# Strength of Materials

**Prof. Dr. Suraj Prakash Harsha**
**Department of Mechanical and Industrial Engineering**
**Indian Institute of Technology, Roorkee**

## Lecture – 11

Hi, this is Dr S.P. Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to deliver my lecture 11 on the subject of strength of material, which is you know like developed under the scheme of national program on technological enhanced learning. If you just refresh your, you know like in the previous lecture, then we discussed many things about the material properties, like if we have you know like the elastic material, the perfectly elastic material, then what exactly the relation is there in between the stress and strain. And that is what you see, you know, like we apply the Hooke's law for that and also we draw the curve, curve in between the stress and strain, like the perfect line was there.

And if you are talking about the perfectly plastic material, then you see there was not there, like the, if you have a stone, you know this kind of structure, then even if we apply the load, there is nothing as such the stress or the strains are there in that kind of material. So, we found that there was nothing in, you know like in sort of the stress and strain curve.

And also you see, as we discussed about the perfectly plastic material. If the elastic with perfectly plastic material or if we have the elastic plastic material, its a elastoplastic material, which is a pretty common material, then what exactly, you know like the characteristic curve is there in between the stress and strains. So, that is what we discussed in the first part of our previous lecture.

Then you see, we you know like moved to that point where you see, if you want to measure the strains, because if we you know like trying to stretch, you know like stretch one, one material. Then you know like there is a kind of extension is there in that material, and there is a, there are corresponding changes are there in the other parts of material, then how to set the relation in between you see, the extension with the other parts of the contraction? So, we were trying to do that part, you see in the previous lecture.

And then you see, you know like we found that if you want to measure, because you see the strain is a measurable quantity, so if you want to measure the strain in other parts, you see, then there are, you see, you know, like the strain gauges were there. And even you see, if you want to measure the strains in the mutually three mutual perpendicular this exercise, then we need at least three strain gauges. So, then how to set those strain gauges to measure the variety of the stresses in the different mutual perpendicular axis?

Then, you see, you know, like we formed the triangle or we can say, you know, like we formed the three different mutual perpendicular axis, the strain gauges that is known as the strain rosette. So, you see, the two types of strain rosette, which we discussed in the previous lecture, one was when it is at the 45 degree. Then, how we can measure and see, you know, like if it is at the 45 degree.

If we set three mutual, you know, like three strain rosette at equally 45, 45, 45 degree, then we found, that the you know, like the two rosette, which is at x-axis and y-axis, the strain gauges, they are simply measure, measuring the normal or the direct strains while you see, the middle one, which was exactly at the 45 degree from both of the axis, the y and x means, this diagonal part of in between, that it simply shows, you know, like measuring the strain gauge of the shear strain.

So, meaning is, that you see, you know, like if you are keeping all those strain gauges at the mutually perpendicular axis. Then we will find, that you see, all three, we can, we can simply measure and we will find, that the strain, strains are there, but in, they are in the different forms of the stresses like you see, that strains are the shear strains.

So, that is what, you see, we discussed in the previous lecture. And also, you see, you know, like we discussed, that actually if we have the stresses, direct stress or normal stress, direct strains or normal strains, then how we can get the principle stresses, principle strains and where is the maximum stresses are there, where is the maximum strains are there? And they have, you know, like a kind of common analog is there.

So, that is what you see we discussed in that and also we defined, that what is the generalized Hooke’s law means. Actually, what exactly the parameters are there, which we can define in the Hooke’s law. If you see, a material is subjected by, you know, like the mutually perpendicular normal stresses with the shear stresses.

So, we found that there are the total nine components are there. But due to the symmetricity, it only, you know, like if six parameters are there, like you see, you know, like we define that gamma, this epsilon 1, epsilon 2 or even we define, you know, like with respect to that if we have epsilon x, epsilon y or this gamma xy, then how we can correlate those things in the form of the Hooke’s law. Or you see, we defined the two main elastic constant, one is for within the elastic region, one is for the direct strain component, that is the Young’s modulus of elasticity and one is for the shear strain and the shear stress.

So, we can set up the relationship in between the stress and strain for all three, you know, mutually perpendicular directions with using of either first, the modulus of elasticity, that is, e and second is the Poisson ratio. Because you see, Poisson ratio will give you, that actually if you are, you know, like pulling this one circular bar, then what exactly the relation is there that if you are pulling in one bar, then there is an extension and there is a contraction on the other side. Then, what exactly, you can set the relation between those extension and those contractions. So, you see, we were trying to do all those kind of, you know, the Young’s modulus of elasticity, elasticity for the direct strain and direct stress.

But if you are talking about the shear strain, shear stress and shear strain, then we need shear modulus of rigidity, which will clearly give, that actually if we have shear stress and due to the shear stress if there is a distortion in, you know, like the total element in x and y direction, then how to compute that. Always you see, we need the shear strain and once we set the relation between the shear stress and shear strain within the elastic region, the shear modulus of elasticity is always an important, this, parameter. So, you see, these are all the elastic, you know, like properties, which we define. So, we would like to start from that and then we will try to set up the relationship between these elastic properties.

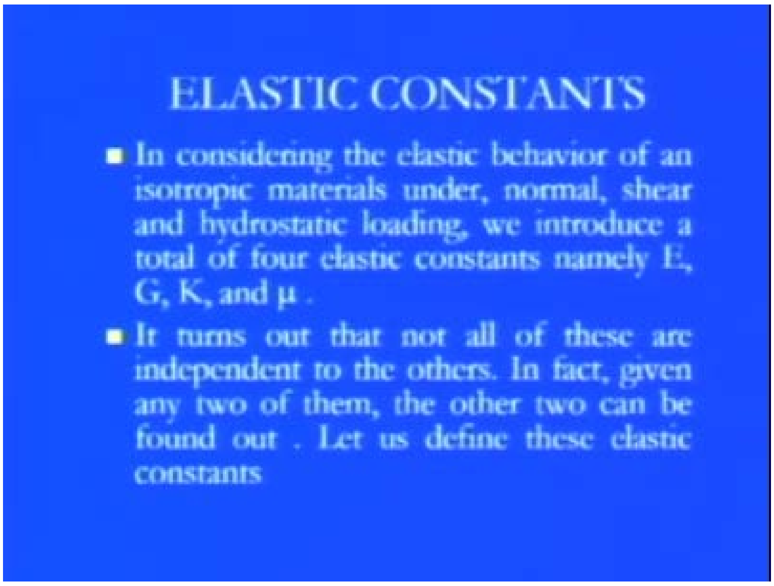
But also, we discussed in the last part of that lecture, that if you see this Hooke’s law or generalized Hooke’s law is valid only when there is, you see, the elastic properties are there. That means, you see, if you are working within the elastic region or the elastic limit or proportional limit, I should say, then only these, whatever the equations are there or whatever the constants are there, they are working or they are applicable.

