Strength of Materials
Prof. Dr. Suraj Prakash Harsha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 15

Hi, this is Dr Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to deliver my lecture fifteenth on the course of the strength of materials, and this course is developed under the national program on technological enhanced learning. As we have you know like discussed in the previous lecture that if we have a bar irrespective whether it has a uniform process or it has you know like the linearly bearing cross section that means to see if we have the tapered bar. Then what kind of you know like the stress, this generation is there due to the application of load or what the strains are there in that. That kind of you know like discussion which we have already even like made in the previous lecture.

And we found that if bar which is a composition of the different segments then instead of you know like going to check the different deformation, we can simply go for the integration. So, you see you know like even we can adopt the two different approaches in that; one you know like just take the individual segments as we discussed, and for individual segments since you know like at the extreme two corners the tensile loading was there. So, due to loading what the deformation is there, and since this deformation we assume that it is the elastic deformation. So, what the individual deformations are there in these special components calculate those deformation by this particular formula delta is nothing but equals to load P into L divided by A into E; P is nothing but the applied load, L is the effective length of that particular segment divided by this effective area for that particular segment and the Young's modulus of elasticity.

So, if you are calculating these delta for individual segments like delta 1, delta 2, whatever the segments are there, and if you sum up of those things delta 1 plus delta 2 plus those things it will give you the total deformation in that bar. Instead of doing that because you know like it is not the perfect you know like the method; what we are doing here we are simply taking a segment small which as you know like the depth or the width of the dx in particular x direction, because we are applying the load in the x direction only, because these are the uniaxial loading.

So, if we are just taking the width of dx, and if we know that whatever the deformation is occurring in that dx segment is small delta, okay, just calculate that and then integrate for the entire length of beam 0 to L, and because you see we have a uniform structure and whatever you see you know like. And we are assuming that the stresses which are developing these are uniform all across the body, and whatever the deformations are there it is symmetric towards the body you see, because we are applying the load symmetrically. So, with those assumptions we can simply put the integration and by taking you know like integration of all those things right from 0 to L we can get the final deformation.

So, this was you know like this kind of you know analysis which we have done for a uniform cross section bar or the tapered section bar. In the tapered section bar also we assume that we have a small segment in between those you know like the small and the bigger diameter. And you see you know like we assume that it is at a distance of x from the smaller diameter, and we calculated the deformation for that. And once we have the deformation, simply by taking the integration part of all those you know like kind of analysis from 0 to L we are getting the deformation. And that too you see know like we can easily get the deformation, because we are assuming here. And this is the biggest assumption in those analysis that whatever the deformation is occurring in those you know like under the application of this tensile loading is the elastic deformation, and that is why the Hooks Law is valid within this particular part.

So, with those you know like kind of analysis we assume that this bar is putting horizontally; that means there is no weight, we are not considering any weight. Let us say if I am keeping this bar vertically; we say if I am hanging that bar irrespective whether it has a uniform cross section or it has a tapered cross section, then you see we need to consider not only the external applied force which of course it is you know like we are applying this particular load. Apart from that we need to consider what is the weight and what is you see where this weight is applying, means where is the center of you know like the gravity of these things. Where is the cg you know like of this particular body.

Once you locate the cg of the body you need to apply the weight and you see you know like then only we can simply calculate the deformation with the application of both you know like the loads simultaneously, means the applied load plus the self weight load and divided by if you divide the area that which is the effective area for the this load concerned. Then we can get the stresses, and once you have de the stresses you also have the strain because you see whatever the extension is there you can measure that strain in this particular deformation and you can get the strain. So you see you can simply set up the relation.

Meaning is pretty simple that actually if you put whether you say know like the uniform cross section bar or the tapered cross section bar. Always you need to consider the self weight of the bar, and then with the consideration of the self weight you need apply both load external applied load and the self load simultaneously to calculate the final deformation. And once you have the deformation you can easily get the other parameters because here also we are assuming that whatever the deformation is occurring that is uniform and this is under the elastic weight means elastic region.

So, that kind of discussion you know like which we have made in the previous lecture. And in this now then we found that since you know like we have a single bar, but if we have more than one bar then what will happen? So, for that you see again as we have discussed in the previous cases that actually okay one bar is there in the horizontal or vertical with the consideration of external applied load or the self laid all those kind of stresses and strains are you know like occurring within those structure and what exactly the relations are there within those things. So, apart from that now what we are going to discuss that, okay, we have a composite beam in which there are more than one bar of a uniform cross section.

Again in that bar may be you will find that more than one diameter is there of the bar, but if you are simply going for a individual bar then we will find that it has a uniform cross action all across the length right from 0 to L. So, now you see in this leg and then we are going to discuss about the thermal stresses that what will happen if we have a thermal stresses. As we have discussed in the previous cases that you see you know like if we have you know just if you remind that that if we have a constant load; under the constant load if you know like if you are increasing the temperature then there is a phenomena is occurring and that phenomena we discussed as the creep.

So, this kind of relations we have already made and we have already set up the relationship in between you know like stress and strain if the different kind of materials are there or if the different kind of loadings are there means what kind of interactions are there between the loading. So, here it is now the thermal stresses. As far as the thermal stresses are concerned, thermal stresses can come you know like under the application of the normal kind of loading in which it can be of tensile stresses or it ca can be of compressor stresses.

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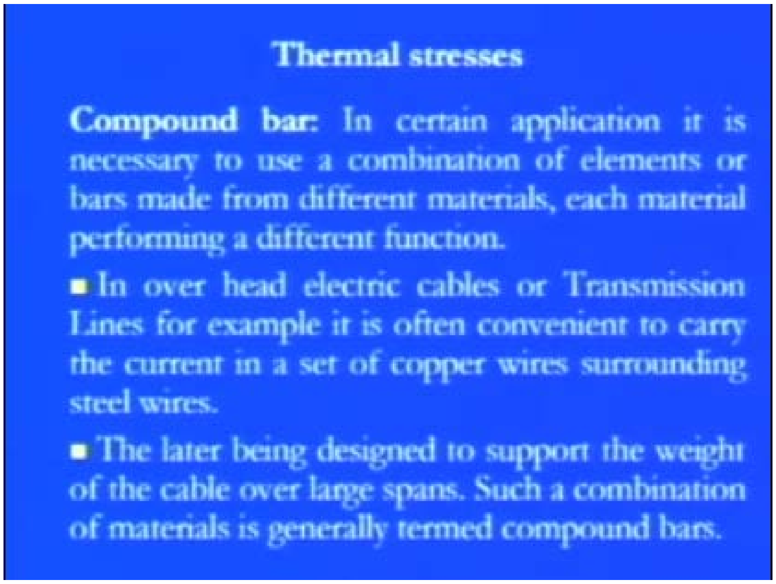

So, if I am taking a compound bar, compound bar means you see it has more than one bar but you know like it is in the composite nature. So, in a certain application it is necessary to use the combination of the elements or the bars made from the different material and each material perform a different function. Because whatever the extensions are going on or we can say whatever the deformation or distortion will happen in the material it is due to the property of the material. Let us say if I have a two different bar; one is mild steel and one is copper, and if we apply let us say 10 Newton bar then definitely whatever the deformation will come in the extension part will be surely different. Because it depends on what the modulus of elasticity is there and what the ductility is there, and we are always measuring the ductility by percentage elongation.

So, how much percentage elongation is there in those kind of material corresponding ductility's are there, and then we can say that you know like all different materials they are exhibiting the different kind of deformation in those you know like due to these modulus of elasticity. So, here if I am having a composite bar, which is you know like having more than one bar and even all bars which are existing in this composite or compound bar have the different materials. So, if we are applying the load then again we need to assume certain thing and definitely you see you know like if you want to calculate certain different elastic properties or elastic constant as we have discussed like the shear modulus of elasticity, bulk modulus of elasticity or this Young's modulus of elasticity or even Poisson ratio, then we need to put the cross bars all across you know like two ends and then we need applied the constant load and then kind of analysis will happen.

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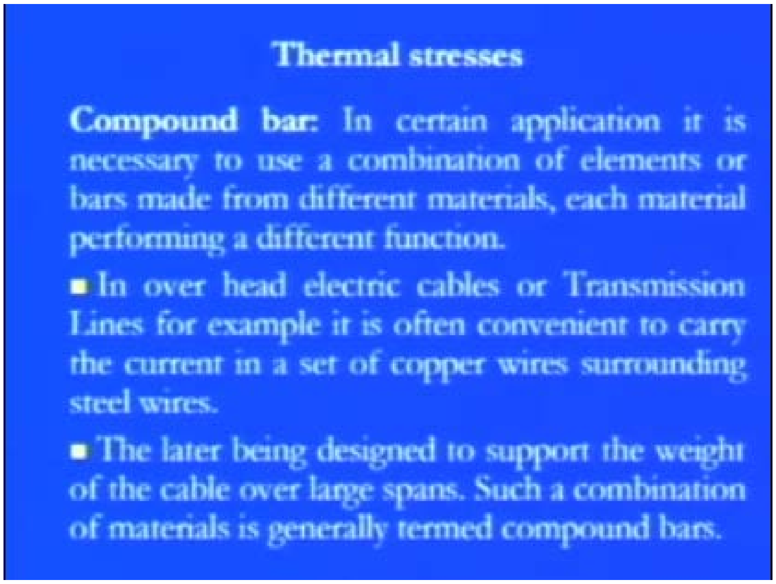

So, here you see you know like in the overhead you know like the electric cables or the transmission lines are there which are you know like flowing from overhead, then for example it is often convenient to carry the current in a set of copper wires surrounding the steel wires. That means you know like always we are just taking the good conductors so that whatever you know electricity is flowing it has to flow perfectly without any sort of distortion in that and later being designed to support the weight of the cable you know like or the large spans what will happen you see when those transmission lines are passing through. Then we are keeping you know like the breaks in between so that we can maintain the constant gap in between these two wires and such a combination of material is generally termed as the compound lines.

