**Strength of Materials**
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**Lecture – 17**

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department IIT Roorkee. I am going to deliver my Lecture 17 on the subject of the Strength of Materials and this course is developed under this national program on technological enhanced learning. So, before starting of the new chapter, we are going to discuss about the previous things, which we you know like discussed on the stress and strains about the thin cylinders that if we have a thin cylinder.

And also we also defined about the thin cylinder that, if we can simply categorize the cylinder or we can say cylindrical shape particular according to the thickness. Like, if the thickness is there which is less than or equals to 0.1 millimeter or even you can say, you know like if it is somewhat more than that we are always categorizing that this is the thin cylinder. And if it is you see know like somewhat more than this is there then it is a thick cylinder and there is a clear difference is there about the stresses development, because of the pressure excursion, in thin as well as the thick cylinders.

So, you see we discussed that actually if we have a thin cylinder, then you know like because of the steam pressure or any kind of pressure, the fluid is you know like, flowing from this thin cylinder or the steam is flowing from the thin cylinder. This whatever the inside pressure is there, it always acted in three different directions and that is what you see, we are saying that when it is you know exerting on the this inside surface or in terms of the circumferential, in terms of the longitudinal whatever like that.

So, we can characterize these whatever the stresses, which are inducing due to the applied these internal pressure in to 3 main categories and these three stresses are always mutually perpendicular to each other. So, if I am saying that one stress which is you know like along the length of the thin cylinder this is nothing but, equals to the longitudinal stress. Longitudinal stress, you know like and also we discussed that actually how to calculate the longitudinal stress.

So, which force is you know like, dominating along these things and which area we need to consider. Because, the main thing is that actually what is the effective area under, which the you know like the pressure is exerting, this is the only difference which you creates, you know like all three you know like types of stresses. So, if you are talking about the longitudinal stresses, then we found that the concerning area was pi d t and if you know like multiple with the p times, the internal pressure times area that is nothing but, the you know like the stress.

So, we can simply this nothing but, the force exerting, so we can found that if we equate the force balance. That means, you see under the exertion of these forces or under the application of these forces, if our object is in a equilibrium condition, we can simply equate those things. And we found that the longitudinal stress is nothing, but equals to p d by 4 t in which you see the main influencing parameter was the applied pressure, then the diameter concern of the thin cylinder and the divided by 4 t what is the thickness is there.

So, you see in that the two main dimensional parameter was there d and t both are you know like the ratio part and always you see is the influencing pressure is always there, that how much you know the stress can be induced within that part. And second stress, which we discussed that is the main stress and if you want to design, you know like the thin cylinder then we have to consider that is the circumferential stress or the hoop stress. So, as per it is name the circumferential whatever the pressure, which is exerting inside the cylinder it is always along the circumference of the thin cylinder.

So, we need to consider the effective area that where you see this pressures are exerting and which is the effective area under that we need to consider that, this is the area which is bearing or which is having a the kind of hoop stresses or the circumferential stresses. So, when we equate the force balance, then we found that the circumferential stress was nothing but, equals to the p d by 2 t, so there was you know like again the similar kind of even, if we are considering the mutual perpendicular means actually if I am saying that this is my longitudinal way.

So, the longitudinal stresses which are acting towards this direction, then we have the mutual perpendicular means that is the top of side, we have one more stress that is known as the circumferential or we can say that is the hoop stress, which is nothing but, equals to p d by 2 t. So, only the difference in between these two stress components are nothing, but a just half, so one is the longitudinal, one is p d by 4 t and one is the circumference it is p d by 2 t. So, we have you know like as per it is magnitude also we found that, the circumferential stresses are always in the dominant way.

So, as I told you that actually these stresses are always, the key stresses and if you want to design any thin cylinder you have to consider carefully and accordingly you see the factor of safety is to be considered. Third stress component was there that is radial stress; that means, the just perpendicular to the radial direction, though you see as compare to these two stresses, means the longitudinal and the circumferential, this stress component is not, so significant. So, you know like just neglect that part or the contribution of this radial stress is very, very negligible.

So, we can simply skip that part and only if you want to design the thin cylinder, then we have to concern, we have to you know like consider the two main stress component as one is the longitudinal one, along the longitudinal direction and one is the circumferential one, along the circumferential of the thin cylinder. But, as we discussed that actually you know like in these two, they are along you like one is along these x axis, one is along y axis.

So, you see when one stress is dominating definitely you see there is you know like, if I am saying that one is the along the longitudinal one and it is having a tensile in the nature. So, definitely on the other side there is a contraction, but we just try to make the balance of that, so what the exact you know like relationship is there in between these longitudinal as well as the circumferential, we just want to set up the relation. So, as we defined in the previously that, if we are considering that these stress components are under the Hooke’s Law.

That means, you see you know like we are saying that only the elastic deformation is happening, under the application of these forces or the stresses, then we can apply the Hooke’s Law. So, here also we are considering that these stress component, which are inducing under the application of these perpendicular or the parallel forces, always they are under the elastic deformation and then we can apply the Hooke’s law.

So, whatever the components are there; that means, the Young’s modulus of elasticity, bulk modulus of elasticity, this Shear modulus of rigidity and the Poisson ratio, they are all applicable for this kind of stresses. So, you see now what we are going to do, we are going to define the relationship between the longitudinal stress and the circumferential stress. So, here we are again we need to define the Poisson ratio and based on the Poisson ratio, you see we can simply make the relationship.

So, here in this lecture we are starting from the Poisson ratio, we just try to set up the relationship in between these two stresses. And then you see will go for the principle stress and principle strains, that what will happen exactly along this, that if we you know like, right now these two the normal stress means you see you know like both are the normal stress component, the longitudinal as well as the circumferential. So, if we are considering the normal stress component, then definitely you know like the principle stresses are there and these stress components are always along the principle planes.

So, we need to define that actually what are different components of the principle stresses like Epsilon 1 and Epsilon 2 for the principle strain and sigma 1 and sigma 2 over the principle stresses. And then you see we will try to set up the relationship for these things, that actually if cylinder you know like subjected by the different kind of pressures, then what will happen. Not only use you see, you know like only when a flow is there if a different kind of pressure is there; that means, if point of force of application of you know like, this pressure is different all together, then what will happen. So, these kind of relations which we are going to discuss in the next lecture, so here we are starting when you see there is a change in dimension when we apply the pressure then what will happen?

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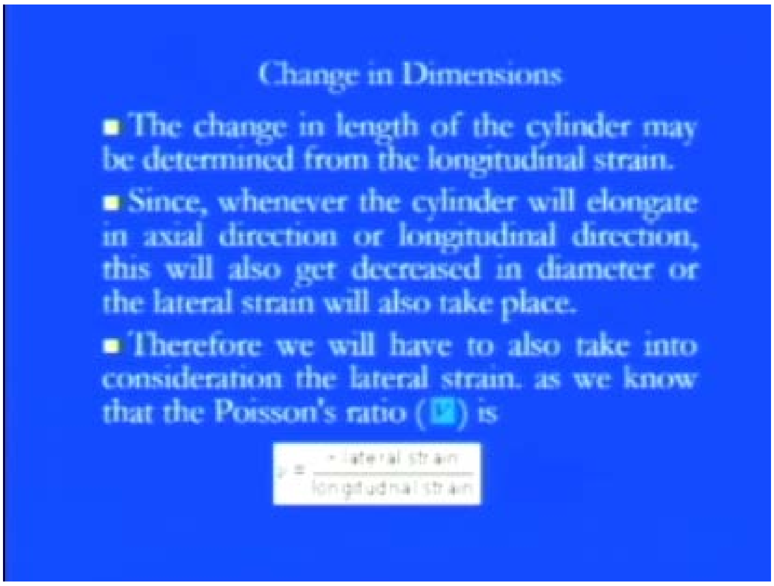


So, if we discussing that when there is you know like, the flow is flowing in the thin cylinder and change in length is there of the cylinder along that path, then definitely you see we have along the path of the longitudinal direction, then we will be having a longitudinal deformation. And if you want to measure the longitudinal deformation; obviously, you see the longitudinal strain is the main component through which, we can say that how much deformation is there.

So, since whenever the cylinder will elongate in axial direction s or we can see the longitudinal direction, this will also get decrease in the diameter or we can say that there is a lateral strain is there or we can say the circumferential strain is there. And if you want to measure that you see, you know like always what the deformation is there in the diameter it will give you the lateral strain. So, you see as we are discussing about the flow is there and this flow is striking on the two extreme ends of the cylinder.