But you see, in generally, the realistic condition says, that if we need to go with the non-linear relationship in between the stress and strain, that means, you see this is not the final part where the stress is proportional to strain. If you apply the load and you see, you know, like if you are going up to the failure part, then there is, you see, you know, like there are two regions, one is the elastic region, one is the plastic region.

So, we need to define both regions separately and we found, that in the elastic region there is a proportionality, but if you go to the plastic region, then there is no proportionality, there is a non-linear relation is there. So, we need to define the non-linear relation and in that you see, these all elastic constants are not at all valid.

So, you see, here we need to go and we need to analyze all those things, but first our main aim is that what exactly the relations are there in between those elastic constants. So, here it is, you see, in this lecture we are going to, we are going to start with those coefficients and then we will analyze all the part.

(Refer Slide Time: 07:30)




So, first the elastic constant. In considering the elastic behavior of an isotropic material, you know, like we also define, that you see the isotropic material what it is there, that if material is showing the property uniform in all the direction, then it is this isotropic material and anisotropic material just like wood or any ((Refer Time: 07:45)) if they are. So, in the different, different characteristics with the different directions.

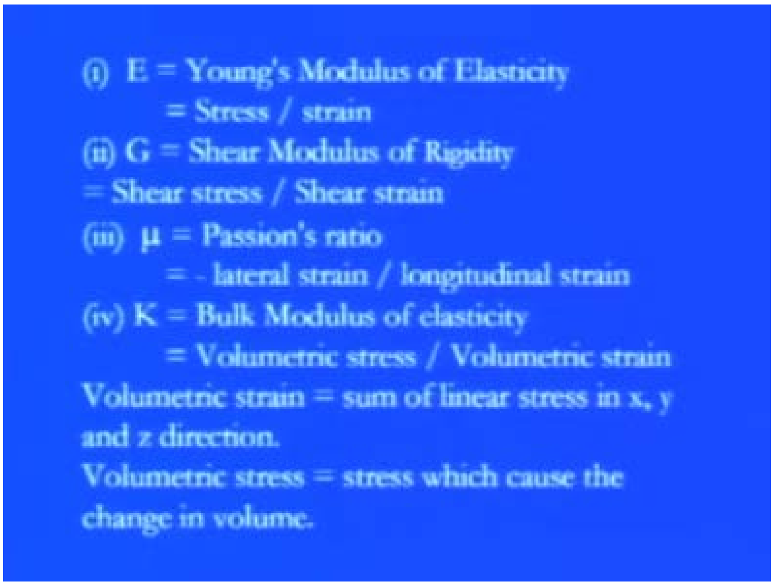
Also, we define the homogeneous material, which is, that means, you see or the, in, in the material all points are showing the equal property. That means, you see the directions are not the properties, are not same in the all directions, but at individual point the properties are same.

So, here we are talking about the elastic behavior of an isotropic material, the linear isotropic material, I should say, under normal, shear and hydrodynamic loading. We can say, that you see, you know, like whatever the elastic properties are there, they are always giving you some sort of constant and these constants are known as the elastic constant.

So, we introduce, you know, like the total four elastic constants in the previous lecture, that is: one, the Young's modulus of elasticity, that is E; the shear modulus of elasticity, that is G; bulk modulus of elasticity, that is K and mu is the Poisson ratio. So, it turns, it turns out, that not all, you know, like all of those independent, you know, like these properties are to be, you know, like there, but they are also dependent on each other.

So, in fact, you see, you know, like we, we just want to find it out, that what exactly the relations are there in between that. So, first of all, you know, like what the meaning is, that is, the Young's modulus of elasticity, which is always coming out from the Hooke's law, that is, the stress is proportional to strain. Then, if we equate those things, then there is a constant and the constant is known as the Young's modulus of elasticity.

(Refer Slide Time: 09:06)



So, E is nothing but equals to stress by strain. Second is the shear modulus of elasticity, shear modulus of rigidity. So, it is coming due to the shear stress and shear strain. So, again, you see, you know, like we can straightaway define for in a specific form of the stress, that is, the shear stress. So, G is nothing but equals to shear stress divided by shear strain. So, you see, either E or G, both are, you see, working in the elastic region with the stress and strain.

Mu is nothing but the Poisson's ratio. Poisson ratio always trying to correlate the contraction or the extension in the other directions, like you see, here mu is nothing but minus lateral strain divided by longitudinal strain. Lateral strain is, you see, if we have a like just the circular bar and if you are just pulling towards the x direction, there is an extension in the x direction. So, whatever the strain is there in the x direction that is the longitudinal strain. But whatever the contraction is there in other two mutually perpendicular direction, like you see in y direction there is a contraction, in z direction there is a contraction. So, this strain in y or z direction is known as the lateral strain.

So, if you want to make the relationship between, in between the contraction and the extension, if we are pulling towards the one direction, then you see, we can say, that the mu, which is Poisson ratio is nothing but equals to minus lateral strain divided by the longitudinal strain.

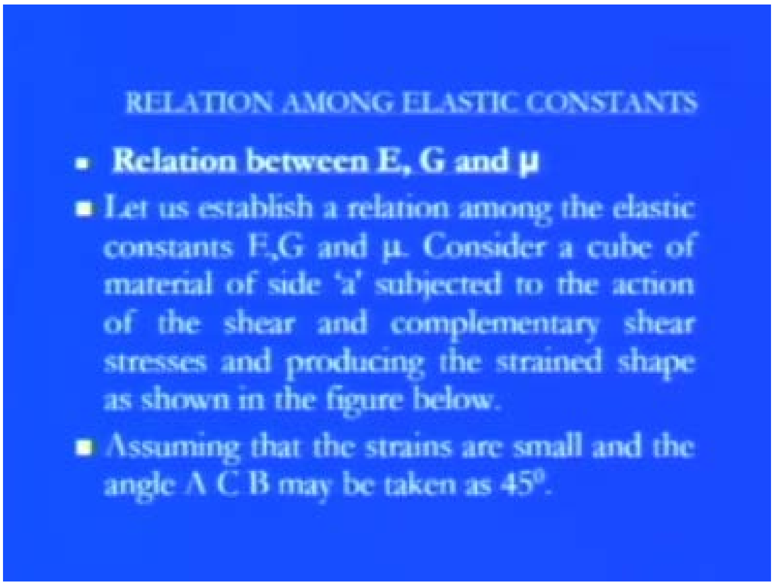
And the last coefficient of the elastic, you know, like constant is the bulk modulus of elasticity, which is nothing but equals to, you know, like this volumetric stress divided by volumetric strain means, you see, this bulk modulus is always working in the particular volume. That means, you see, if we have, you know, like the direct impact of both, mutually perpendicular stresses and the shear stresses, then you see, there is combined effect is there and due to this combined effect there is a change in the volume, and we can simply compute the change in the volume with the use of bulk modulus of elasticity So, K is nothing but equals to volumetric strain divided by volumetric strain.

