

**Strength of Materials**  
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**Lecture – 16**

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to deliver my lecture 16 and the course is you know like the Strength of Material, which is developed under the National Program on Technological enhance learning. If you refresh those previous lecture, then we found that if a material is subjected by the tensile load and if this material is having a composite bar kind of, if we have two, three different kinds of bars are there, in if they have the uniform across section.

But if they have the different, the area as well as if they have the different modulus of elasticity, and if they have even the different lengths or together, but if we assume that if both ends of the composite bar is sustained by the two supports. And if we apply the load, then we got this uniform extension and we can say that the uniform strain is there whatever the strains are...

So, this kind of analysis which we have made in the previous lecture and we found that, if we assume that there is the deformation is same, then how it gives an impact on the different components of this bars. And how the deformation is taking place individually in these components of this composite bar is. So then you see you know like we found that actually how we can calculate the equivalent this Young's modulus of elasticity, because whatever the deformation is there under the application of this force is the elastic deformation.

And you know like if we apply the Hooke's law, then we can easily calculate the equivalent the Young's modulus of elasticity for a composite bar. And after analyzing those things, we found that if only this excitation is there or we can say the external applied load is there, then we can simply compute that this is the uniform strain is there.

And since we have uniform strain, then how this stresses are there, this individual stresses are there, in these individual components and the individual areas are there and how they are interacting to each other with this application of force and the stress verses

strain. Then in the next case we found that if we have a composite bar and the temperature is there, means if any effect of the temperatures are there, then we have again kind of the normal stress know as the thermal stress.

And then you see this thermal stresses are well set up that they are acting in the normal strain, like the normal way, like the tensile is there or the compression is there, because if we are rising the temperature definitely is going to be extension in those bars. And we can simply compute as a thermal strain in a positive direction, and if you see you know like kind of cooling is there, then definitely there going to be the contraction is there; and we can say there is a negative thermal strain is there.

Then how we can compute those kind of strains that we discussed in the previous bar and then also if bar is there and if it is subjected by a normal pulling. And if the temperature effect is there, then what the interaction is there in between the thermal stresses with the normal stress component. And then how we can compute the equivalent part of these two stress components that kind of discussion, which we made in the previous a lecture.

So then we found that if we have, let us say the composite bar in which the two different metals are there, like we have the steel bar which we discussed and we have a brass tube. And if you apply the same temperature, this extension is absolutely depends on linear thermal expansion of the coefficient; means we have a coefficient which has a linear expansion. So, depends on and this alpha which we defined so called is absolutely a function of material.

So, as we apply the temperature, since whatever the alpha is there that means, whatever the thermal coefficient is there, corresponding changes are there or we can say the corresponding expansion is there at the free end of the bar. Because, if we the composite bar is rigidly fix up at these two point, then we can simply assume that there is a uniform deformation in that bar or we can say simultaneously the uniform thermal strains are there in that bar.

But, since here we are saying that if we have a cantilever kind of the element in which one end is fixed, one end is free and at the free end, if we apply this kind of temperature variation. Then definitely we are going to get the different expansions or we can say the contraction for the different kind of material, so in that if you want to make an equivalent kind of composite bar.

Then there is a kind of contraction in a material, which is having the more or we can say high thermal this expansion and there is an extension in the less thermal expansion. That means, if you want to make the same this extension in both of the bar even they have the different thermal expansion. Then, we need to put or we can say there is an interactive effect is there from the shorter member on the higher member and we can say the higher member on the shorter member.

So, if a shorter member is there then there is an extension, so there was kind of extension in the steel bar and there was a contraction in this brass bar was there and then if we made the equivalent the extension and based on this equivalent extension formulize those things. And we found that, if we have again we found that even if we have different materials and though they have the different alpha.

That means, the coefficient of linear expansion apart from that also, if there is a composite bar or we can say the compound bar, then we can get the uniform extension and we can get the uniform thermal strain in that. Or we can say whatever this stress into the area is there, which is equals to the stress into area in the another way or the stress into area divided by the Young's modulus of individual part, will be equal to the stress into area divided by the Young's modulus of elasticity for that particular material.

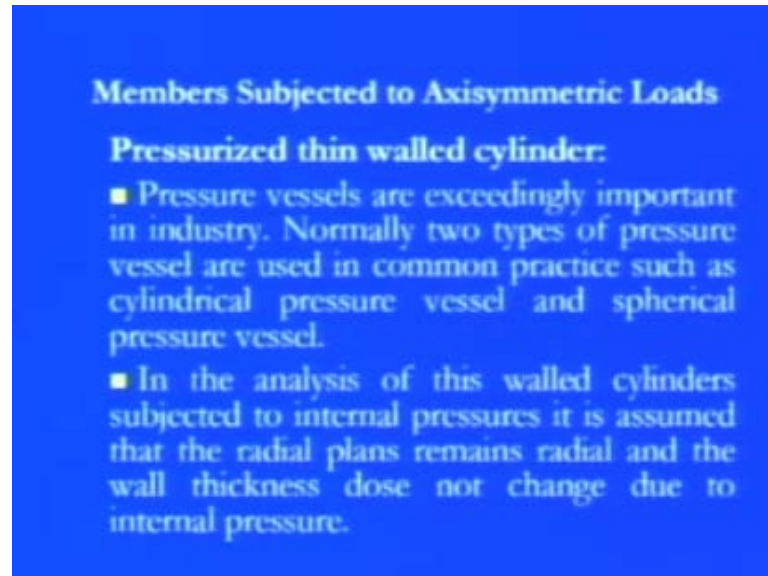
So, the meaning is pretty simple that, if we are talking about a composite bar, then the things are pretty symmetrical and if you want to calculate the individual parameter, then how they how they can get. And how the interaction is there in between these other parts of a bar under the application of either the temperature or the external applied load, this kind of discussion which we made in the previous lecture.

So, now in this lecture again we are going to continue the previous bar that now we have the thin cylinder, then how we can calculate. Because, if we have a kind of cylinder, then probably whenever we apply either the externally applied load or we have a temperature variation, then definitely there going to be the extension is there in the longitudinal direction, but what about the lateral direction.

So, there going to be contraction is there in the lateral direction, then how we can formulate that means, which pressure is dominating towards the axial direction. And which pressure is dominating towards the circumferential direction, these are the two

main factors on which we are going to concern and then we are going to analyze those part.

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So, here it is when we have a members which are subjected to axisymmetric loads that means, if we have a pressurized thin walled cylinder, we have a cylinder which has the wall of less thickness. So, thin cylinders are there, so pressure vessels are exceedingly important in the industry, because everywhere you will find that the pressure vessels are there just even if we are talking about a boiler or even if we are talking about to generate the pressures always a big vessels are there.