And due to that there is elongation in the, you know like the length of the cylinder then corresponding change is there in the diameter. So, you can relate both the things by Poisson ratio, so you see we will have you know like just taking in to consideration of longitudinal. That means the change of length and lateral; that means, the change in diameter, only we can correlate by with using of the Poisson ratio.

So, here it is you see we have the Poisson ratio mu is nothing but, equals to as per it is own definition minus because, there is a reduction in diameter a as there is a increase in the lengths. So, minus sign will be there, lateral strain, which is nothing but, equals to the change of diameter divided by original diameter because, there is you know like as we move in the further this longitudinal direction, there is a contraction or the squeezing is there in the diameter side.

So, minus sign into lateral strain is there, which we can simply compute by taking the diametrical changes. So, change in diameter divided by original diameter will give you the lateral strain divided by we have a longitudinal strain and that is you see the tensile form is there. So, we have a perfect elongation in that, so change in length is positive divided by what is the original length is there. So, you see now what we are doing here we are simply comparing the results with the diameter as well as the length by taking into consideration of a mu, mu is the Poisson ratio.

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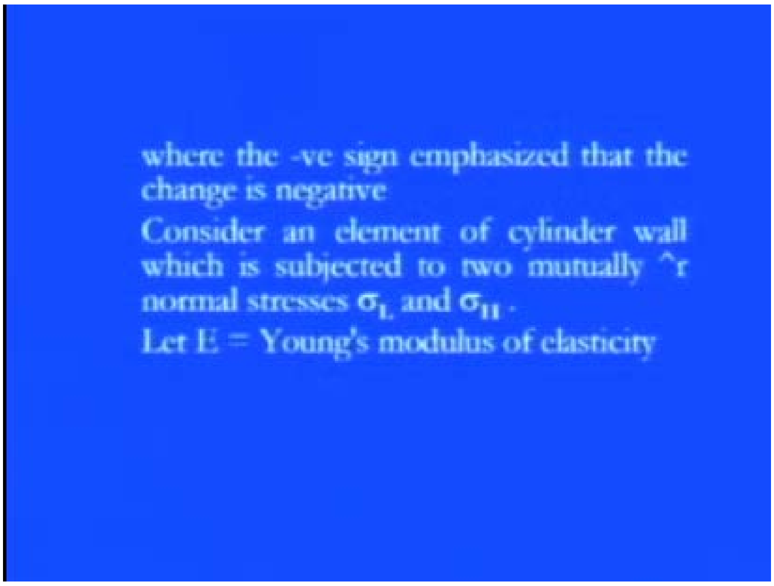


So, you see whenever the negative sign emphasized that, you know like we have the change of diameter, whatever the change is there irrespective of the if there is a compression, then change in length is there. So, instead of you are saying in particular dimension we can say there is a change, whatever the change is there is negative. So, here we are clearly mentioned that, the negative sign always gives you the emphasis on what the change is there and whatever the change is there, that is you see in terms of the negative ((Refer Time: 11:08)) squeezing part is there.

So, here you see we have this negative sign towards the diametrical way or circumferential way, consider an element of the cylinder you see a cylinder wall. Because, it is you see the pressure is exerting on the cylinder wall, which is subjected to a two mutual perpendicular stresses, as I told you this both stresses are normal to each other or these are the normal stresses, they are mutually perpendicular to each other, one is along the length.

So, that is the sigma L, that is the longitudinal stress one is along the y axis; that means, the perpendicular to this along the circumference or it is hoop stress sigma H is there. And since, you see as I told you that we are taking the deformation only under elastic deformation, so all those you know like the Hooke’s law concern is valid to this kind of deformation. So, we can say that the E Young’s modulus of elasticity is very well valid for this kind of deformation.

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So, you see now you know like, if you want to see that what exactly the resultant strain is there. Because, now what will happen there is a change in length, there is a change in diameter, so what is the resultant deformation is there in the cylinder due to these application of fluid. So, we just want to measure the resultant deformation, so always you see resultant deformation is coming in terms of the resultant strain. So, if we are talking about longitudinal direction, then you see the resultant strain in the longitudinal direction is nothing but, equals to the sigma L.

Because, we are considering the dominant way in the longitudinal one, so the sigma L has to be come in the first component. So, sigma L by E this is nothing but, equals to the deformation stress by modulus of elasticity is our strain is there, so the strain in the longitudinal direction Epsilon E which is nothing but, equals to sigma L by E minus mu times of sigma H by E. So, mu is nothing but, the Poisson ratio because, we are considering along the longitudinal direction, so if you are going for other direction, then as I told you in the previous slide that the Poisson ratio has to be come.

So, Poisson ratio into sigma H that is the hoop strain, so Epsilon H is nothing but, equals to sigma H by E. So, you see here, if you compute those things then we have 1 by E which is you know like, the common segment in both of the terms, so 1 by E into bracket of sigma L minus mu times of sigma H. So, you see here, if you want to compute the resulting strain along you know like the length of the tube or cylinder you can say thin cylinder.

So, we will find that both are contributely, you know like we have any contribution from sigma L, as well as we have contribution from the circumferential stress. So, now you see considering both the stress, we have already derive the derivation that the sigma L as well as the sigma H can be derived with the using of you know like, the pressure, diameter and the thickness. So, again we are considering those things the sigma L which is nothing but, equals to p d by 4 t sigma H, which is the hoop stress is nothing but, equals to p d by 2 t.

So, by applying both the things in the first equation what we have, we have Epsilon 1 because, this is you see what the longitudinal strain, means the strain or the deformation in the length direction or we can say along the longitudinal direction. So, we have Epsilon 1 which is nothing but, equals to if you put those things, then we have p d by 4 E t. So, you see you know like I am just taking four also as a common in denominator, so p d by 4 E into t, which is into 1 minus 2 mu.

So, now what you have, you have the change in the length, so if would like to see that what exactly you know like, you have the total deformation. So, if you want to correlate that how much change is there in the length along this particular direction, we can again take you know like that delta L or we can say the change in length, which is equals to you know like, the longitudinal strain. Because, it is in along the longitudinal direction into the original length because, if you want to compute the strain, strain is nothing but, equals to change in dimension divided by original dimension.

So, here since we are taking the length as a main dimension, so here the change in length, which is equals to longitudinal strain into original length. So, original length we are already you know like put that it is L capital L, so we have Epsilon 1 which is coming from p d by 4 E t into 1 minus 2 times of Poisson ratio. So, this is Epsilon 1 into L is the total original length. So, now you see similarly if we are going for the hoop strain; that means, the second strain, means you see if we are going along that longitudinal direction, then we have sigma L by E in minus mu times of sigma H by E.

Because, our dominancy is along the longitudinal way, but if we are talking about the hoop stress that, now we are going for the next strain, that the hoop means what the exact deformation is there, along the circumference of the cylinder. And if you want to measure that then definitely we have a strain component and that strain component is known as the hoop strain or we can circumferential strain. So, hoop is strain is Epsilon 2, so Epsilon 2 will be nothing but, equals to sigma H by E.

Because, here the dominant parameter is sigma H and due to that you see whatever the deformation is coming that is Epsilon H. So, Epsilon H is nothing, but equals to sigma H by E, so here you see if we are you know like see this particular part, then you will find that Epsilon 2 is nothing but, equals to sigma H by E minus mu times of sigma L by E. Because, now you see what we are considering that the elongation is there along the diametrical size, so you see this is the dominant parameter now. So, if you are saying that the dominance parameter along the diameter, so whatever the contraction is there.

So, now the tensile load is there along the diameter, so; obviously, we have an contraction along the length directions. So, what we need to concern we just want to set up the relationship and we want to find it out that what is the resultant strain is there in the cylinder, if the dominancy is there along the this y direction or the hoop stresses. So, that is what you see in the first segment it was their sigma H by E and there is a contraction is there along the length and if you want to measure on the same domain, then we have mu times sigma L by E.

So, you see here since the contribution is there from both of the segments, so again we have you know like the 1 by E which is a common segment sigma H is there. So, sigma

H as usual p d by 2 t, so it is p d by 2 t minus mu times this sigma L is there, so sigma L is p d by 4 t. So, you see here we have Epsilon 2 which is nothing but, equals to p d by 4 this E t into 2 minus mu, so only you see if you are comparing the longitudinal strain and if we are comparing the this hoop strain, then we will find that the p d by 4 E t is the common segment, which will you know like deform this thin cylinder.

