Strength of Materials
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Lecture - 14

Hi, this is Dr S P Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to deliver my lecture fourteenth on that subject of strength of materials, and this you know like course developed under the national program on technological enhanced learning. if I just you know like refresh on the previous concept, then in the previous lectures we have discussed about the main concept the stress and strains, and you see you know like when they are all the both components they are applying on this material, then you know like how the material is reacting, and you know like what the stress components are there and where we can get the maximum and minimum stress is that you know like we discussed analytical and the graphical away.

And then you see like we discussed about the two main categories of this material as one if the ductile material is there, and we also discussed about that what the ductility is there and how ductile material is behaving when you know like we apply the tensile test. So, whenever you see you know like the tensile pulling is there, then you see you know like we define that the stress versus stain curve and there are two main reasons; one is the elastic region, and the plastic region. And also we define about if you are talking about the elastic region then what exactly the elastic constants are there like the modulus of elasticity, the shear modulus of rigidity, bulk modulus of elasticity, the Poisson ratio.

All these are you know like the main constant which are you know like useful and which can be defined in the elastic region only. And once we go you know like the nonlinear the stress versus strain relationship; that means we are going the plastic region a permanent self deformation is there, and in that also we can get the maximum you know like the strength which is known as the ultimate tensile strength before fracture.

And then we are approaching towards the fracture and if it is a ductile material like we have if we have the mild steel or high speed steel or high carbon steel or we can say aluminum or whatever you see this kind of ductile material which are you know like exhibiting a good tensile strength. Then probably we can say that actually at the end of the fracture, we have a very specified shape generally as we termed as the coco pine cone structure in which you see we have a cup; that is a dip is there, and there is a cone kind of that where you see you know like there is a extension gradually reducing portion is there.

So, this is you see you know like as well as the ductile material is concerned under the tensile test. And if we apply you know like the similar kind of tensile test in the brittle material, then we would find that you see you know like that we have you know like the linear region and between the stress and strain where you see the proportional limit is there and epsilon is valid for that, but the plastic region is very, very small. So, once you know like leave from the yield point, then immediately you know like we found that the fracture is there, and there is no specified shape as far as the brittle material is concerned under the tensile test.

So, this kind of you know like the information which we captured form the tensile test and if you are conducting the compression, you know like the compressive test on you know like the tensile on this particular ductile material or on the brittle material. Then we found in both of you see under compression if the ductile material is there or brittle material is there, the linear region is a similar as the tensile test; that means you see even if you apply the compression or even if you apply you know like the tensile, then there is no change in the linear region. And all those parameters is well applicable for both of you know like both material under all conditions.

So, that is why you see in the numerical problems if you found that, that okay this material is under consideration of the tensile test or the compression test. Then always you can apply you know like if it is going beyond you see the yield point, then we cannot apply all those constant which we discussed like the Young’s modulus or shear modulus or bulk modulus or Poisson ratio. But if it is within this range in which the elastic region range, then it would be preferable to use those elastic constant to get the value of the stress versus strains, and we can say you see we can get all those you know like the values of maximum or minimum stresses within that object.

So, this kind of you know like the relations which we set up for the ductile as well as the brittle material if it is you know under the application of tensile test or under the application of compression test. And also we would find that actually you know like the mild steel or we can say the ductile material if you want you know like capture the ductility of property, then always the elongation will be the key feature. So, how much elongation is there and always you know like we put the limit that if you want to go for ductility, then it should be starting from 10 percentage to 40 percentage of the limit is there. So, that kind of you see you know like the ductility if you want to see in any material we can use those information.

Then another point which we discussed about the elastic ray is that what will happen under the elastic action. So that is what you see there were four graphs which we discussed in between this stress versus strain. So, this again you see you know like the important information was there as far as the ductile property or the brittle property of a material is concerned. Then also you see apart from the stress strain ductile brittle, then there are some other properties which are very very useful to choose a material for engineering design. Because if you want to design the things then only the material is ductile or material is brittle is not at all sufficient, then we would go for the another property and those properties are the hardness. Then you see you know like to measure the hardness we define the hardness; that it is you know like the resistance against the penetration.

Then there are some methods like you see the brittle hardness testing machine was there, the local hardness testing machine was there, even the Vickers hardness testing machine was there. So, all three are applicable, but they are applicable in the different different regions. Because you see in the Brinell we had used you know like that steel ball was there as a penetrator and the material is you know like a specimen of any kind of you know like either the steel or the less harder material as compared to the indenter. But there were some you know like disadvantages were there.

So, probably you know like it is not acceptable that because of any truncation error or because of any you know like the geometrical deviation of indenter if you are not getting exact value of the hardness, then it is not preferable. Then always we are using the Vickers hardness in which the diamond point is there, but again you know like the biggest disadvantage in that was you know like diamond is there. And we just want to avoid the damage of the diamond because it is very expensive and even it is not safe to keep all those diamond points you know like for this kind of application to check the hardness.

So, you see preferable one is you know like the Rockwell hardness is there in which you know like the conical shape of indenter is there. And you see since it is conical shapes of 120 degree of you know like the shapes are there. So, it is preferable to use in industry, and since you see we are only measuring the depth of penetration and based on that actually we can calculate the hardness. So, it is preferable to use you know like this Rockwell testing machine to check the hardness. So, these three kinds of you know like that hardness testing machine was there, and then we discussed about the toughness.

Toughness is nothing but you know like if you just refresh those things, then it is nothing but whatever the cracks are there in any material where the strains concentrations are there. So, it is the property of a material through which material the you know like the strength against the impact loading. So, whatever the energy which material can absorb against the impact loading, that is nothing but the toughness. So, it is always you see preferable to get the value of the toughness if we are using the Izod Charpy test or even the Izod test is there.

Then there was another you know like the property was there that is the creep. Creep is nothing but you see when you apply the load and if it is a constant load, then again you see because of the surrounding temperature or the high temperature region like you see if you are using turbine blade or nuclear reactor or this kind of you know like this generator is there or any turbine part is there in which the steams are there, boilers are there. So, wherever if you are talking about the material which is under the application of high temperature region, then even the load application is same or the stress formation is similar. But due to the temperature variation, the thermal stresses are being set up in the material are so high that even you know like no load condition, but it will start from the strain hardening, and it will end up with the fracture of the material.

So, this kind of failure always comes under the creep phenomena and even actually we have shown you know like the previous lecture that actually there is a standard creep curve is there which is you know like drawn in between the strain versus time. So, how you see you know time place an key role in that, and there how we can display all three regions of the creep, three types of the creep like first creep, second creep and third creep. And how they are you know like behaving; we simply want to calculate you know like any strain or strain hardening or you see within that actually the stresses, then how the tangent and how we can draw the tangent, and how we can get those value. This part in particular we have discussed in the previous lecture.

So, you see you know like if you found that, then we had enough information now to analyze the problem. So, now we would like to go for you see if a material which is not the idealistic way; that means you see if you have material which is under the application of tensile load, and you see more than one load is there or even you see you know like in intermediate part o any specimen if different kind of loadings are there then how we can analyze those parts specially.

So, that kind of you know like the information which we would like to discuss in this lecture.

But again you see the key part is that actually first which material is there and what are you know like the application of the load is there within those materials and corresponding stresses and strains are coming.

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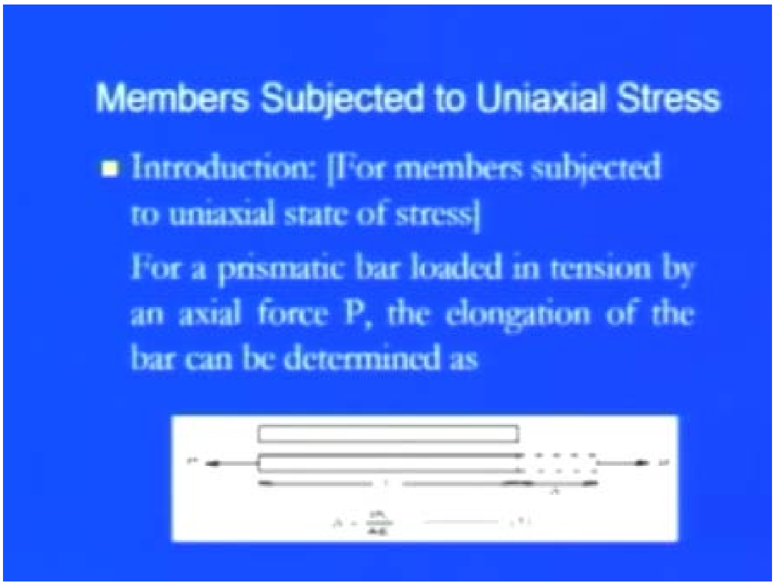

So, if you see that actually there are some members which are subjected by the uniaxial stresses we would like to see that actually how the stresses are being formed if more than one load is applied on a particular metal, and then how the stress and the strains are there at the different locations. So, here you see you know like we have the members which are subjected by the uniaxial state of stress and for prismatic bar as we can see in this particular diagram, this particular we have a prismatic bar and at the two extreme corners of the bar, the tensions are there; that means the tensile axial forces are there towards the outward direction. And due to this load application we have an extension, and this extension can be measured by you see the delta.