So, stress by strain always gives you Young's modulus of elasticity, shear stress divided by shear strain will give you the shear modulus of rigidity and volumetric strain divided by volumetric strain will give you the bulk modulus of elasticity.

So, you see, volumetric strain is equal to sum of the linear strain or we can say, that you know, like in corresponding xyz direction because you see, we are, you know, like trying to capture the deformation in all three directions in, mutual direction under the effect of any loading. So, here you see, if you had, if you were, if you just want to, you know, like define the strain in terms of the volume, we need to go within corresponding direction xyz and we need to compute separately, that what exactly the deformations are there, like extension, contraction, whatever it is and we need to compute and multiply those things to get the exact volume. So, volumetric strain or volumetric stress, we can say, that actually whatever the changes are there we need to concern in the volume domain.

Or you see other, as far as this Young's modulus of elasticity is concerned, only it is based on the axial stress and axial strain. Shear modulus of elasticity is, shear modulus of rigidity is concerned, it is simply based on the plane stress because you see, you know, like it, it is simply related to the shear modulus of elasticity. So, shear stress is, direct part is there in that.

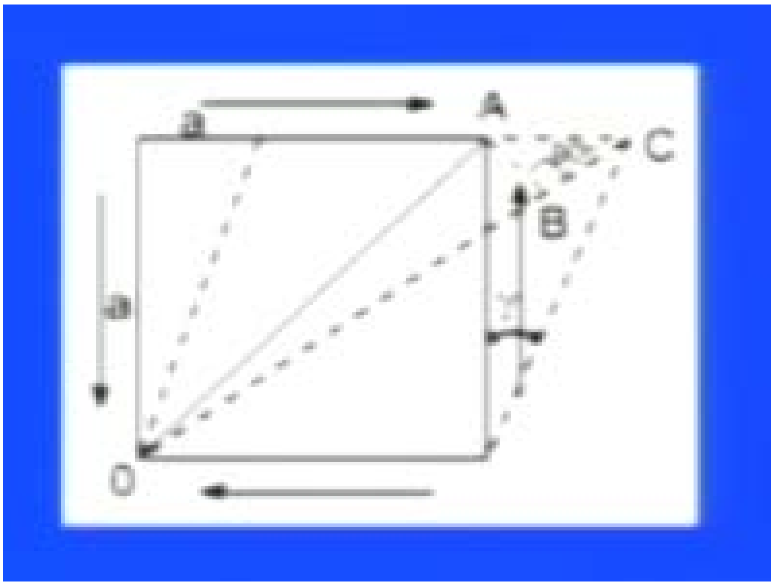
(Refer Slide Time: 12:22)




So, now you see, we just want to set the relationship in between those elastic constants. So, you know, you see, here first the relation between E, G and mu. E is the Young's modulus of elasticity, G is the shear modulus of rigidity and mu is the Poisson ratio. Let us just establish a relation among the elastic constant, these things and consider a cube.

So, again you see, you know, like we are going for a cube, which has a unit weight, the unit depth and unit height is there of a material, which has a side a subjected to action of shear and correspond complementary shear stresses and producing the strained shape as shown in the figure.

(Refer Slide Time: 12:53)



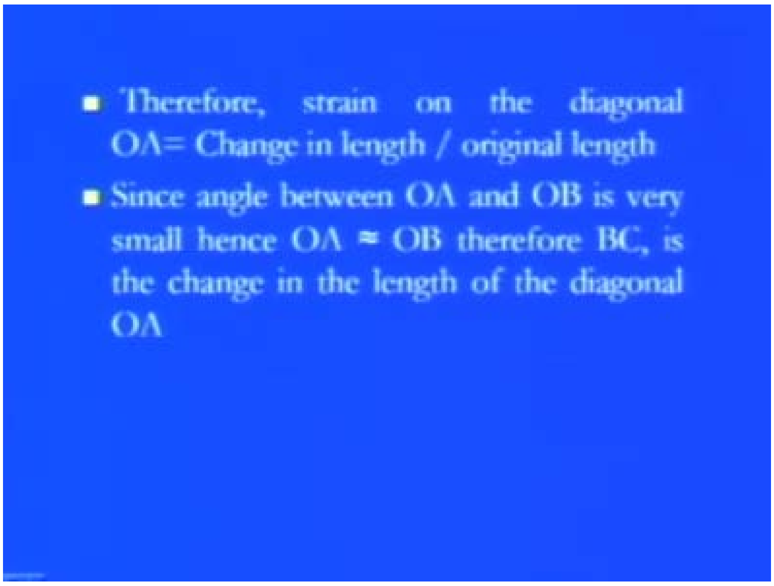

So, you see, if you have, you know, like see this figure we will find, that this is a perfect square is there, which has equal side on four. This is a; this is a. So, we can say, unit cube with which has a unit, you know, like length in amongst all six faces. So, what is there, you see, if you are just trying to, you know, like put the under the application of shear. So, you see, here this is the shear, first part of the shear stress is because we know, that the shear stresses are always parallel, you know, like to the surfaces. So, you see, here these are the parallel forces. So, due to the shear there is a distortion and distortion is there in the, you know, like in the angle.

So, you see, earlier if we have the right angle, that is, this you see, if you are talking about this point to this point in this figure. So, this point O and this point you see, this is the initial part and due to the shear stresses there is a distortion and then, you see, you know, like this A point, which, which is at the extreme corner of that. So, A, OA is nothing but now, it is my diagonal. So, this diagonal is now shifted. So, this diagonal is now shifted from A to C. So, we have the new diagonal that is OC and earlier was this OA.