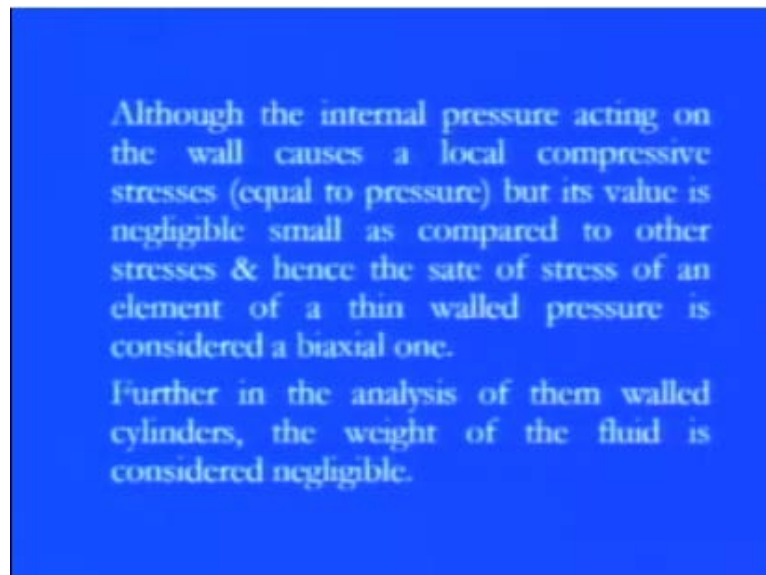
And those vessels if you want a compute the thickness of the wall, you will find that they are very thin, because they are always playing in a axial way. So, normally we are going for the two types of pressure vessels in an common industries, like one is the cylindrical pressure in which they have a cylindrical shape of a kind of vessels. And those inside the pressures are either they are the pressures are being formed by the steam or we can say these are the different kinds of chemicals are there through which of the pressure can be generated.

And they are simply acting towards the axis as well as the circumferential direction and other thing is that spherical pressure vessels. So, in the particular analysis of thin wall cylinder which is subjected to a internal pressure, because the steam generation or any kind it is assumed this is a big assumption is there in that particular thin wall cylinder

analysis. It is assumed that the radial planes, whatever the radial planes are there on which the steam is putting the pressure remains radial and the wall thickness does not change due to internal pressure.

That means, there is not the stress concentrations are there all across this pressure vessel, so there is no change in the thickness due to the internal applied pressure. So, thickness remains same and also whatever the radialness is there, whatever the all across the circumference it remains it is original shape due to the action of the internal pressure. So, these the two big assumptions which we need to put to analyze the pressure distribution, or we can say the stress or we can say the strains in the thin cylinder thin wall cylinder.

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Although the internal pressure acting on the wall causes a local compressive stresses (equal to pressure) but its value is negligible small as compared to other stresses & hence the state of stress of an element of a thin walled pressure is considered a biaxial one.  
Further in the analysis of thin walled cylinders, the weight of the fluid is considered negligible.

Then although the internal pressure acting on the wall causes the local compression, stresses that means, always we have due to the internal pressure always we have a different segments, where this compressive and tensile stresses are there, which is equal to pressure. Because, as I told you the pressure is not only, this is the externally applied kind of a force, because force per unit area it is there, so where the effective part is there on which these externally applied forces are there, this will give you the pressure.

So, you can say that whatever the local compressive stresses are coming at the different, different segments they are always equal to pressure. And even we have discussed that actually there is an analogy in between pressure and the stress, pressure is force per unit

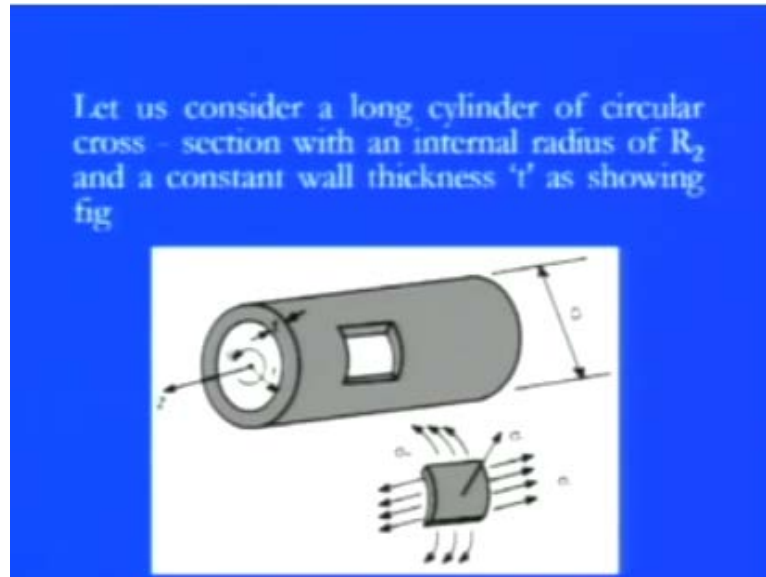
area that is Newton per meter square or even it is the Pascal. And the stresses are also the same, pressure per unit area or we can say Newton per meter square or the Pascal.

But, there is a difference, the difference is simply that the stresses are always being induced due to the application of force while pressure can be applied. That means, we cannot apply the stress if stress are always inducing in an object, while pressure is always applying on an object. So, here come to the main point, it is value is negligible, because those whatever the local compressive things are there, whatever the deformation due to these stresses they are very small. Or we can say we can simply neglect, they do not have the real dominancy all across the body, so that is what we can simply compare those things and hence, the state of a stress of an element for a thin walled cylinder pressure vessel is considered to be biaxial one. That means, as I told you we can always go for the axial, that means whatever the longitudinal part is there and this normal to that towards the circumferential part that is this y direction.

That means, we have x direction, y direction and these to biaxial stresses are always there in the thin walled cylinder and these the stresses are always coming due to the internal pressure. Further in the analysis of this particular cylinders are always we need to be consider that what is the weight of the fluid is there in side that, if you are considering the weight, then again there is there has to be we need to consider what the dynamism is there.

So, we need to assume that whatever the weight is there, irrespective of whatever the liquid or the steam or whatever due to which the internal pressure is coming we need to consider the negligible. So, even if you are considering that the pressures are there and this pressure are acting all across the body, but we need to neglect those kind of weight. So, that whatever the stress formations are there it has to be uniform all across the circumference of this thin pressure wall.

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So, now just consider a long cylinder, as you can see in this figure on an a screen of a circular cross section within internal radius  $R_2$  and a constant wall thickness  $t$  is there, that means what we have, we have this cylinder is there thin cylinder, which has a thickness  $t$ . So, once you have the thickness  $t$  and which has the diameter capital  $D$  is there and the  $Z$  direction you can see here towards this direction we have  $R$ , we have theta.

So, it is a Cartesian coordinate in which there are three main coordinates  $R$ , theta and  $z$ , so whatever the stress components are there, if you just go previously, then you find that if you have the polar coordinates, then we have  $R$ , theta,  $z$  and if we have the Cartesian coordinate, then we have the  $x$ ,  $y$ ,  $z$ . So, if we have a  $R$ , theta,  $z$ , then you can easily calculate the stress component at the oblique plane that means, the sigma, theta like in the normal component as well as in the shear component tau theta.

So, it is pretty simple to calculate the normal stress component and the shear stress component due to the combination of this interactive part, like the sigma  $R$ . Because it is always normal to these things always we have the normal stress components sigma  $R$ . Sigma  $L$  is there, sigma  $L$  is nothing but equals to towards the length of those things, so these are sigma  $L$  direction; so what we have, we have if we are talking about the plane which is perpendicular to this particular cross sectional area.

So, if we are talking about this radius, then we have the  $\sigma_R$  towards this direction, we have the  $\sigma_{\theta}$  is there towards the circumferential direction, because this  $\theta$  is there. So, we have  $\sigma_R$  in this direction, we have  $\sigma_z$  and we have  $\sigma_{\theta}$ , so all three mutually perpendicular direction gives you the three kinds of stress formation here.