So, this is the delta 1; this is you see the total. I should say the gauge length is there, and whatever the elongations which are coming it can be easily computed by the stress versus strain. So, you see we have the delta which is nothing but equals to the PL by AE. P is the applied load is there, L is the total length divided by the, A is the area of the prismatic bar, and E is the Young's modulus of elasticity. So, whatever the load application is there, we are going up to you know like the elastic deformation we apply the Hooke's law. With the using of Hooke's law you see you know like just put the relationship, the sigma is the stress is equals to E into strain; that is epsilon. Put the sigma value F by A equals to E which is the Young's modulus of elasticity into this epsilon which is the strain which is nothing but equals to the delta by capital L. So, you can compute this delta L which is nothing but the deformation due to this application of load axial, okay. So, this is there.

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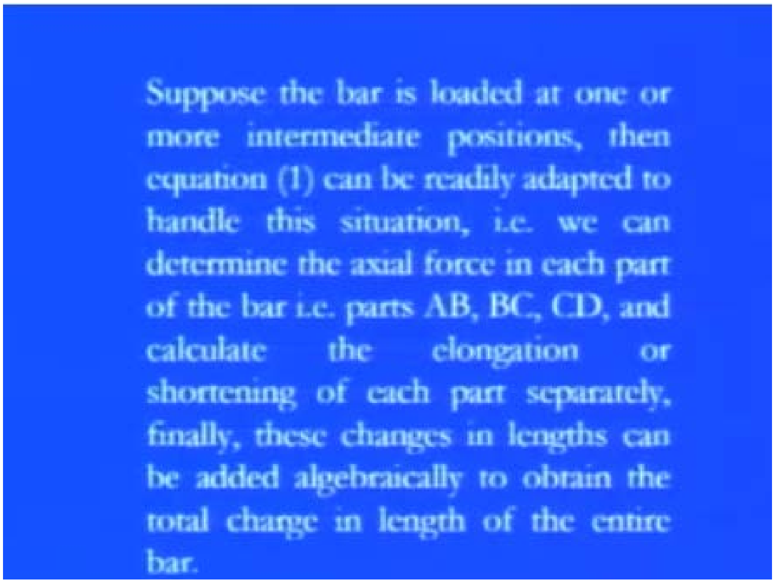

Now suppose you see you know like if this bar is at one or more intermediate position; that means you see if we have a bar which has a different segments, then you see this equation can be easily adopted to you know like just to handle this particular kind of situation. It means you know like if we have as I will show the figure that actually if we have you know like the kind of total bar in which there are the parts we can say AB, BC, CD and you see you know like the another some elongations are there; that means you see what we are doing here we have a uniform bar as you see in the previous figure that was the uniform cross sectional bar, no separation is there. But if we have a bar in which there are some segments, and in the segments you see there is a close connection is there and if you are applying the load.

Then whatever the elongation or you know like the shortening if you are applying you know like the extension or if you are applying the compression, then we need to take the elongation as well as the compression separately, and then you need to you know like simply if I am saying that in the AB part if we have a delta 1, in BC part if we have a delta 2, in CD part if I have a delta 3. So, what I need to? I need to calculate the deformation separately in separate segments, then some delta 1, delta 2 and delta 3 together, the total deformation in terms of elongation or shortening of the total deformation of bar.

So, you see here the meaning is pretty simple that actually it has the same algebraic part as you see you know like we have if we have a uniform bar. So, we can either segregate those part in terms of the uniform deformation or we can simple sum up you know like what is the total load is there at the end and how we can sum up those things and how we can get the total deformation out of the entire length of this prismatic bar. So, you see here we have you know like the kind of as we discussed that whatever the prismatic bar is there the total segments are coming in terms of DL.

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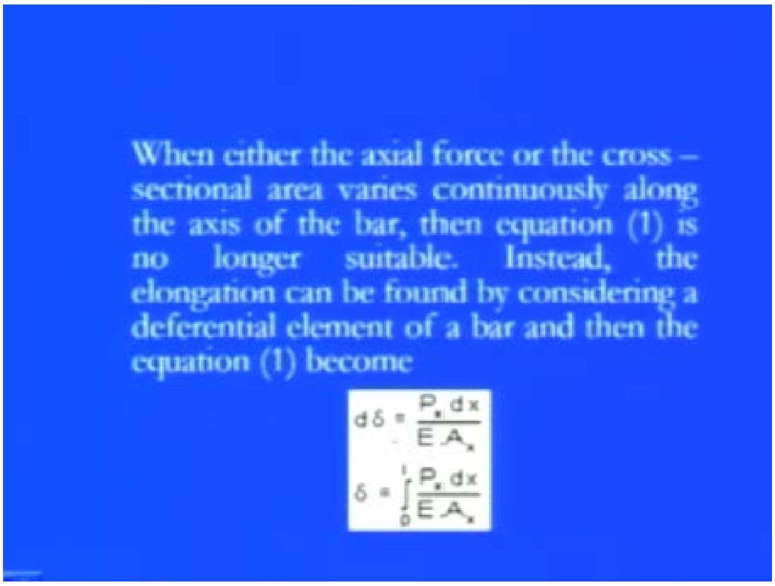

So, we will see here the d del which is nothing but equals to P x dx divided by E times A x. Meaning is pretty simple that if we have a small segment even if it is the tensile loading is there, it can be easily you know like the load applications are there in the axial part because it is a uniaxial loading is there. So, P x is there, and then you see whatever the small segment is there that is the dx segment in terms of the area A x is there in terms of Young's modulus we have E.

So, if you want compute for entire region that means you see if I want to just get that what is the total deformation is there for whole prismatic bar is integration is the perfect method. So, delta is nothing but equals to 0 to l which is the total length starting from 0 to entire l equals to P x into dx divided by E into A x.

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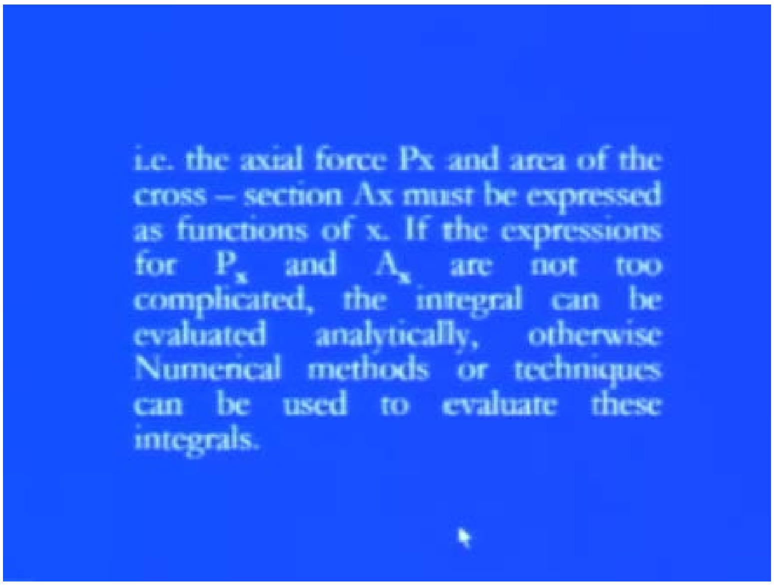

So, meaning is pretty simple that actually whatever the load is there axial force is there and whatever the cross sectional area is there under the area of concern means whatever the matter is there in that and the load application is there what we need to do we need to go for individual you know like the load application, and then we need to sum up those things. So, if the expression for you know like P x or A x are not too complicated, then the integral can be you know like found out and through integral we can get the analytical solution or else you see you know like some of the numerical techniques are there just to you know like if we have any complicated part is there.

So, what we can do here either we can apply the Runge-Kutta technique or even if we can apply the numerical, because there are tons of numerical techniques are there through which we can get exact you know like the deformation or we can say we can get the exact stress and the strains at a particular location. So, these are the key you know like the features in that which we would like to you know like maintain for a uniform prismatic bar. But now here you see if we do not have the uniform prismatic bar; means you see we have a bar which does not have the uniform cross section a non-uniform cross section is there. Then how we can get you know like the stresses and the strain at the different section that we would like to discuss.

