of hoop strain. And if you put those things like we have the longitudinal strain is $\frac{p d}{4 t e} (1 - 2 \mu)$, and hoop strain that is $\frac{p d}{4 t e} (2 - \mu)$, if you put to both together then we have a volumetric strain which is nothing but equals to $\frac{p d}{4 t e} (5 - 4 \mu)$.

That means, you see we have both, you know like all three components are there, as well as the strain things are concern that means, you see you know like that what exactly the deformation is going on in terms of length in terms of diameter in terms of volume we

can simply compute. So, that part we discussed when we have a thin cylinder with no specified shape at the end, but you see if we put, the specified shape that means see if we have a spherical corners. And you know like both spherical corners are having different thickness as compare to the wall thickness of the cylinder.

That means, you see if you are considering that wall thickness is t_1 and, if then definitely you see we have a spherical wall thickness at the extreme corner is t_2 , but if you are keeping the diameter is same. Then we found that definitely we have you know like the hoop stress component in the cylindrical portion is, different than the this hoop stress component in the spherical part. And corresponding changes are there that means, this hoop or circumferential strain in both of the portion cylindrical as well as the spherical portion were different.

So, you see we computed both the thing that if we have a cylindrical portion, then the hoop strain is nothing but equals to $p d$ by $4 t_1$. If t_1 is the thickness of the wall of cylinder then t_1 times e into 2 minus μ was there and if you are computing, the this hoop strain or we can say the circumferential strain, along you see you know like the spherical portion. Then we found that it is nothing but equals to $p d$ by $4 t_2$, and t_2 is the thickness of the spherical cell into e multiplied by 1 minus μ , that means the Poisson ratio.

So, you see here we found that there are you know like significant difference is there in cylindrical portion as well as the spherical portion. And then you see now, you know like if you want to compute, the common portion that what exactly the relation is there to avoid the distortion at the junction portion. That means, you see you know like because, there is a spherical portion at the end there was you see you know like the cylindrical portion is there, so obviously there is a very good chance to have bending as well as the shearing stresses at the junction point.

So, to avoid those things we set up you know like, we discussed that actually there is a ratio of t_2 by t_1 and in that you see if, the wall thickness of cylinder is 2.4 times wall thickness of the spherical portion. Then you see we have a continuous structure from cylindrical to spherical portion and you can simply, avoid the distortion at the junction point. So, this kind of relations which set up but all those discussion in the previous part

you know like was set up when, there is no shearing portion only these two mutually perpendicular normal stresses were there.

So, now you see we just want because, the cylinder when there is a rotation means if we have you know like shaft is there and if it is rotating and then, the kind of internal pressure or the kind of different pressures are exerting then what will happen. So, now you see in this lecture, we are going on discusses that actually, if we have a sphere or if have the rotating shaft is there. Then, what the kind of stresses are being dominating and how we can set up the relationship in between the stress component, this kind of discussion is going to be there in the next lecture.

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Thin rotating ring or cylinder

Consider a thin ring or cylinder as shown in Fig below subjected to a radial internal pressure p caused by the centrifugal effect of its own mass when rotating. The centrifugal effect on a unit length of the circumference is

$$p = m w^2 r$$

Here the radial pressure ' p ' is acting per unit length and is caused by the centrifugal effect of its own mass when rotating.

So, here you see in this lecture 18 we have the first portion that is, the thin rotating ring or cylinder and considering a thin rotating ring or cylinder which, I am going to show you in the next slide. It just subjected to a radial internal pressure p that means, you see now, we have a internal pressure towards the radial direction of this thin ring or we can say the thin cylinder caused by the this centrifugal effect.

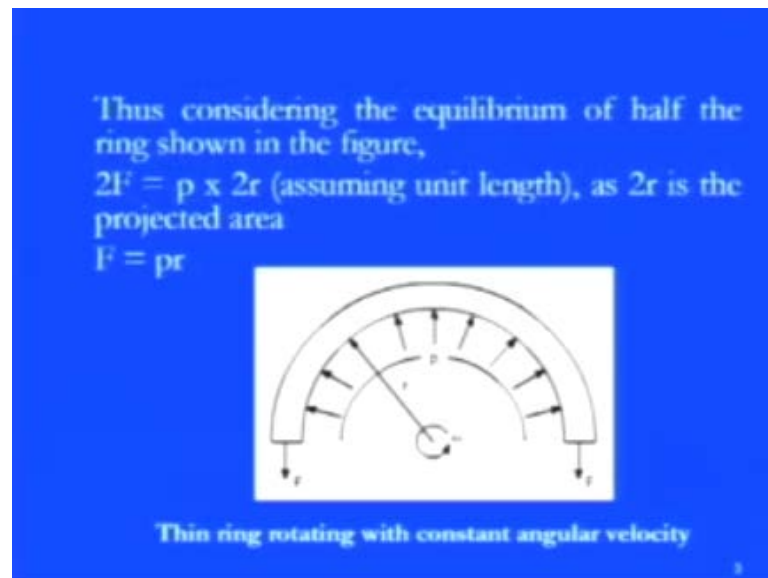
Because you see now, this is rotating, so definitely we have a centrifugal center through which the centrifugal forces are exerting, on you know like the this radial direction of, this rotating ring or we can say the this cylinder and that due to the centrifugal effect of, its own mass when it is rotating always we have a kind of centrifugal stresses. So, the

centrifugal effect on the unit length of a circumference, because it is always acting the in the circumferential part of this particular rotating ring.

So, we have now, p which is the internal pressure is always acting towards that is equals to mass is mass is nothing but the mass of this, cylinder into ω square that at what point it is rotating, because now you see we have a rotational speed so $m \omega$ square into the radius. Where you see the radial pressure p is acting per unit length, so here you see we are saying that at per unit length, how this p pressure is exerting on the wall of the this cylinder, so this you see p is always as the, this force per unit length.

So, you see corresponding the change is there and it is caused by the centrifugal effect because, it is rotating if its own mass is you see having a rotation. And this rotation is there at the ω that is the rotational speed is there, so corresponding we can simply calculate that what will be the pressure is there and pressure is $m \omega$ square r .

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So, now you see by considering this particular figure as I told you that, we have you know like this thin cylinder or the rotating ring, so this is rotating and as I told you we have a centrifugal center. So, from this center now, since it is rotating at ω , so obviously, you see whatever the forces are coming that will be coming as $m \omega$ square into r will be given there is a pressure and thus, you see the considering the equilibrium portion.

Because, you see what we have a same the symmetric section is there, so we are simply cutting the portion, so you see this is the half portion which we are cutting and the half portion we can simply see that, this is the radius of the thin cylinder. And since you see the internal pressure is exerting all along the circumference of this particular, this specify specified shape of the cylinder will find that these are the internal pressure which are always going towards the outer circumferences.