Because you see what will happen there are two bars, okay, and you see at both extreme ends of the bar there are the cables which are simply associated from the two extreme point. So, you see here that these two copper lines are there and they are simply passing from fixed distance, and they are having a similar kind of you like the current is passing; that means whatever the load application is there in these two bar they are same. So, how they are behaving under the application of this force or the bar or these currents this kind of you know like analysis, which we are going to discusses in this.

So, consider therefore a compound bar consisting of a member and members let us say more than two members, each is having a different length and the cross sectional area and each being of a different material. So, you see now what we have? We have the three different kinds of parameters. First we have a different material and since it is different material then an elastic deformation is there. So obviously, the Young's modulus of elasticity for different material is different. So, one is this. Second, we are also taking the different cross sectional area. So, whatever the stresses will come though the force is constant because you have applied the force, and we are assuming that this force is uniformly distributed all across these parts of this compound bar, but this cross sectional area is different for different bar.

So obviously, we need to be you know like taken care that the stresses whatever the stresses are inducing in these bar they are different, because again the Young's modulus is different. And then also here what we are considering that since we have a different length we have a different cross sectional area and we have a different material. So, if you want to calculate the deformation delta, delta is nothing but equals to P into L divided by A into E. So, here L is different, A is different, E is different. So, that is for sure that actually whatever the deformation will come individually in these compound bar the bars of these you know like the compound one are definitely different.

And then you see let all the members have a common extension. If you are bounded that they cannot go beyond this limit, then definitely you can say that they have a common extension, and if I am say saying that this is x; that is that means you see the load is positioned to produce the same extension in each member. Even you see apart from all those things like they have different length, different area and the different Young's modulus though if you put the rigid be you know like extreme corners; that means they cannot go beyond certain limits or you see they have if they want to expand they can expand up to this certain limit. That means whatever the extension are they are in all you know like the parts of this particular composite bar, they have to be same.

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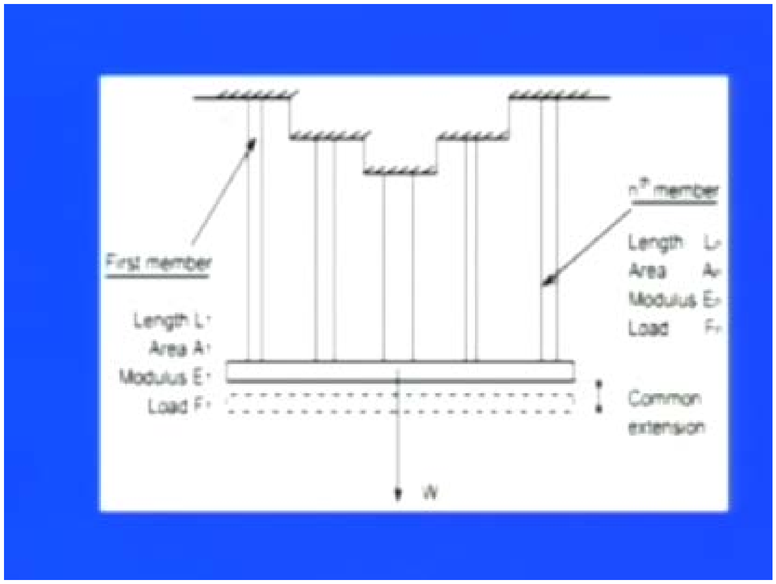

And if you are assuming that this is x, then you can see in this particular figure what we have. We have this is you know like the particular this one extreme end which is simply you know like the rigid base, and this is you see the datum is there. Though all of these bars if you see here this bar the first bar, the second bar, even the third bar, fourth and fifth bar; all five bars are having different cross sectional area, different lengths and since they are having a different material. So obviously, they are having a different Young's modulus of elasticity. So, apart from those things what I am doing here at another corner if I am keeping; if you see this figure then you will find that at the another extreme corner if I am keeping a rigid base.

That means you see this rigid base is there, and at this base if I am applying the load that means whatever the extensions are there this is a common extension as you can see here, though you see we have n member but all n members are having different cross sectional area with different lengths and different material. But they have a common extension, and if I am assuming the common extension is x then we can simply analyze the total deformation of the compound bar. So, meaning is pretty simple that we need to assume that whatever the these two extreme ends are there, and if we apply the load on these two extreme ends whatever the deformation or the extension is coming out from these compound bar it must be same and you can simply compute those things. And once you compute those deformation you have the strain component from that.

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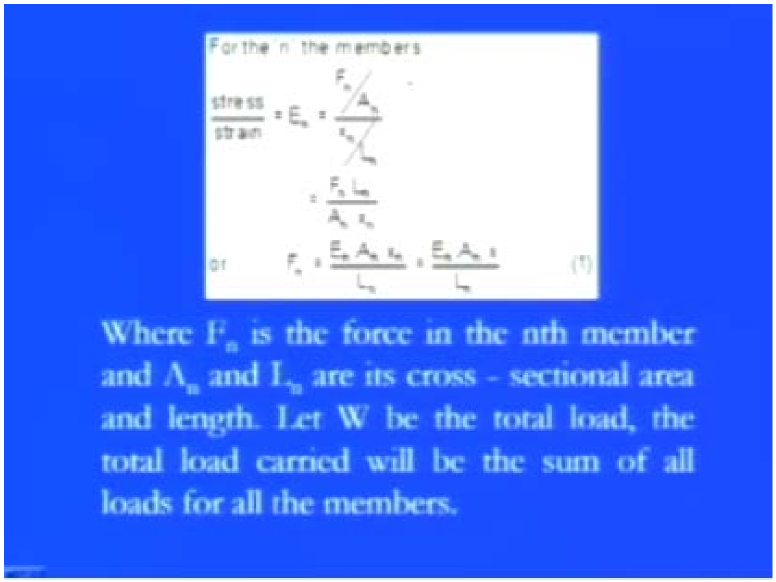

So, come to the real point that we have nth member and since you see you know like this is elastic deformation is there for all these members and they have common extension x. So, you can simply calculate the Young's modulus of elasticity for nth member and this is nothing but equal to stress by strain. And the stress is nothing but since the applied load is there and the area is there the n divided by whatever the extension is there, and if I am saying that the extension or whatever the deformation is there that is x n divided by the L n, or we can say that the nth member will have you see you know like the Young's modulus of elasticity is nothing but equals to F n into L n divided by A n into X n or you can simply you know like computer science we are assuming that in all bars the deformation is same.

So, again you can simply put the deformation equal for all this kind of bars we can say that the force at the nth member is nothing but equals Young's modulus of elasticity of nth member area, the cross sectional area of nth member into the deformation as we assume x divided by L n. So, here you see we have F n which is nothing but the force on the nth member as in all the L n are nothing but the cross sectional area under the length. Then whatever the radial load or we can say the total load which is towards the download direction is there, it is nothing but the sum of the all loads which are applying to all the members.

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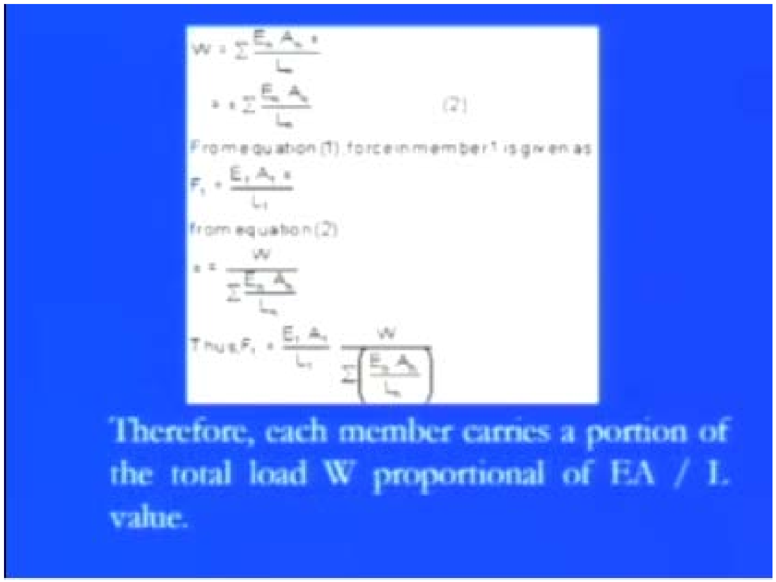

So, now you come to this radial load which is you know acted towards the downward direction is nothing but equals to sum of the all you know like the loads of these bars, then we can simply sum up those you know like summation E and A and x divided by L n. So, here you see you know like after keeping those what we assume that the length is different for individual bar, a cross sectional area is different for individual bar, Young's modulus is different for individual bar. So, if you sum up these let us say for first two segment we have E 1 A 1 L 1, for second segment we have E 2 A 2 and L 2. So, individually see you know like we are computing the properties, and after computing these property from all these segment if you compute and multiply by x, because x is only we assume that the common deformation is there from this particular substance.

So, by multiplying the x you can get the total load you know like form all those bars which are the part of this composite beam. So, now you see from you know like the previous equation if we are keeping the force member for first, then F 1 is nothing but equals to E 1 A 1 into x by L 1 as I told you and if I am keeping those things then what I have, I have x which is deformation is nothing but equals to w divided by that summation of all these components that is summation of E and the Young's modulus of elasticity cross sectional area for nth member divided this length of the nth member, or we can say that the force in the first member is nothing but equals to E 1 A 1 by L 1 or you can you know like multiply by these whatever the x deformation is there; that is nothing but equals to w divided by summation of the E n A n divided by this L n.