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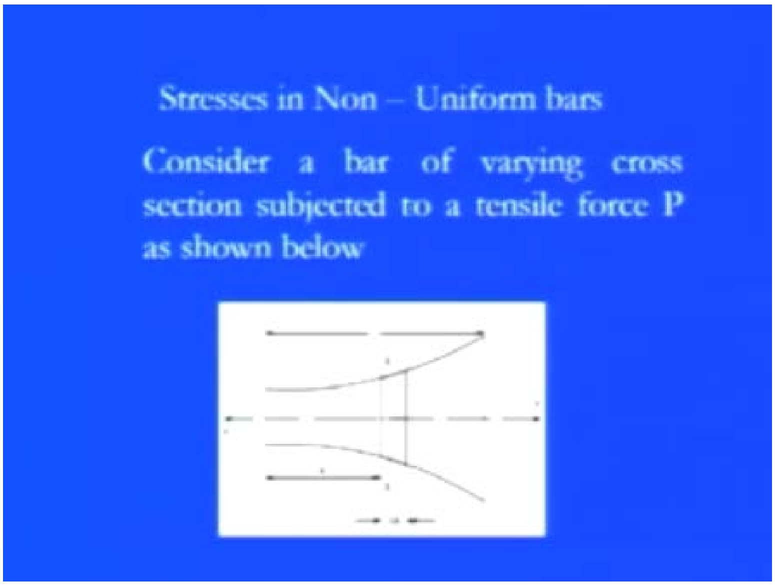

So, now you see here we are considering a non-uniform bar, okay, which has a cross section as shown in this figure, and the tensile load is applying at the extreme end. So, you see here what we have. We have you know like this prismatic bar, which has a non-uniform you know like the cross section. So, the cross section at this; this is the tapered section the smallest area is there and as you move further it has a symmetric extension towards the x direction. So, we are ending up to this part. So, total length is L again similar, and if I am just cutting a section particular at let us say x distance from the left end.

So, from this x distance if I have this section which has the total you know like the width is dx, and if I am just simply making the section of that and I just want to find it out that what exactly the stress or what exactly the deformation is there under that you know application of this load. Then what I need to do because here this area is not at all you know like the constant throughout this extension. So, here you see this length is you know like small and this length is bigger height. So, what I need to do here? I need to maintain you know like because the stress distribution is also not exactly similar as we have seen in the previous uniform prismatic bar.

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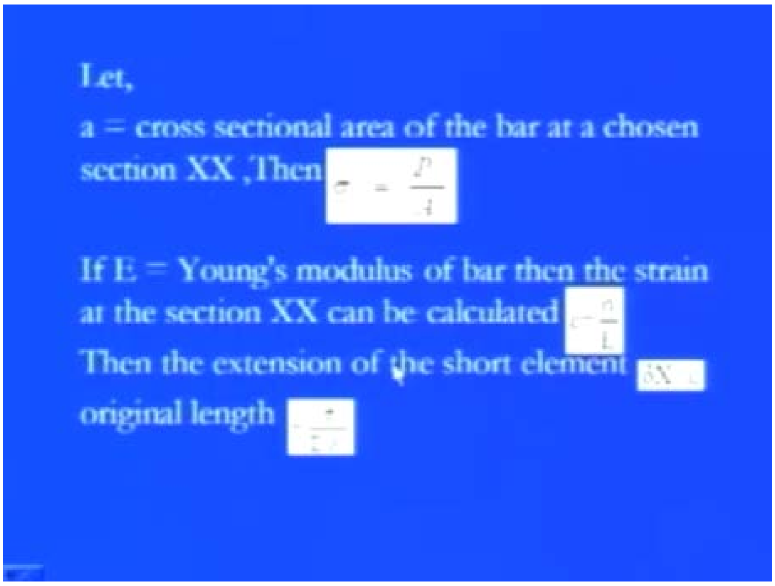

So, here I need to apply some other technique here to get those values. What is here a is which we have discussed here nothing but the cross sectional area of bar chosen at XX sections. So, sigma is nothing but equals to P by A where here A which we need to calculate separately for the different shape of this prismatic bar. So, here E is nothing but as usual you see the Young's modulus of elasticity and since we are applying the load within the elastic region. So, obviously this E is well defined for those you know like section, and if I just cut the section at XX it can be easily calculated with using of the stress calculated in the strain measure. So, strain is nothing but equals to sigma by E. So, it can be easily coming through this particular formula.

Then you see whatever the extensions are there in that due to the application of load, it can be also be measured because if you know the strain you can get the strain is nothing but equals to the change of length divided by the original length. So, what kind of deformation is there under axial load. So, this formation into X the del X is nothing but equals to this epsilon, because epsilon will give you the real feeling about the strain that what exactly the distortion of the deformation is going to on in that particular prismatic bar.

And then you see you know like if you know the original length then you can easily get that sigma divided by E into epsilon or whatever the undefined parameter is there you can easily put those things and get the another value. Meaning is very simple; only the change is there, the effective area under which this load application is there. Once you know like the area, you can easily get the sigma. Once you have the sigma you can get the value of E if you have the v E value if you have the sigma value. E is nothing but the property of the material. So, you can easily get from the any of the standard graph and then you can calculate the strain.

Once you have the strain you can get that the deformation that how much deformation is there; once you have the deformation you can easily correlate that actually what exactly you know like the distortions are there under the application of the load. So, this kind of analysis can be easily made once you have all those parameters with you.

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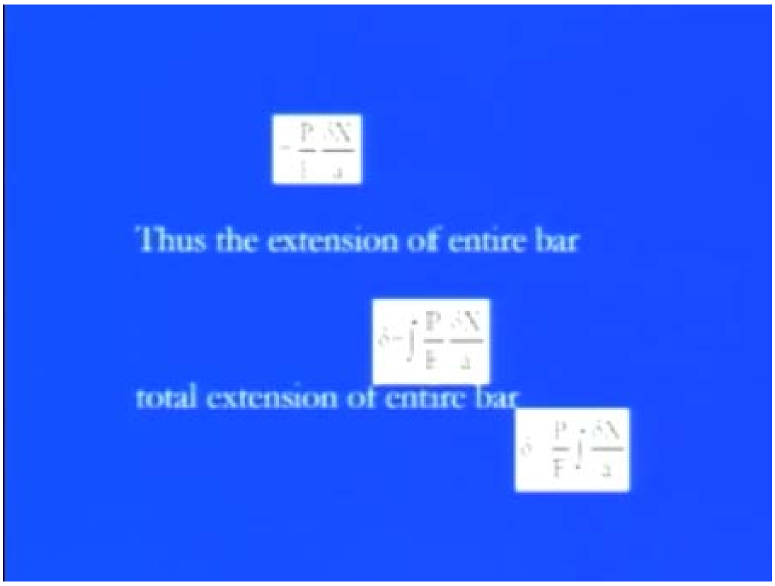

And then you see once you apply the original length then again it is nothing but equals to P by l into delta X by a. And if you want to get the extension you see the total bar, then nothing but what you need to do just the small small as we assume that actually this total bar is the summation of all the small small segments. So, again delta is nothing but equals to integration, because integration is a perfect way to get the final exact solution using this analytical method. So, again we are trying to applying you know like these kind of technique to get the final solution. So, delta is nothing but equals to integration 0 to l P by E into del X by a.

So, if I want to get the extension of the entire bar, it is pretty easy and it is equals to P by E because P is load, E is the modulus of elasticity, and they are constant in nature. So, they will come out and integration because what the changing is there change in only the delta X. So, delta X delta X delta X. So, all those small small segments they have the different width is there because it is a non-uniform bar is there. So, what we need to do here. We need to calculate integration 0 to l delta X by a. So, in corresponding you see you know like the deformation will come of the whole hole.

So, instead of doing you know like the small small portion and get you see what the delta 1 plus is there, delta 2 plus is there, what we can do here we can straightaway apply the this integration method. So, either direct method or integration method both are applicable if you see you know like the shape is not too complex it is pretty easily to apply the integration. But if shape is complex, then you see we need to divide the shape into the proper regular you know like the shapes and then you see calculate the different different deformation for these value specified regions or we can say the well defined region and then sum up those things, and that is known as the summation method or the direct method. So, both methods are applicable depends on what kind of shape is there and what kind of load applications are there. So, now you see come to the main point again.

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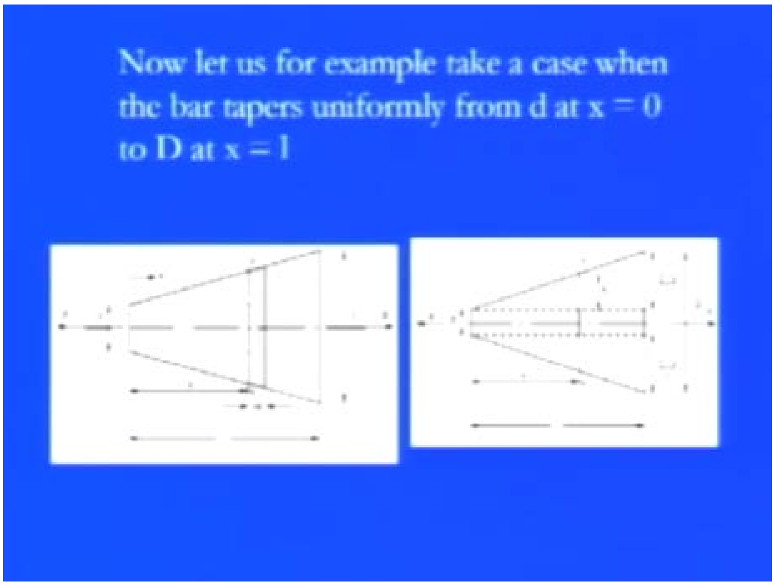

Then let us you see the example is there when you like the bar is tapered as usual you see this some tapered bars are there uniformly from the d at x equals to 0 and capital D at x equals to 1;It means you see at initial point we have you know like the tapered bar that means the small diameter is there that is the small d and at extreme end; that means at x equals to l we have the diameter is capital D. So, you see the total length is L as it is showing and then again we will go the previous section that actually in towards the x direction, because the load is applied along the x axis.