Because you see we have a centrifugal action, which is always dominating in this kind of rotating devices, so due to that we have this kind of the pressure exerting on those things and we have the force, which is perfectly there on these particular cutting sections. So, you can see that we have a different forces, which are dominating at different sections. And we need to be very carefully that actually, when this forces are acting on the different portions of this body what is the effective area under which this forces are acting.

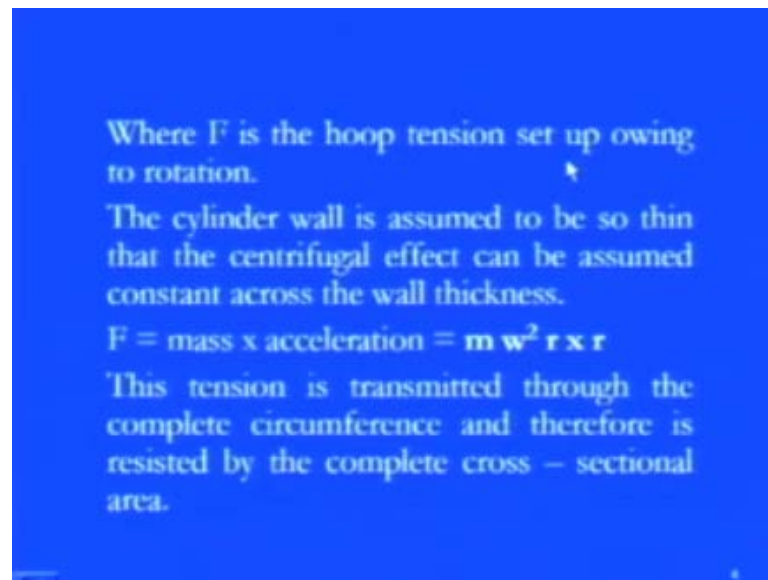
So, this you see by considering the equilibrium portion of the half of the ring which is showing here in this figure. We can simply say that the force which is you see at these two extreme corners you can simply see this these two extreme corners, so both you see by considering both portion two times of F is nothing but equals to p times of 2 times of r . Again you see here we are assuming that, we have you know like the unit length that means, whatever the pressure which is exerting is always pressure per unit length.

And $2r$ is the projected area because, you see this r is there and the total r that is a diameter, because this pressure is exerting all along this diameter, so this p into $2r$ is the effective portion of a this cylinder is there. So, it will give you 2 times of force which is exerting on the, this cylinder, so we have force which is exerting pressure is nothing but equals to p times of r , so you see here we have considering that whatever the rotation is there of the cylinder is constant.

That means, it is constantly rotating at ω and ω is nothing but equals to the rotational speed of this shaft or we can say the cylinder, so corresponding the forces are always dominating one due to the internal pressure, so you see circumferential pressures are there and one is along that you see the force, so this F is there. So, we can say that 1 p is $\omega^2 r$ this is one part second you see we have the f which is nothing but equals to p times of r .

So, you see by considering the F as in the diametrical portion, so what we have since it is you see the diametrical part is there along these the this perpendicular direction or the y direction. We have the hoop tension or we can say the hoop compressions are there along with that.

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So, F is the hoop tension set up according to the rotation and the cylinder wall which is you know like assumed to be, so thin that the centrifugal effect can be assumed constant all across the wall thickness. So, here you see what we are considering, we are considering the first thing that whatever the wall thickness is there it is the pressure, which is exerting on the circumference of wall is always uniform and it is the uniform in terms of pressure per unit length.

And second assumption which we assume that if the cylinder wall is, so thin that whatever the centrifugal effect is there on those wall, it is having a constant all across the wall thickness that means, whatever the centrifugal forces are coming there are uniformly acting. So, whatever the deformation is there they are also uniform that means, there is no stress concentrations are there or there is no distortion are there, which are different and different regions.

So, as you see we discussed that the hoop tensions are there at the diametrical portion, so this is nothing but equals to mass into acceleration. And you see here if you are considering the acceleration that is $\omega^2 r$ into r and if you multiply with the

mass we have you see m into a , that is the applied force, so $m \omega^2 r$ into r or we can say $m \omega^2 r^2$ is there. And since this is a as a tensile force, due to that you see always there is a chance of extension is there towards, the circumferential direction or we can say the hoop stresses which are inducing, due to this particular rotation.

Under the application of this internal pressure always, we have that tensile hoop forces are there and that this tension is transmitted through the complete circumference, because we are assuming that these circumferential effect is uniform all across in the wall thickness. So obviously, you see whatever the transmission is there this tensile forces transmission, this is also you see the uniformly distributed.

So, this tension which is transmitted through the complete circumference is always uniform. And therefore, this rested by the complete across sectional area, because it is not that actually as I told you that, there is no stress concentrations are there which are different and different regions, so they have the uniform way. So, whatever the cross sectional area is coming under the section of this tensile forces, it has to be uniform and it has to be considerable equally.

So, hoop stresses as I told you the force the hoop tension divided by area, so you see the force, which we computed previously is mass into acceleration, so mass m into $\omega^2 r^2$ divided by area. Because, this area is the effective area under the impact of the circumferential the effect plus whatever the forces are applying due to p .

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$$\text{hoop stress} = F/A = m \omega^2 r^2 / A$$

Where A is the cross-sectional area of the ring.

Now with unit length assumed m/A is the mass of the material per unit volume, i.e. the density ρ .

$$\text{hoop stress} = \rho \omega^2 r^2$$
$$\sigma_{11} = \rho \cdot \omega^2 \cdot r^2$$

So, now this unit length assume m by A , because we are assuming that the pressure is applied per unit length, so in this unit length assumption is always coming in to the picture an m by A is nothing but equals to mass of the material per unit volume. That is now, with those kind of the density we can say that, the hoop stresses are nothing but equals to ρ into ω square r square or we can say σ_H is nothing but equals to this density into ω square into the r square.