So, what the influencing parameters are there, through which the deformation is there, in the resultant deformation is there or the resulting strain is there, these parameters are the applied pressure. Then we have the diameter that how much diameter is there, if more diameter is there, more deformation is there, if more pressure you are applying then definitely there is the thin cylinder is going to deform more. And then you see we have the Young’s modulus of elasticity, so what type of material you are using accordingly the deformation is there.

If we have you know like more elasticity is there, then definitely you see the percentage elongation is more. So, E as we are comparing E with the stress and the strain, so accordingly you see the value of E will come that which material because, E is a property of the material. And then how much thickness is there, more thicker less strain is there, less thicker more strain is there; that means, more deformation is there. So, it is you know like quite obvious things are there, but the difference is one is 1 minus 2 mu.

So, you see here whatever the deformation is coming or whatever the change of the relation is there in between the lateral and the longitudinal, this is coming while computing the Poisson ratio. So, 1 minus 2 mu and here it is 2 minus mu, so this is the only difference is there in between Epsilon 1 which is the longitudinal strain and Epsilon 2 which is the hoop strain.

So, now you see here you like we just want to you know like compute that actually what exactly is happening, if we are talking about Epsilon 1 that only the change of length is there. And we found that the change in length is nothing but, equals to Epsilon 1 into L; that means, p d L divided by 2 E t into 1 minus 2 mu. So, here you know like the length is also contributing, if we have more length then definitely you see you know like the Epsilon 1 is also means effected by the whatever the original length is there.

So, Epsilon 2 as we you know like measuring the Epsilon 2 on the basis of circumference. So, here length is not contributing here the diameter will contribute, so you see Epsilon 1 as you see in this particular way hoop strain, if we just going with the this definition of Epsilon 2 then it is nothing but, equals to what is the change of dimension. So, since we are taking the circumferential dimension, so diameter is dominating here, so change in diameter divided by the original diameter.

So, change in diameter is nothing but, equals to Epsilon 2 into the original diameter, so you see here, we have change in length which is nothing but, equals to Epsilon 1 that is p d by 4 E t into 1 minus mu into L. So, if you multiply with this you have the change in length, if you want to compute the change in diameter, then it is nothing but, equals to p d by 4 E t into 2 minus mu into original diameter. And d is you see you know like that we are always measuring that actually what exactly the diameter is there and how it contributed towards the elongation in the hoop stresses part.

So, you see if we are interested find it out that what the change in diameter as we found that, then exactly it is nothing but, equals to p d by 4 E t into 2 minus mu into d as we discussed. So, now if you know manipulate those things then what we have, we have p which is the internal pressure exerting by steam or water into d square divided by 4 times t, which is the thickness and t into 2 minus mu or we can say delta this del d that is the change in diameter is p d square by 4 t E 2 minus mu, which is a very good formula to be learn.

Because, you see if we are saying that if you want find it out any other parameter and if something is given to; that means the change of diameter is given to you or the change of length is given to you, corresponding changes are there in the longitudinal as well as the lateral strains. And then you see you can find out that actually what the remaining parameter is, so just these formulas are very, very important as far as the these thin cylinders are concern. So, now you see here as we discussed that these two mutually perpendicular stresses are there.

So, if you want to represent on a unit cube, then you see you know like as usual it is there, there is no shear stress component. So, whatever this stresses are coming, you can say that actually these are the principle stresses and corresponding strain, which we discussed this Epsilon 1 that was the longitudinal strain and Epsilon 2 that was the hoop strain.

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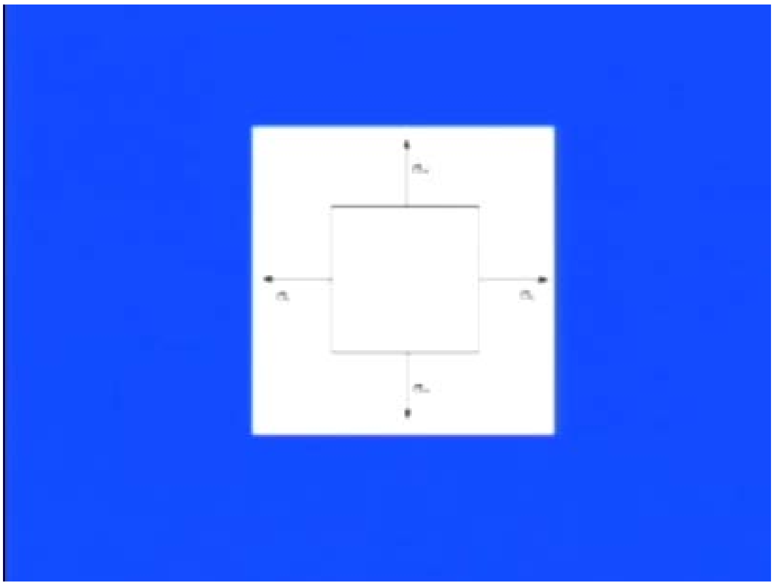

So, these are nothing but, the along this principle strains component, so here you see we have this unit cube and these two mutually perpendicular stresses are there and we can easily represent in this particular unit cube. So, we have you know like the unit cube in which we have unit depth, unit height and unit width is there, so along that in the x axis we have the longitudinal stresses and due to longitudinal stresses we have you know like the longitudinal strain.

And we can calculate the longitudinal strain, as you know like in the previous slide we discussed and the mutually perpendicular direction we have the hoop or we have this circumferential stresses. So, circumferential stresses are nothing but, in the y direction and corresponding strains are nothing but, the circumferential or the hoop strain and we calculate as an Epsilon 2 which was nothing but, the sigma H by E minus mu times of sigma L by E or you can say that it is nothing but, equals to if you compute both the part as far as the thin cylinder is concerned.

Then we have the sigma L which is nothing but, equals to p d by 4 E t into 1 minus 2 mu or if you are talking about the sigma H, then it is nothing but equals to p d by 4 E t because, these are the influencing parameter into 2 minus mu. So, you see you know like both can be easily computed irrespective of which one is dominating, but only we have to very much, you know like concern about that what is the change in dimension is because, if you are talking about the dominancy of sigma L.

That means you know along the x axis, then definitely you see the length is the dominant the influencing parameter. And you see, if you are talking about perpendicular direction; that means, the hoop stresses, then the diameter is the influencing parameter under which you see the a major impact is there. So, you know like while considering both the things just we need to focus on the dimension, that which dimension is affected under the influence of these pressure or we can say the forces.

So, now you see you know like we were discussing about the normal stress component in both of the direction mutually perpendicular directions. So, you see if you remember in the previous slides, then you know like in the second lecture we found that, you see if we have only the two mutually perpendicular normally stress component is there, then how this you know like the change in dimensions are there. So, now, again we would like to you know like put those kind of theories here in the thin cylinder.

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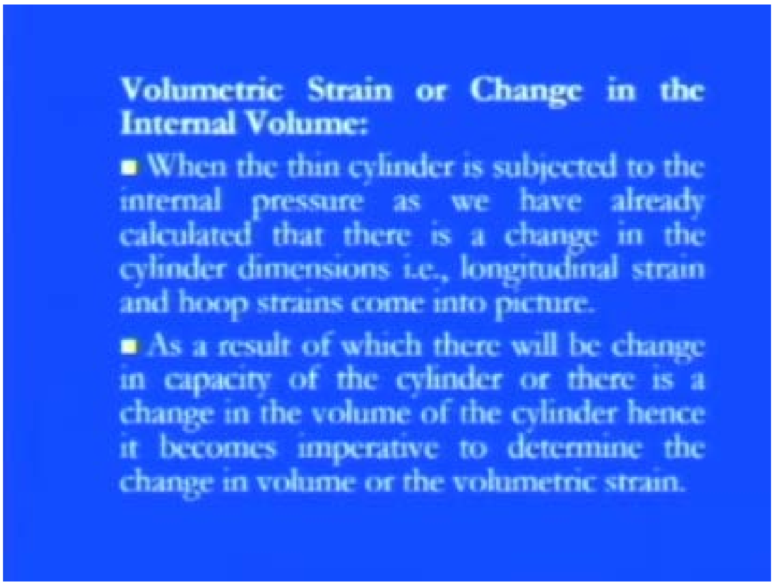


So, now again first thing will come that what is the volumetric strain or due to the changes you see because, if there is change in the length, direction or the longitudinal direction, then there is an impact of the this the diametrical way. So, what exactly happening with the total volume, so now you see for measuring those that what the net change is there in the volume, if you want to measure that kind of deformation, always you see there is a strain component.