So, what we are doing here? We are just cutting the x section, okay, along you see you know like the x axis. So, what it is there at the x axis right from the D this D or we can say x equals to zero, we just cut the section which has the width is dx and it is you see the total length right from this is this you see as we can simply shown by this shaded area. So, what it is there you see if you just you know like trace those points then you will find that this area in this particular figure the curser is showing this is the diameter small d. So, as it is you know like cutting those things and probably you see we will end up here is D by is this capital D which is this final diameter. So, capital D minus small d by 2; so this is this portion. This portion is again you see we have the distances capital D minus small d by 2.

So, once you have these two portions this is the standard right angle triangle. So, you can get you see in the required information, because this distance is nothing but the L, this distance is D minus d by 2, and this is the ninety degree, okay. So, you can get you see with the using of the trigonometric relations one can easily get the required extension. So, now you see as you apply the load what will happen the extension will be you know like there and extension will be there along the x axis. So, here you see this at these particular directions you will get the extensions, and if I am saying that this is the k distance; k is nothing but you see you know like here I am cutting this plane whatever you see this particular you know like the structure is there. If I am cutting from these things then this distance from the centre region you see.

If I am saying that the k then what the impact of the k is because if I am cutting from here if I am cutting from the here the k will vary. So, what I need to do here instead of you know like cutting this sec cutting this section at its small small segment, what I am doing? Simply taking the uniform you know like region you know like just take a small segment, find out what the deformation is there, apply the integration right from x equals to 0 to x equal to a and you see here at x equals to 0, what we have? We have the small diameter d at x equals to l, we have the capital diameter this D, so all those. Either a small diameter or bigger diameter both information which we have at x equals to 0 and x equals to l. And also what we have? We have two triangles right angle triangles at these two corners. So, with the using of this information we can easily extract the total deformation by integration technique.

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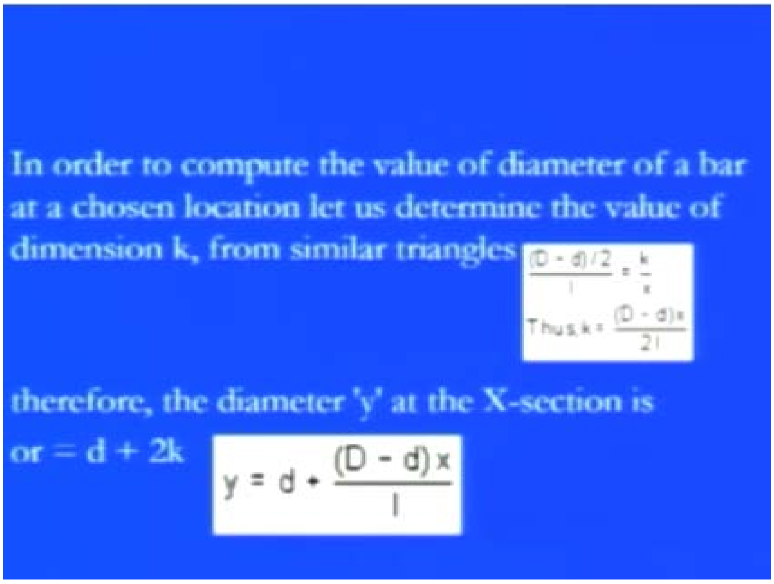

So, you see here. So, in order to compute you know like the value of the diameter of a bar or any other undefined parameter what we need to do here, simply you know like using those similar triangles as I shown you. So, with those diagrams D minus d by 2 was the base of the diagram which was pretty same, length of you know like the triangle was again similar, so it was l. So, D minus d by 2 divided by l it should be equals to k by x or we can say that actually k is nothing but k was as I told you the distance was there from the central distance to this particular tapered bar at a particular distance which we have taken the x region. So, we can say that the k is nothing but equals to D minus d divided by 2 l into x.

So, now you see what you have? You have a distance, which is known as the intermediate distance, which is you see you know like as you move further this distance will vary accordingly, and how it will vary? It will vary by D minus d you see here. If you see the formula it will vary that how much difference, what is the exact difference is there in between the major diameter and minor diameter and also it will vary with the x; as you move further with the x it will vary accordingly. So, you know like the diameter y whatever you see it is there in the x section we can get the d plus 2 k or we can say y which is we are saying that the diameter is nothing but equals to d plus 2 times of you see. So, two two will cancel out. So, d plus D minus d into x divided by l. So, now you see what we have. We have a diameter in terms of this 2 is small diameter. So, the total we can say the average diameter is nothing but equals to d plus D minus d into x divided by l.

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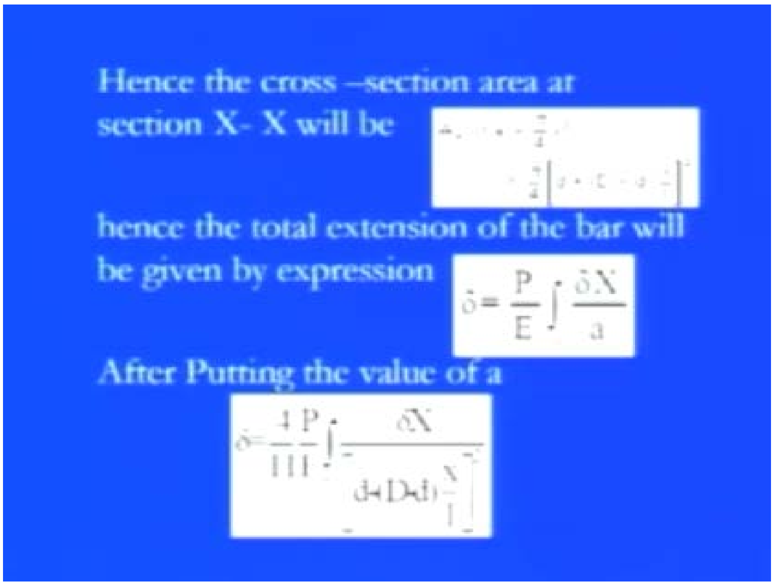

So, you see as we move further then we will find that since we are talking about the x you know like XX cross section. So, for that the area is nothing but equals to pi by 4 y square and you see since we know that what the y is. So, we can simply say that it is nothing but equals to pi by 4 d plus D minus d into x by l whole square. So, once you sum up those things. So, now what we have? We have the effective area that exactly which is you know affected by the application of load and how the deformation will take place; now we will go for that. So, the total extension of the bar will be given by delta which is P by a as we have seen in the previous derivation that P by E into integration of zero to l d del X by a.

So, now you see if you keep those values there then probably we have you know like delta is nothing but equals to 4 P divided by pi E because the area is there pi by 4 square. So, this 4 will go on top of that this E value, okay. So, 4 P by this pi E into 0 to l because at X equals to 0 we have a small diameter at X equals to 8 at we have bigger diameter. So, integration of 0 to l into del x divided by this whole you know like this this area; what are the area will come, so it is we have this small d plus D minus dx by l whole square. So, now what we have? We have the total deformation in the bar if you apply the load in towards the x direction.

So, instead of going for the small, small segments and instead of the calculating the different deformation and then sum up those things, it is better you know like once you have a symmetric geometry, cut the x section, take portion. In the portion just see that how the variations is going on; once you have the variations within that structure integrate those things at from beginning to end and then get the value of the total extension and that is what it is you see here.

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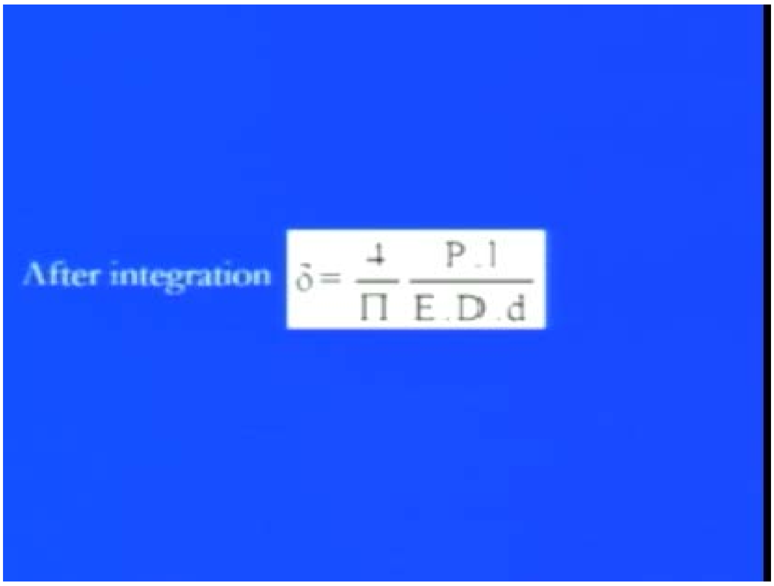

So, now you know like once you have you know like after the integration what we have because we know the values at both the points 0 and l. So, we can simply get the final value of the deformation that is 4 by pi into P into l divided by E is Young’s modulus of elasticity, D is the capital diameter of measure and d is the small diameter. You know like the thing is that whatever the information is given in the problem like you see the load is given to you, the deformation is given to you or Young's modulus is always there as per the material selected, and if any of the diameter is given to you then you can calculate the remaining parameters that exactly.