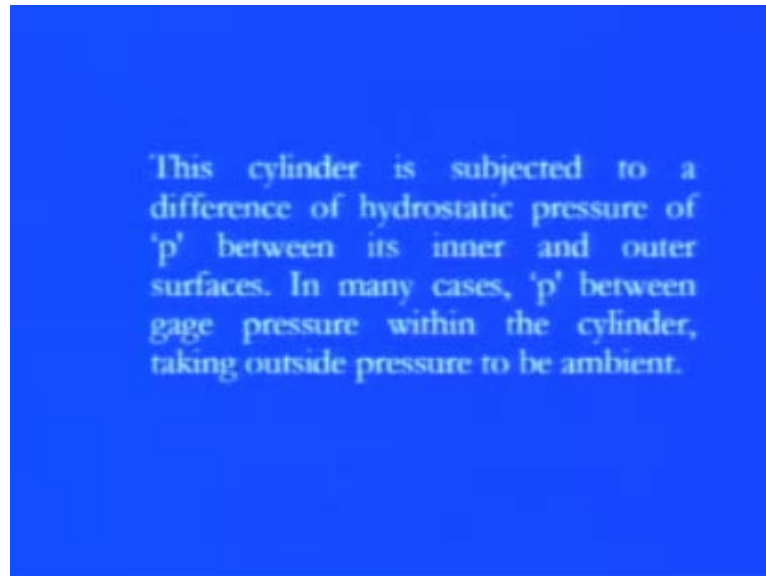
But the key thing is that actually since we are considering here the biaxial form of stress, then we have a thin cylinder, this is the  $t$  is very, very small in that and this is the cylinder in which if the pressure is flowing all across this body, one due to this pressure there is always a kind of extension is there. So, always we are considering that this body can be extended in the  $x$  direction and then another part will come as the circumferential part like in the  $\theta$ .

So, in that you can say this  $\sigma_z$  and  $\sigma_{\theta}$  will play a dominant role in those kind of thing, because both are the normal stress component. And since, any fluid is flowing or any steam is there it place in a forces, it always offers an forces towards the circumference as well as the in the  $x$  direction. So, this cylinder will give you a clear feeling about those thing that wall thicknesses the constant and it is very tiny.

So, always whatever stresses are coming in the circumferential one, they are always dominating and they are depending on what the radius is there and the thickness is there. And what the load application or we can say the force application is there from the steam side inside like that.



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So, the cylinder is subjected to a difference of hydrostatic pressure, like if I am saying that if it is  $p$ , so hydrostatic pressure  $p$  between the inner and the outer surfaces. So, here we have the thickness, so the if the inner surface if we are talking about and outer surface we are talking about, then whatever the radius difference is there that is thickness. So, if we are saying that the hydrostatic pressure is applying, so hydrostatic pressure is always applying uniformly all across the circumference of the body.

So, in many cases we can say that the  $p$  is between the gauge pressure within the cylinder, which we are always taking outside the pressure to the ambient. That means, whatever if we are talking about the pressure, then again I just want to refresh your concept that whatever the datum pressure is there, that is the normal pressure or we can say the atmospheric pressure is there.

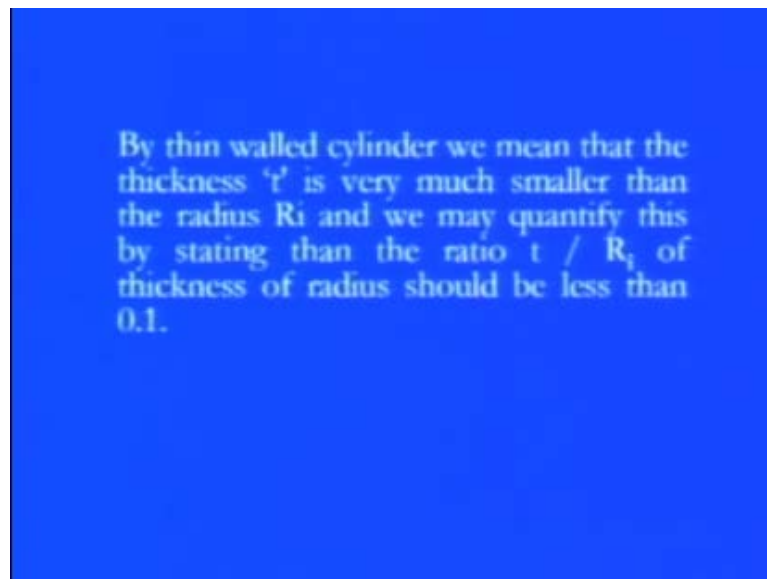
So, one is the ambient pressure which is the realistic pressure, which is always greater than the atmospheric pressure, because atmospheric pressure plus gauge pressure will give the absolute pressure. Or atmospheric pressure minus vacuum pressure, whatever the vacuum is there will give you the absolute pressure. So, if you are talking about a general domain condition in which it is open to atmosphere, then gauge pressure is the real applied pressure is there.

Because, it is more than the atmospheric pressure and it will give you the clear feeling that, since this gauge pressure is coming this forces are there due to this atmospheric

pressure plus this we can say the gauge pressures are there. That means, whatever the ambient things are there that means, whatever the room temperatures are there these pressures are always applicable.

But, if you are saying that there is some external source is there, that means or we can say if there is external excitation there, it always comes through some more positive pressure or technically we can say it is a gauge pressure. So, this is a pressure phenomena if we are talking about anything pressure cylinder, then always pressure this gauge pressure is playing as a key role. Because we are not going in the negative pressure, like a vacuum pressure always we are going for the positive pressure.

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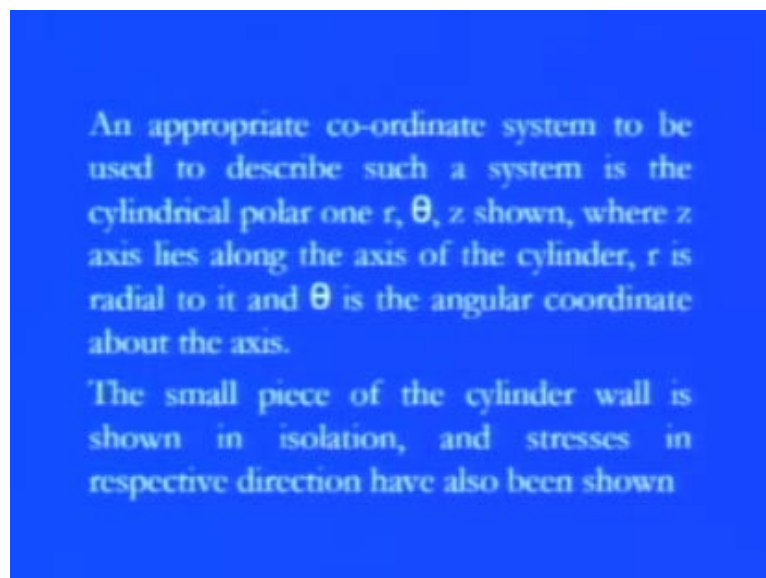
So, by thin walled cylinder means, that how much thickness is there, because as it is clearly stated that we are taking about the thin walled, so how thickness will vary and what the relation is there in between the thickness and the inner radius or outer radius that we just want to focuses. So, here the thickness of this thin walled is very very smaller than the radius  $R_i$  or we can say that actually if you want to quantify those things, then we easily quantify by the ratio of the thickness to the inner radius.