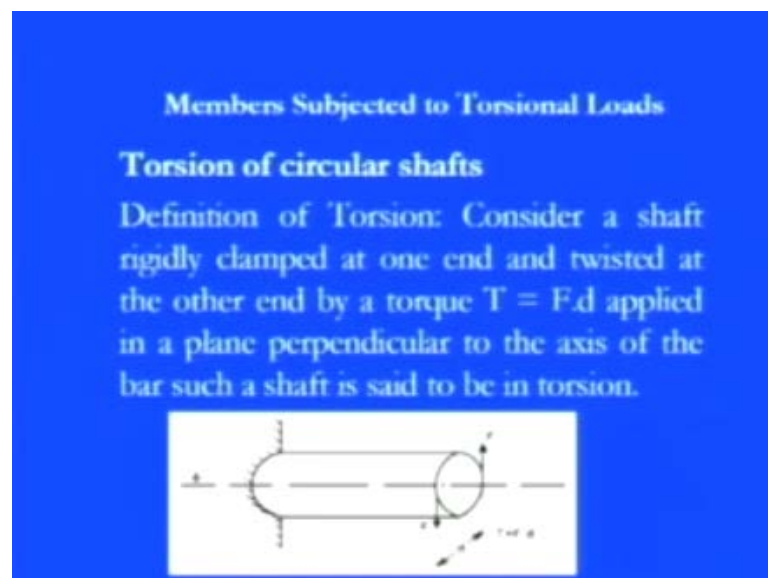
So, we can say that since, the mass we are considering that this mass of material per unit volume always density will come, so you see here what we are doing here we are simply considering the density, density ρ into ω square r square, so this is our hoop stresses. So, with the consideration of the hoop stresses always we are assuming that this hoop stresses are along the diametrical portion and it is in the tension, so whatever the deformation is coming, these are the tensile we can say the percentage elongation is positive, so these are always in the positive directions.

So, now you see you know like with those kinds of conditions, we can say that the hoop stresses are there, and this hoop stresses are due to the circumferential part. Due to the this centrifugal portion, along the circumferential part and this is nothing but equals to this density of the material ρ into ω square r square, so this is the hoop stresses. Now, you see here, if you are saying that, this thin cylinder members which are subjected

to the tensional load, that means if any kind of shearing is there till now, whatever we discussed only about the normal stress component.

That, we have normal stress component, they are acting along these two directions, so along the longitudinal or along the hoop stresses and that is what we were discussing even, if it is rotating on the hoop stresses are dominating and these are coming due to this density into omega square r square. But, if there is shearing portion that means, if a kind of couple or moment is acting on this particular area or particular direction, then what will happen then what kind of stress are there.

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So, this here we need to concern, the torsional effect on a circular shaft, so the definition of torsion is pretty simple that considering a shaft which is shown here, that we have a particular shaft which is rigidly clamped on a one end. That means, we have a shaft which is you can say a kind of cantilever part one end is rigid and one end is free, so we can simply apply the kind of load couple moment at the free end, but one end is absolutely rigid, so we cannot apply any kind of action on the fixed part.

So, if we consider the kind of shaft which is having a cantilever feeling, and which is clamped at one end and twisted at the other end by the torque and always, when we are applying a torque, always there is a force at the extreme corner into the diameter. So, this torque is F into d which is applied in a plain perpendicular to the axis of a bar such that

the shaft is said to be torsion, that means what the forces is always applying in such a way that they will just try to tend this shaft in a some direction.

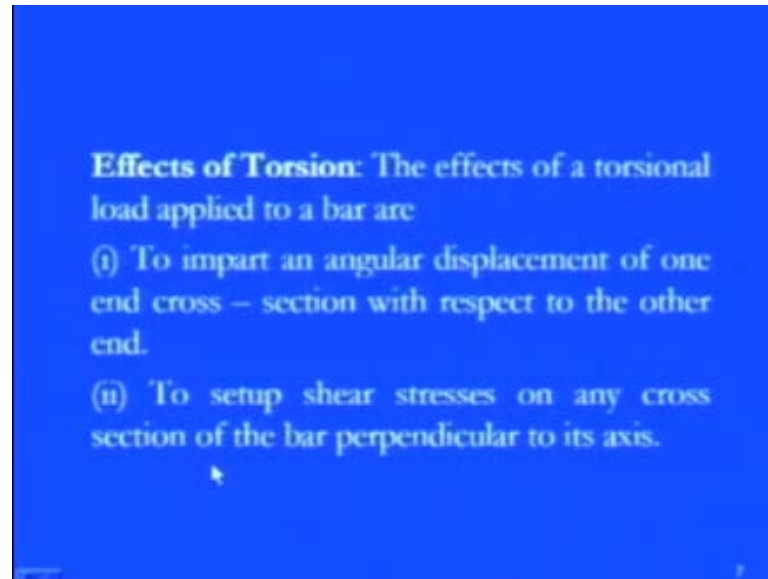
So, if I am saying that actually, if they tend to move this shaft in a clock wise or anti clock wise direction, always there is distortion is there and this distortion is not along the axis this distortion is there along a plane. So, if I am saying that this is my plane and if I am apply a torque is there the kind of distortion is there along this line, so here in this shaft if I am saying that this, these forces which are applying along these things with they will just try to tend this shaft into a anti clock wise direction.

So, whatever the distortion, because and this will tend to resist this kind of motion like that, so now we have a shearing action here, so one end is opposing and one end is in action and what will happen then, we have a torque which is F times of d , this is the diameter of this particular shaft. So, what will happen this kind of the this rotation or the twisting will come into the shaft, and the twisting in the direction, you can see this particular slide that the twisting will be there along this direction.

That means this direction will be effected more as compared to this x axis, so we can simply compute the kind of deformation and this deformation is computed by FE , so FE is nothing but the distortion in the shaft and this shaft is under, we can say rotating part or the couple is there. So, whatever the shearing action will come this will computed as a torsional, so torsion part is there, so now, what the effects are there of the torsion the effect of torsion load, at which is applying on a bar is to impart an angular displacement of one end cross section with respect to the other.

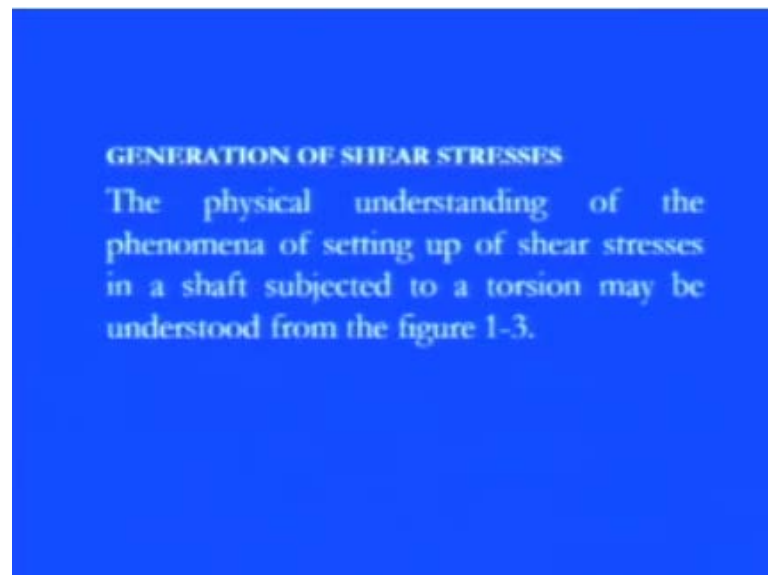
That means, as I told you in the previous we have a FE , so how FE is coming because of the distortion and this distortion is since along the circumference of the shaft. So, this FE is always known as the angular displacement, not the uniaxial, it is a angular displacement and angular displacement is always coming for a particular plane direction, so that is what we have a shearing part.