So, this information is very, very you know like fruitful to design any of you know like the material for the application when the tensile loading is there, because we have the total load. We also have you see the specified shape of this any material. Once you have those things once you have the material, then you can calculate that actually if you apply this amount of load then probably you will end up this kind of deformation and then you see you can safely design those things. So, an interesting problem is to determine the shape of a bar which would have a uniform stress in it and under the action of its own weight and the load P.

So, now you see in the next segment what we are going to discuss; till now we didn't select what the weight effect is there. So, now if we go further then it would be very interested to see that actually if a weight is there and due to its own weight what kind of extensions are there and with those extensions how the stress formations are there with the interaction effect of an external applied load. Because you see what will happen? We have the segment due to its own weight you see it is going towards the centre of gravity is always towards the downward direction. So, the stress formations are there, load application is there at the extreme ends. So, because of that stress formations are there, then what the interaction effect is there of these two segments we would like to see and that is you see you know like in the next figure we would like to see.

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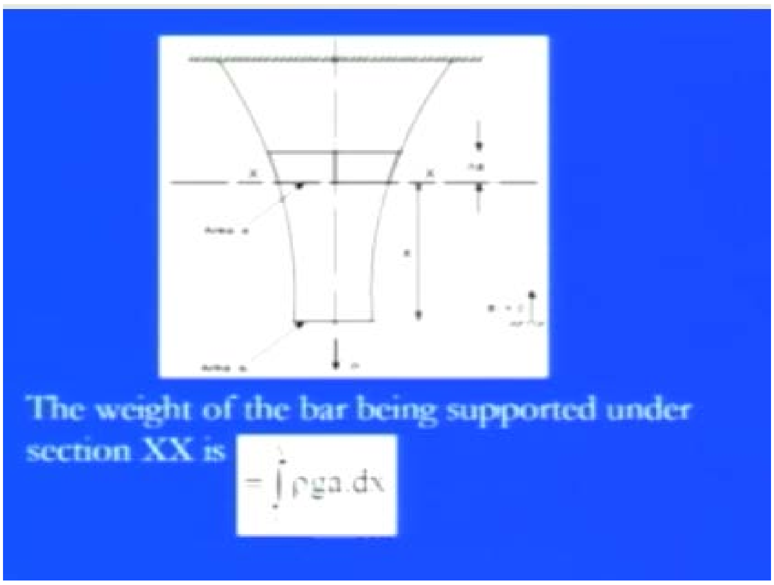

So, this figure clearly shows uniformly you know like the tapered bar which is hanged vertically. So, now we would like to consider the weight of that. So, in that what we are going to do this you know like this is hanging at this particular position, this is you see the symmetric geometry is there and at the extreme corner you see this load is applied. So, once you apply the load you see we have two different components. One we are considering here the weight and due to the weight you see you know like this area is also affected; that means you see the centre of gravity as it somewhere it this particular location is there where this see these two the central lines are you know like crossing. And now at this particular section we would like to see that what the impact of the weight as well as the external applied load is there.

So, this form can be easily continued by taking again the similar kind of analysis that just cut this particular section by the XX. So, now we have this particular effective area and we would like to say that how this effective area is being distorted or the deformed shape will come due to the application of load. So, what we are doing here? Again we are taking the same delta X as the unit width of those things and it is occurring at right from this particular corner which is a tapered corner is there. So, if you are taking this X distance right from this we have the delta X and those things this area. This is the effective area, which is shown here, and this is the effective area where this particular load application is there

So, these two areas are clearly showing that at one point we have the self weight at one point the external applied load is there. So, now if you are considering the weight of the bar which is you know supported under the section of this XX here clearly you know like this arrow is showing here. It is nothing but equals to integration of 0 to X, because what we are taking here? We are taking the small segment of that. So, now we would like to see that actually how these you know like the variation is there of the stress or any kind of you know like the extension within this particular structure 0 to X.

So, 0 to X you know like the integration of 0 to X into the density because now we are considering the weights density into the gravitation aspiration because the weight is acted towards the downward direction due to the gravitation aspiration into the area into dx. So, now you see what we have? We have now the weight, and due to this weight we have an extension. So, we would like to see the impact of that. So, this rho is the density as I told you.

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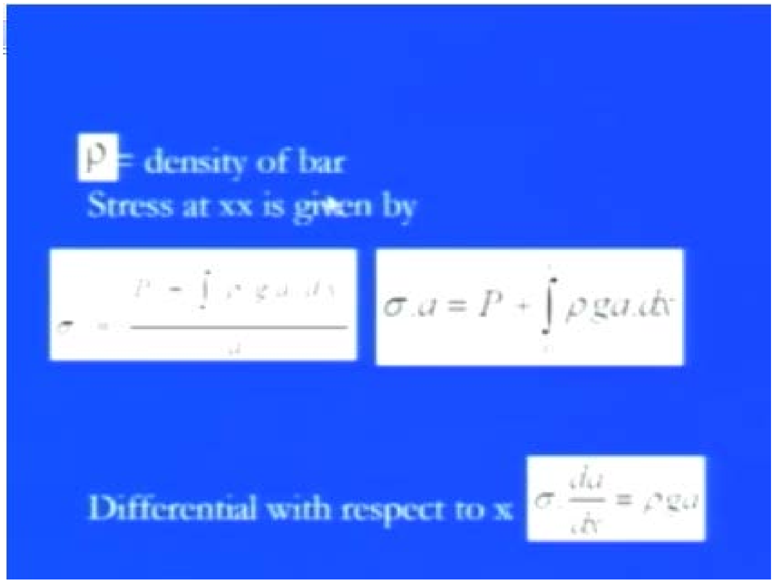

So, now we can see that what the stresses are there due to this. The stresses are nothing but equals to P, which is the external applied load. It has a uniform you know like the magnitude is there plus the weight. So, weight is nothing but as we have discussed in the previous part that actually the rho into g into this the area, because you see know if you are considering the mass; mass is nothing but equals to density upon volume. So, whatever you see from those components we can easily computed that what exactly the weight is there. So, rho g a into dx integral 0 to a divided by whole area. So, now this is the total load external applied plus weight divided by area will give the stress, or we can say that if you multiply those things by the manipulation what we have the stress due to the combine you know like the applied load and the applied weight, sigma into a is equals to the applied load P plus integral 0 to x sigma g a into dx.

So, now you see you know like if you differentiate those parts you know like by taking dx into 0 to x. So, what we have? We have sigma into da by dx by differentiation. So, once you apply the differentiation, this will gone because it is a constant value; there is no variation as such with the X distance unlike towards the vertical direction. Then what we have? We have sigma g a, okay, because dx will be integrated. So, now we have an equation which will clearly show that actually how the area is varying because you see da by dx is nothing but the variation of area in the X domain. So, you know like area intersection is there all across the body due to you know like those load application we would like to see. So, sigma into da by dx always gives you this rho into g into a.

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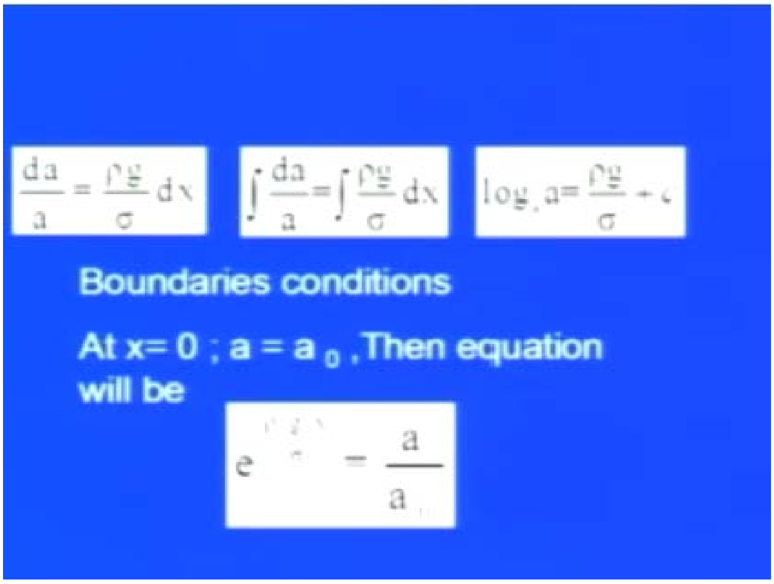

So, now you see like with the same previous equation what we have, we have da by a. the change of area divided by original area which is equals to this rho, which is the density of bar whatever the material which you are preferring; you know like here. Since it is a extension is there, tensile load is there so always we prefer to use the mild steel component or any ductile material. So, tau is nothing but equals to you know like it will come in the denominator side, so sigma is there you see so what we have. We have rho into g divided by sigma into dx from the previous figure which will give you da by a or if I now integrate those that you see for a small small segment which we have taken for a small XX segment or other segments also.