So, now it is nothing but it is a parallel piped, which has you see, the distortion right from this. So, we can, you see, if you see this figure, then you will find, that this is nothing but the distortion measurement and this we can say, this is the shear strain. So, we can measure the shear strain and we can, you know, like get the shear stresses at these points. So, we have both the shear formula, but if you project this thing, means if you put, you know, like the projection from A to this C, C line, this OC line, then we get the new point, that is, the B point.

So, here it is, you know, like this is, the total angle is 90 degree. So, we can resolve these forces on this particular diagram and we can get, that you see, if it is at 45 degree or at 90 degree because of the two angle and what exactly the resolution is there of the forces we can get. Or this is, you see, the gamma, which is simply, you see, so you get the shear strain at these points.

(Refer Slide Time: 14:47)




So, therefore, a strain on the diagonal, whatever the diagonal OA was there, as I shown you, you see, the OA, which has now changed to OC, so this OA is nothing but equals to change in length divided by original length. So, you see, it will give you the strain because whatever the changes are there in the length due to the shear stresses divided by original length will give you the measurement of the deformation and this is known as the, shear stress, shear strain.

Since the angle between OA and OB is very small as usually, you know, like hence OA is, you know, like almost equal to OB, therefore BC is the change of the length of the diagonal, which will give you the diagonal OA.

(Refer Slide Time: 15:29)


Thus, the strain on diagonal OA, because our matter of concern is OA, OA is simply shifted to some point, so that strain on the diagonal OA will be BC divided by OA. Or BC can be resolved by, you know, like dropping the diagonal on OC feature. So, we have AC cos 45 because the 90 degree was there, so 2 theta was 90. So, the angle is 45 degree. So, AC cos 45 will give you the BC. So, we can keep here, AC cos, AC cos 45 divided by OA or we can have the OA, you know, like from the triangle, which is equals to A over sine 45 or we can say, A into square root of 2.

So, now if you are keeping those things in the previous formula, then we have the strain is equals to AC divided by A root 2, which, which has come from OA in the previous figure, so AC over, AC over A root 2 into 1 by 2, which is there from the cos 45.

So, if you resolve these forces, then we have the strain, which is equal to A to C, you see, that how the distortion is there from A point to C point. So, what exactly you see in the shifting is there from A to C. So, this is AC divided by 2 times of A. A is, you see, the common side of, you know, like the total length of any common side.

Now, you see, we know, that the AC, AC was nothing but equals to, you know, like the distortion, the distorted part from the extreme corner A to C. AC is nothing but equals to A times gamma from, you see, you know, like if you have the curvature part, then we can simply get this relation. So, if AC is equals to A times gamma, gamma is nothing but equals to shear strain, you know, like, so we can put those things, you see, you know, like in, in to the previous formula, then we have the strain on the diagonal is nothing but equals to, we can simply replace the AC by A gamma.

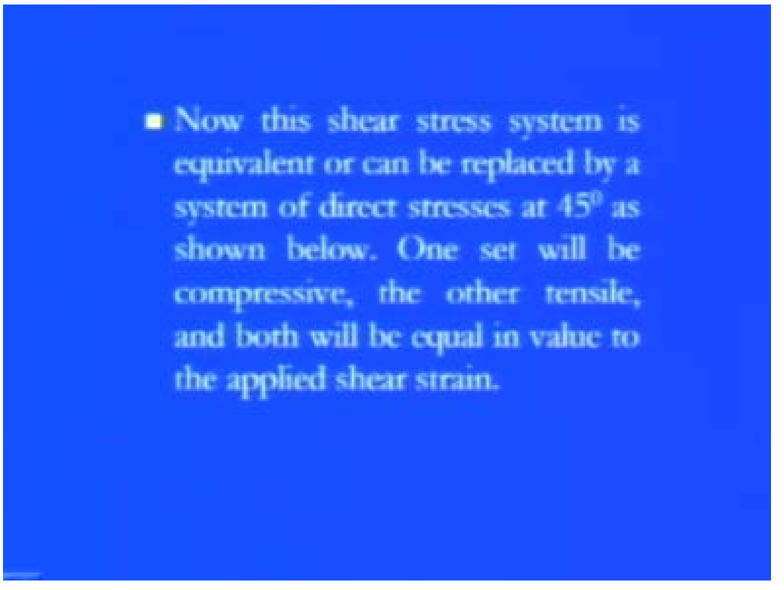
So, this AC, this A gamma divided by two way or we have the strain on the diagonal, which is earlier OA was there and then OC was there. So, whatever the change is there, the strain is there, which can be get by, you know, like gamma by 2. Gamma is nothing but the total distortion or we can say, the shear strain. So, from, you see, the definition we can say, that now we have the strain here.

So, from the Young's modulus of the shear modulus of rigidity, is nothing but equals to the shear stress divided by shear strain or we get the shear strain is nothing but equal to shear stress divided by this one. Or if you are going for our condition where the diagonal is stretching and we have the strain on the diagonal, which is the shear strain divided by 2, so we can get the strain on diagonal is nothing but equals to gamma by 2 or it is equals to tau by 2 G.

So, now you see, here we have the total strain on the diagonal, which is absolutely dependent on shear stress divided by, you see, whatever the shear modulus of rigidity. Shear modulus of rigidity is the property of material, that, which property is there if it, whether it is a perfectly elastic body or whether it is this elastic plastic body or what kind of, you know, like the body structure is there, correspondingly you see the change of strain is there along the diagonal. Or we can straightaway compute that since we are, you know, like if we are measuring, if you just try to remind those things, we measure those, this Mohr's circle for the strain. If the abscissa is the normal strain component on this y-axis, we have gamma by 2.

So, here also if you want to measure the strain, always it comes as the gamma by 2. So, here again you see, even if you want to measure the strain in along the diagonal, we have the similar kind of, you know, like the structure, that is, gamma by 2 or we can replace by manipulation of the simple formula in the elastic region, which is equals to tau divided by 2 G.

(Refer Slide Time: 19:00)




So, this kind of, you see, you know, like relations, which we can set up easily for any kind of, you know, like the structure is there and when it is, you know, like subjected by a pure shear because here there is no axial term is there, only shear stresses are being applied and we are getting the shear strain. And then, we can set up these relations with the using of a shear modulus of rigidity.