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So, to set up the shearing stress, any cross sectional bar is always perpendicular to its axis that means, it is not the axial part it is always perpendicular to one axis. So, now these two major impacts are there, on the kind of rotating shaft wherever the couple is applied on that.

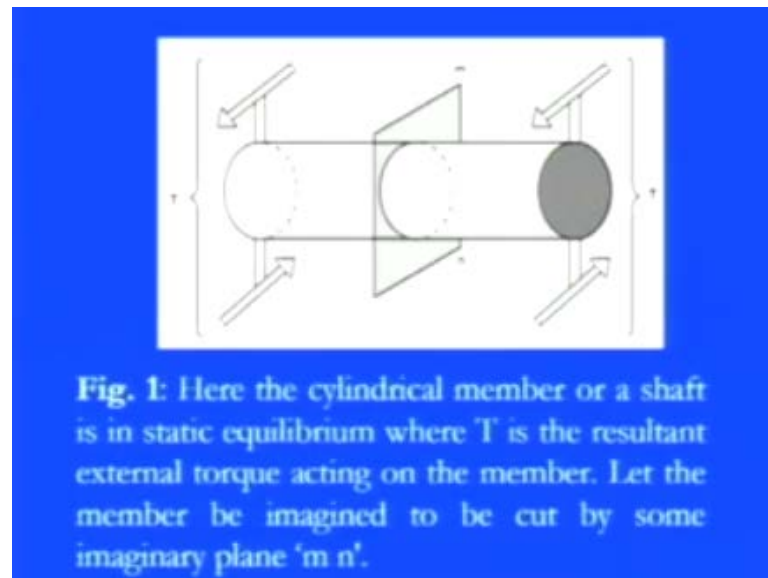
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So, now we just want to see that, how this shearing stresses are occurring on those things, so we have the physical understanding of the phenomena, of particular this set up of the shearing stresses in a shaft subjected to a torsion, can be easily understanding by

taking a different applications. That, how this forces are been applied and how this cross sectional area is to affected under the application of this, the torsion or the couple or any kind of the moment is there at some of the corners. So, now we are taking the three different cases and we would like to analyze that actually how these stresses are being set up in those things.

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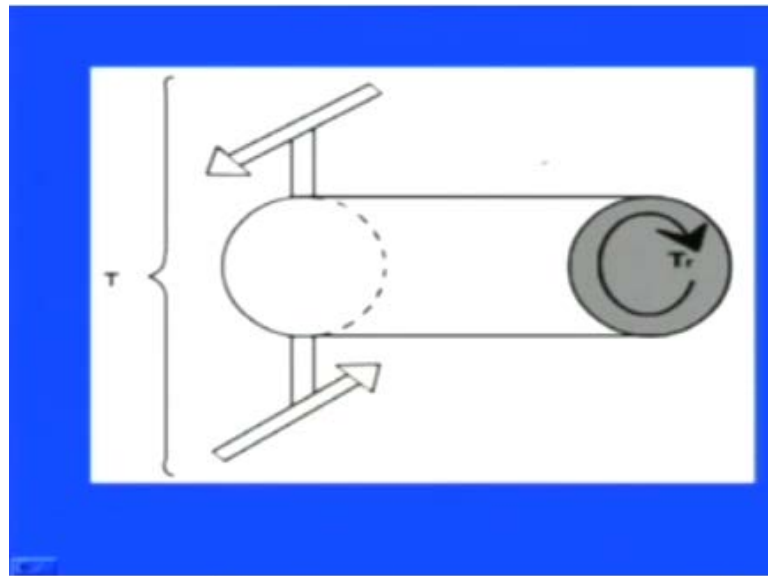
So, first what we have, we have a cylindrical member or we can say a shaft, which in the static equilibrium and this particular torque, you can see this torques are simply applied in a resultant external torque, which is just is a member. Because, whatever the members are there of this particular shaft they are acting in such a way that they have just tried to act and react in a particular direction.

And, let the member be imagined to be cut at some section, on a plane m cross n that means, what will happen, now just look at this particular figure, we have a simply shaft and at these two corners, we are simply trying to apply the torque. So, the corresponding torque how it will come, it will come you know like in this side, if we are talking about the right side of this particular portion, it will just try to tend this shaft in a clock wise direction.

Then, we need to apply in opposite way or we can say these kind of force, so that we can say that this is this under the application of this torque, now our shaft is in equilibrium position. So, what will happen, if you are considering that if we have a one segment, this

m n segment, all we just want to see that how this shafts the torsional portion is to be acted that means, how these torsional stresses are being set up within this shaft and since I told you that it is a circumferential portion. So, always it is you know like acting all across this particular circumference of this rotating shaft and this is always going as a diagonal part, not along the axis as I told you in the previous section, that it will be perpendicular to the this axis.

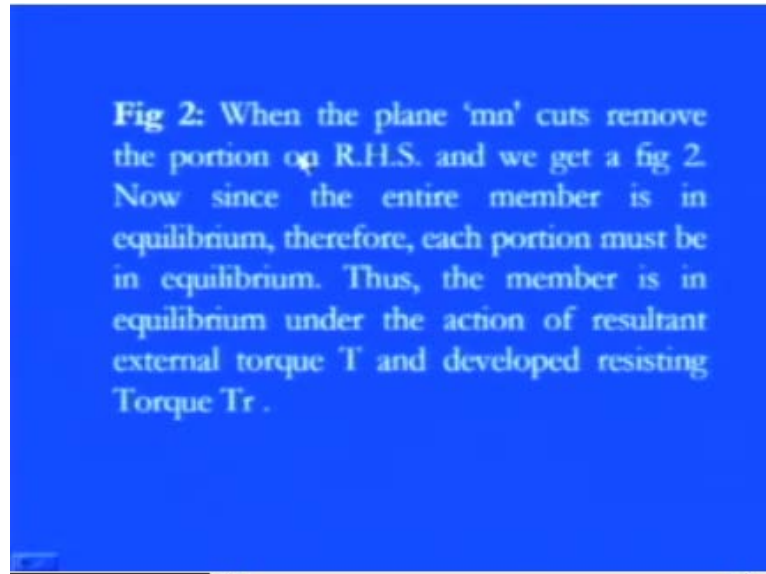
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So, here, now this is the great impact, so this the torque is you know like trying to rotate in this clock wise direction, as I told you and you see, now we have the another portion, if we just want to make this segment as an equilibrium that, we need to apply in a anticlockwise direction. So, this is the one significant effect is there middle portion is also affected by the real cause of rotation, so we have this circumferential part is there, which is and the layers of this circumference of this particular cylinder will always tend to move in this direction

So, here there is no elongation towards x direction or y direction, only they are simply tending in the circumference of those things, so on the other end what we are doing here we are just trying to put the this opposite torque is there, so there is net effect is there on this kind of thing.