So, if you integrate those things what we have integration da by a is equals to you know like again density into gravitation aspiration divided by sigma which has a constant value into integration of dx. So, now if we are doing those things it is pretty simple that actually logarithmic is there. So, log of a to the base a to the base e is equals to density into gravitational aspiration divided by sigma which has a constant value plus c. So, if you put the condition as usual like that if you have you know like at x equals to zero at the bottom you see where the load application is there, we have a equals to a 0. So, what we have? We have the equation a by a 0 is nothing but equal to e to the power density gravitational aspiration into whatever the total distance which we have taken you see if you see the previous part x divided by sigma. So, now you see you know like the original area or the effective area now we have both the area and the ratio will give you the exponential component.

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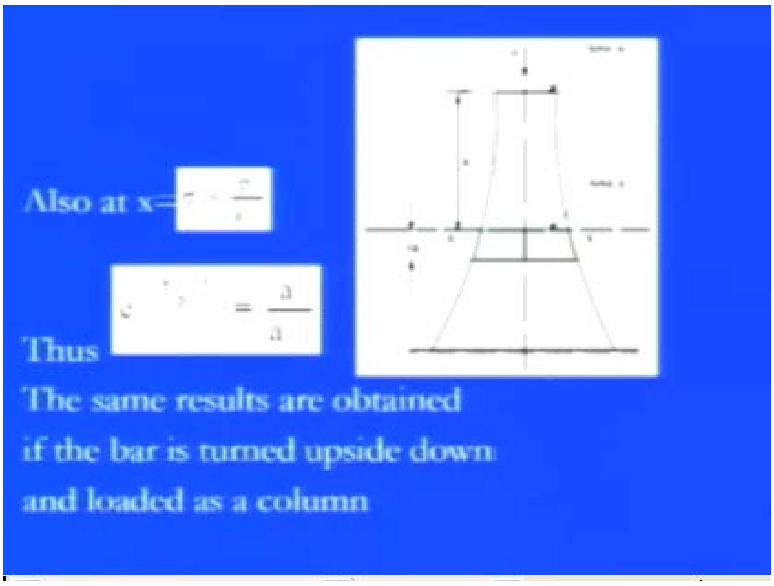

So, now again if I just reverse that component that you see you know like just hanging at the below and it is simple you know like when towards the top then it is all just reciprocal parts are there, the load application is compressive now. In the previous case it was tensile, compressive load is there and we are considering at a this area is a 0 as I discussed and this area which is you see you know like at the self weight this area is effective area and the a by a 0 we have discussed that actually. It was nothing but equals to exponentially this density into you see the gravitational aspiration into x divided by P.

So, now if I have this region that if it is just you know like reverse this phenomena then I have the compressive load and due to that you see you know like we can say that the sigma is nothing but equals to P by a 0. Because the a 0 is the effective you know like the load application is there. So, since this is the effective area under the compression load so I need to take the compressive stresses at this particular stress is nothing but equals to this P by a 0 while if I am going for the XX section then we have you see this ratio of the area a by a 0 which will be nothing but equals to. In this case the exponential terms e to the power this density of the material whatever the material which we have taken here gravitational aspiration which is going towards the downward direction here into you see the x this distance, which we have taken the x from the applied load to this centre, and you know like the a 0, which is you see this effective area at that particular end divided by the load.

So, you see the same results even if you are getting you know like if I am just doing like putting the downward direction. If I am putting you know like the upward direction, there is no change, only the load application will change here and that term you see what we have the nature of the self weight and the applied load will be in the extension part or we can say the tensile loading is there and in this case you see we have the compression loading. So, it is pretty easy to calculate both in respective of only plus sign will come in the tensile, minus sign will come in the compression of load. So, you see here now we would like to calculate some of the numerical value so that actually this refresh you know like the basics of the formula and how to apply those formulas into the real problem.

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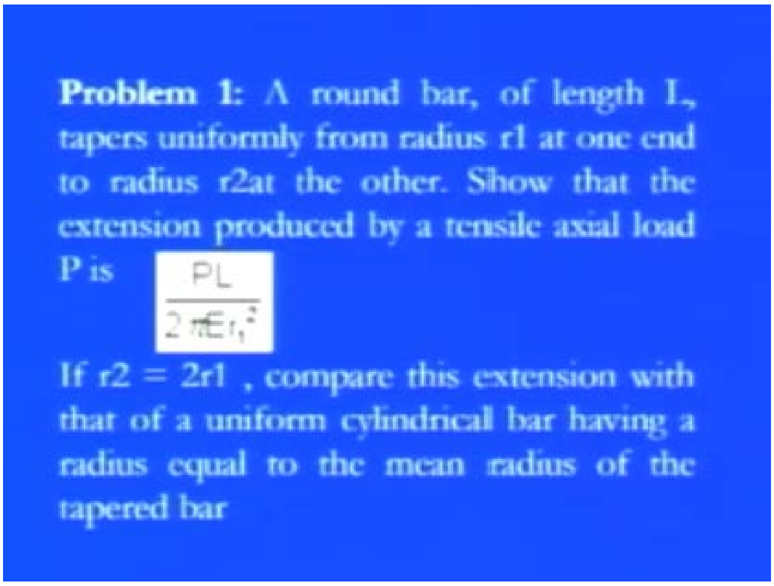

So, first of all the statement of the problems says that we have a round bar of length this you see and it is tapered uniformly from you see the radius r 1 at 1 end to the radius r 2; that means you see you know like we have the two different radius as usual, because it is uniform you know like extensive bar has a taper shape at these two extreme corners. So, now you show that the extension of you know like due to the tensile load protection, we have you see some sort of extension, and we would like to find out that this extension is nothing but equals to P L divided by 2 pi E r square where you see nothing you know like this r 2 is equals to two times of r 1 and you know like means this final extension r 2 is twice of the previous r 1.

So, you see what we have? We have nothing but you know like at small segment this is bigger segment and this you see if I connect those things doubled of those things we have this kind of uniformly tapered bar and we are applying the extension. So, we would like to see that actually how this extensions, okay, will take place and what the magnitude of this extension is.

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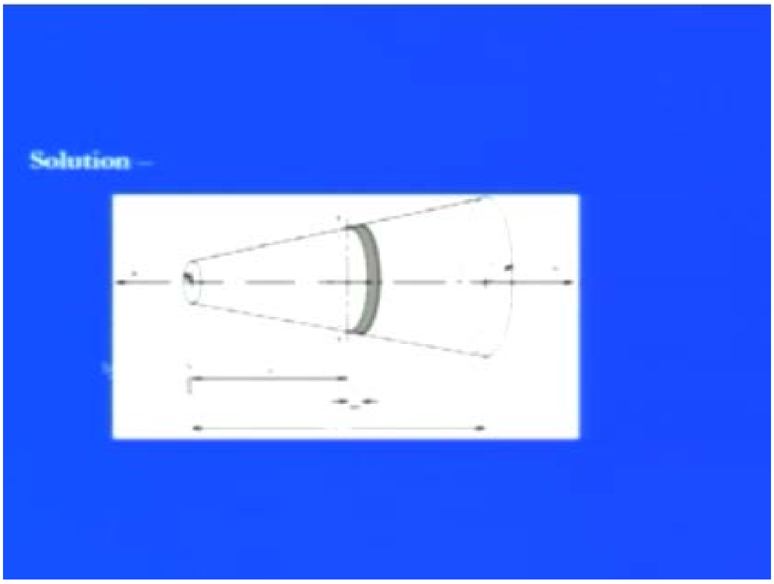

So, now you see you know like same you know like if you trace this problem in the real figure then you would be having this the small r as I shown you; this is the bigger R which has doubled of this R. Now the same this extension is going on right from this particular you know like the central line. So, as you are simply extending these parts then probably you will end up with you know like kind of extension. So, if I am taking that at x equals to 0 this part is there like this base is there. I just you know like I need to take the x section which has you know like the uniform this thickness is there that is the dx. So, this dx will be calculated here.

So, I have a uniform you know like this sort of thickness is there all across the x axis. So, this dx is there, and now what I am doing here? I am taking you see the distance right from here to here as the total length is nothing but the L is there. Now I would like to see that actually what exactly the relation is because this is if I am saying that this r 1 is there and this is my r 2 is there, then the relation of r 2 and r 1 is nothing but equals to it is the doubled of the small radius.