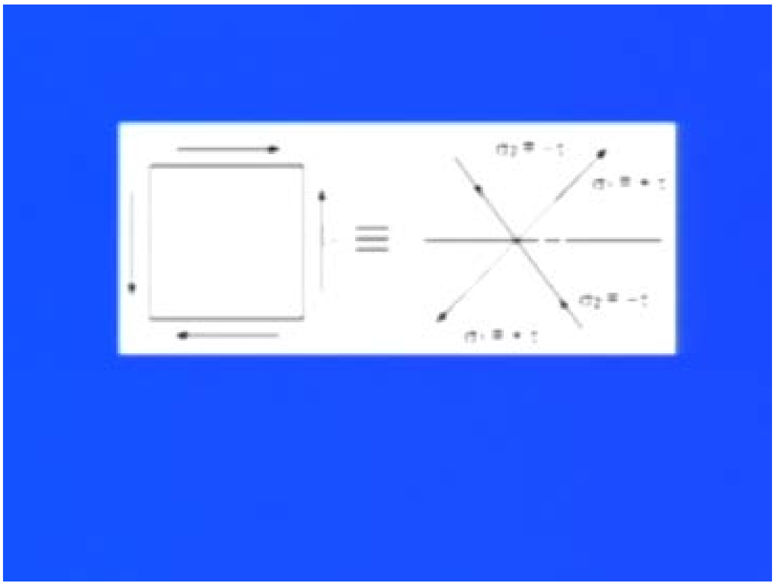
Now, the shear stress system is equivalent or can be replaced by a system of direct stress at 45 degree. Because you know, like at 45 degree we are having the similar kind of, you know, like the results if we apply the direct stress at 45 degree, we have the similar kind of planes, the planes are, so in the similar kind of nature. So, you see, instead of analyzing by shear stresses, we can also put the direct stress at 45 and get the same result.

So, once that will be, you know, like compressive and other one will be tensile. Obviously, you see, because one end is going to compress and one end is going to extend. If you see the previous figure, then you will find, that one is compression, just going towards the compressive side and one is extending towards, you know, like A to C. So, all in all the total effect is, you know, like we can observe easily. So, both will be equal and you know, equal in the value to the applied shear strain. So, we can simply measure.

As you see, the shear strain is nothing but equals to OAB. If I am saying, that O is the center point A and B, they are the two extreme points and these, you see, this is A, this is O, this is B. So, we can measure the angle AOB. And if it is distorted, then you see, we can simply, you know, like AOB minus, if I am saying that the distorted one A dash and B dash. So, we can measure the shear strain is nothing but equals to AOB when they are straight and when it is distorted, minus A dash OB.

So, whatever the change of the angle is there towards the plane, we can simply measure in the shear strain. And we can say, that even if we apply instead of shear stress, if we, if we apply the mutual, you know, like direct stress is mutually perpendicular at 45 degree angle, means if it tilted, that part, the similar kind of results are coming in terms of tensile or compressive and also they will exhibit the equal in value to the applied shear strain.

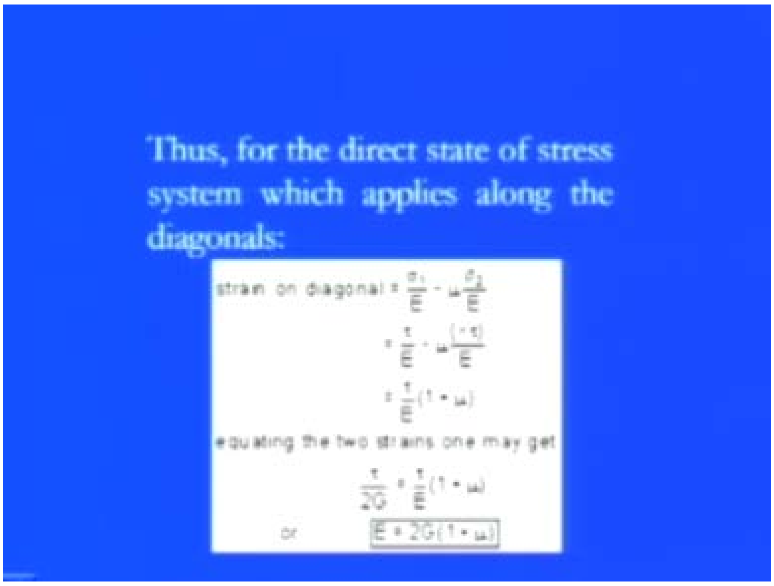
(Refer Slide Time: 20:55)



So, now you see here, the similar kind of, you know, like whatever the discussion, which we, you know, like discussed the similar kind of things, we are being shown here in this figure, we have a uniform structure. The shear stresses are being applied at these two corners and we have the complimentary shear stresses just to maintain the balance of this particular element. So, we have this complementary shear stresses. So, now this element is perfectly balanced under the application of pure shear.

So, now you know, if I just want to replace those things, what I did, what I need to do only, just you know, like apply the direct stresses at 45. So, here you see, these are the, you know, like the two main planes are there at the 45 degree angle. And you see, here when we have sigma 1, which is positive shear, so sigma 1 sigma 1 is the one direct stress and sigma 2 is the other direct stress and they are always applied at the 45 degree. So, they can also, you know, like exhibit the similar kind of behavior as it is being you know, like exhibited by this behavior on the left side. So, you see, here sigma 1 and 1 sigma 1 is positive positive tau, sigma 2 sigma 2 is negative tau and they are exactly at, you know, like the 45. So, all in all, the total angle is 90 degree to each other perpendicular.

(Refer Slide Time: 22:09)



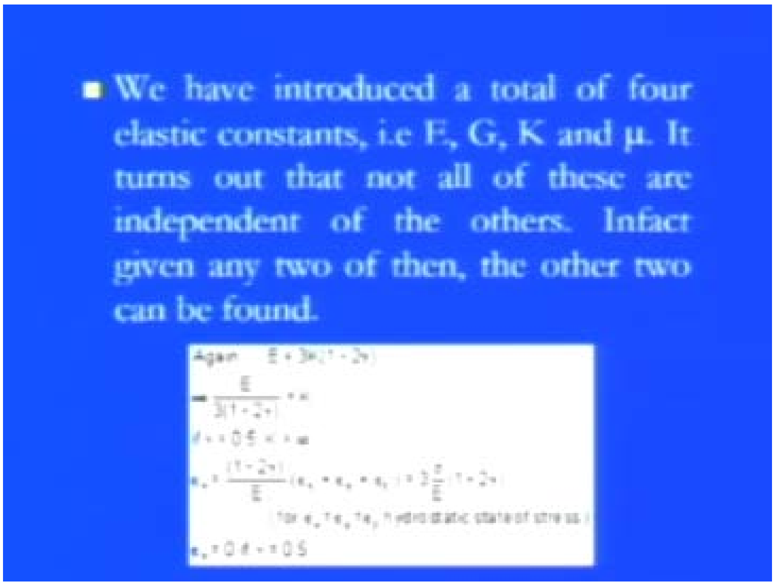

So, thus you see, for the direct state of stress system, which applies along the diagonal we can get, you know, like the strain at the diagonal. As we discussed in the previous case, that the strain on the diagonal is equals to sigma 1 by E minus mu times of sigma 2 by E. Because you see, if we apply the load in the one direction there is an extension. So, this extension is created by this sigma 1. So, sigma 1 is always, is the dominating parameter, so sigma 1 by E. And in other direction there is a contraction, so minus mu times of sigma 2 by E