So, whatever the ratios are there and correspondingly these values are we are quantifying those things, and even these kinds of the tables are to be made based on that. Now, we have the  $t$  by  $R_i$  is 0.05, 0.1, 0.15, 0.2, 0.25 and corresponding grades are there, so let us

say if I want to make the thin cylinder of this nature, like let us say this  $t$  by  $R_i$  is 0.15 it has a the specific nature and it has a specific applications altogether this.

So, if we are talking about the thin wall cylinder, then it has somewhat laser ratio in between the thickness to this inner radius; so here in this particular thing we can say that, we are computing this  $t$  by  $R_i$  of the thickness of the radius it should be less than 0.1. So, if you are talking about 0.05 as I told you or 0.75 or even 0.1 or even lesser than that always they have a different kind of application. So and even we always try to keep whatever the internal pressures are there, so that there should not be the extension or the deformation is going on, it is going beyond a certain limit. Or we can say permanent set of deformation will be existing, if we go certain value of the load application.

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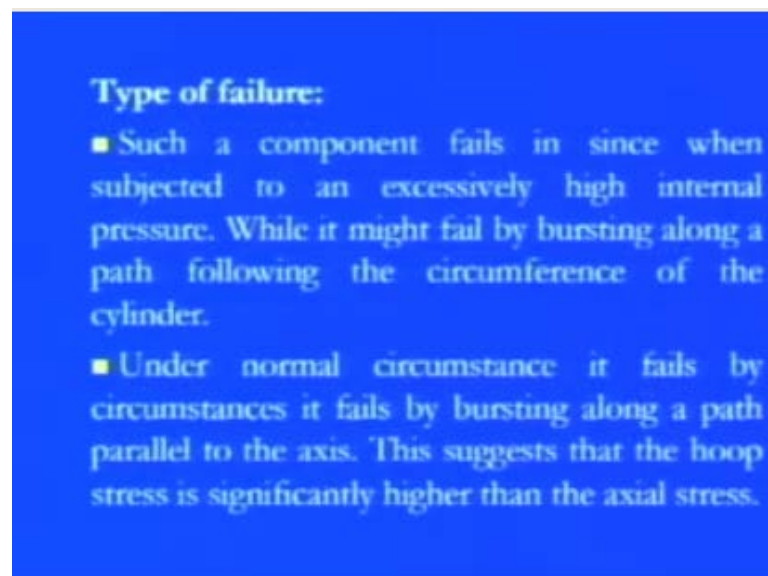
So, an appropriate coordinate system as we discussed that actually the  $r, \theta, z$  is to be taken which can be easily describe, because what we have here is cylindrical, the pressure vessel. So, again we have to choose the cylindrical polar coordinates, like  $r, \theta, z$  which we have shown in the previous diagram that you see towards the  $r$  and  $\theta$  is towards circumferential and  $z$  is towards the longitudinal part.

So, when I am talking about the  $r$ , then it is always normal to the plane concern when we are talking about the  $\theta$ , where all the circumferential or the different kind of stresses are to be occurred due this pressure application. So, here if I am talking about  $r, \theta, z$ ,

then the  $z$  lies along with the axis of the cylinder as I told you towards the longitudinal direction  $r$  is the radial towards the outside of the plane.

And  $\theta$  is the angular coordinate as usual where the circumferential part is coming, and if I am talking about the previous figure, then we have taken out the small piece of a cylindrical wall. And also since this piece is symmetric towards all the direction  $r$ ,  $\theta$ ,  $z$  direction, so we can easily show the  $r$ ,  $\theta$ ,  $z$  stress components all across of this front side and the back side of these small segment.

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So, now if I am talking about this kind of the pressure exerts on this thin cylinder, then there is a type of failure can also be there, because we just want to limit our load condition. So, if we go beyond the load condition, then as I told you that the kind of plastic deformation may occur, so we would like to keep the load condition below certain level.

And if we go then actually what will happen a type of the ultimate tensile strength will come, because it is a tensile loading and then it appears to be a fractures. So, what type of failure can come if we go up to a certain level, then such a component which fails in the this kind of pressure vessels, when subjected to an excessive high internal pressure always gives you a real feeling. That it may fail towards in a excessive force in this  $z$  direction in which the longitudinal one, it may fail towards the normal direction in this  $R$  direction.

Or it may fail due to this more circumferential stresses, because stress is uniformly distributed and if it just pushing towards the outward direction, there is a fairly well chance to have an expansion in those direction and there may be a fail. So, here if you are talking about the fail in this kind of thing, then all three possible cases are there, while it might fail by bursting along the path following this the circumference as I told you towards the z direction of the cylinder.

Under the normal circumference it fails by the circumstances, it fails like the bursting along a parallel to the axis like as I told you longitudinal 1 means the z direction, so this suggest that the hoop stresses is significantly higher than the axial stresses. So, here the new term will come as the hoop stresses one is the longitudinal stresses, that means we have the kind of hoop stresses the technical term is there in the z direction.

We have the lateral stresses, that means we have the circumferential stresses in the theta direction as we discussed in the first point, so it can be burst in the circumferential one it can be burst in the z direction the hoop one. So, here which stresses are the dominating, it has to be taken care of an accordingly the factor of safety is to considered just to safely design the kind of thin cylinder. So, if we are talking about the application, then there are the tones of applications are there, where this thin cylinder walls are usable only, where thickness by internal radius is always smaller than 0.1.

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#### **Applications :**

Liquid storage tanks and containers, water pipes, boilers, submarine hulls, and certain air plane components are common examples of thin walled cylinders and spheres, roof domes.

**ANALYSIS ;** In order to analyses the thin walled cylinders, let us make the following assumptions :

- There are no shear stresses acting in the wall.
- The longitudinal and hoop stresses do not vary through the wall.

So, what the applications are, the applications are the liquid storage tank, where always we are keeping all those the liquids or which has a lower temperature or we have the containers in which all these kind of steams which we can keep. Or we are just using the water pipes we have the boilers in which the steam is to be generated for power generation, submarine hulls are there which always operating below the water level.

So, the kind of hydrostatic pressures are there right from the external and the internal sides, these are the common components which we are using or generally sum of the airplanes are there, where the kind of drag forces are there right from outer side as well as inner side. So, we have to be design safely that which forces are dominating in what direction and if we are talking about the hoop or if we are taking about the longitudinal, this lateral one than how we can designs safely.

That because, if we are talking about the underwater or the air kind of situations are there, then the whatever the subject the pressure variation is there, it depends on where it is landing or where is flowing and what kind of drag forces are there and how the lift forces are coming from these internal parts. So, we have to be very careful, if you want to design, the thin wall cylinder and based on either the spherical one or the cylindrical one.

So, as we have seen that actually there are lots of applications, so for this kind of thin cylinders where all the kind of load or we can say the pressure variations are there. So, now if you want to analyze those components, then for analysis first of all I would like to make certain assumptions prior to analyze the kind of stresses and because of the kind of internal pressure.

How the stresses formations are there and what kind of strains are there and how we can set up the relations between stresses and strain this kind of analysis, which we going to discuss here. So, here first of all certain assumptions prior to do the analysis, the first assumption is there are no shear stress acting in the wall, you means it is pretty simple that when we are talking about. Let us say a boiler or if we are talking about a water pipe, the water is flowing or though this steam is flowing in the boiler is in very streamlined manner.

So, wherever if they are striking on the wall, there are always trying to go towards the outward direction, that means either we have the compression or we have a extension,

there is no shearing is there. That means, whatever the force application is there this is the point of force of application, that means if the point of application of this kind of forces or the pressure on these outer surface is just normal to the surface concern.