Meaning is pretty simple. If you know this deformation that how much deformation is there the combined deformation and if you know that actually you know like what the elastic modulus's are there for that particular kind of thing, you can easily get that how this force is distributed and this force distribution in the different bars of the composite, it depends on that how you see the ratios of you see, because this is the total radial load W and it is you know like divided by the nth member and it is even multiplied by the first member. So, what we are doing here that the first member what the total components are there like E 1 A 1 and L 1 and how they are you know and what is the ratio is there in the nth member.

So, once we have the ratio like E 1 by L 1 A 1 by A n L n by L 1. So, if you have this ratio that actually how what the variations are there, you can simple say that, yeah, this much fraction of W will be given to the F 1. So, even it is it is pretty logical that actually you know like that what exactly the ratios are there in between these geometrical parameters as well as the material parameter. Once you have the ratio the relative coordination in between these two components you can simply say that, yeah, this can carry this much percentage of the total load. Therefore, you see each member carries a portion of that total load for sure which is proportional to E A by L value. That means you see whatever the portion will go it simply depends on the proportion of the Young's modulus of elasticity, this cross sectional area and the length of these two members, and whatever the proportional things are there corresponding W will be shared by these members.

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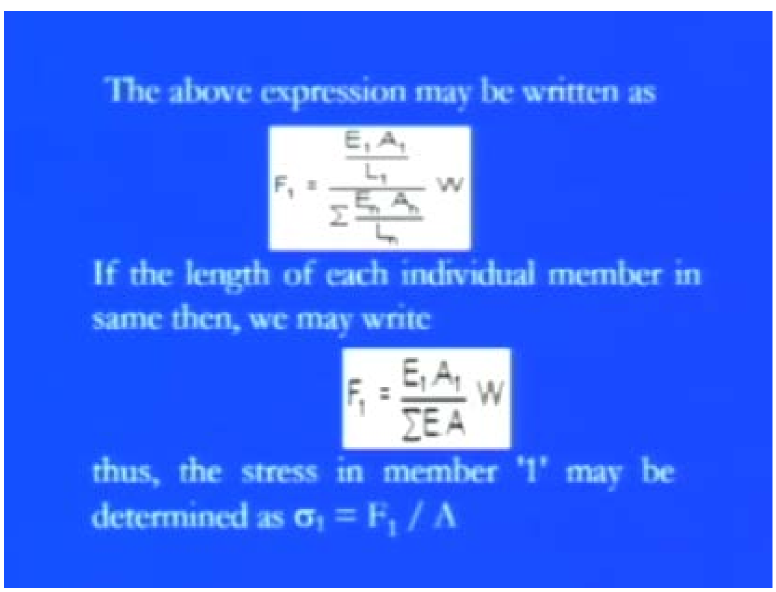

Now you see here as we can see whatever the expressions it came you found that actually there was a ratio for calculating the force. So, again if we you know like rewrite those equations then we will find that as we discussed F 1 is nothing but equals to you know like E 1 A 1 by L 1 divided by the summation of all the nth part E n A n by this L n into the radial load. So one should know those you see these critical parameters it is pretty easy to find the F 1, because W is well known to us that external applied load and if the length of individual member is same. Let us say if all these members are uniform then you see you can simply share like how many members are there divide by the five, because if they are same then you can say that F 1 is nothing but equals to E 1 A 1 divided by summation of E n A n, but that means you see it is summation of combined into W.

So, either you can write E n A n or it is the summation of E in to A. Since the length we assume that it is same that means all the bars are using the compound as we have seen in the previous figure there were five different select members are there. So if they are equal in the length we can simply say that whatever the force will come it is simply the ratio of E 1 by An and A 1 by A n into the radial load. Thus you see you know like the stress member it can easily you know like determine sigma 1 is equals to F 1 by A. So this is for first member you know like this first member.

Now you see you know like correspondingly you can also calculate that what will be the shared force ratio the force is there in the another corner, but again the critical part which we have discussed here is what exactly the Young’s modulus of elasticity is there, because Young’s modulus of elasticity is a property of material. So, until unless if we cannot compare those things then we cannot say that actually what exactly the expansion is there of a material and then how these loads are to be shared. It means how the distribution of load is there because until unless if you do not know that there are you see the five different segments are there in a compound. And if you do not that which a bar is carrying, which load then we cannot design safely, because we need to give that different factor of safety for the different bars for just to design safely.

So, that is what you see this analysis is very, very important in terms of you know like expansion that we are applying the same load; that means you see whatever the external load is there it is same but inside those bodies which are resting this force, how they are in like shared these different proportion of the load and if we can you know like design perfectly these individual you know like components by giving different material or you know like sufficient support. Then only we can say that whatever the design is there it is quite safe and corresponding factor of safety will come in the real design.

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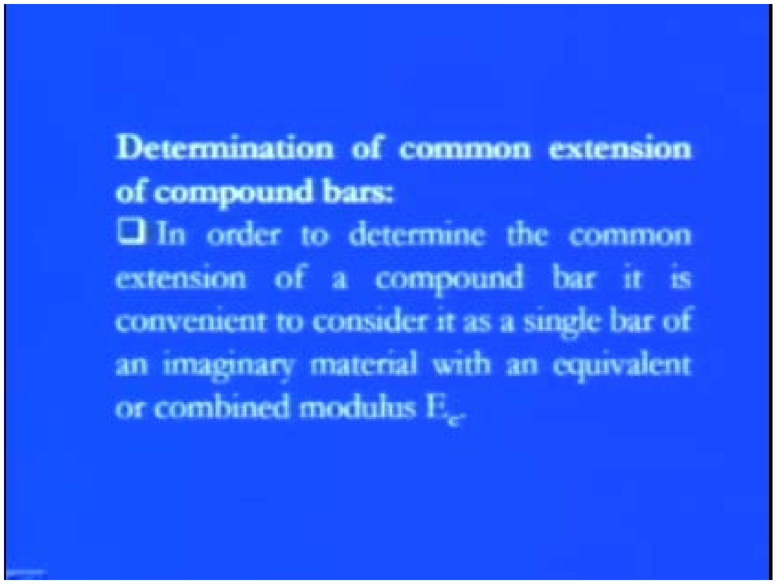

Now we come to the determination of a common extension of compound bar. Always you see in order to determine these things what we need to do always we need to check that whether it is convenient to consider it as a single bar of an you know like this imaginary material or not actually. That means you see whether if we are considering that we have one ductile material and other side we have a brittle material and if you know like combining a bar then definitely whatever you see under the application of this tensile force whatever the extension will come in the bar it will not be uniform. So, if you are assuming the same thing like we have discussed in the previous section that the different you know like bars are there and you see you know like even they have a different Young's modulus the common extension is there.

In this section we have the two different materials and one material is ductile and one material is brittle; we cannot say that the same you know like this kind of analysis is valid for that or we can say that whatever this x is coming that is the deformation is coming it cannot be same. And if you apply the same theories there then whatever the design will come it has to be fail because the ductile material cannot exhibit the similar kind of you know like the extension as the ductile material is considering; means the brittle material has their own brittleness in those kinds of thing. That means they cannot you know like share this deformations or we can say you know like the percentage elongation under the application of force.

So, if I am saying that you know if I just want to you know like consider that if I have a single bar with the imaginary material with an equivalent or we can say the combined modulus E c then how we can you know like go for the deformations.

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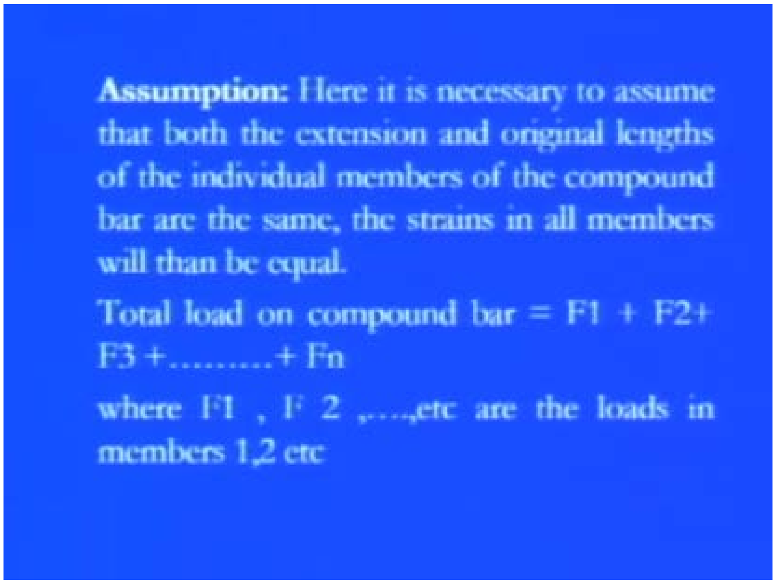

So, here it is you see you know like certain assumptions which we need to make if you want to analyze those kind of structure in which the imaginary materials are there and we are assuming that we have a single bar. So, here it is necessary to assume that both extensions as well as the original length of the individual members are compound bars are {FL} they have to be same, then only you can go for the analysis. And the strain in all member will also be equal, because you see if you know that let us say if you are taking the copper bar and the steel bar or another is aluminum bar they are you know like though they have the different Young's modulus but they are perfectly you know like the ductile material.

And you can assume that they can exhibit you know like the common deformation if you know like put the compound bar and if you apply the load at the extreme ends of this particular bar. So, this assumption is perfectly valid for this kind of combination, but if we have a combination of brittle and ductile material then I am pretty suspicious to get this kind of deformation from both of the material equally. So, you see here if I have you know like this kind of bar where you know like all these materials are ductile and we have you know like the common this deformation. So, if you have a common deformation the length is same; obviously, the strain must be same. So, we have to use this kind of you know like the constant phenomena's.