Or we can say, by replacing sigma 1 and sigma 2 by, you know, like the sigma 1 is positive tau and sigma 2 is negative tau, so we can, you know, like get the final equation is tau by E minus mu times of minus tau because sigma 2 is there A by E or we can say, tau by E into 1 plus mu. Or by equating these two, you know, like strengths, what we can get, you know, like because the strain on the diagonal is nothing but equals to this sigma divided by 2 G because this gamma by 2 was there on the diagonal side AC, A, this O to A to O to C, what we had? We had gamma by 2 or sigma by 2 G. So, if I am replacing this sigma by 2 G equal to sigma by, you know, like this tau by E into 1 plus mu what I have, you know, like this tau tau will cancel out, E equals to 2 times of G into 1 plus mu.

So, now you see, we have a relation in between the Young's modulus of elasticity and the shear modulus of rigidity, just you see, by equating these two terms like, you know, like these two terms what we have? We have E by G is nothing but equals to 2 times 1 plus mu.

So, you see, here we have the direct, this Young's modulus of elasticity due to direct shear and direct, this direct shear, this direct stress and direct strain. We have the G, which is shear modulus of rigidity, which is due to the shear stress and shear strain. So, you see, we can simply set up the relation in between direct stress and direct strain component, the elastic component, just if we know the mu, means the Poisson ratio. So, we have E equals to 2 times of G into 1 plus mu. By keeping those values we can get the another value. So, this is G, you see, you know, like the relation one.

(Refer Slide Time: 24:16)



So, now you see, we, we have, you know, like introduced a total of four elastic constants, like E, G, K and mu or rather I should say, that actually this E, G and mu, it turns out, that not all of these, they are independent to each other. But in fact, any of two or you see, you know, like two or three, they are, you know, like can be easily dependent on each other, like you see, in the previous case we found, that if we want to calculate E from G, then we should know mu. So, you see, all three are having dependency on each other for calculating.

So, again you see, we just go back to that. We have E equals to, you know, like 3, we just want to, you know, like we, we have one more relation, E equals to 3 K into 1 minus 2 gamma. So, by using this K what we have? We have E over 3 into 1 minus 2 gamma. Or I can say, that actually if we have gamma is half means, you see, this 50 percent, you know, like the distortion is there in that. The K is infinite means, if whatever the distortion, as I told you, gamma is nothing but equals to AOB minus A dash OB dash.

So, if it is distorted 50 percent, then the bulk modulus of elasticity is, you know, like is infinite. Or I should say, if I just want to, because the bulk modulus of elasticity is always defining on the basis of volumetric strain and volumetric stress, so this volumetric strain, this epsilon v, which is nothing but equals to 1 minus 2 gamma divided by E into, if we have all three components, epsilon x plus epsilon y plus epsilon z.

So, you see, here by keeping those things what we have? We have this, you know, like E, 3 times of sigma by E into 1 minus 2 mu or I should say, you know, like if I am comparing those things, then what I have I have? E v equals to 0 if, even if the gamma is equals to 0.5.

Meaning is pretty simple, that if we have the hydrostate of stress, means, if all three, you know, like in all three directions if the stresses are uniformly distributed to each other and if we have epsilon x is equals to epsilon y is equals to epsilon z. So, by keeping those things what I have? I have simply divide it by 3. So, 1 minus 2 into gamma, divided, into 3 divided by E is equals to E into sigma y for minus 2 mu.

Or I should say, actually if I am keeping at this point gamma is half, so half, half. So, 1 it is gone 0. So, we have, that is, no volumetric strain. Meaning is, that the change in volume divided by the total volume is 0. That means, if even if there is, you see, you know, like the distortion is there due to this kind of, in all three directions. But if they are equal to each other, then we can say, that there is no, the net change, the net summation of the volume, the net change in the volume is 0 or there is, no volumetric strain is there in that component. So, this is one special condition as far as the volumetric strain is concerned.

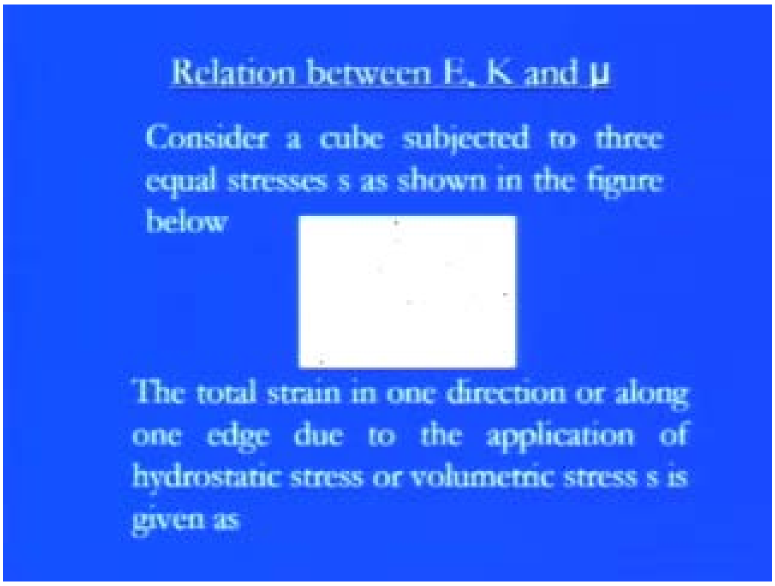
Irrespective, you know, like of the stresses, the material is incompressible. Incompressible means, even you see, even we are, you know, like applying the different kind of loading, different stresses are there in the different directions. But even irrespective of those stresses we have the volumetric strain is 0. That means, material is incompressible or we can say, when the mu is 0.5, the value of K is, you know, like is just going beyond certain things rather a 0 value of E or volumetric strain is 0. In other words, the material is incompressible.