That means, whatever the stresses are forming due to these loads are always this normal stresses, there is no parallel forces means whatever the layers are there of this particular thin cylinder, this forces are not exactly parallel to these layers. That means, there is no shearing is there and that to also there is no eccentricity is there where the load application is there.

So, there is no chance where the shear stresses can be formed in a pressure vessel, so again we need to assume that this has to be uniform all across this pressure vessel. So, whatever the stresses are forming, they are always in the form of the normal stresses, no shear stress is there on an wall, so this is the first assumption. Second assumption is the longitudinal as well as the hoop stress do not vary throughout the wall, that means there is no stress concentration is there all across the wall.

And this wall is well, this surface tuned is there that means, whatever the surface roughness is there it is highly super finished, even we can say the honing or lapping whatever the process are there, we need to apply to make the surface roughness perfectly. So, that whatever the flow or this pressure is going on, it has to apply a uniform pressure, that means there is no variation of the pressure is there and there is no displacement of the micro structures are there inside the wall, so that we have different kind of pressure.

So, this pressure is uniform redistributed all across the body irrespective of whether we are talking in a axial part or irrespective of whether we are talking about the longitudinal or we can say circumferential part. So, in the longitudinal and the hoop part there is no change of the stresses, if we are moving towards the wall thickness or if we are moving towards the outward direction. So, here what we have the next one is the radial stresses this towards the circumferential directions, which acts normal to the curved plane like the  $\sigma_r$ .

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- Radial stresses  $\sigma_r$ , which acts normal to the curved plane of the isolated element are negligibly small as compared to other two stresses especially when The state of stress for an element of a thin walled pressure vessel is considered to be biaxial, although the internal pressure acting normal to the wall causes a local compressive stress equal to the internal pressure.

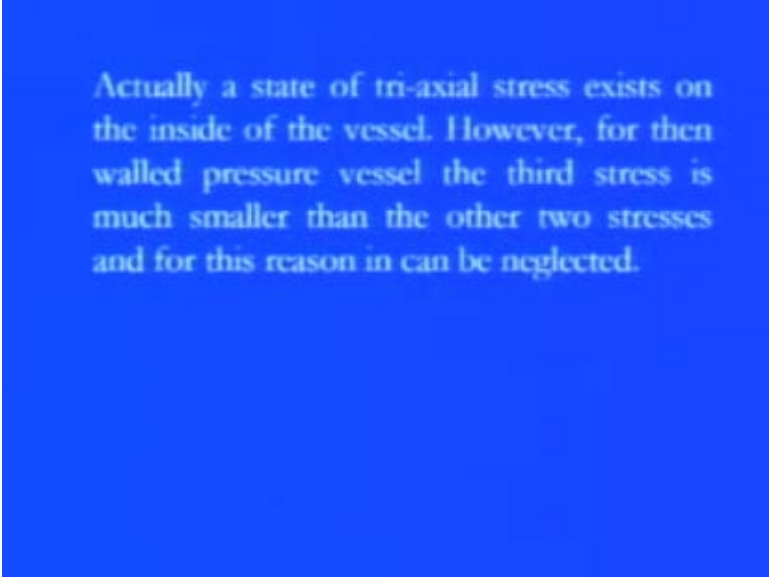
We are talking about of this isolated element are always negligible, that means because it is just we have a confined structure. So, whatever the pressures are coming which are normal to the surfaces always they are not in the dominating way as compared to the other two forces like the hoop or the longitudinal one. So, if whatever the stress components are coming all across the curved plane for an this particular isolated element, it can be this neglected, because if it is less as comparable to other stress component.

And the state of the stress for an element in this particular thin walled cylinder pressure is consider to be biaxial, as we discussed it is always biaxial, one is along the x axis, one is along the y axis. Or we can say along this particular normal to this x axis in x or y z that means, it is in the circumferential style. Although the internal pressure acting on the normal to these kind of loads will cause on the particular wall especially, a local compressive stresses which is exactly equal to the internal pressure.

That means pretty simple, if you want to analyze any kind of stress analysis and the corresponding strains the deformation I should say, then always we need to consider the biaxial state of stress. That means, one is along the x axis, one is perpendicular to this axis in any of the direction; and both are playing an important role to carry out that how the interactions are there and what exactly the kind of deformation will come in the structure.



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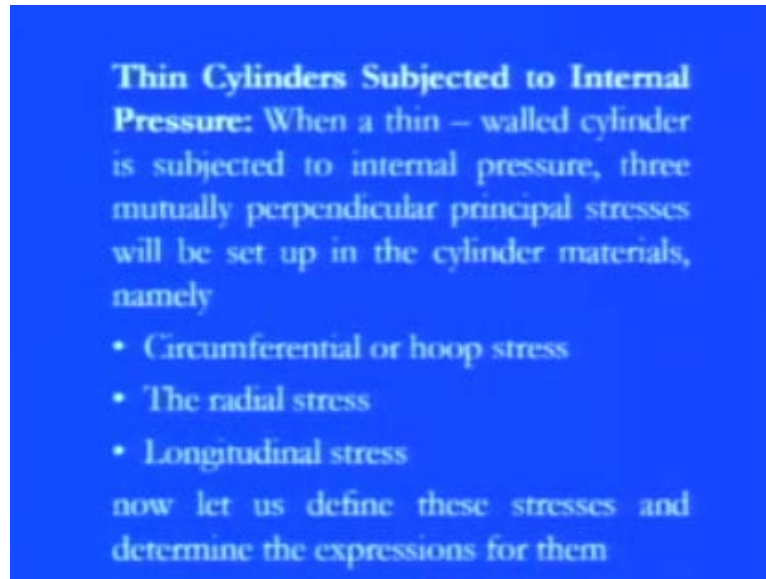
Actually a state of tri-axial stress exists on the inside of the vessel. However, for thin walled pressure vessel the third stress is much smaller than the other two stresses and for this reason it can be neglected.

Actually a state of tri-axial stress exists on the inside of the vessel, but since we know that the one component means which is the normal to this plane the  $\sigma_r$  is not dominating as compared to the other two surfaces, like hoop or longitudinal one. So, that is what we are not going to analyze these pressure thin vessels on the basis of this tri-axial part, so biaxial state of stress, is stress will give you the sufficient knowledge for this kind of stress this structure.

However for an thin walled or this thick walled structure, always the third stress is playing some sort of the key role where the less pressure analysis is there, that means where we are not going for huge pressure. But, if a huge pressure is there, then all stress tensor always play an key role and all nine components or we can say all three mutually perpendicular directions with the using all nine components of the stress, three normal stress and six the shear stress component are considerable.

And we need to be very careful that actually how these stresses are interacting to each other and how we can get the corresponding strains are there with those tensor stress. So, here we will find that actually these are the certain assumptions, which are really fruitful to make an analysis for thin cylinder. So, now here as we move the thin cylinder which his subjected to the internal pressure as we discussed, now how we can make the analysis.