So, if I want to calculate out the total load in the compound bar it has to be F 1 plus F 2 means the total summation of the force on the individual bar. So, F 1 plus F 2 plus F 3 up to F n where you see all these loads are shared by these different different segments of a bar. So, F 1 is shared by first element, F 2 is shared by second element, and le let us you see if I am saying that the total five elements are there as we have seen in the previous figure. So, you see F 3, F 4 and F 5 will be shared by the different segments of a bar; that means you see you know like the total force is coming out from the summation of the bar as we have seen in the deformation also, the deformation as we have discussed in the previous section that deformation is coming by delta 1 plus delta 2 plus delta 3. So, if you sum up those deformations you will get the total deformation of this bar.

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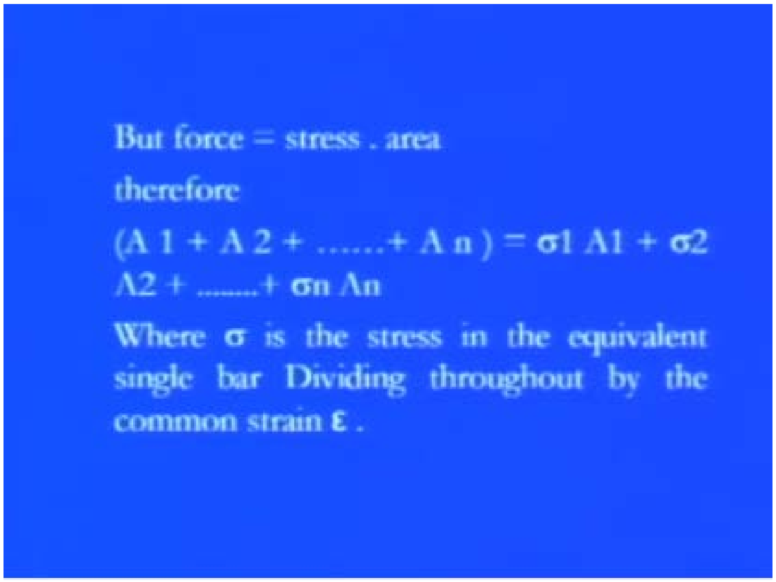

So, here you see this assumption is quite valid but you seem like the force is nothing but equals to stress into area. So, again you see we need to keep that what exactly the area is. So, A 1 plus A 2 up to A n; that means you see what are the individual cross sectional areas are there of the bars we need to calculate and the corresponding you see you know like the forces are coming as this because we are having F 1 plus F 2 plus F 3 up to this one. So, F 1 as I told you this stress into area, so sigma 1 A 1 plus sigma 2 A 2 plus sigma 3 A 3 sigma 4 A 4 and sigma 5 A 5; that means you see even if you can go up to nth but here if we are using the five bar, then the total force will be of stresses. Whatever the individual stresses are coming goes again the stress will never be same because of the material is different, area is different.

So, corresponding the stresses are different, but we are assuming that the strain whatever the strain is coming that has to be same. So, after applying this particular equation what we have? We can simply get that whatever the forces are coming they are the summation of the individual forces or whatever the stress components are coming they are nothing but equals to sigma 1 A 1 plus Sigma 2 A 2 plus these things, where sigma is the stress in the equivalent single bar and you know like this whatever the strains are there is the same but whatever the cross sectional areas are there they are different for the different bars of a single unit of this. So, now you see here what we have done? We have done that we have a bar in which there are simple different segments are there.

So, if you want to calculate that what exactly is happening with the Young's modulus of elasticity because you see for all individual areas and for individual this Young's modulus of elasticity, what we have? We have the different extension; you know like we have to discuss that the different deformation but due to the certain assumptions that you see by keeping those blocks it cannot go beyond certain length, and it is going up to this particular length and this length has to be uniform from all those bars; that means you see you know like we are assuming that the strain must be same.

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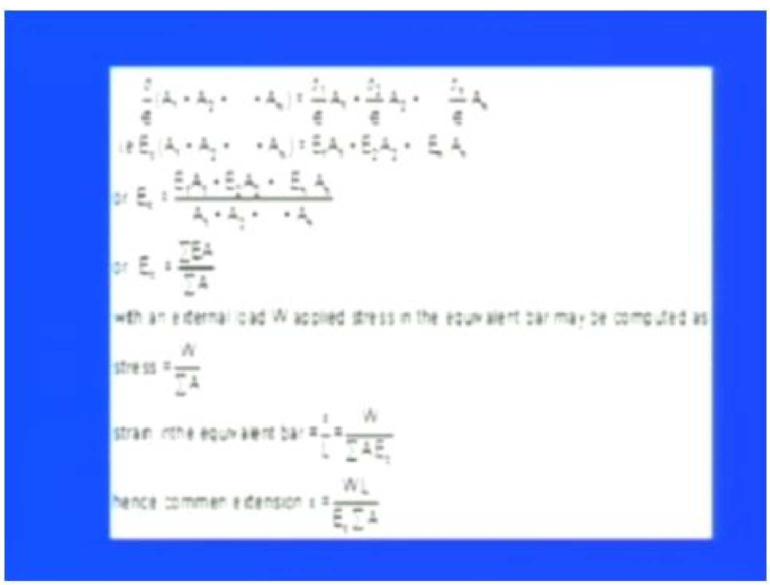

So, after applying this condition here what we are getting here? We are having the sigma divided by this epsilon; that means to see we have you know like the stress divided by the strain into whatever the areas are there, they are different. So, we can say that the sigma 1 A 1 as we have discussed in the first case but the epsilon must be same; that means the strain is same. So, what we have for different segments of a compound bar? Sigma 1 A 1 by epsilon plus sigma 2 A 2 by epsilon plus whatever the components are there, the sigma nth into A nth divided by epsilon. You can say that that whatever the stresses are there they are different, areas are different but strain is same.

So, this equation is very much valid and for numerical problems we are going to use this equation very much, and you please keep to remember that actually how this equations are playing in a important role to find out the different parameter in the compound bar. So, in this compound bar we assume that whatever the strain component is coming that is constant. So, even we can calculate the equivalent Young's modulus of elasticity, and for that you see you know like as we have discussed in the first line that it is equals to those components. So, if you are saying that the equivalent Young's modulus of elasticity is E c. So, just by keeping sigma by this epsilon the stress by strain this E c it will give you the E c into this all summation of the area into this kind of you know like E 1. This sigma 1 by E is E 1 into A 1, sigma 2 by epsilon is E 2, sigma n by epsilon is E n and corresponding area is there.

So, you see here though we have you see the different Young's modulus of elasticity and we know that we have only one condition which is same that is the common extension we can calculate the equivalent Young's modulus of elasticity for that compound bar and which is nothing but equals to E 1 A 1 plus E 2 A 2. That means you see the first bar the Young's modulus into area concern plus for second bar Young's modulus into the area concern up to you see sum for nth bar divided by if you divide by the area A 1 plus A 2 plus A 3 plus up to A n, it will give you know like the equivalent Young's modulus of elasticity for a compound bar or we can say that it is nothing but equals to summation of E into A; that means you see the Young's modulus of elasticity into the cross sectional area.

If you sum up for individual component divided by sum of the area individual area it will give you the equivalent Young's modulus for the compound bar. So, with then you see the external load, because we are applying the external load at the extreme end, the applied stress in the equivalent you know like bar can easily be computed with you know like the stresses are nothing but equals to W by summation of A. That means you see the total load divided by the total area and total area we can simply compute by computing the A 1 plus A 2 plus A 3 plus whatever the number of bars are A n.

So, correspondingly the strain will come in this equivalent bar, because we are assuming the same. So, it is the deformation x because it has a common deformation is there. So, x divided by the L, but you see here we have already assumed that whatever the strain component is there W by you know like the summation of area and if you divide by you know like this stress by Young's modulus of elasticity equivalent means E c then you will get the common we can say the strain or whatever the same strain is there for the compound bar which will be nothing but equals to that W divided by summation of area into the E c.

So, you see with that you can simply get the common extension for all you see the compound bar though the individual extensions are different, but the common extension for this compound bar will be equals to W into L divided by E c into summation of area. That means you see the load applied the same length because we are assuming that the compound bar has some length; that is L divided by the E c. E c is nothing but equals to you know like that is an equivalent term Young's modulus of elasticity for compound bar which can be you know like coming by the individual contribution of the E 1 A 1, E 2 A 2 divided by A 1 A 2 A 3 like that and then summation of the area. Because you see you know like different are will play a different roles in that. So, we need to go for a summation of area.

So, here you see the extension if we are concerning about a compound bar, then only the key feature is that though the length you see we are assuming the same of the this particular compound bar, the applied load is same, summation of area is same, but the equivalent this Young's modulus of elasticity will play an critical role to calculate the deformation. So, if you want to you know like make the common deformation for a compound bar always we have to be carefully choosing this particular material of the bars. So that whatever the Young's modulus is coming for all those bars it has to be you know like showing the common type of phenomena, so that you see there should not be any ambiguity is there like we are assuming one as a brass, one as a concrete and other side you see we have copper, we have you know like the aluminum or we have the mild steel.

So, you see this combination will disturb your deformation and whatever the deformation or we can say the extension is coming that does not have to be you know like the same as we have you know like discussed here in this section. So, that is why you see the equivalent Young's modulus of elasticity is you know like critical path in this particular common extension. Then you see you know like if you are discussing about a compound bar as we have discussed due to you know like whatever the compound bars are there and whatever the changes are there and if these changes are taken place due to the temperature change.

That means you see the thermal stresses are being set up in the compound bar, and as I told you see you know like due to this temperature change only the normal stresses are being induced in an object like it has to be you know like extension or the compression. Then we have to be very careful as we discussed in the previous case that actually if you are simply applying the tensile load or if you are applying the compressive load then what will happen with the compound bar. So, again you see here we have taken the compound bar; we are keeping in a high temperature zone, and you see since the extension is there. And if you are keeping you see the extreme ends of the bar is a constant or some heat flow is going on from the tubes as you see in the industrial application we will find that this you know like the heat whatever the hot water or any this coolant with the cold temperature. If they are passing then certain extension or the contraction will take place you know like in the different different tubes and if you are saying that these tubes are simply bounded from two extreme ends, then what will happen.