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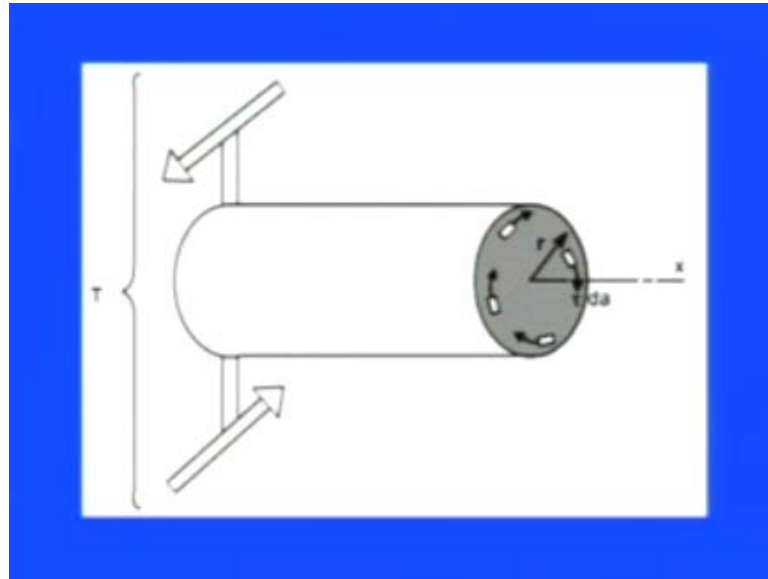


That means, the first thing which we discussed that actually, if we cut the portion in the middle one then, we found that actually this middle one is in a equilibrium portion only under the application of those things. But, if we the figures the previous figure, which I shown you when a plane is m n means, if we cut that particular portion and if we see that the portion on right hand side what we get, we get that actually the entire member is subjected by the applied force and the remaining force are just trying to settle down the layers of this thin shaft.

That means, what are the distortion is there and if you want to restore those portions, we need apply the equivalent m , the opposite direction of this kind of torsions just to make the layers in a perfect way. So, since the entire member is in equilibrium position as I told you that is what therefore, an each portion must be in equilibrium that means, whatever the layers or the distortion of the layers are there, they have to be there in a their original position by under the application of this action and reactive torsions.

Thus a member is in equilibrium under the action of resultant torque T and developed the resisting torque T_r , so always you will find that they are just trying to tend in a real manner. So, that whatever the torque which are applying, they can simply form in resultant torque T_r , in such a you know way that it can simply appose and they will tend to you know like this portion in an equilibrium way.

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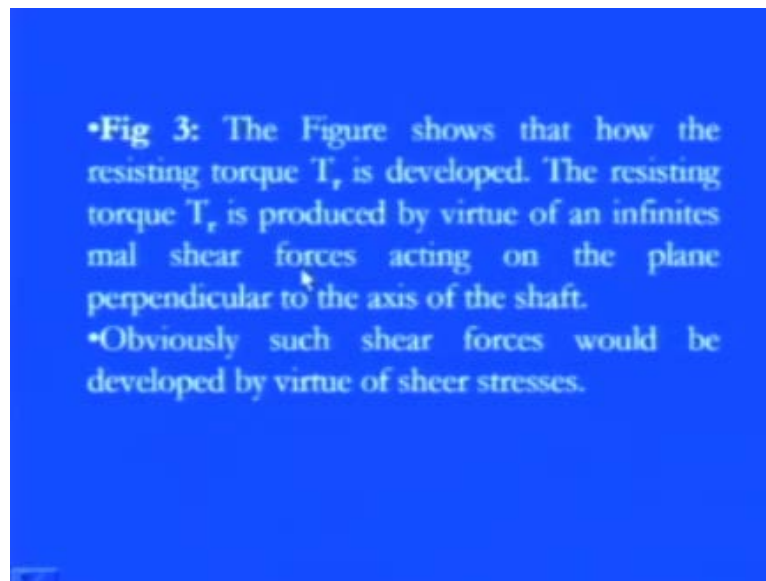


So, now here this one more figure, this third case which we can simply see that, now these the portions are just like that, when you are simply applying the load the kind of distortion as I told you, so these are the rotational part if I am saying that these are the small segments are there. So, the all small segments are just trying to tend in a particular direction or we can say these are the layers if you are considering, then they are layers are just tending to move in this direction under the application of this load.

So, what will happen we will have a distortion, so if I am considering those distortion or the ((Refer Time: 26:45)) which I told in very first figure, that we have your angular displacement is there. So, if I am considering the angular displacement and if it is the radius of r is there then, this τ into da will give you that what exactly the distortion is that means, how distortion is taking place in a shaft under the application of this torque and then and all those, even not only since we are cutting this section along the x axis.

So, this distortion is there, but if all along these the entire portion you will find that the entire the layers of the entire portion is equally affected, under this particular application of these torques.

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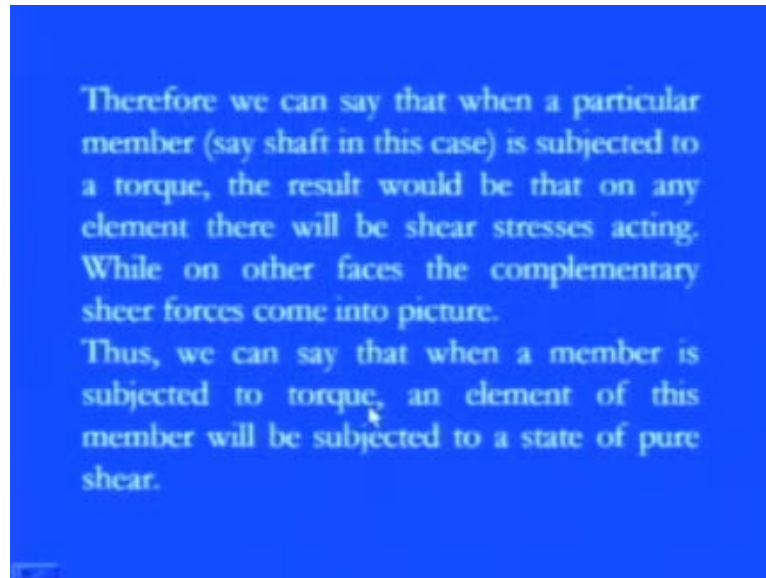


So, we need to consider this things the previous figure, which I shown you is simply shows that, how the resultant torque is simply developing, because if until unless if resulting torque is not applied, then probably this permant distortion will take place and then there is a the failure will start along the diagonal portion. So, equal and this resisting forces are always dominating, wherever the couples are there along any axis.