And since our domain is the whole volume of the thin cylinder, then definitely the kind of strain is known as the volumetric strain. And you see for the volumetric strain, we define modulus and that modulus is known as the bulk modulus of elasticity, again you see whatever the deformation, which is coming due to the application of this pressure, we are only thinking about this elastic deformation. So, k is very well applied to this kind of theories and we are discussing about the volumetric strain, so; obviously, there is a change in the internal volume.

So, here you see in this slide when the thin cylinder is subjected to a internal pressure p as you know we have already calculate the p in the two different dimension that if there is a change in the cylindrical this dimension along the x axis. That means, the longitudinal strain is there and if it is along the y axis, then we have the hoop strain is there. So, you see both are mutually perpendicular as previously we have seen and both are you see having a different kind of magnitude also a sigma L is nothing but, equals to p d by 4 Et and sigma H is nothing but, equals to p d by 2 t.

So, corresponding you see you know like the changes are there, so as a result of these two changes, you know like there is a capacity of cylinder will also be going to change or we can say there is a change in the volume of the cylinder. As I told you that, since we are talking about the change of length, we are talking about a change of diameter at the same time; that means, there is a corresponding changes are there in the two dimension. So; obviously, it has a great impact of the change of the volume, so it becomes imperative to determine that what exactly the change in volume is there and if you want to compute that the change in volume always as I told you that volumetric strain is there.

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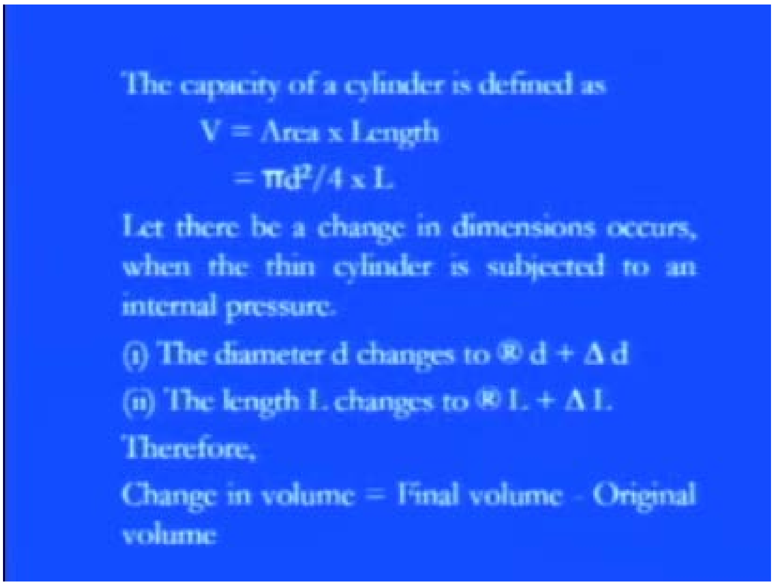


And now you see would like to you know compute the volumetric change, so the capacity of cylinder is nothing but a the area into the length and since you see we are considering the cylinder. So, area is pi d square by 4 along the length into since you see here we are considering that actually along the length what will be the area. So, area is pi d square by 4 into L and let there be a change in the dimension occurs along the length when a thin cylinder is subject to a internal pressure.

So, what the corresponding changes are there in that dimension, we have you see the this diameter is changes d. Since, there is a change in this diameter, which is delta d we you see here at that the particular point, so that total change is d plus delta d and if I am saying that there is a change in I am simply saying change, we do not know whether it is a contraction or extension there is a change is there. So, we are measuring the change irrespective of whether it is a positive sign or negative sign, so if you are saying that there is a change is there.

So, the change is d plus delta d along the diameter and L plus delta L along the length, now you see if you are saying that actually the change of volume is there, means if there is you see you know like whatever the deformation and the extensions are there. So, what is the change of volume, the final volume minus original volume and whatever the change will come, we can simply compute the change of volume divided by original volume will give you the volumetric strain.

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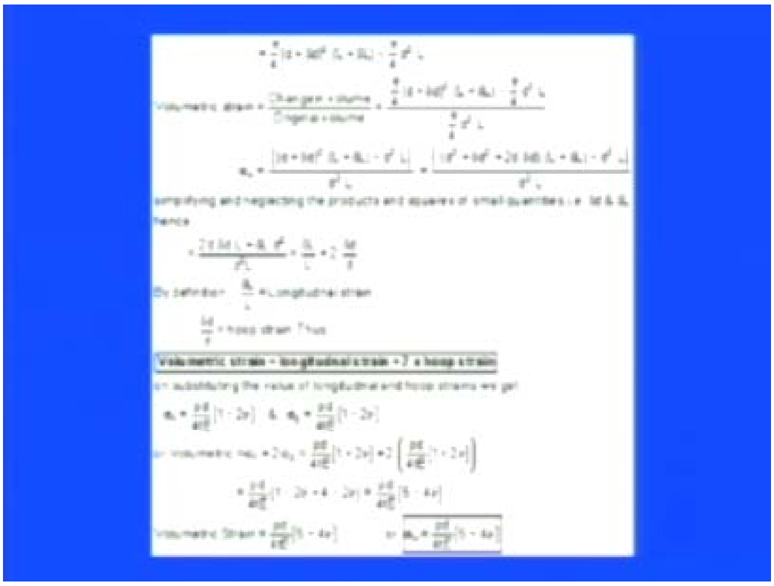

So, since it is the difference between this original volume and the final volume, so final volume always consist the net change in the diameter as well as the net change in the length. So, final volume is nothing but, equals to pi by 4 into what is the change of the diametrical way d square into the length, so you see here pi by 4 d plus delta d is the net change of the volume plus the original diameter. So, d plus delta d whole square, so this will be the area into the length L plus delta L.

So, you see n minus the original diameter is nothing but, equals to pi by 4 d square into the length will give you the original volume. So, now you see what we have, we have the change of volume, now you see if you want to compute as I told you that the volumetric strain is nothing but, the change of volume divided by the original volume. So, change of volume is as we are computed in the you know like the top of one is pi by 4 d plus del d whole square plus L plus delta L is the final volume minus original volume pi 4 d square L divided by original volume is pi by 4 d square L.

So, now you see here what we need to do, we need to expand this square term, so we have pi by 4 the squaring is d square plus del d square plus two times of d plus del d. And then you see you need to multiple with L plus delta L, so you see you know what we have, we have the final term in those things is this del square whatever the diameter is there.

So, d square plus delta d square plus two times of d into delta d, so this is the whole square term plus L plus delta L minus, this since it is you see you know like pi by 4 is already being taken common and it is been canceled out from denominator as well as the nominator side. So, now you see only we have the dimensional parameters into the consideration, then you have this minus d square L divided by d square L.

So, you see here since we know that due to the applied pressure inside, we have a negligible change in the diameter as well as means there is a very small change, not I can I cannot say that negligible part, but change is very, very small. So, you see if there is a multiplication of these two changes like, delta L or delta d if you multiply, then again since it is in the microns or it is in the very you know like 0.01 millimeter kind of that. So; obviously, you see if you multiply these two kind of terms, then probably you know like we are end ending up with the very, very small term.

So, you can neglect that part because, they are not contributing as such, as you see you know like the there is a significant part is there from the their own term, rather than the squaring or the cubic part. So, by considering this effect that we are ignoring square term or multiplication of del d and delta L what we are doing here, just we are considering the other effect. So, here we have you know like the del square, so this d square into L will be cutting you know like the d square L, so this will cancel out and then you see it is del d square.

So; obviously, you see you know like we are neglecting this term, so there is no impact of that, then what we have, we have you know like the delta L d, this d square into delta L this one component is there. And another component what we have, we have two times of d delta d into L because, delta d and delta L again we are you know like neglecting. So, at the end what we have, we have two main terms, one is we have this d square into delta L, so d square into delta L is this term plus we have this two d delta d into this L, so two d into delta d into L, so these two terms are there divided by this d square L.

So, if we you know like divides separately to these two terms into these categories, then what do we have, we have delta L by L because, this d square will cancel out. So, delta L by L plus you see here, this two times of two times will definitely come here and this d d will cancel out, so we have delta d by d. So, in this L L will cancel out, so at the end you see you know like we have a volumetric strain is equals to delta L by L plus two times of delta d by d.