So, this kind of analysis you see we are going to discuss in this particular section. So, ordinary material expands when heated or contract when cool, okay. Hence, increase in temperature produced in a positive thermal strain as you know like we discussed that always due to you know like these thermal changes, if high temperature region is there, it is simply expanding so we have a positive thermal strain. If contraction is there that means if a low temperature region is there always materials try to the squeeze in its own part. So, obviously we have a negative thermal strain. So, thermal strain usually you see are reversible in the sense that the member returns to its original shape when temperature you know like comes to its original value or we can say if you are keeping you know like from high temperature region to the room temperature, then body comes to its original shape. Then we can say that actually it has a revisable nature like we have the other processes as we have discussed in the thermodynamic process like we have isotropic process, isobaric isentropic process.

So, all those processes which we assumed generally they have you know even isothermal process like that. So, they are you know like the reversible nature. So, here also since we are assuming here that the elastic deformation is there and even you see due to this temperature change if you are going up to the elastic deformation. So, even if that object is there in the high temperature region and if you see we are cooling that up to normal room temperature body you see whatever the thermal stress are there in that and body deformation if they come to you know like if this body comes to in original shape, then we can say that whatever the deformations are there it is in the reversible nature, and it is the elastic deformation.

However, there are some materials, which do not behave in this manner. It is because you see some of the materials are highly sensitive towards the temperature you know like, because the micro structures are changing so rapidly due to the temperature change that we cannot say that whatever the strains are coming they are either the positive negative or they are exactly the same or we can say sometimes which the strain you know like hardening is there or sometime due to the stress concentration, the cracks can be also developed on the surfaces of this material. And then we cannot you know like get the similar kind of behavior which we are you know like observing in for the ductile material.

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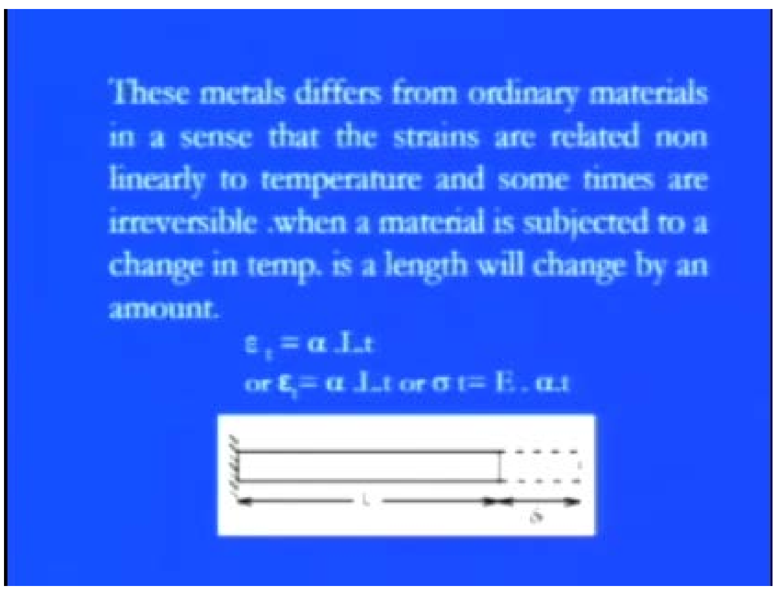

So, you see these you know like the material differs from the ordinary material in the sense that the strains are related just related nonlinearly to the temperature, because sometimes you see you know like the different material is showing different different extension due to the temperature. So, we cannot say that it has a linear relationship with the temperature just like we have seen in the elastic deformation that the stress is you know like the stress is having a linear relationship with the strain. So, you see now we just try to relate the nonlinear behavior between the stress and strains or even the strains inside of a composite bar due to the temperature, sometimes really it is highly irreversible in the nature.

And that is what you see sometimes you know like when we are discussing about the realistic way like the turbine blades are there or you see you know like the different regions of a boiler where you know like the steam is passing; always you will find that at certain you know like at the no points we have a different kind of a extension while you see you know like at some heated reason we have different kind of reasons. And if you know this particular kind of high pressure streams or you see the superheated streams, if it comes to the saturated one, then you see sometimes it is not coming to the original shape of a boiler.

So, this kind of analysis will always be there in terms of two categories; one is the reversible nature, one is the irreversible nature. When a material is subjected you know like to a change of the temperature always there is an extension or there is a contraction; that means you see what exactly happening only there is a change of the length. So, you see we can say that whatever the deformation or the extensions are coming in an object this is you know like depends on this epsilon t is equal to alpha times L into t, where you see epsilon is nothing but the thermal strains. So, you see that means the strain due to the temperature change. So, this epsilon t is equals to alpha that is the thermal you know like expansion of the material that how this material is sensitive to the temperature and how you know like if we are talking about the room temperature or what is the melting temperature. And if we are going from the room temperature to the melting temperature of material, then how you know like the micro structure of this material will behave.

So, this is alpha, L is the length, and t is the temperature that actually we are talking about which temperature and if you are increasing the temperature then what will happen to the strain at a particular temperature. So, t is the temperature or we can say that you see if you want to calculate once you have the strain then you can calculate the stress. So, sigma t is nothing but equals to if you multiply the Young's modulus of elasticity with the strain you will get the stress. So, sigma t is nothing but equals to e times alpha t. So, you see here meaning is very simple that once you have the thermal strain you can go for the thermal stresses and you can simply you know like calibrate those things.

So, if you see this diagram what you have? You have this kind of cantilever; one end of this beam is simply rigid so that we cannot allow the expansion or the contraction of this bar you know like to expand or to squeeze form this end. So, we have this free end and now if you apply you know like the temperature change whatever the extensions are there, this extension will take place along this particular length. So, we have this delta t and whatever the delta t is coming due to that, we can simply measure the strain, and this strain is coming here; this is epsilon t. So, you see epsilon t is absolutely based on how much you see the deformation is there in an object divided by the original length of this.

So, you see delta t divided by L will give you this t or we can say you see again. So, this is one way of calculating the strain or another way is you see alpha t whatever the deformation will come, it is due to this epsilon t is nothing but equals to alpha into L into t. So, this can also be calculated since you see the alpha is coming and alpha is the material property, L is the length of that and t is the temperature that at which temperature this beam is operating.

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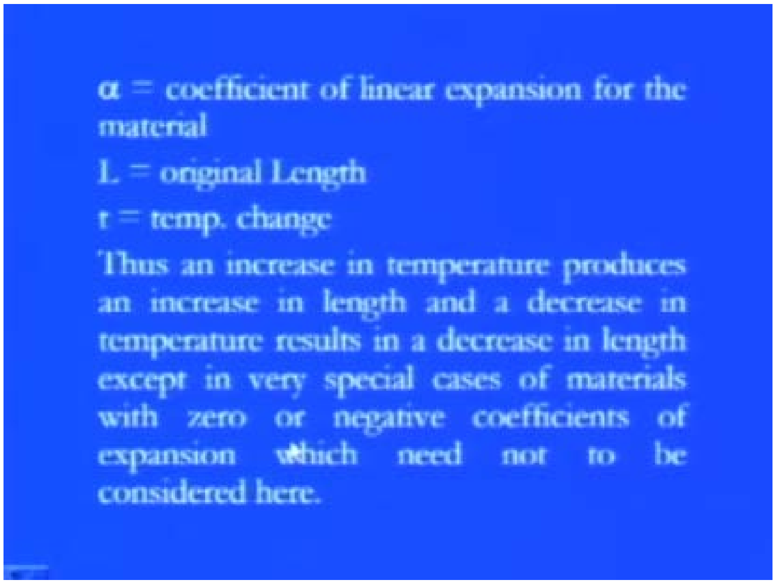

So, you see here as we discussed that alpha is nothing but equals to the coefficient of the linear expansion for a material. Again you see here we are assuming that since it is elastic deformation so even the temperature due to the temperature effect also whatever the expansion or the contraction is there in the object it is to be elastic. So, that is why you see we are using the coefficient of the linear expansion of a material. L is the original length as I discussed and t is the temperature change. So, you see with an increasing in the temperature always produce an increase in length as discussed or a decrease in temperature always decrease in the length except in a very special cases where the material with the zero negative coefficient of the thermal expansion is. Because if we have at alpha is zero or negative then we cannot exhibit this kind of behavior due to the change of temperature.

So, but generally you see if you are talking about ductile material or the different kind of brittle material, than they are having some sort of you know like the alpha which is positive. So, if you have a positive alpha then we can say that if the temperature is increasing definitely there is an extension in the bar and if there is a temperature decreasing then we have a contraction or the decrease in the length of a bar. So, with that you see you know like we need to consider alpha very carefully because whatever the temperature applying is there, it absolutely depends on the alpha because alpha into L into t is there. So, how alpha will be taking place or what the material property is there like you see Young's modulus of elasticity. So, correspondingly you see alpha is also as you know like playing a dominant role while you see for temperature expansion is there.