The same meaning is, that material is always incompressible or it is irrespective of the stresses when you see, we have the shear strain is, you know, like just half of those things or we can say, if whatever the K value is there, that is, the infinite at half of the shear strain as we decide, you know, like derived in the previous case.

So, now you see, here we have the next relation. So, in previous, you know, like relations we had two main relations. One, we, you know, like make the relationship in between this E into this Young’s modulus of elasticity, shear modulus of rigidity and the Poisson ratio. So, we will find, that actually you know, like E is nothing but equals to 2 times of G into 1 plus mu. And then, we found, that actually even we can, you know, like relate E into K, means the Young’s modulus of elasticity to bulk modulus of elasticity because both are working under, working into, the elastic, elasticity.

So, we can relate with the using of even mu also, but in that you see, since it is the volumetric strain, so gamma is also the shear strain, is also taking an important place. And if we are keeping shear strain because it is total distortion is there, so if you are keeping the shear strain, half of the value, then we found, that this K is going beyond certain limit. There is no meaning in that; material is highly incompressible. So, the net change in volume, the volumetric strain is giving the 0 value.

(Refer Slide Time: 28:52)




Here, now you see, here we just want to make the relationship in between the Young's modulus of elasticity, this bulk modulus of elasticity and this mu, that is, the Poisson ratio. So, consider again the same object, which we, you know, like comes in the previous figure, subjected to the three equal stresses. So, you see, here now we would like to make the relationship between the Young's modulus of elasticity, bulk modulus of elasticity and Poisson ratio.

So, again you see, we are going to concern the same cubic part, which has, you know, like all three equal, all six equal sides are there and also, we apply the condition, that actually in all three directions, there are equal stresses are being applied to that. So, we can see this figure, that the sigma, sigma, sigma is there in all three mutually perpendicular directions and they have the equal magnitude. And you see here, these sides a, a, a is there. So, all the six sides of this particular cube have the equal dimensions.

(Refer Slide Time: 29:52)


So, the total strain in one direction or along one is due to the application of the hydrostatic stress because this is the unique, you know, like the equal stress or the volumetric stress. This is also given by, you know, like the sigma by E, you know, like the tau time because this sigma by E or minus this tau times this sigma by E. So, if you compare all these things, because it is the volumetric strain, or I should say, it is the hydrostatic stress, because it has the equal magnitude in all these three directions with the equal sizes, so we can say, that the total volume, this total strain is nothing but equals to sigma by E into 1 minus 2 times of gamma.

So, you see, here volumetric strain is nothing but equals to 3 times because it is equal, so three times of linear strain. So, we can say, the volumetric strain is also equals to epsilon x plus epsilon y plus epsilon z or we can say, that epsilon x is equals to epsilon y is equals to epsilon z if we have the unique features with the uniform stress distribution as well as the uniform sizes.

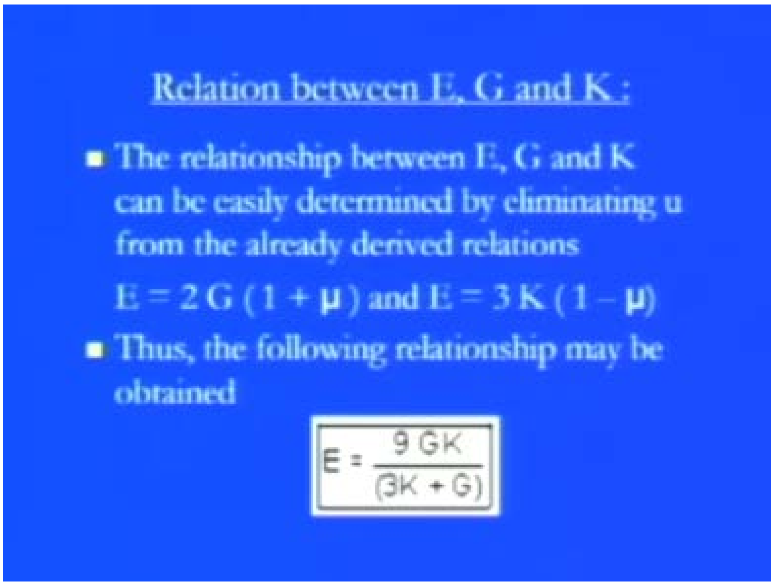
So, we can simply apply those conditions to get the volumetric strain, which is nothing but equals to 3 three times of whatever the strain, which we got here, the sigma by E into 1 minus 2 2 times of gamma. So, we have this volume strain is equals to 3 times because it is nothing but the 3 times of linear strain. So, 3 times of sigma by E into 1 minus 2 gamma.

So, by the definition of the bulk, you know, like modulus of elasticity we know, that it is absolutely based on the volumetric part in the, we are always playing with the, volume, volumetric domain. So, the K is nothing but equal to volumetric stress divided by volumetric strain. Or we can say, this is equals to this sigma by K volume, whatever the volumetric strain is there, or we can get easily the sigma by K, which is the volumetric strain. So, at one side we have the volumetric strain sigma by K. So, from sigma divided by K we can get the volumetric strain. So, volumetric strain is equals to sigma by K is equals to the first one, 3 times of sigma by E into 1 minus 2 gamma.

So, if you are equating those things, then the sigma sigma will be gone out, whatever you see, you know, like the stress component is there. So, what we have at the end E equals to the Young's modulus of elasticity is equals to 3 times of K, that is, the bulk modulus of elasticity into 1 minus 2 times of gamma, which is the shear stress, so you see, shear strain.

So, we can see here, that we can easily relate those, you know, like the components if we know, you know, like the exact relationship, that actually what kind of material is there, how these stresses are being formed and what kind of, you know, like the strain, strain components are there. And in this case also we discussed, that actually if we have the gamma, that is, the shear strain is 50 percent, the half is there, we have the K value is infinite. That means, pretty simple, that if we have this kind of thing, then obviously, the bulk modulus is infinite or the material is highly incompressible. So, that is what you see, we discussed about that.