So, the resisting torque T_r is simply produced by a virtue of infinite decimal that means, the small portion of shear stresses acting on a plane perpendicular the axis of shaft. And that is what I told you like wherever, these torques are applied these the layers are deforming in a perfect way, because the shearing the shearing action is there on individual segment of the layer.

So, and if we are just combining those segment, will find that the there is a complete distortion is there all along from, the resisting portion to the extreme corner and then these are at a this rigid corner always it is very small. But, as you go along the way the point of application of this torque is you will find the more of distortion is there, because the layers are, so set up in such a way, that shear stresses can be easily acted on that and the resultant torque can easily equally applied in a resisting way. So obviously, such shear forces would be developed by virtue of these shearing actions or we can say the shear stresses.

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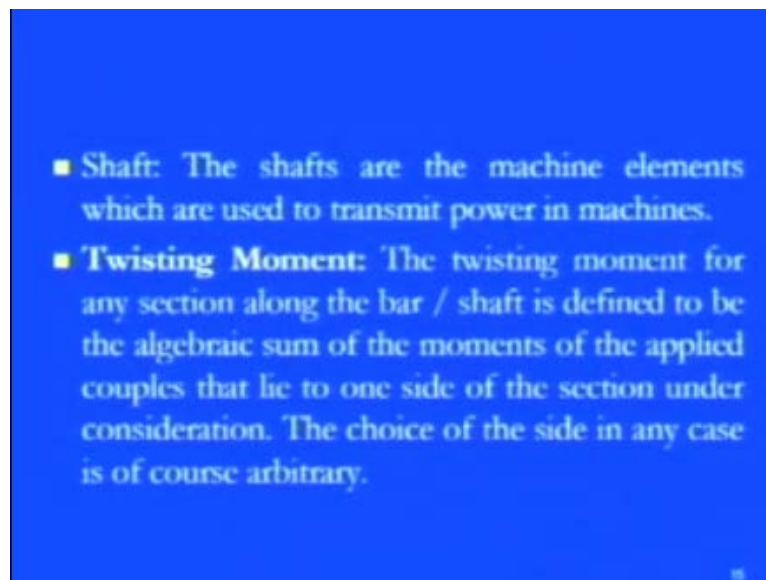
So, we can say that actually whenever when a particular member or let us say, we can say a shaft is subjected to a torque the result would be that on any of the element, there will be a shearing action is there, we know which is acting. So, whatever the layers are there as we form that actually we found that the shearing action or the shearing action is there on these individual elements, and the they are tending to move in a opposite direction.

But, if we are saying that the object is in equilibrium position always, we have a equal and opposite part and the technical term for the equal and opposite shearing stress is the complementary shearing stresses or complementary shear forces. So, if I am saying that, any object which is under the rotation and there is a couple is acting, the shearing is there all elements which are you know like under the influence of this couple, they are simply experiences the shear forces. And they will just tend to move along the circumference part not along the axis just towards the plane part.

And then we have equal and opposite portion, which is coming to balance or to equilibrium that portion and these shearing forces are known as the complementary shearing forces. Always comes in the picture whenever the action of this nature is there, thus we can say that when a member is subjected to a torque an element of this member will subjected to a state of pure shear, always keep this thing in your mind if you are saying that this a member is there and the torque is applying on that member any the shaft beam or any rotational portion.

If we are saying that torque is applied, couple is applied on any of the power portion it is simply applied, under the application of this torque these, this whatever the portions are there or the elements are there or the microstructures are there or we can say the layers of these portions are always, under the action of pure shear. So, shearing is coming in that way, so if you are saying that the shafts are you know like the machine element in which we are you know we are using for transmitting power.

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And if due to the action of this torque they are coming with those kind of forces, whatever the stresses are inducing, due to the action of the couple or the torque on this kind of shaft rotating shaft, they will be under the action of pure shear. But, now we have the different kinds of, so moments are also there means we cannot say the only the torque will give you the kind of feeling of the shearing stresses.

But, if have a twisting moment, what is the meaning of the twisting moment the twisting moment for any section, along a bar or shaft can be easily defined that the algebraic sum of the moments applied just couple, which we which we are applying the couples that lie on one side of the section under consideration. That means, if we are saying that this is a rigid bar just like the cantilever bar as I shown you in the previous figure you know like and the couple is acting.

So, these kind of twisting moments are there they will just try to twist them, either the bar or shaft and the kind of distortion will started from the action starts like that... So, if you are saying that one section which is under the kind of these moments and if you are

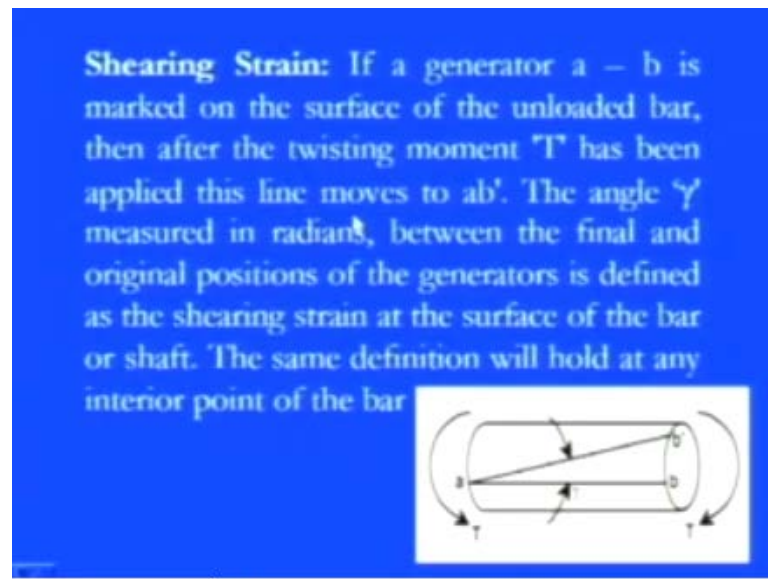
summing those applied couples or the moments, we are saying that the choice of that side in any case is always the arbitrary is there, but in any case what we have we have a kind of distortion in all those kinds of segments.