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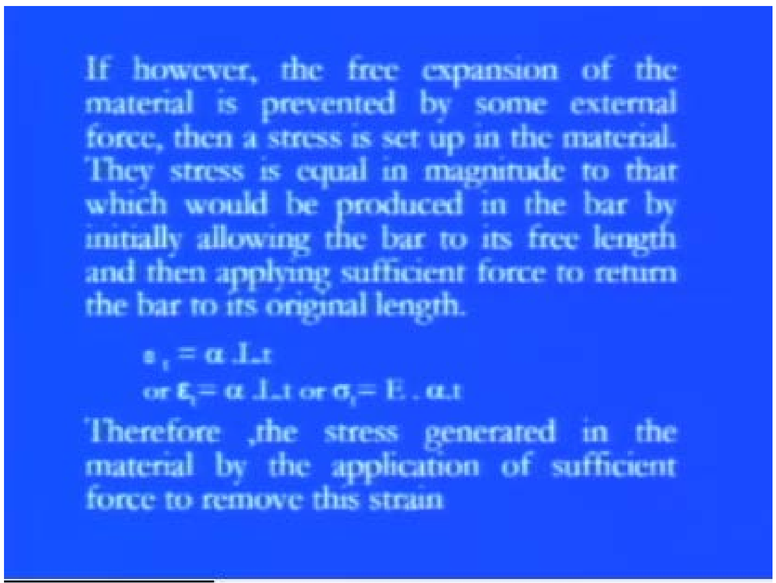

So, you see here you know like if you are talking about a free expansion of material which is you know like prevented by an external force, then a stress is to be setup in the material always, because you see as we apply the load from external side; if one side is free or one side is busy or you know like applied by that, always some sought of the stresses are there, and these stresses are always equal in magnitude to which you see you know like which are produced by you know like a bar which is allowing towards the extension and always extension is there towards the free you know like length. So, whatever you know like the extension is going on at the extreme corner we can simply calculate you know like the extension by the deformation that how much deformation is there at the free length as we have seen in the previous diagram.

And then by computing the deformation towards the free length always what we are having? We are having the total strain that how much you know like the strain is there, what is the change of length is there divided by the total length or if you are keeping this strain here and if you are having the L and if you are having t, you can simply calculate the alpha and you see you know like the alpha since alpha is the material property. Once you have the alpha then you can say that, yeah, if you apply this much temperature then this much extension will be expected. And if you go beyond this thing then we have a permanent set of deformation, we cannot you know like then we have a non linear relationship between the stress and strain, and this is the plastic region is there.

Therefore you see the stress generated in the material by application of the sufficient force to remove the strain is always you know like suggested that you see if you want to apply the load you see and at outside you see some temperatures are there. Then it always plays and you know like the equivalent role as compared to you know like you are applying the load and it is a room temperature is there. So, that means you see whenever these two conditions are there and if they are simultaneously applies on this bar, then you have to consider the deformation by both ways. One is by an application of the force you know like another one is due to the temperature. So, you see we need to apply the sufficient force to remove the strain if you see you know like the stress are being generated in the material due to the temperature change.

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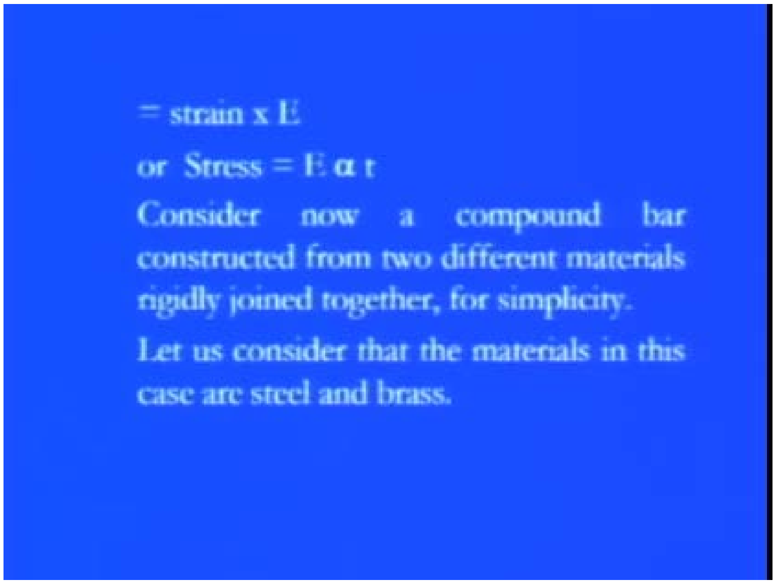

So, now you see you know like if we are talking about the stress which is nothing but equals to you know like the strain into Young's modulus of elasticity. So, again you see we can say as far as the deformation is there due to temperature change. So, stress is nothing but equals to E into alpha t. Consider now a composite bar you see you know like the compound bar we can say constructed from two different materials rigidly you know like are joined together.

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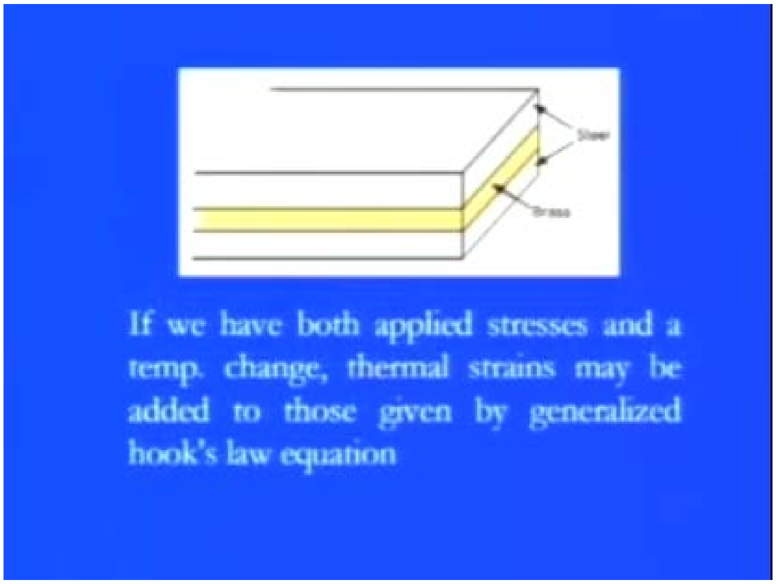

And for simplicity what we are doing here we are simply taking you know like this kind of a structure in which these two extreme ends are there. So, both of you see; both the outside material is steel and middle one is a brass material. So, you see if we apply the stresses and the temperature change is there then always we have a thermal strain because whatever the temperature changes are there, there is certain deformation because if you raise the temperature extension is there and this extension is to be computed in terms of the thermal strain. And this thermal stresses if you are saying that this you know like if it is compounded in a perfect way, then whatever the deformation is coming this is same. So, thermal stresses are also the same.

So, it may be added to those given by the generalized Hook's Law because of whatever the deformation which we are concerning here that is nothing but the elastic deformation. So, for that you see you know like again if you are applying like the two mutually perpendicular stresses are there.

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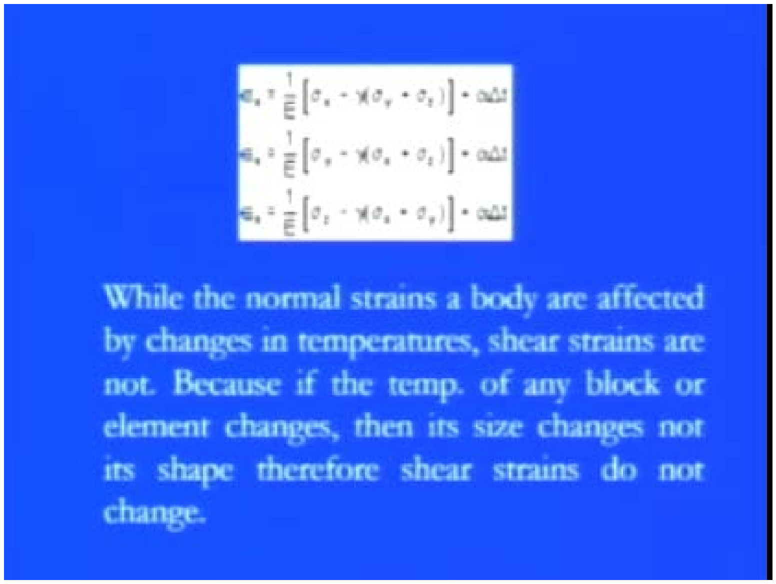

And if they are applying in all three directions then we can simply compute the strain component in the x y z direction correspondingly you see if you are saying that if the deformation is there the x direction only due to the temperature change that means you see towards the expansion is there; it is in normal stress component. So, the corresponding strain is there; the thermal strain is nothing but equals to 1 by E into sigma x minus tau times of sigma y plus sigma z. So, you see here what we have? We have a bar in which you see all three in the mutual directions, the load is applying and due to that what we have? We have all three stress component normal stress component sigma x sigma y sigma z, and if you want to compute under the effect of this that, okay, what will be the stress the equivalent stress component, then you see the extension is there in the x direction and in other two direction there is a contraction.

So, with the using of this Poisson ratio we can compute the total impacts of the you know like the deformation or we can say the strain, and if you have see you know like the stresses. So, once you have to multiply with the Young's modulus of elasticity you have the stress. So, stress you see divided by like the Young's modulus will give you the strain. So, sigma x minus this Poisson ratio into sigma y plus sigma z which will give you the kind of you know like the deformation divided by which will give you the kind of deformation due to the application of load plus you see we have alpha times d t. That means this deformation due to the temperature change and this delta t will be the temperature change.

So, correspondingly you see we can simply calculate the sigma x in the sigma y or epsilon y or epsilon z in the corresponding direction but again temperature this alpha d t alpha d t will play in a same role. So, you see while in a normal strain, body are affected by the change of temperature shear strains are you know like always to be notable, but because of the temperature change if any block or element changes then its size changes not in the shape of you know like perfectly but then shear strains are there. And if you are saying that they have the same you know like the width, then we can say that the shear strain cannot be changed. So, shear strains must be same for a compound strain, because we are assuming that whatever the deformation is going, it cannot go beyond a certain depth or whatever deformation is there it is same for all those components like as we have discussed in the x y or this sigma x sigma y or sigma z bar because of this particular alpha.