So, delta L by L is nothing but, equals to we have a longitudinal strain; that means, you see if any change is there in the length along a longitudinal direction, it can be easily computed as I told you in the longitudinal strain way. So, this is delta L by L and corresponding you see we have two times of delta d by d here, so delta d by d is nothing but, equals to the corresponding change in the diametrical direction, which can be computed as an strain form is hoop stress or the circumferential strain.

So, this hoop strain is nothing but, equals to delta d by d, so now, you see in terms of mathematical form what we have, we have the volumetric strain is equals to the delta L by L plus two times of delta d by d or in other terms, you see if you are going for the technical term. So, called what we have, we have volumetric strain is equals to longitudinal strain that is delta L by L plus two times of hoop strain or circumferential strain that is delta d by d.

So, now you see you know like we have a strong relation, that since if you apply the load on you know like the thin cylinder, then the resultant strain is nothing but, equals to the net change in the length direction as well as the diametrical direction. And you see we can simply set up the relation in the volume domain, that volumetric strain is nothing but, equals to there is a contribution from longitudinal strain plus there is a contribution from hoop strain twicely.

So, you see this is a main contribution in the change of the volume, so delta L is there, delta d is there and even two times of delta d is there as far as the volume is concerned or the change of volume is concerned of a thin cylinder. So, now you see by keeping those values of a longitudinal strain, as well as you know like the hoop stress we are getting the new term of the volumetric strain. So, you see as we discussed or as we already derived the volumetric strain this Epsilon 1 is nothing but, equals to p d by 4 t E into 1 minus 2 mu.

So, this is you see we have a one term that is the longitudinal strain or Epsilon 1 and the second strain that is the hoop strain or circumferential strain is Epsilon 2, which is nothing but, equals to pd by 4 E t into 2 minus mu. So, by keeping both values in the you know like the volumetric strain formula what we have, we have volumetric strain is nothing but, equals to Epsilon 1 plus two times of Epsilon 2 or what we have, we have p d by 4 t E into 1 minus 2 mu that is Epsilon 1 plus two times p d you know like by 4 t E into 1 minus 2 times of mu that is the Poisson ratio.

So, you see now you know like compute though both the things what we have, we have you know like the common segment here is p d by 4 t E 1 minus 2 mu plus, since it is twice, this twice is there. So, two times, so 2 minus 2 into 4 mu, so what we have inside, inside we have 1 minus 2 mu's of this contribution plus we have 4 minus this two times of mu, so we have this 4 minus 2 mu. After multiplying both the things, we have you know like the equate part p d by 4 t E into 5 minus four mu.

So, now what we have, we have the volumetric strain which is nothing but, the you know like the combination of longitudinal strain plus hoop strain is p d by 4 t E. So, this is again you see, you know like equally contributed p d by 4 t E is contributed in longitudinal strain p d by 4 t E is contributed in hoop strain. So, again you see p d by 4 t E is also contributed; that means, you see as for as the dimensional parameter, the contribution is there from diameter as well as the thickness in this relative phenomena.

And we have also the Young's modulus of elasticity, this is also contributing in the volumetric strain, as you see you know like it is a dimensional parameter like that. So, what we have p d by 4 t E times 5 minus four times of mu, so you see volumetric strain is always since it is a strain component. So, Epsilon V is the volumetric strain and just remember the formula that p d by 4 t E is a common segment in all of those, you know these strain component longitudinal hoop as well as this volumetric into only the squaring this bracket term is different here it is 5 minus 4 mu.

So, starting from again I just want to give a special type about how to remember this thing, starting from longitudinal strain we have p d by 4 t E. Now, only the bracket is 1 minus 2 times of mu, then you see we have the hoop strain which is nothing but, pd by 4 t E, now bracket is 2 minus mu. So, you see 1 minus 2 mu was there in the longitudinal, here it is 2 minus mu and here you see you know like, if you combine both the thing in the two mutually perpendicular direction, we have volumetric strain which is nothing but, equals to p d by 4 t E into 5 minus four times of mu.

So, you see here what we have only the brackets are changing, so now, since we have the volumetric strain that is the Epsilon V, this V we can again find out the what the change of volume is there. So, with what the change of volume or we can say what is the change in capacity is there is nothing but, equals to you know like what is the delta V is this volumetric strain is change of volume delta V divided by original volume or we can say the increase in volume or decrease in volume.

Whatever the change is there the increase or decrease in volume is nothing but, equals to p d by 4 t E as usual into 5 minus four times of mu into the original volume.

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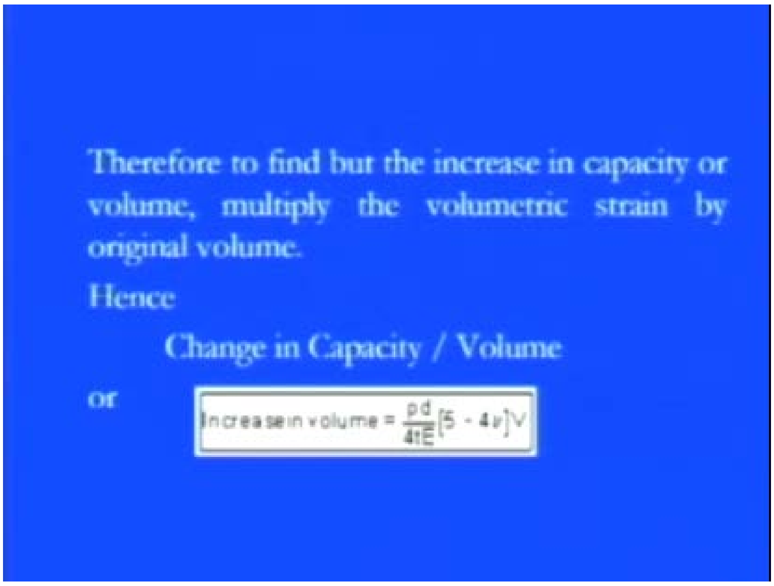


So, now you see what do you have, since under the application of any kind of load irrespective whether the load is you know like, tensile in longitudinal direction and compressive in this lateral direction or just vice versa that the load is in tensile direction in that the diametrical direction or we can say the lateral direction and compressive nature in along the length direction, we have the net change of volume. And the net change of volume is nothing but, equals to you know like p d by 4 t E into five times of four mu into the original volume.

So, you can compute on the basis of volume or we can say in a volumetric domain, so this is you see you know like, the new you know like the phenomena is there that we have three kinds of strain. And there is a straight relationship is there in between these strains like, this volumetric strain is nothing but, equals to longitudinal strain plus two times of diametrical or hoop strain or circumferential strain. So, now you see you know like this whatever the relations, which we set up in those you see only what we had, we had the ends that was simply a cylindrical part.

But, if I am saying you see that you see you know like this rectangular part was there, the cylinder of that kind. Now, you see if you are talking about we have a same cylindrical vessel of for the thin cylinder, but instead of you know like the straight corners or the ends, we have a hemispherical ends are there.

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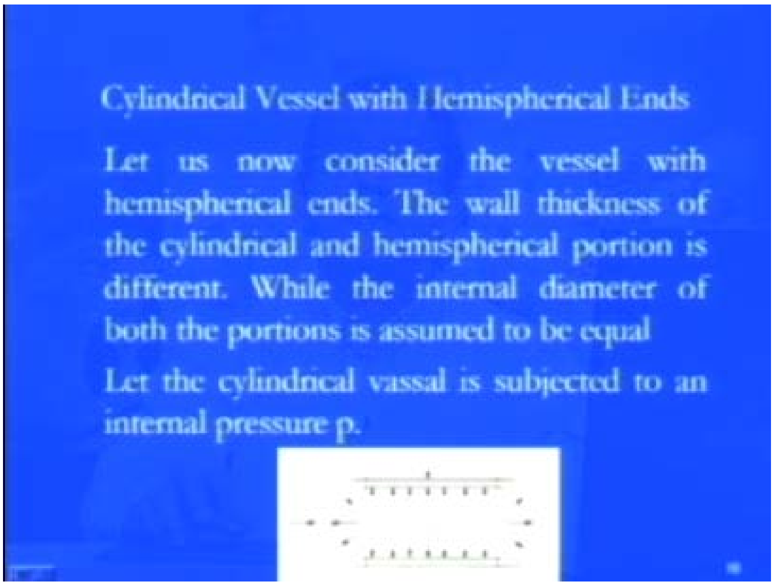


That means, you see now again the elliptical part is there, you see and at the end, so these you see in this particular figure you will find that, we have the similar kind of you know like for these two structure it is pretty similar this thin cylinder is there, but these two extreme corners are having a this hemispherical you know like the ends. So, now after considering the vessel with this hemispherical end, now what we have the wall thickness of the cylinder as well as hemispherical portions are definitely different.