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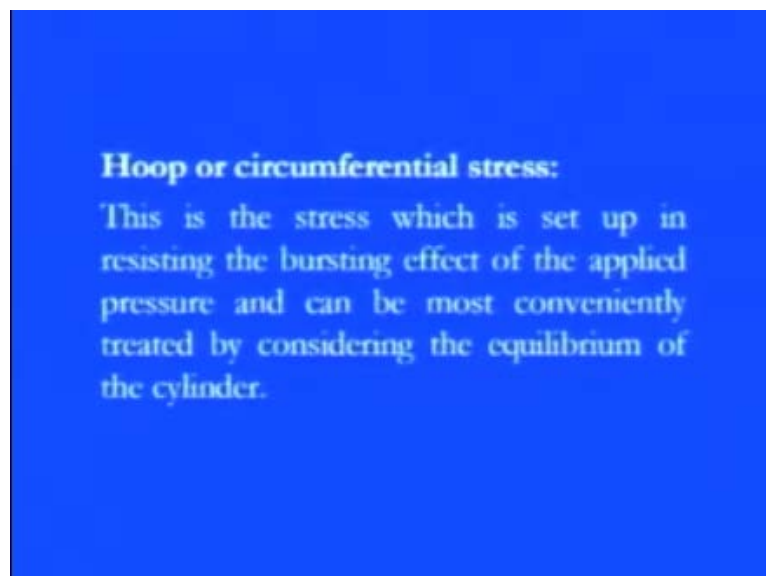
**Thin Cylinders Subjected to Internal Pressure:** When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely

- Circumferential or hoop stress
- The radial stress
- Longitudinal stress

now let us define these stresses and determine the expressions for them

So, when a thin wall cylinder which is subjected to internal pressure, three mutual the perpendicular principle stresses are well set up within the cylindrical materials. And those stresses are nothing but, first as I told you the circumferential one, that is hoop one means in the theta direction. So, as the steam pressure is generated, they are always act towards the outward direction and which will always create the stresses and these stresses are known as the circumferential stresses or we can say the hoop stresses.

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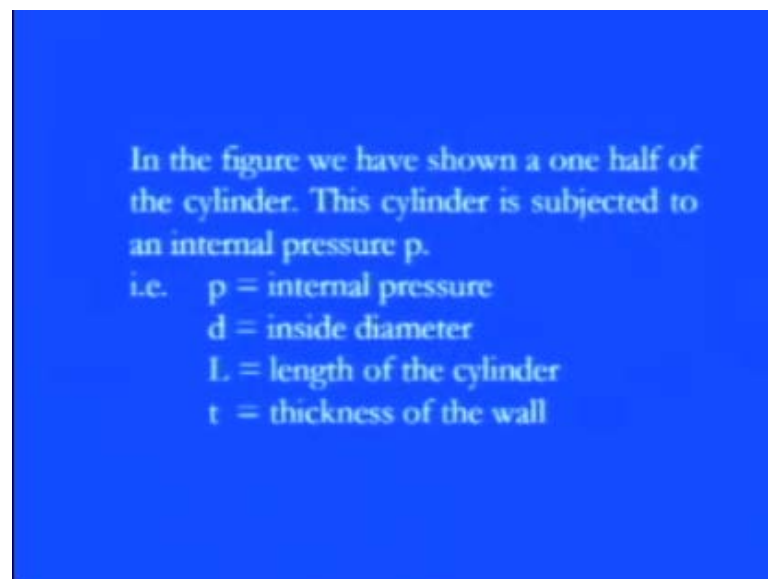


**Hoop or circumferential stress:**  
This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.

So, the hoop stress is the main stress which is set up in the thin cylinder, just to resist the bursting effect of the applied pressure and can be the most conveniently treated by considering the equilibrium of a cylinder. Because, when it is just applying on the circumferential part or we can say the lateral part always it just tend to move towards the outward direction.

So, you just try to given a impact of the bursting on the wall of the thin cylinder, so we need to consider that wherever it is going towards the outward direction, what will be the resisting effects are there against these bursting effects. So, whatever the stresses, which are inducing those things they are known as the hoop or the circumferential stresses. And always we are treating these stresses after the equilibrium that, they are well setup on the circumference of these thin wall, thin cylinder wall. So, it can be easily evaluated on that basis and as we discussed that these are always coming on the circumferences.

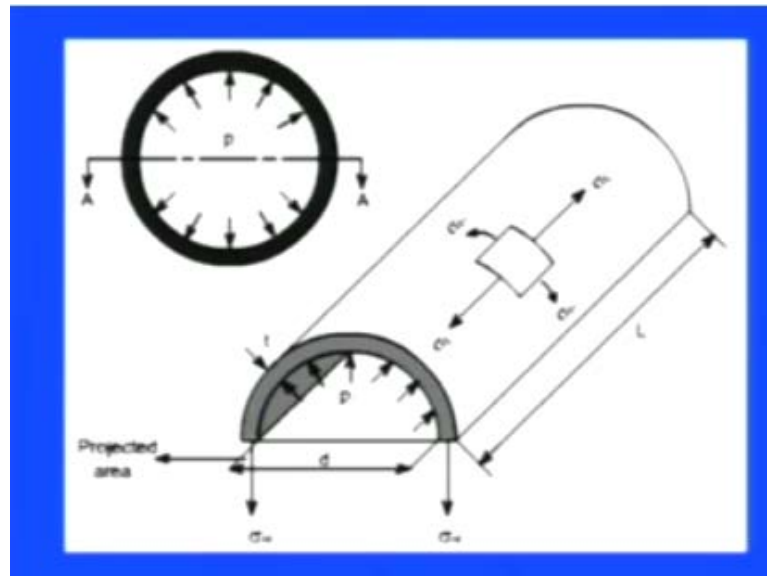
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So, in the next figure we are going to show you that, the half of cylinder that how these the formation of these hoop stresses are coming on that particular the upper part, so the cylinder is subjected to the internal pressure as usual. And due this particular internal pressure  $p$ , on that circumference of this thin cylinder this induced stresses are there which are hoop stresses lateral stresses or you can say circumferential stresses, so  $p$  is the internal pressure,  $d$  is the inside diameter.

So, because now we are considering the diametrical way  $L$  is the length which is the longitudinal part and  $t$  is thickness of wall. And since, we have already discussed that what will be the limiting conditions are there for the thickness to consider the thin cylinder wall.

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So, now this figure, this is the thin cylinder which has the thickness  $t$  and the internal pressure is acting on this, so if you just see the elevation, then you will find that we have the only this portion a circular portion. And this is the cutting section  $A, A$ , if you just cut the section at  $A, A$  we have the half cylinder and we would like to see that what kind of stresses are there which are inducing in these cylinder. And in this cylinder and then what the variation is there along the length of the cylinder.

So, this is the direction of the pressure which is exerting on just inside the wall thickness of the cylinder and then you can find that just they are simply going towards the outward direction, so we have a bursting effect on the wall. So now, due to that always we have an a kind of resistances are there from the wall side and then it will just going against the bursting effect due to this pressure.

Because, this will going just going towards the outward direction, so if you cut the section here we have this side view, so this is the side view of these things where this is the entire length of this thin cylinder wall. And now you can easily visualize that there

are two main types of the stresses are one, at this particular circumferences or we can say at this particular direction or at this diametrical way is there not along the length.

But, along the diameter of that and these stresses are the hoop stresses, so these are the resultant of these things and the projected area is along this part, so this the diameter is there along with these stresses are acting on this particular wall and this is a thickness  $t$  is there. So, if we look at, if you simply cut the portion on this particular thin cylinder, then you will find that we have the two main types of the stresses, because radial stress component has a minimum contribution to the overall stresses.

So, probably we can simply ignore this part, so by considering the two major effect of the stresses, one is the hoop stress, so hoop stresses all along the diameter. So, this is the diameter and these are the hoop stress direction, which is all along the circumference of this thin cylinder and then we have the other portion which is just the longitudinal way. So, what we have we have this longitudinal stresses just along the length of this thin cylinder.