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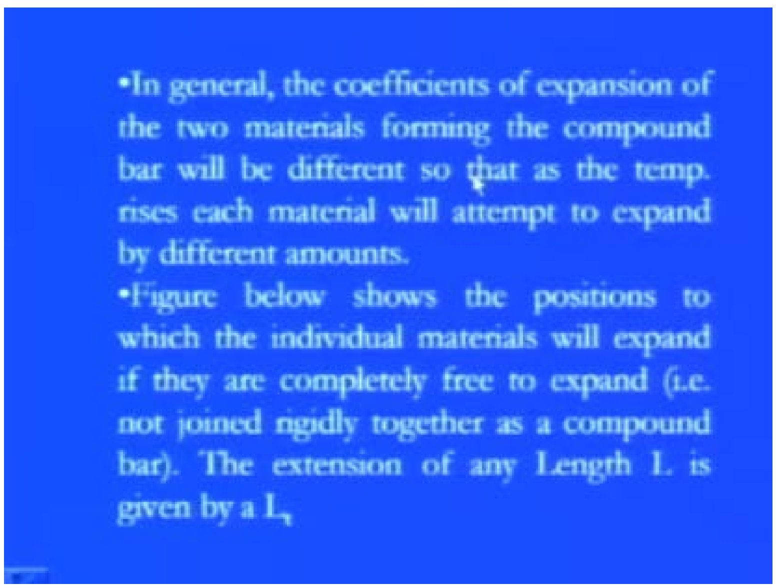

So, you see in general the coefficient of expansion of any two materials forming the compound bar will be different, so that as the temperature rises each material will attempt to expand by different amounts. That is pretty simple as I told you that alpha is nothing but alpha is a function of that material. So, as you change the material like if you are using aluminum, if you are using copper, even if you are using steel and brass; all materials are having different kind of microstructures as we discussed BCC FCC or HCP. Then definitely they are going to you know like exhibit a kind of different expansions or the contraction due to the temperature. So, we are getting you know like the different alpha's for different kind of material. And whatever the figure which I am going to show you it shows the position to which the individual material will expand if they are completely free towards the expansion. So, that means you see you know like there is no rigid boundations are there at these two corners and if these corners are free and if you apply the temperature, definitely there is an extension is there.

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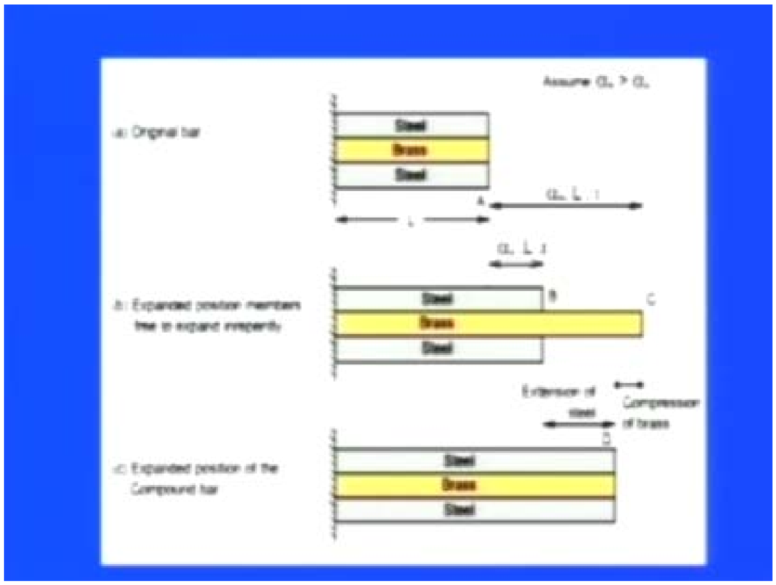

And if I am saying that this extension is Lt due to the temperature, then probably you can see this diagram that we have you see know like this extreme end this is the original bar. In this bar you see we have the cantilever kind of thing, and one end is absolutely rigid; we cannot allow any extension or the contraction from this end but one end is free. So, it can go up to any extension of this bar, and we have two different material as I shown in the previous diagram that outwards two bars are from the steels and inner bar is from this brass. Now you see if we change the temperature and because of you see the linear thermal expansion alpha of brass is more than the alpha of steel. So obviously, the extension in the brass will be more, because you see this alpha is more. So, it will you know like expand fastly or rapidly due to the temperature change.

So, as you see we are going for a certain temperature change, then we will find that after you see expanded position members you know like towards the free direction, then you see it is showing that the steel is moving from this corner to this corner. That means due to this alpha s it is expanding up to that, but the brass is right from this point to this point. It is expanding rapidly fast because of this alpha b. So, alpha is you know at the linear coefficient of expansion is always playing in a key role for you know like kind of expansion. And if you see you know like if I just want to go for a compound bar then I need to put you see the extra you know like this expansion of the steel just to make the common deformation. So, this is a compound bar.

So, in a realistic way the deformation is different as it seems because of that different alpha, but if you want to make a compound bar or a composite bar then the deformation has to be same. So, you see this kind of information is always essential for making a bar that actually how they are expanding how they are behaving and since this behavior is absolutely based on this alpha.

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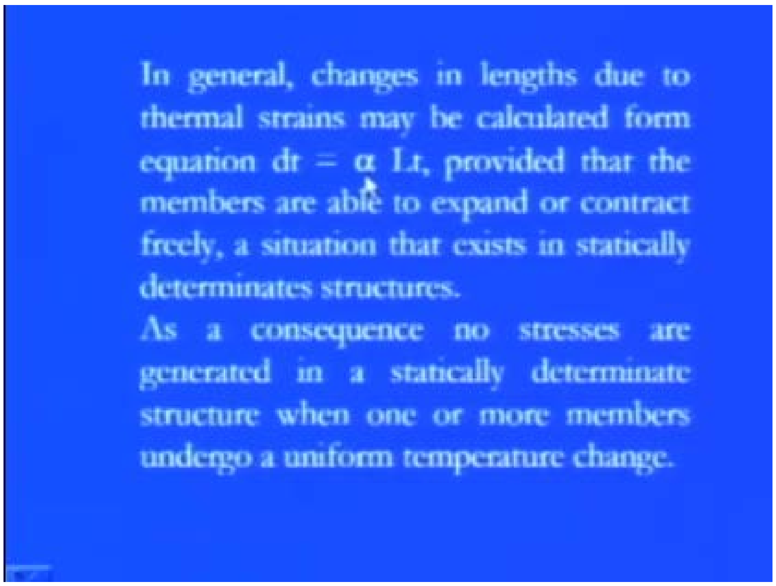

So, we have to be very careful that actually how this alpha is to be chosen or how material is to be chosen. So, as a consequence you see there is more stresses being generated in a statically determinate structure when one or more members undergoes uniform temperature change. So, if you see there is a uniform temperature change is there then we can say that whatever the thermal strains are there, they are well said if they are well equal, and since this alpha is you know like because as we have seen that whatever the change of temperature is there, this alpha is a key role. So, if you are saying that it is a uniform temperature is there we can say that this whatever the statistical determine structure are there, there is no stresses are being generated in those kind of structures.

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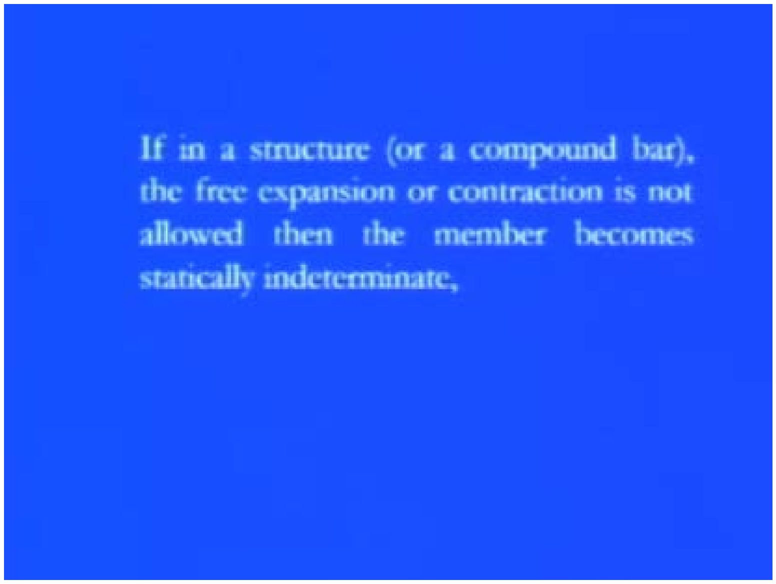

If in a structure or a compound bar the free expansion or contraction is not allowed then member becomes statically indeterminate; means you see you know like if statically determinate is there then you see in those kind of structure we are allowing to expand or contract from the free end. But if you are talking about a statically indeterminate structure then always you see it is simply rigidly bounded and there is no free space is there for a kind of expansion or this contraction.

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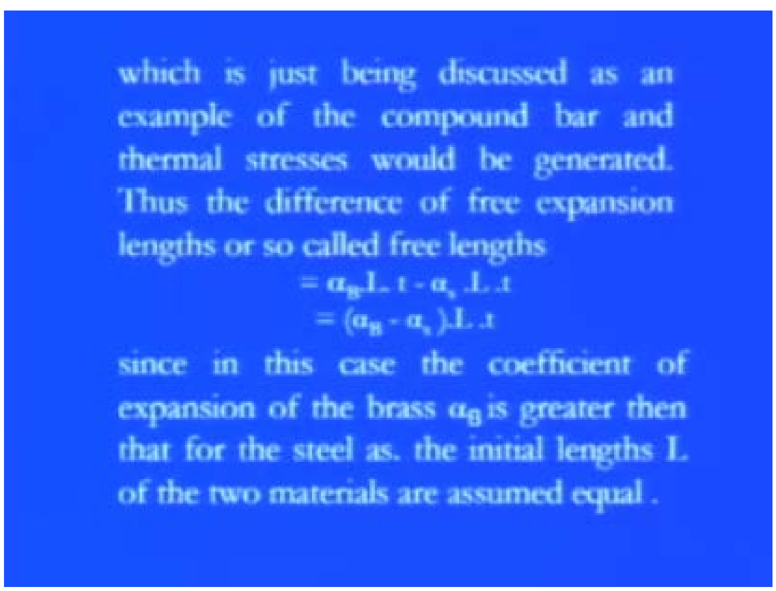

So, now you see as we discussed that if we have you know like this whatever the expansion is there and if you want to compute the difference of the free expansion lens or we can see the free lens, because the expansion is there towards the free length only; we can go easily for you see this alpha b into L, whatever the change of length is there in to t minus alpha s, because whatever the steel components are there into L into t or we can say that alpha b minus alpha s into L into t.