So, if I am saying that the t 1, this t 1 is thickness of cylindrical wall and t 2 is the thickness of the hemispherical ends. So, both are you see you know like the different thicknesses are there for the different portion and while the internal diameter of both portions are assume to be equal. So, only we are considering that thickness is different and if we are considering the hoop stresses or longitudinal stresses, these you see you know like thickness will definitely contributed from both the side. So, now you see if I am saying that the cylindrical vessel is subjected by a same internal pressure with hemispherical end.

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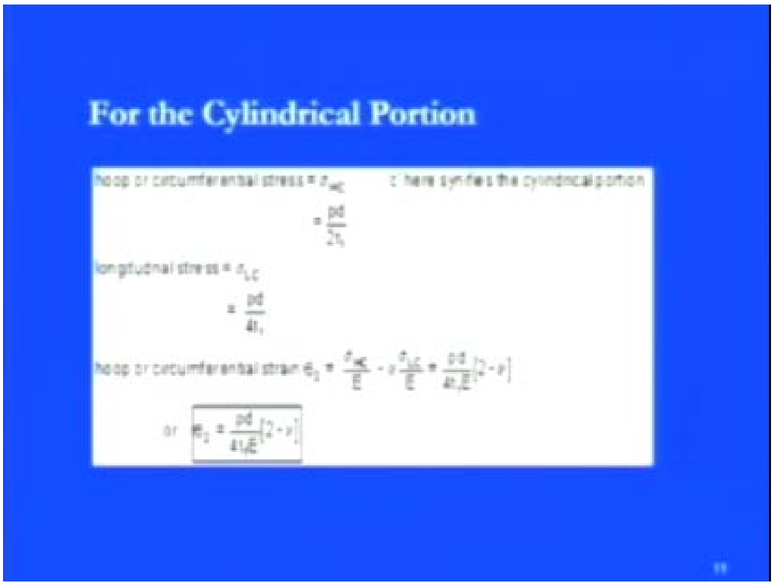


So, what we have, we have you see for as far as the cylindrical portion is concern where the thickness is t 1. So, what will be there the hoop stresses are nothing but, equals to sigma H times c, so now here you know like c which we are saying that only for cylindrical portion, we are not going for the spherical portion only we are right now focusing on the cylindrical portion. So, we have sigma H c which is nothing but, the this hoop stresses in the cylindrical part is nothing but equals to p d by 2 t 1 as we discussed.

And in that also we have the longitudinal stress in this cylindrical part, again we are the key feature is cylindrical portion. And that is why you see at the subscript, you see we are adding c part, so either sigma H c for hoop stress and sigma L c for longitudinal stress only the effective portion is the cylindrical portion. So, it is p d by 4 t 1, so now, you see what the you know like Epsilon 2 which is nothing but, the this hoop strain is sigma H c by E, which is you see you know like the volumetric strain or this longitudinal strain is sigma L by c into mu will contribute and we have this hoop strain that is sigma H by E.

So, it is also contribute, so if you see the effective combination of this hoop strain as well as the longitudinal strain, then we have as usual which we discussed for even we have a this same two extremely straight corners p d by 4 t 1 E. Here, you see now we need to be very careful that actually, which portion is to be affected and what is the corresponding thickness is, so p d by 4 t 1 E into 2 minus mu.

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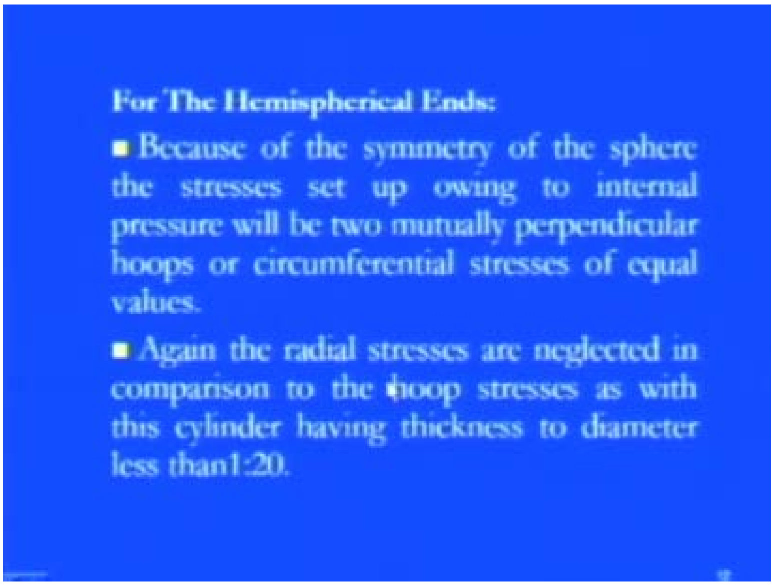


So, this is my Epsilon 2 as far as this cylindrical portion is concern, but now you focus on the hemispherical ends, because of the symmetricity you see of the hemispherical ends, the stresses set up owing to the internal pressure will be the two mutually perpendicular hoops or we can say the circumferential are there. And they have the same equal values; that means, because you know like at both ends we have a symmetricity.

So, whatever the pressures are exerting, you know like either on this end or at that end and since they are carrying the similar thicknesses and you see the equal pressure is exerting. So; obviously, whatever the stresses which are forming due to the application of these pressures, they must be equal, so again you see in these kind of things same this assumptions are there, that the radial stresses are always you know like giving their emphasis in a such a small portion or the contribution of this radial stresses are very, very small in comparison of the hoop stresses here.

So; obviously, we can simply neglect that part because, they are perpendicular to these you know like the hoop stresses. So, straight way you neglect and again here the limiting condition as well as the this thickness to diameter ratio is concern, that is always if it is less than or equal to 1 is to 20. That means, you see even if it 1 is to 19 is there; that means, if thickness is 1 and the diameter is 20 times always you can go for this kind of theory, that you know like the hemispherical ends will always be given an equal portion,as well as you see you know like the cylindrical portion is... So, you need to concern the effectiveness of cylindrical thickness, as well as the this hemispherical thickness of these two ends.

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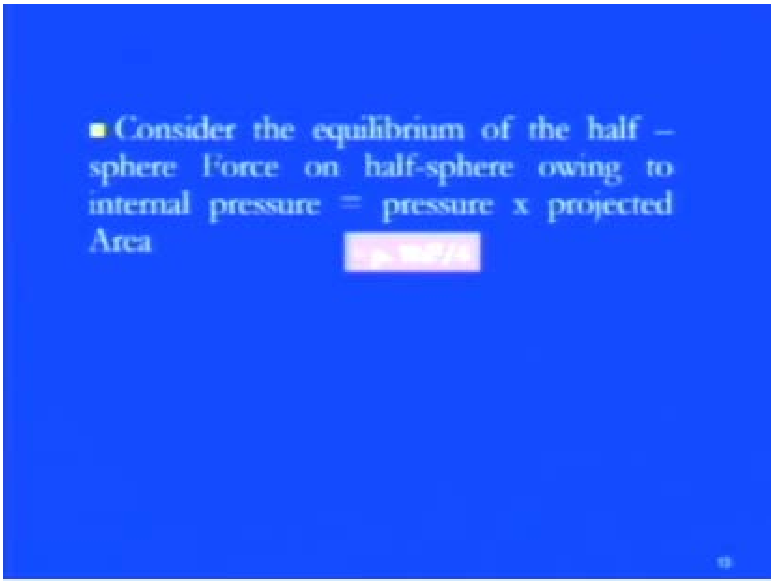


So, you see here by considering the equilibrium portions because, as we have you know like discussed in the previous case also. Whenever you see there is a you know like the longitudinal directions are there, consider the half portion as you know like the previous slide you must be knowing that half portion is there. And we are saying that since the half portion is similar analog is there or there is a symmetricity is there along the center line of both the portion of the cylinder.

So, again similar you see you know like considering the equilibrium of spherical, you know like spherical part here, the force on a half sphere owing to the internal pressure that is pressure, whatever the pressure which is exerting on the spherical cells into the projected area. Since, it is a diameter is here and it has the same diameter as the diameter of the cylinder, so we have p times of this pi d square by 4.