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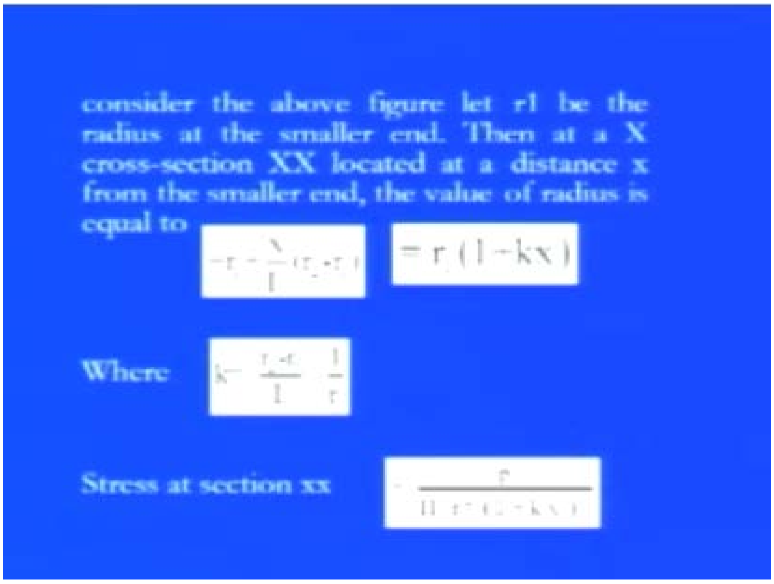

So, now you see you know like if I just apply concept here then what I am having just you see you know like considering those figures I have r 1 you know like will be the radius of the smaller end and r 2 will be the radius of the bigger end; it is doubled of those things. So, now just taking the linear relationship because it is you know like uniformly distributed all across the entire length of this particular uniform bar. So, if I am just taking those things then I have the radius at this smaller end is nothing but equals to you know like this r 1 first smaller end r 1 plus this you see the intermediate position which is nothing but equals to x by l; x is the distance, l is the total distance of that.

So, x y l is the non uniform you know like this non dimensional parameter is there into what the difference of these 2 radiuses. So, r 1 plus x by l because x is the distance of that small segment l is the total. So, x by l into r 2 minus r 1 or I can say that if I am taking that this k is nothing but the constant which is equals to r 2 this r 2 minus r 1 divided by l into 1 upon r 1. Then I am having this r 1 into 1 plus kx. So, now you see if I am keeping those things and if you want to find it out that what will be you know like the stresses are there, then it is pretty simple that I have the radius the intermediate radius where you see I am considering the delta x as a portion, so P by pi by 4 a square.

So again you see if I am going for this that what the diameter is there, then it is easy or if I am going for the radius then it is pi r square. So, by considering those things I can easily calculate the radiuses like these things that if I have uniform section there, then what the stress distribution is there, and what the extensions are there. So, stresses are nothing but equals to since it is a tensile stress. So, it is nothing but equals to the load divided by pi into r square which we have calculated r square into 1 plus kx square.

So, you see this will give you a clear-cut picture that actually if we have the two different radiuses and if we are not considering the weight, then how you all different radiuses are giving the impact and also with that actually where we are considering the x. So, right now you see you know the stresses are nothing this is not a total stresses; this is just the stresses are there at the XX component.

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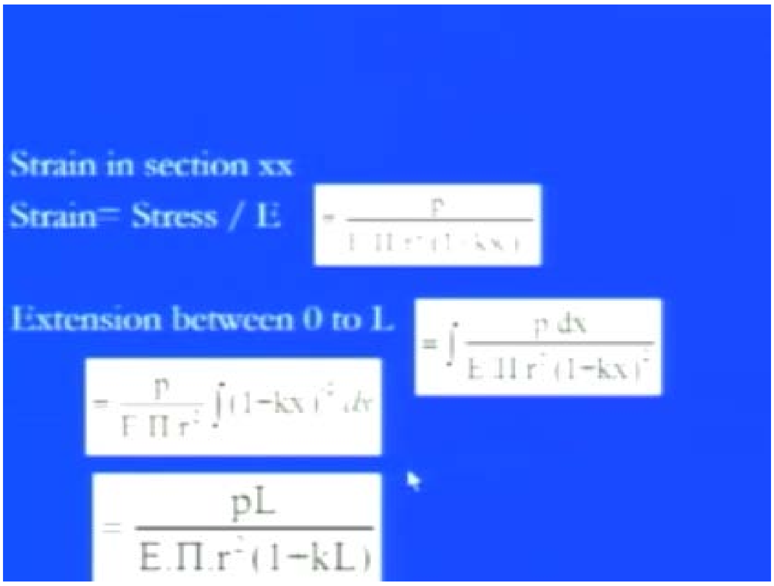

So, now if I you know like sum up those things for the entire region of the bar which has a tapered bar then you see you know like we can again go further that what the strains are there once you have the stress. So, strains are nothing but equals to stress divided by E. So, you see here p divided by E into the entire area or we can say you know like if you want to calculate that deformation for that now you have the strain. So, probably you can if you sum up those things for the entire region of this particular uniform bar, then it is nothing but equals to zero to l because you see started from 0 and we are going up to the end, intermediate position was the x.

So, 0 to l into p times of dx divided by E pi r 1 square into 1 plus kx whole square or we can say you know like if we integrate a simple method then you simply take because this is the multiple of the x and we are you know like integrating with a dx. So, just take it on the top of that so probably you know like we have P by E pi r 1 square into 0 to l 1 plus kx to the power minus 2 into dx or you know like if we have minus 2 then minus 2 plus 1 divided by minus 2 plus 1. So, we will be having you see you know like P into L divided by E times pi r 1 square 1 plus kL because again you see now we need to put the this X in terms of the L. So you see here minus sign minus sign will be cancel out, so we have this much you know like this extension.

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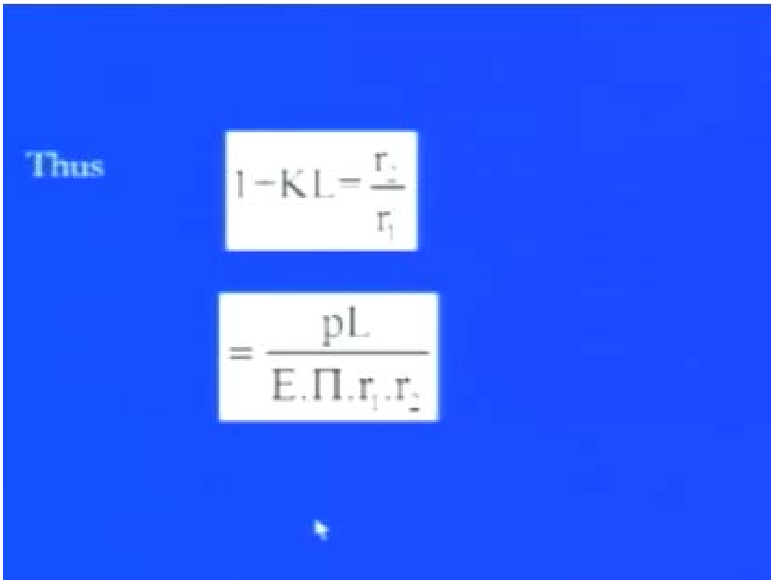

So, you see now go back to the relation then what we have? We have 1 plus KL equals to r 2 by r 1 or what we have? We have the total extension if you are keeping those things. Then it is nothing but equals to pL divided by E you know like E into pi times r 1 r 2. So, that is what you see you know like we would like to just put those conditions and after putting those conditions you see.

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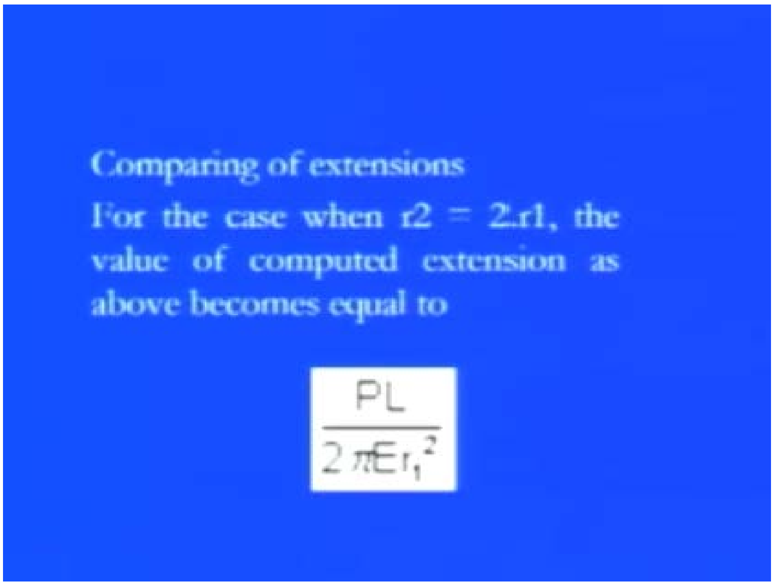

Now if I just want to you know like compare the expansion by keeping the case where you see the special case was given that r 2 is two times of r 1. Then you see what we had we had PL divided by 2 pi E r 1 r 2, and if we are keeping those things, then we have PL divided by 2 pi r 1 E times of r 1 square. So, in that we found that actually whatever the extensions are there it is due to the applied load whatever the extension is there and also the r 1 is there; that means what the smallest region of you know like the segment is. So, these two are the influencing parameters are there and we have to be very careful that actually while selecting those things you know like that what the extension is there and how this extension is taking place due to the applied load as well as the a smallest region of that particular segment is.