And if you sum up those segments will find that where the applied portions are there, we have we are more distortion where the rigid sections are there we have a small distortion and due to the impact of this moments. So, we can simply sum up those twisting moments as such and due to, that the kind of distortion is always coming into the picture and they will be in the form of this circumferential part, that means the circumference is always tend to move in a particular direction.

So, if you want to compute the distortion as such we are saying that or the deformation, always a strain form is coming and since we are saying that, whenever twisting moment or couple or any kind of the torque is applied, on the kind of the shaft or we can say the bar always what we have it is under the state of pure shear. So, if you are saying that due to the pure shear this, deformation is there and which we are measuring in terms of the strain the kind of strain is always known as the shearing strain.

So, you can see this particular figure that, if you know like we have a simply shaft is there and since it is simply unloaded that means, there is no load application is there, there is no torque is there then it will simply show the layer of these shafts are just along the a b. So, now this is a b is there which is pretty simple, so if the if a generator a b is marked on a surface of a unloaded bar, there is no distortion is there, there is no load application the layers of these bars are well you know like stabilized manner and they are perfectly you know like stabilize way. So now, if we apply the torque, so here the torque application is there or the twisting moments are there on both of the corners of this particular shaft then what will happen, then after a twisting moment T has been applied on this particular line.

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And this line will move the resultant of the twisting moment tend to this b portion will shifted to b dash that means, as I told you the layers are, so deform or distorted that these portion these are nothing but this is the layer and this is the deform layer, un-deform layer to deform layer we are gaining some sort of distortion. And the angle this γ which is also known as the strains shearing strain component is simply, measured in radians between the final and the original position of the generator.

And always since, we are saying that it is measuring all along the circumference, circumferential part or all along we are saying that towards the plane. So, this distortion which is measuring in terms of in terms of angles or in terms of the radians, these strains are known as the shearing strains, and always they are acting not along the axis, along the plane or we can say along the circumferential part and the same definition will be you know holding for any kind of interior portion of the bar.

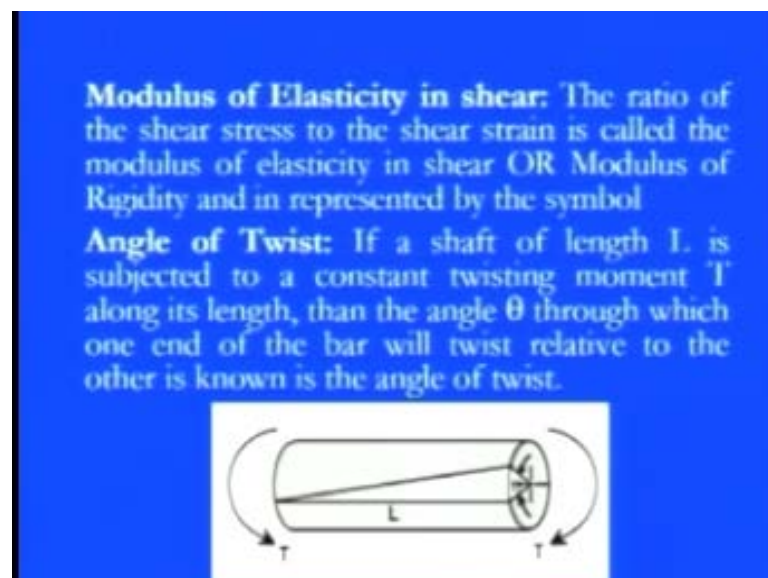
That means, as I told you that there is an individual impacts are there, on the individual component of these bars and they are always try to tend to move to that, has I told you clearly that you know these all these internal portion or along this particular length. They just try to you know like move a to move along the circumferential part, due to the application of this twisting moment, so as a resultant if I am saying that you know like, now this is the stabilized manner.

So, we will find that there is a distortion and this distortion is as we are computing in terms of shearing strain, so this the angle will clearly gives you the kind of distortion and

this angle is known as the shearing strain angle or we can say that this γ will be is nothing but equals to shearing strain. So, this is real significance of shearing strain.

So, whenever a shaft or a bar is under the application of twisting moment or torque, we have a state of pure stress you know which are acting on this particular bars and due to that we have a distortion which can be easily computed in form of the strain, and these strains are known as the shearing strain. And these strains are always acting along you know like the plane or we can say along the circumferential part, which can be easily computed as we can simply you know like shown in this particular figure.

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So now, what we are doing here we are simply saying that we have a stress, we have a strain, so obviously since we are considering the stress and strain, since both stress and strains are the shearing part. So, and we are assuming that these under the application of these you know like, the twisting moment or the torque these deformation or the stress formation is under the elastic one.

So, we can simply apply the Hooke's Law and for which, always we are defining the Young's modulus of elasticity and since here, the shearing moment is there. So, Young's modulus will not be applied, because Young's modulus is only applicable wherever the normal stress component are there, so here we are applying the shear modulus of rigidity because, it is shearing action is there, so as we discussed you see in the previous chapter that we have a G which is the Young's modulus of rigidity.

So, the modulus of elasticity in the shear the ratio of shear stress to shear strain, will give you the modulus of elasticity or you can say the rigidity in the shear or modulus of rigidity means both have the same meaning does not means, since here it is you know like the distortion is there along the plane. So, we are saying that modulus of rigidity, otherwise it compute the same kind of the relation that the shear stress divided by the shear strain will give you the this shear modulus of rigidity G .

And it is always denoted by the G , G is a capital you know the capital G is there and in that if you are saying that we are measuring the angle of twist. So, how we can measure as I told you in the previous section that since there is a shearing action and with the action there is always the distortion is there, so this distortion is more at the this extreme corner where this twisting moment is applied, so these you see, now this is the angle which gives you a clear feeling of the shearing strain.

So, if a if a shaft if I am saying that the shaft is of length L which is subjected to a constant twisting moment T . So, these are the T along its length then angle θ will always gives you that actually, how much distortion is there and through this always one end of bar, will twist relative to the other end and this angle is known as the angle of twist, because wherever the shearing action is there always the angle of twist is forming and if we can measure the angle of twist it will give you that, how much distortion is there and this distortion is gives you a clear feeling about the shearing strain.

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1. Despite the differences in the forms of loading, we see that there are number of similarities between bending and torsion, including for example, a linear variation of stresses and strain with position.

In torsion the members are subjected to moments (couples) in planes normal to their axes.

2. For the purpose of designing a circular shaft to withstand a given torque, we must develop an equation giving the relation between twisting moment, maximum shear stress produced, and a quantity representing the size and shape of the cross-sectional area of the shaft.