So, this difference of the linear coefficient of the expansion will give you that actually what is the equivalent or what is the free expansion is there because of the change of that into whatever the length is there or the temperature applied is there. So, it absolutely depends on you see what the differences are there in between the real property of the expansion of brass as well as this steel component.

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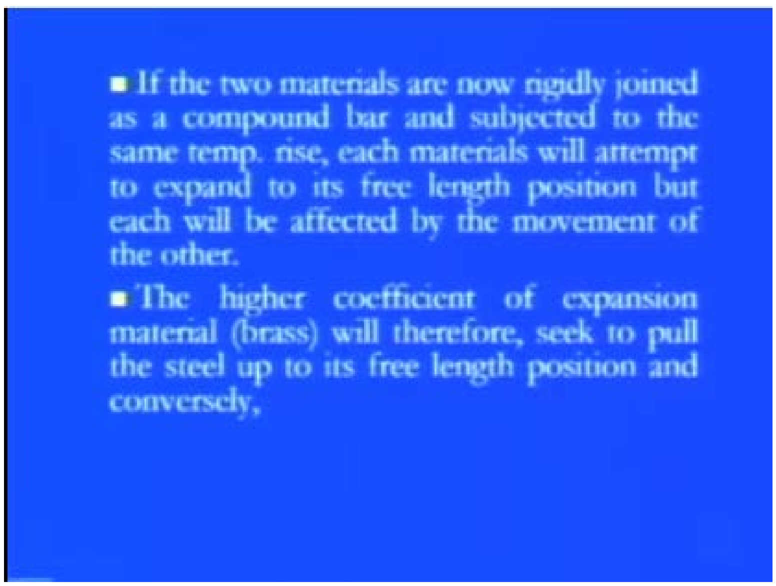

So, now you see if two materials are now rigidly joined as a compound bar and subjected to the same temperature rise each material will attempt to expand its free length obviously towards the outer side but each will be you know like attracted by the movement of each other, because you see you know like the interaction will always be there in between those things and this outside forces or we can say the temperature is same. So, they will always you know be affected by the movement of each other and the higher coefficient or expansion alpha as we have discussed alpha b is greater than alpha s; this expansion material will be therefore sleeked to pull the steel, you know like from up to its free length position and consequence you see you know it will get the composite bar at the end in the same way.

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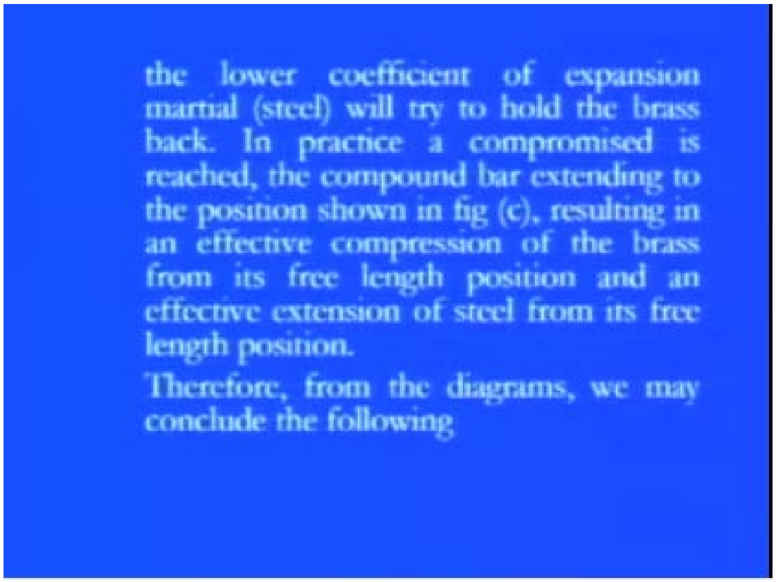

So, you see with the same thing if the lower coefficient of expansion is there like the steel will try to hold the brass back. So, you see there is you know like the action is going on from the brass and the reaction will be there from the steel and they just try to hold in their own position. And that is what we can say that you know like we have a kind of common extensions are there in both of the structure. So, if you want to calculate the free length then always it is there the extension of the steel, because some sort of the extension is there, and if you apply you know like the compression is there, then the compression of the brass which will give the difference of free length as I told you alpha b minus alpha s is there. So, if you do that kind of you know like the analysis always it will give you the real free length.

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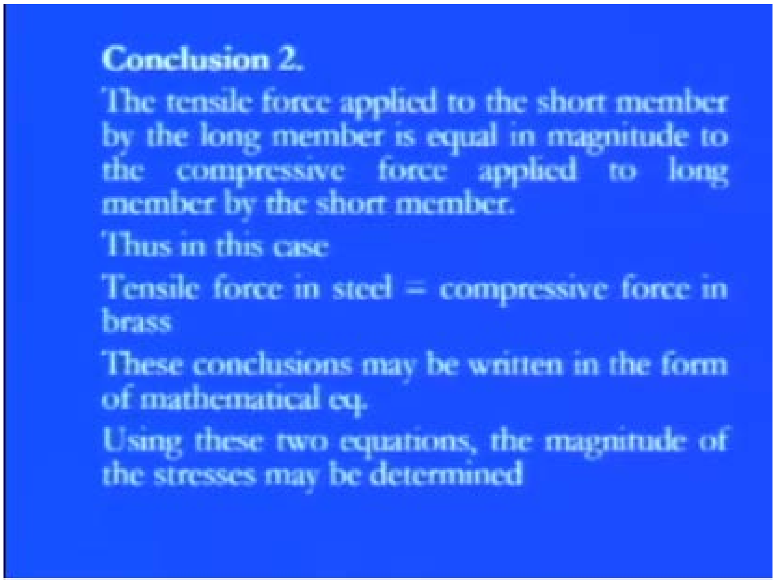

So, you see here in the second conclusion we can say that the tensile force applied to the short member by a long member is equal you know like in the magnitude to this compressive force applied to the long member by a short member. It is pretty you know like the philosophical theory is there that if the compression force is applied by a longer member to a shorter member, it is always having a equal like the magnitude as we are talking about a tensile force, force from the short member to a long member or we can say tensile force in the steel is equal to the compressive force in the bar.

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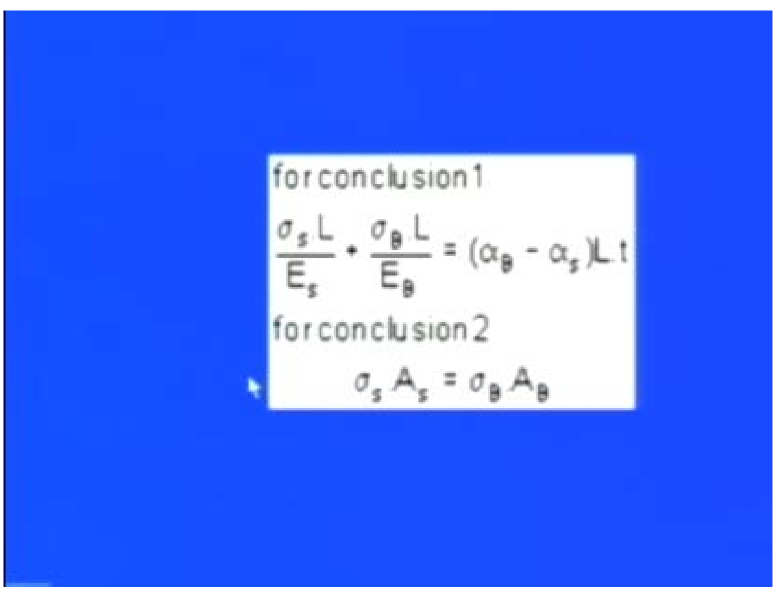

Or as you know like if you see simply go in the later on side it is this sigma s L by Es plus sigma B l by E b; this extension or the stresses in these stresses in brass and steel will give you alpha b by minus alpha s as we discussed into L into t, or we can say that the sigma s. This extension the tresses in tensile part into A s will be equal to the compression in sigma B into A B. So, in this lecture you see we discussed about the real phenomena of the thermal stresses that how thermal stresses are being set up in the structure and how they are playing a key role especially when you see this load application is there and the temperature effects are there and how we can say that if we have a compound bar then we can calculate you see you know like the stress and the strain. If a common strain is there then what this other parameters are there and how this loads are to be distributed in the common you now segments of this particular bar.

So, now you see now you know like this analysis is very, very important as you see in you know like the real application that if we have a compound bar and all the members of the compound bar are subjected by the different kinds of stresses, then how we can say you know how we can analyze those things and what are the real phenomena's with the stress and strain and with this particular Young's modulus of elasticity is there and in that also if the thermal stress are there, then how these alpha the lean expansion of the coefficient is playing a key role. So, this kind of a you know like discursion we discussed in this lecture, and in the next next lecture now we are going to analysis that actually if these temperature effects are there and if we have you know like the composite bar, then how we can go for the individual force component and how we can sum up those things. So, this kind of relations we are going to discusses in the next lecture.

Thank you.