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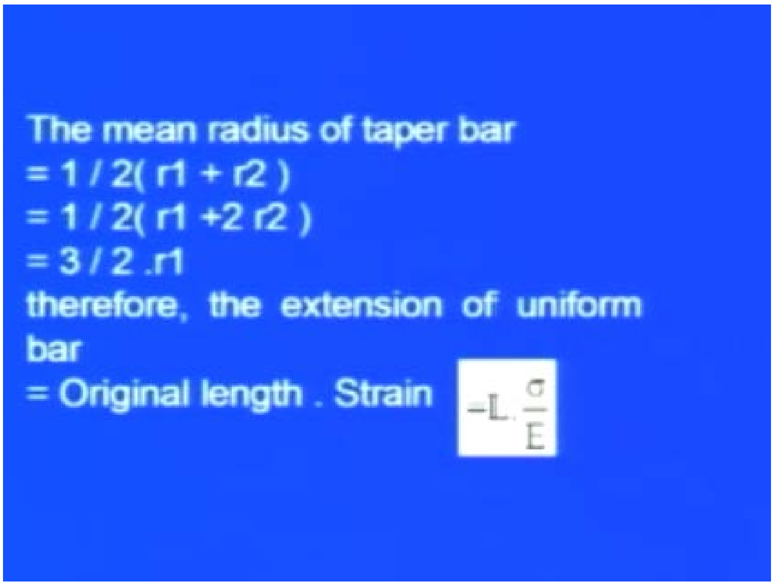

So, now you see if you want to calculate because what we have taken we have taken the x segment in between you know like the tapered bar. So, again we would like to see that what the mean radiation of the taper bar is. So, the mean radius is nothing but equals to 1 by 2 r 1 plus r 2 or we can say now if I keeping r 2 as this thing. So, what I am having? I am having 3 by 2 r 1. So, therefore if we have a uniform bar the extension of the uniform bar is nothing but equals to you know like the original length into strain or we can say if I am keeping original length as L the strain is sigma by E.

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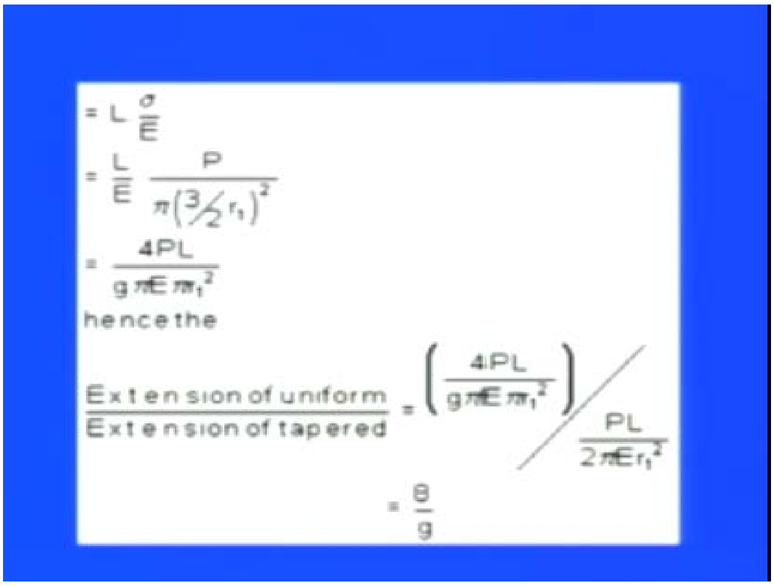

So, now you see by keeping by those derivations in that particular formula would be ending up with the deformation is nothing but equals to sigma by E into L or we can say what are the extensions are there simply keeping those you know like the responsible parameters to see the value or see the significance of those things. Now what we have? We have L by E into P divided by pi which we have calculated 3 by 2 r 1 whole square, because r 2 which we have calculated that is nothing but equals to the mean radius which we have calculated is nothing but equals to 3 by 2 r 1 square 3 by 2 r 1 the whole square.

So, now if I am you know like generalize those things then I am ending up with the deformation is nothing but equals to the 4 PL divided by you know like g times pi E into you know like this pi r 1 square. Hence, if I just want to find it out that if I have a uniform bar as I discussed recently and if I have a tapered bar, then what the difference is. The difference is pretty simple that if I am talking about a tapered bar then the influencing parameters are the load applied as well as the smallest segment of that parameter r 1, and if I am taking about a uniform bar; that means there is no taperedness is there. Simply I am going for the middle portion what the intermediate position is there. Then you will find that you see here, not only these positions you see 4 PL is there divided by you see g times of pi r 1 square.

So, both are the responsible parameters you know like see that if you have a uniform bar and if you have this tapered bar. So now you like if I just calculating you see this comparing those things then what I have? I have this, the extensions in this uniform bar divided by extension of this bar is nothing but equals to 8 by 9; that means you see you know like you can easily compute that what exactly the total extensions are there in the uniform as well as the tapered bar.

So, in this chapter what we have discussed; we discussed that actually if we have a uniform bar then what kind of extensions are there which are the influencing parameter, and if it is you see you know like this uniform bar is extended by its own you know like weight. And if any this applied region the external applied forces are there, then how to get the total stress distribution as well as the deformation.

And also we discussed that if we have a total you know like the elongated bar we can easily calculate by just separating the individual sections and just calculate the deformation for individual sections, sum up those parameter and get the final deformation of the entire length of the beam. So, this was you see you know like we did it for the uniform bar and if I have a uniformly tapered section; that means you see you know like it is uniform cross sections are there all along varying with the x axis. So, in that what we need to do? We need to just take and see the small segment right from you know like taken a specified region that, okay, right from you know like the left corner attach distance if this particular portion is there, cut that portion, see that actually how the deformation is taking place due to the load application and how the stresses are being set up in those you known regions. And due to the stress and load application we have deformation.

And once you get the deformation for the specified region or the small segment sum up those small small segment for the uniform tapered bar because you see in the uniform tapered bar only the variation is the area is there. When the area is changing, load is same. So, stress will be different, and if the stress will be different definitely the strains will be different. So, sum up those small small segments, once you have you know like sum up those things or integrate. So, either we can adopt the sum up positions or we can adopt the integration of that. Once you have the total integration of that you have you know the extension of the entire uniformly tapered bar.

So, in that you see segment we discussed the two main things. If you see it is horizontally put the condition then nothing XL loading is there. If I am keeping that you know like that uniformly tapered bar as a vertical position that means the self weight is there, there are two types of load application self weight and the externally applied load. Then you see you know like you need to add both the component external applied load and the self you know like weight, add those things, see the effective area that had self weight which is the effective area at external load application which is the effective area. And when you sum up those you know like the loads with this effective area you will be ending up with the summation of the two different stresses.

Once you have the stresses and you have the Young's modulus of elasticity you can calculate the strains. Once you calculate the strain you have the total length of the bar, you can easily calculate the change of length that means the deformation. Once you have the deformation now you can either distribute those deformations to different segment or you can simply compare those things, and that is what you see you know like we discussed in the entire chapter. So, you see you know like in this chapter the total orientation was on the stress and the strain with this distribution under the elastic components. But in that you see you know like what we assume that whatever the load application is there it is uniformly you know distributed all across the element.

But if I am saying that this load is you know like distributed even the load is distributed at different different segment. It means if I am saying that I have a uniform bar and in this bar you see you know like for this particular segment the load application is 10 Newton, for this segment 20 Newton load is there, for this segment 30 Newton load is there. So, if the different different segment is experiencing a different load then what kind of deformation is there, and how you can sum up the deformation you know like for calculating the entire you know like the deformation for the entire beam; that is the really a matter of interest that how the you know like the different load will play an important role to design you know like especially this prismatic bar.

And that is what it is happening in the realistic way that you see we have a different you know like in the building also or in any of the uniform bar we have the different different load applications are there, and under the load application how you see the deformations are there which you know like which part of the beam is weakest or which part of you know like the beam is the strongest and how to calculate you see the different deformation at these things are really very important. And for that you see to define the stress verses strain relationship is also very important. So, in that you know like the next segment this all kind of analysis we are going to discuss.

Thank you.