So, despite the differences in the form of the loading, whatever in terms of the twisting moment or couple or whatever, we see that there are number of similar similarity is there in between the bending and the torsion including for a linear variation of the stresses and the strain within that particular effective position. So, in torsion the members are subjected to the moments in a plane normal to always towards the means this actions that means, we are not considering whether the torsional actions this torsion actions are there we are not considering that what is going on with the axial form.

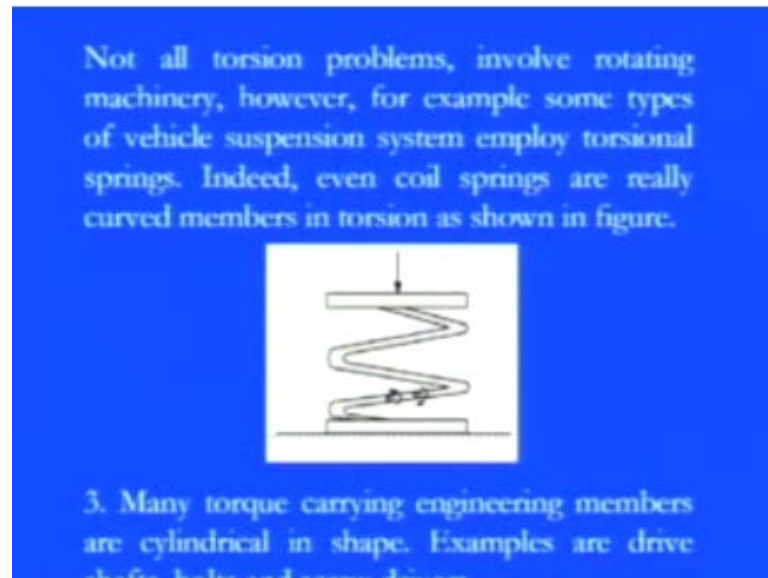
Only we are considering that what is the impact is there perpendicular to this axis or we can say along the plane. So, this is one the clear which gives you a clear feeling about the bending as well as the torsional part, for the purpose of designing a circular shaft to withstand the given torque because, you see whatever the torque is there due to that always there is a kind of shearing forms are there.

We must develop an equation which will give you the relationship between the twisting moment maximum shear stresses produced, due to the action of twisting moment and a quantity which represent the size and shape of a cross sectional area of a shaft. So, you see here, what are the influencing parameter just to design a twisting just to design a shaft or a bar first the main thing is that actually what exactly the twisting moment is applying due to that, so once you know the magnitude of or once you know the nature of the twisting moment corresponding shear stresses are there.

Because, once we can say that, now the twisting moment is applying at the extreme corner or the middle portion or due to this reason like that the couples are there, immediately the shear the impact is there on the shear stresses. So, what is the maximum shear stress which is producing due to the application of the twisting moment, and then the corresponding quantity is there which represent means, this is the clear dimensional parameters there.

Because, if we have a bigger shaft or smaller shaft if we have a thin shaft or the thick shaft then definitely or we have hallow shaft or rigid shaft, what exactly the shape and size is there, this will also you know influence the design of this particular kind of shaft. So, we need to consider this kind of parameter, we want to design the shaft, so not all the torsional problems you know like involves the rotating Machinery.

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However, some of the examples are there like the suspension systems, which employ the torsional the torsional springs they are always under the action of the shearing part. That means, f this particular figure, you will find that all these coils are there of springs there are you know like as a curved surfaces are there and they are under the action of the torsion.

So, whenever we apply the load, always they have the kind of you know like the twisting is there all along that you can see this particular figure that they are under you know like the twisting moment. So, whenever we just want to design those things always we have to concern, that actually how this moments are taking place and what is the effective area under which this moments are there, once you set up the twisting moment then the immediate action as I told you will come that, actually what will be the shearing action is there under the action of the twisting moment.

Once you once you know those things, that twisting moments as well as the shear stress then the next is there that how many number of coils are there, the n number of coils and what is the effective diameter of these coils you can see these things. So, these are the key features and if you want to design those things always in terms of the spring, because spring is nothing but they are simply based on the stiffness. .

So, what you know like the force per unit deformation is there under the action of these shearing part. So, we need to consider the stiffness that how much stiff of the stiffness is there in this the spring that means, how stiff these parts are there how many number of

coils are there, this is very important figure and then what the twisting moment and the shearing stress are occurring into those kind of the springs.

Many torque carrying engineering members are always the cylindrical in shape, so that is what we need to derive that, whether it is you know like shaft is there bolts are there the screw drivers are there. So, how we can see that actually the twisting moments are playing on those specially these shafts or the bolts or we can say kind of the springs, and how the maximum shear stresses are coming in to the picture.

So, here in this particular chapter what we discussed, we simply discussed that if a member is under the action of pure shear that means, if I am saying that if a member is rotating then, how the you know like we can say that the pure shear will come into the picture. And if you are saying that you know like all the elements, which are there within this particular cylinder they are under the action of the pure shear then, what will happen then the complementary shearing component, will come to stabilize you know like the this particular member in a equilibrium manner.

So, once we have this kind of phenomena then immediately it comes actually, if you let us say if we have any kind of circular bar which is under the action of the couple then what will happen, if you want to design those kinds of thing then actually, we have to be very careful that actually what is the affected area or means this rotating planes are there. And then if we are saying that this shearing actions are there then what is the shear strains are there, because when we are saying that this is under the action of couple or twisting moment or torque then, shearing is there straight way it is under the action of pure shear.

And because of, the pure shear we have the shearing strain, once you have the shearing strain or this kind of thing then you need to measure that actually, what exactly the shearing strains are there or how the distortion is taking place under the action of pure torsion. And once you stabilize those things we have a angle θ which is known as the this angle of twist and corresponding you see, since we have the stress shear stress component, we have the shearing strain component obviously, we have this new constant that is the since it the under elastic deformation.

So, we have the new constant, which are measuring the relationship between or we can say which are setting the relationship between the stress and strain that is the shearing modulus of rigidity. So, shearing modulus of rigidity G is nothing but equals to shear

stress τ divided by ϵ we can say γ that is the shear strain, so we can measure all these kind of things and it is applicable equally this twisting concept to the springs also.

Because springs are always you see, wherever some twisting is there this all coils are simply subjected by the twisting moment or the torque and corresponding, if you want to design those things how this stresses are being set up within that structure we have to be careful. So, in this chapter we discussed those things, now in the next chapter our main focus is that actually once we have the rotational shaft then, how we can compute those things.

We still only we set up that these stresses are there the strains are there, but how we you see, how we can compute those part if we have a moment at the middle portion or extreme corner or if we have a simply supported beam or if we have a cantilever. So, what will happen and what the real impact is there of the shear stresses, so this kind of discussion which we are going to discuss in our next lecture.

Thank you.