So, this is the longitudinal stress directions are there and this is the hoop stress direction are there, they are both mutually perpendicular only just keep this thing in your mind, both stresses are just the form of normal stress component. So, whatever the strain or whatever the other properties are coming, they will just come due to this normal stress component, only there is no shearing effect is there with that, because the loads are just perpendicular to the active plane. So, now the total force on 1 half of the cylinder going to the internal pressure  $p$ , because the exerting pressure  $p$ , so it is nothing but,  $p$  into this projected area, so what we have, we have  $p$  into  $d$  into  $L$  the total length.

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$$\begin{aligned} &\text{Total force on one half of the cylinder owing} \\ &\text{to the internal pressure 'p'} \\ &= p \times \text{Projected Area} \\ &= p \times d \times L \\ &= p \cdot d \cdot L \quad \text{----- (1)} \\ &\text{The total resisting force owing to hoop} \\ &\text{stresses set up in the cylinder walls} \\ &= 2 \cdot \sigma_H \cdot L \cdot t \quad \text{----- (2)} \end{aligned}$$

So,  $p$  into  $d$  into  $L$  is nothing but, the total force which is exerting on a 1 half of the cylinder, so this total resisting force owing to the hoop stresses along the diametrical is well set up within the cylindrical cell. So, with those things within the  $t$  thickness, we can simply say that two, because it is a half so total is double. So,  $2$  into  $\sigma_H$  is the hoop stresses into  $L$  into  $t$  will give you the total force in this whole thin cylinder, which has the thickness  $t$ ,  $L$  is the total length and  $\sigma_H$  is the hoop stresses which are inducing due to this internal pressure  $p$ .

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$$\begin{aligned} &\text{Because } \sigma_H \cdot L \cdot t \text{ is the force in the one wall} \\ &\text{of the half cylinder.} \\ &\text{the equations (1) \& (2) we get} \\ &2 \cdot \sigma_H \cdot L \cdot t = p \cdot d \cdot L \\ &\sigma_H = (p \cdot d) / 2t \\ &\text{Circumferential or hoop Stress} \\ &(\sigma_H) = (p \cdot d) / 2t \end{aligned}$$

So, now we have both the force component and because of this  $\sigma_H$  into  $L$  into  $t$  is the force on the 1 half as I told you, so we need to multiply two. And if we equating both of the forces we have the real value of the  $\sigma_H$ , which is exerting in the thin cylindrical wall due to the internal pressure  $p$ . So, it is equals to two times of  $\sigma_H$  into  $L$  into  $t$ , which is equals  $p$  into  $d$  into  $L$  or we have a  $\sigma_H$ , which is the hoop stress or circumferential stress.

Or we can say the lateral stresses is equals to  $p$  into  $d$  divided by  $2 t$  or we can say that is a circumferential this is nothing but, equals to  $p d$  by  $2 t$ . So, here the main contribution of  $\sigma_H$  is always along the circumferential and in that the dominant parameters are first. The internal pressure, because this is the main cause for any kind of stresses that what the pressures are there, how they are exerting.

Second is the diameter, if more diameter is there, more this  $\sigma_H$  is there, but the important thing is that how the pressure variation is if we keep on increasing the internal pressure always the stresses whatever the inducing stresses are there, they are always on the higher side. But, the other thing is thing that it has a reciprocal relation with the thicknesses, so now  $\sigma_H$  is proportional to  $d$   $\sigma_H$  is proportional  $1$  by  $t$  and  $\sigma_H$  is proportional to  $p$ . So, now this is the one form of this stresses, which are inducing in a cylinder and that is the hoop stresses which are always along the diameter or we can say always along the circumference of the thin cylinder; second which we discussed is the longitudinal stress.

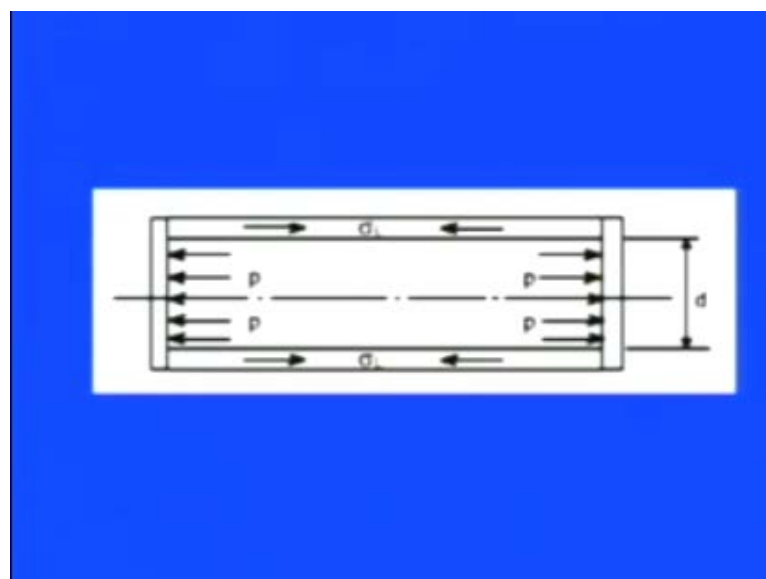
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**Longitudinal Stress:**

- Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure  $p$ .
- Then the walls of the cylinder will have a longitudinal stress as well as a circumferential stress.

Considering the, now the effect of the same figure, now we are again going for the same figure in which the cross sectional areas were in that way half of the cylinder and along the length we have the total length is  $L$  along longitudinal axis. So, the vessel could be considered to have the close ends, always it is close end and contain the fluid undergoes a gauge pressure  $p$ . So, we have a positive pressure due to the steam or any kind of the internal, whatever the liquid or gases are there. And this pressures is exerted on this wall whatever this thickness of  $t$  is there, then the walls of the cylinder will have a longitudinal stresses as well as the circumferentially stresses.

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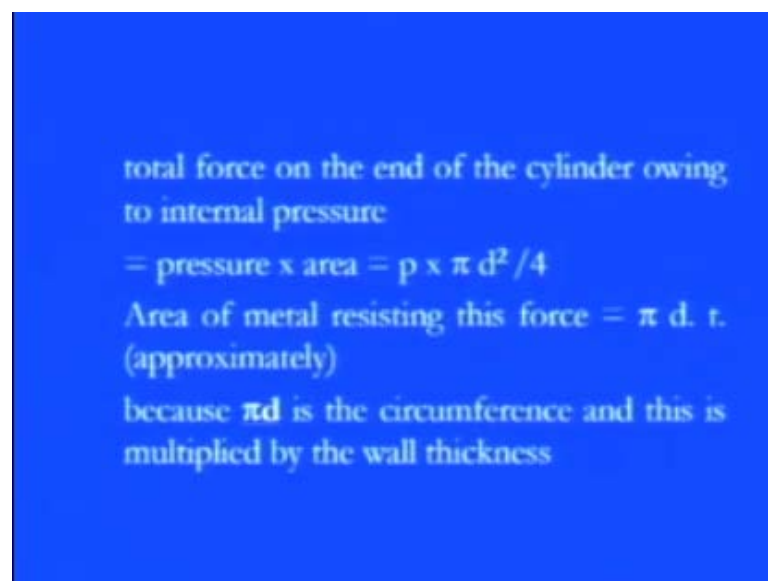


Circumferentially stress as we discussed is always along the diameter longitudinal stresses are always along the length of that. So, see in this diagram that we have a thin cylinder and this is the pressure is exerting on along this x axis and due to this particular load application we have the sigma L which is nothing but, the longitude stresses, which are inserting on the this thin cylinder along the length of these beam.