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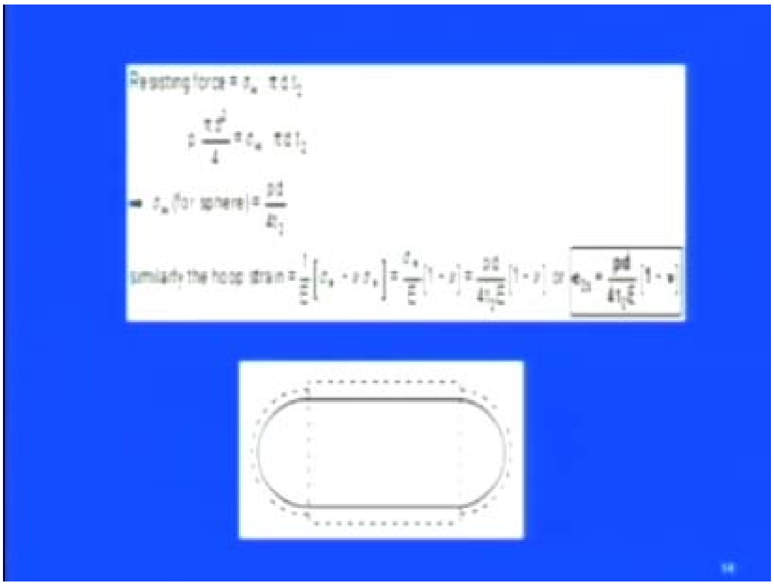


So, now you see if you, you know like considering the resisting forces for in terms of the circumference of these kind of thing. Now, you again our focus is on spherical part only, just keep this thing in your mind that, whatever the diameter is coming, the diameter is same as the cylindrical portion, but thickness is changing. So, now if you are considering or if you want to compute the resisting force by consideration of the spherical portion what we have, we have the hoop stress sigma H into pi d into t 2.

So, now you see you know like since this curvature part, this spherical part is there, so what the circumference pi into d into thickness the t 2 is there. So, now you see you know like the total if we compare the equilibrium portion, so p d into pi d square by 4 is equals to sigma ends, which is the hoop stress into pi d into t 2. And now you see the sigma H for sphere now, this is the you know like sigma you can say it is sigma H into s.

The spherical portion is equals to p d by 4 t or if you compute the hoop strain as such, then you will find that here it is 1 by E into sigma H minus mu times of sigma L or we can say that p d by 4 t 2 E. Here, now the main difference is only the effected portion is t 2, so p d by 4 t two times E into 1 minus mu will give you the hoop strain in the spherical direction. So, you can write this Epsilon two times E, so here you see here you know like on the screen you found that, we have Epsilon which is nothing but, the hoop strain component in a spherical portion, so that is what it is you know like the two times of s.

So, since it is Epsilon 2 is there because, we are consider again 2 y 2 is there because, Epsilon 1 is the longitudinal strain and here only we are considering the hoop strain concern. So, we have Epsilon 2 as gives you the indication that only we are considering the portion, where the spherical region is there, so Epsilon 2 s will be nothing but, equals to p d by 4 E into t 2 times because, now effective thickness is t 2 into 1 minus this Poisson ratio mu or you can, you know like visualize this picture that what exactly going on with the spherical ends of the cylinder.

Then this picture clearly you know like signifies this part, that we have this square term, you see with these two firm line is our original diameter and since there is a increase in the diameter. So, what the net change of the diameter will be simply shown by these dotted lines, so these dotted lines are like that, so we have a diametrical change there and since this cylinder is having, you know like the spherical outlets. So, this spherical out this firm lines are clearly showing that there is only the original you know like, the spherical diameter.

So, here we have a spherical diameter is this and after putting the expansion what the total dotted part is the net change of you know like the spherical ends. So, these dotted portion irrespective of these either, you know like these kind of things they are simply showing that what exactly change is there in the in the hoop of the cylindrical portion and these changes are in the hoop direction in the spherical portion.

So, these dotted lines clearly show you that, how we can compute the hoop strain, you know in the cylindrical part this Epsilon 2 c which was nothing but, equals to p d by 4 t 1 you know like t 1 times E into 1 minus this 2 minus mu. And here you see we have Epsilon 2 s which is nothing but, equals to p d by 4 t 2 E into 1 minus mu, so it is a Poisson ratio. So, these are the corresponding you know like the changes are there, if you are considering the hemispherical ends.

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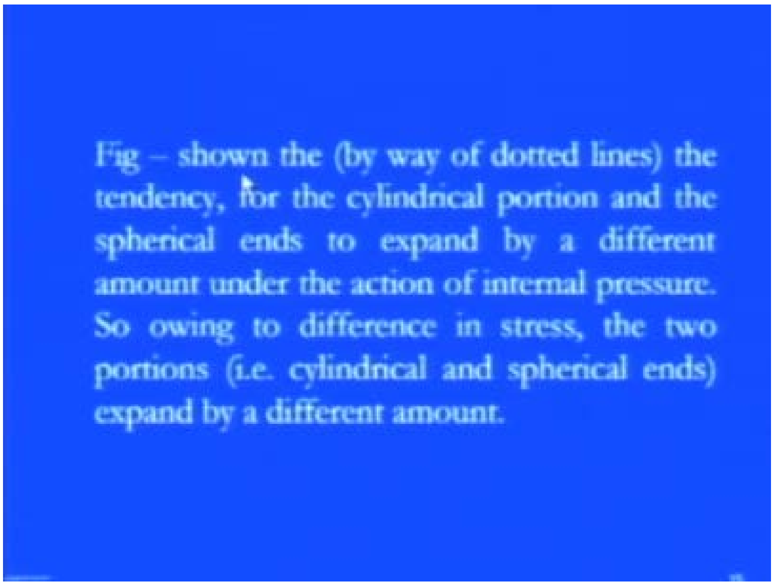


And the previous case which we consider that if we had you know like, the simple case means there is no hemispherical ends are there. Then how these strains are contributing means you know like, how the sigma H by E and sigma L by E, these strain components will contribute towards the final change. So, figure you see you know like, which we shown in the previous the dotted part, the tendency this simply giving you the tendency for the cylinder portion that how they are expanding towards the hoop direction.

And the spherical ends to expand in a different amount under the action of the same internal pressure, which is a you know like applying in the this cylindrical as well as the spherical portion. So, owing to difference in the stresses the two portion that is one the cylindrical and the spherical ends expanded by a different amount and that is why you see you know like, this t 1 and t 2 are equally contributed. Here you see, if you compare the hoop stresses for c and if you compare the hoop stresses for s.

That means, the cylindrical portion and spherical portion you find that they have a different value s compare to you know if we are not considering the hemispherical portion. But, here since we have two different thicknesses again the big assumption which we made that they have the same diameter, if the diameter is different at the extreme ends of these spherical regions. Then definitely again there is a huge difference is there in between the shares component of cylindrical portion as well as the spherical portion.

So, by considering those things what we are assuming that, if they have a same diameter and if they do not have the same thickness in the spherical as well as the cylindrical ends, you can simply analyzing you know like what exactly the differences are there and how we can you know like compute both the impacts on that. So, this in compatibility of the deformation, which causes a local bending and the shearing action in these you know like the two different portion, always you see you have to be considered that actually, wherever these join points are there how they have to be consider.

That means, you see what the impacts are there because, as well as the uniform directions are concerned, there is no problem you see you can simply you know like measure the deformation or the displacement in the corresponding direction. But, if you see wherever the spherical points are there and they are joining at the cylindrical points always shearing action is always dominating in that and due to even sometimes bending it may have to fail, so these points have to be concern very seriously. And since there must be a physical continuity between the ends of this cylindrical portion for this region, properly curve ends must be used for pressure vessel and you need to consider that what is the factor of safeties are there for these two kinds of portion.