So, there is no meaning with the diameter, but we have a clear meaning at this particular entire length, so if you visualize those figures, then you can easily say that we have both the component of the normal stresses and both are just normal to each other. That means, both are mutually perpendicular and that is why we can say that, if one is along the length other one is definitely along the diameter.

And that is why the meaning is, if we are saying that one is along the length it is the longitudinal stress and other one definitely along the diameter or we can say the circumference, so it is the lateral stress. So, both the stresses are exerting together when there is in a few fluid pressure  $p$  is there which is exerting on the thin cylinder wall. So, now coming to the main point that if we have a longitudinal the pressures are there on particular this axis, we have a longitudinal stresses or we can say just along this x axis we have stresses.

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total force on the end of the cylinder owing  
to internal pressure  
 $= \text{pressure} \times \text{area} = p \times \pi d^2 / 4$   
Area of metal resisting this force  $= \pi d \cdot t$   
(approximately)  
because  $\pi d$  is the circumference and this is  
multiplied by the wall thickness

So, the pressure into area will give you the force that how much force is there and since, we are saying that it is a cylindrical portion is there on along this particular

circumference and the total diameter is  $d$ . So, the projected area or we can say the affected area by that is  $\pi d^2$  by 4, so  $p$  into  $\pi d^2$  by 4, what  $p$  is applied pressure which inside into  $\pi d^2$  by 4 will give you the total force.

Now, this area of the metal which is resisting this force is along these things and have the effective portion is the thickness  $t$  also we considerable under that part. So, we have  $\pi d t$  where you see, because the  $\pi d$  is the circumference and this is we know multiply by the wall thickness will give you the real feeling about that what is the affected part is under that particular force.

So, this  $\sigma_L$  is nothing but, our this longitudinal stress into  $d$  into  $t$  is the force in the 1 half of the cylinder and this is an uniform in all across this particular, this length of this particular beam this thin cylinder. So, what we have  $\sigma_L$  into  $\pi$  into  $d$  will be equal to the first equation, which is coming in the total force due to this applied force in the affected area  $p$  into  $\pi d^2$  by 4.

So, if we equate both the things then what we have, we have a  $\sigma_L$  which is the longitudinal stress is nothing but, equals to  $p d$  by 4  $t$ . So, if we compare the longitudinal stress and if we compare the circumferentially stress at the same domain, then we find that the  $\sigma_L$  is half of the this  $\sigma_H$ , that means if you multiple  $\sigma_H$  by 1 of the half the it is exactly equals to  $\sigma_L$ . And but, the key feature is though it has a different magnitude altogether in the hoop as well as the longitudinal stresses.

But, the clear feeling is that the main inducing parameters, so we can say the dominating parameter in both of the stresses are same, that means the applied pressure. If you increase the pressure, the longitudinal as well as the circumference stresses are increasing at the same time if you increase the diameter also it has a same impact on the longitudinal as well as the circumferential stresses.

And if you change the thickness also it has a similar effect on the longitudinal as well as the circumferential stresses only there is a change in the magnitude. So, the meaning is that, if you are changing any of the dimension both of the normal stress component, longitudinal as well as the circumferential will be affected simultaneously. And then we can say that if we want to consider the thin cylinder on that basis always what we need to do, we need to correlate these two forms of the stresses.

Because, it is only the normal stress components are there in these kind of stresses, so in this lecture what we discussed that, if we have the thin cylinder first of all we have to design the thin cylinder on the basis of their thickness. And then actually if any internal fluid pressure is there then how we can say that these three forms of the stresses are forming, then what exactly the relations are there in these particular stresses.

So, we found that there are main three components of the stress formations are there in the thin cylinder, one is the radial, one is the longitudinal and one along the circumference. But, the circumferential and the longitudinal are always dominating in the thin cylinder, but radial component though it shows its presence, but its contribution is pretty small as compared to both of them.

And then we have calculated that what will be the hoop stresses are there along the diameter of the thin cylinder and we found that it is nothing but, equals to  $\frac{pd}{2t}$ . And then we also discussed about the longitudinal stresses and also we found that, it has the similar component which are there in the hoop stresses only the change in the magnitude value is there and it is nothing but, equals to  $\frac{pd}{4t}$ . So, we can easily compare both of the terms, in terms of the longitudinal as well as the circumferential stresses.

But, these are the individual parts, so we must correlate that actually if the fluid pressure is there and both the longitudinal as well as the circumferential stresses are there, then what is the correlation. And for that in the next lecture, we are going to discuss about that we have this longitudinal one, we have the mutually perpendicular lateral one and if you refresh your ideas, then we discuss the Poisson ratio.

Because, whatever the load application is there in this thin cylinder, again we are assuming that this is just under this elastic deformation or this Hooke's law is valid. So, for that the Poisson ratio is well defined for this reason, so if you include the Poisson ratio you can easily correlate the longitudinal and the lateral ratio. Then you can say that whatever the changes are there in the length as well as in the diametrical side, we can easily correlate.

So, in the next chapter, first we are going to discuss about how to relate these two dimensions and then the most important part is coming that, when there is a change a significant change is there the length as well as the diameter, then what will be net change is there in the volume. So, this can be easily computed by the volumetric stress,

so when we are talking about the stress component, then always there is a deformation is there and to measure the deformation what we have, we have the strain component, so volumetric strain is there.

So, in the next lecture we are also going to discuss about the volumetric changes and then we can easily define the value of  $k$  that is the bulk modulus of elasticity, that if you know the volume change and if we know the real volume. So, whatever the changes are there in the volume which is computing the volumetric strain, we can also find out the value of the  $k$ , which is the bulk modulus of elasticity.

And then once you know the volumetric strain also, then our main focus is there that if we there is change of whatever the pressure or whatever this longitudinal as well as the circumferentially stresses are there. Then, what exactly the impacts are there, on these design of the thin cylinder, so for that what we are going to do here, we are simply designing that first of all you have the this longitudinal stress and you have circumferentially stresses.

And then you are correlating the Poisson ratio and that also apart from that you have the volumetric strain then what, then you need to define the principle stresses, you need to define the principle strain component. And then you need to find it that we have both of the means, we have this longitudinal as well as this lateral stresses are there on a simple this cubic form, then how to analyze those component. So, these all we are going to discuss in the next lecture, so just remember those formula for longitudinal stresses and the circumferential stresses.

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Because  $\sigma_L \cdot d \cdot t$  is the force in the one wall of the cylinder.  
the equations (1) & (2) we get  
$$\sigma_L \cdot \pi d \cdot t = p \times \pi d^2 / 4$$
$$\sigma_L = (p \cdot d) / 4t$$
**Longitudinal Stress**  
$$(\sigma_L) = (p \cdot d) / 4t$$
**Circumferential or hoop Stress**  
$$(\sigma_H) = (p \cdot d) / 2t$$

And what are the dominating parameters are there and then what will be the formulas are there for Poisson ratio  $\mu$  and what it has a clear impact just to calculate for whatever the deformation or we can say the stresses strain diagram. Or we can say the two major the deformation all those things will come, because we are applying the load under the elastic deformation only where the Hooke's law is valid.

Thank you.