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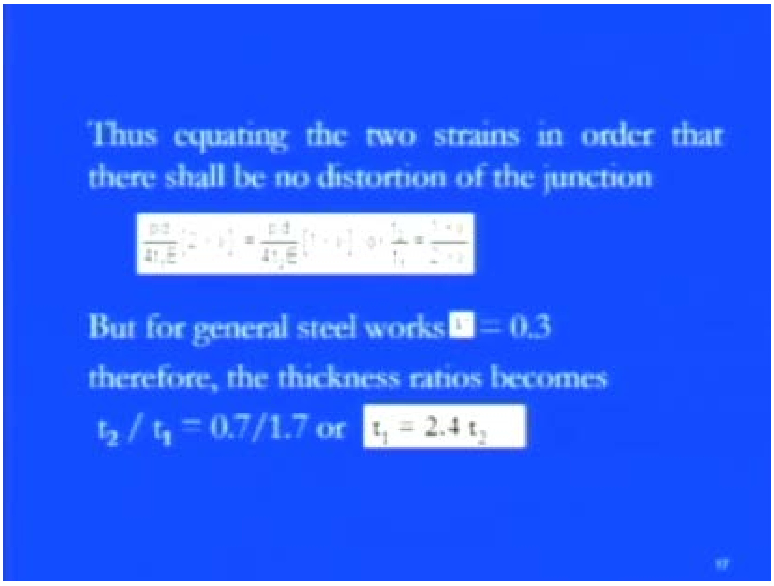


So, as we do not want any kind of distortion in that we just want the continuous structure along the junction point also. So, you see we can simply calculate that actually what the ratios are there in between these thicknesses just to avoid that distortion, so for that what we are doing here, we are simply equating two strain components as we got for the cylindrical as well as the spherical portion and by equating you see you know like p d by 4 t 1 E into 2 minus mu, which was you know like the strain component and the cylindrical portion.

And if you are equating this p d by 4 t 2 E into 1 minus mu the strain component and the spherical portion what we have, we have t 2 by t 1 is 1 minus mu divided by 2 minus mu. Now, you know like the mu is as I told you know like this is the Poisson ratio and it is very much subject to the material concern, so if I am considering here the mild steel or common steel here, the mu is always 0.3. Therefore, in the thickness ratio the t 2 by t 1 is always equals to 0.1 by 1.7 or t 1 is equals to 2.4 times t 2.

That means you see, if the thickness of cylinder walls must be approximately equal to 2.4 times of the hemispherical ends, then there want be an distortion issue is there. So, this is a critical issue that actually, if you want a continuity in these two portions the cylindrical portion and the hemispherical portion, there want be an issue if you consider the thickness of wall of the cylinder 2.4 times of thickness of the hemispherical ends. So, this is a key issue is there and we have to be concern that which material you are using and corresponding the values there of a mu accordingly you can calculate the thickness ratio.

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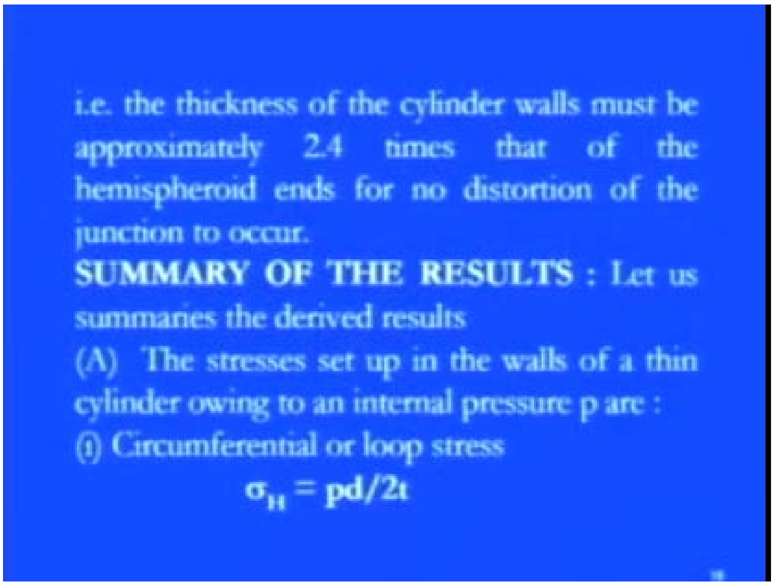

So, now you see whatever we discussed now we just what to summarize in the these kind of thing that, you know like we have first part the stresses set up in the walls of the thin cylinder owing to the internal pressure p, which are you now like the two many stresses were there, one is the hoop stress or we can say the circumferential you know like the stresses, so sigma H which is nothing but, equals to p d by 4 t.

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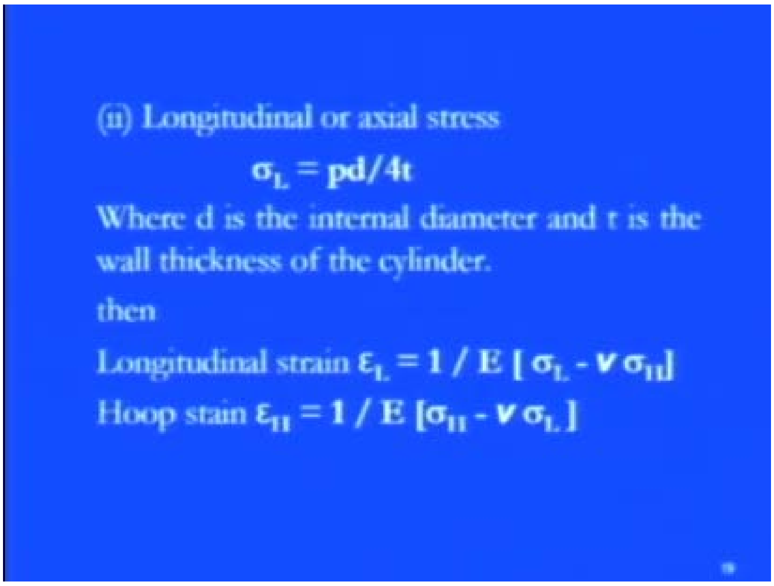


And we have the another component that is the longitudinal strain or the actual the stresses are there that is nothing but, the p d by 4 t, where the d is the diameter and t is the wall thickness of the cylinder. And corresponding you see you know like we can compute that how much deformation is there, in these corresponding direction, so we have you know like the longitudinal stress was there.

So, corresponding longitudinal strain Epsilon L is nothing but, equals to 1 by E sigma L minus mu times of sigma H or we you can say that this is nothing but, equals to pd by 4 t E into this 1 minus 2 times of mu or same you see hoop strain also we can calculate that is nothing but, equals to 1 by E sigma H minus mu times of sigma L or it is also be equal to p d by 4 t E 2 minus mu. So, you see you know like we can simply calculate these two components, so this was the first part.

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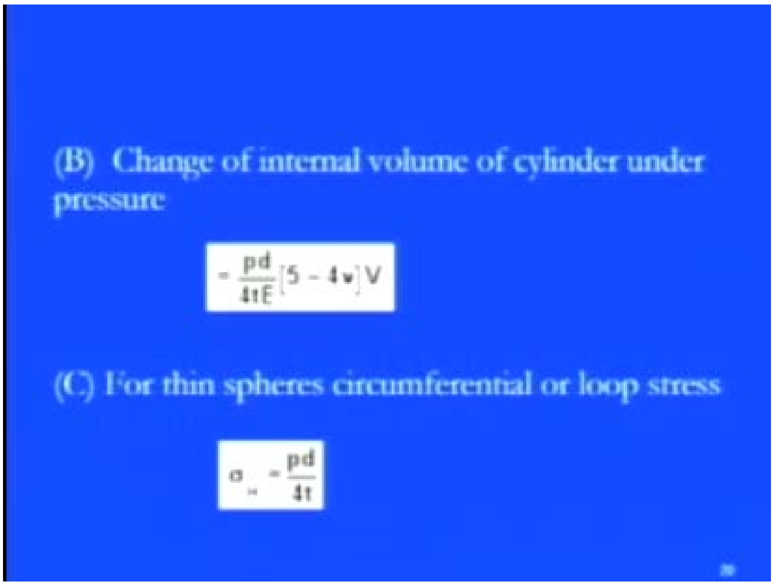


And second part which we discussed, that since there is a change in length as well as the volume. So, there is a corresponding change in the volume, so what that corresponding change is there that is to be computed on the volumetric strain, so change in volume is nothing but, equals to p d by 4 t E into 1 minus four times of mu into the total volume final V. And if we have the thin cylindrical this circumferential stresses are there for you know like the spherical things, then sigma H is p d by 4 t.

So, in this chapter you see our main concentration was there, that if we have a thin cylinder, then what this stress ratios are there and how we can calculate you know like the different stresses and the corresponding strains. And in the next chapter is you see our main focus is that if this cylinder is in the rotating part, then what will happen then how the shearing and the twisting part will be dominating and you know like the because, these two normal stresses are very much static. Now, you see we would like to see that if there is shearing part is there or we can say if there is a twisting part is there, then how you know like it will dominate. So, you see in the next lecture our main consideration is the rotating thin cylinder or the rotating shaft.

Thank you.