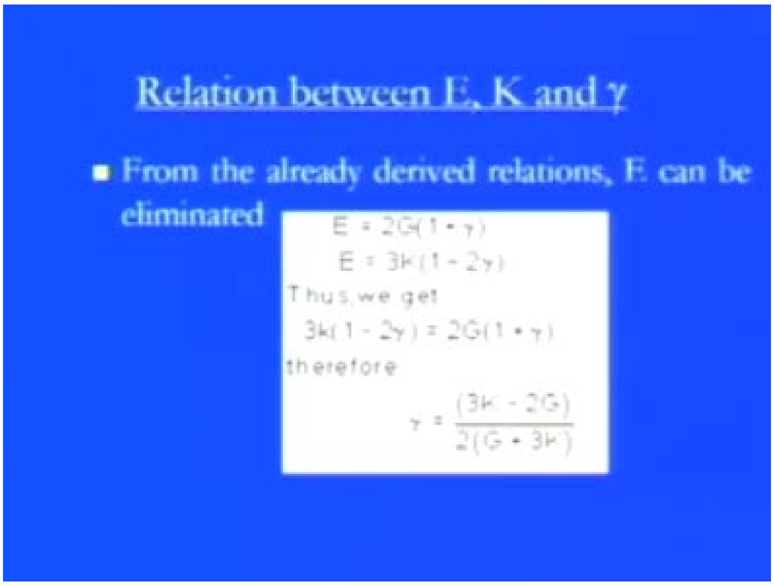
(Refer Slide Time: 32:48)




Now, we want to, you know, like again set up some relationship in between, you know, like these elastic constants. So, we have now the three new elastic constants, that is, the Young's modulus of elasticity E, the shear modulus of the rigidity G and the bulk modulus of elasticity.

So, now by just taking the two earlier relationship, we have E equals to 2 times of G into 1 plus mu and we have E equals to, you know, like E equals to 3 K into 1 minus mu. So, and if, if I am correlating those things, then we can, we can make the relationship E equals to 9 times of GK divided by 3 K plus G. Means, you see here the Young's modulus of elasticity can be compute easily if you know the shear modulus of rigidity. And if you know the bulk modulus of elasticity by computing this relationship. So, we have this kind of relationship.

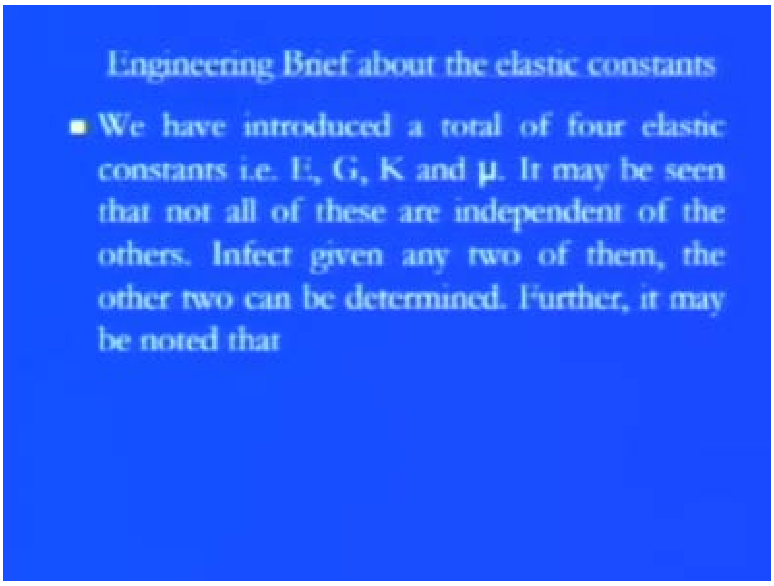
(Refer Slide Time: 33:40)




Now, you see, again we would like to relate this E and K with the using of gamma as we discussed. So, what we have from the earlier relationship, that modulus of elasticity is nothing but equals to 2 times of shear modulus of rigidity into 1 plus gamma or we have, you see, E equals to 3 times of K, bulk modulus of elasticity, into 1 minus 2 gamma. Gamma is the shear strain.

So, now you see, if I am, you know, equating both the side, then what we have? We have 3 times of K into 1 minus 2 gamma is equals to 2 times of G into 1 plus gamma. So, if, if I am, if I just want to relate those things, then we have the gamma, which is the shear strain, you know, is equals to 3 K minus 2 G. You know, like if I just make the set up of those things divided by 2 times of G plus 3 K. So, we have the new shear strain component with the use of this, either the bulk modulus of rigidity or we can say, the shear modulus of the, you know, like the elasticity. So, by computing all those things we can easily form the new value of the shear strain.

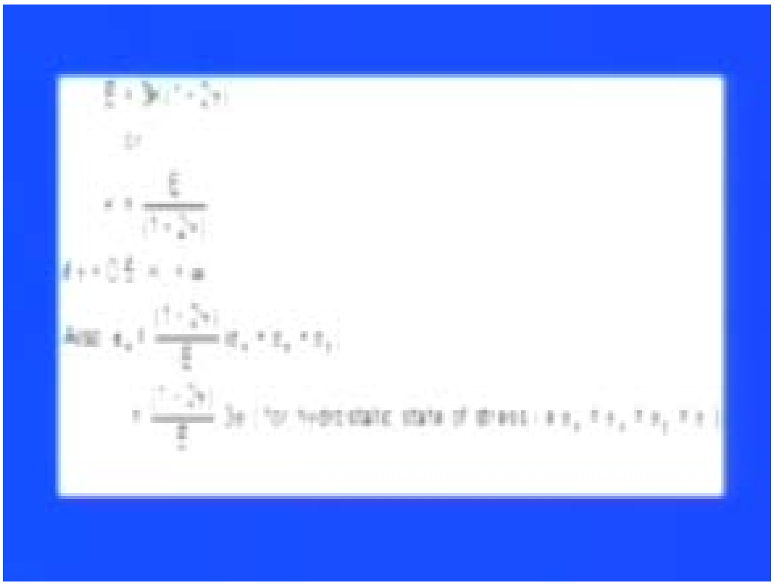
(Refer Slide Time: 34:32)




So, as we move further, we can get the engineering brief, you know, like about the elastic constant. We have already introduced, you know, like the total number of four elastic constants, that is, the Young's modulus of elasticity, the shear modulus of rigidity, the bulk modulus of elasticity and the Poisson ratio.

So, it may seem, we, you see, you know, like all of not are independent, you see, of the others. But they are, you see, if, if you know the two values we can get the two other values or vice versa, you see, you know. But the key feature in that you will find, that the Poisson ratio, because the, due to the Poisson ratio or we can say the shear strain, we can get the other three moduluses like Young's modulus, bulk modulus or the shear modulus. So, these are the key main, you know, like the moduluses are there within the elastic region of an object.

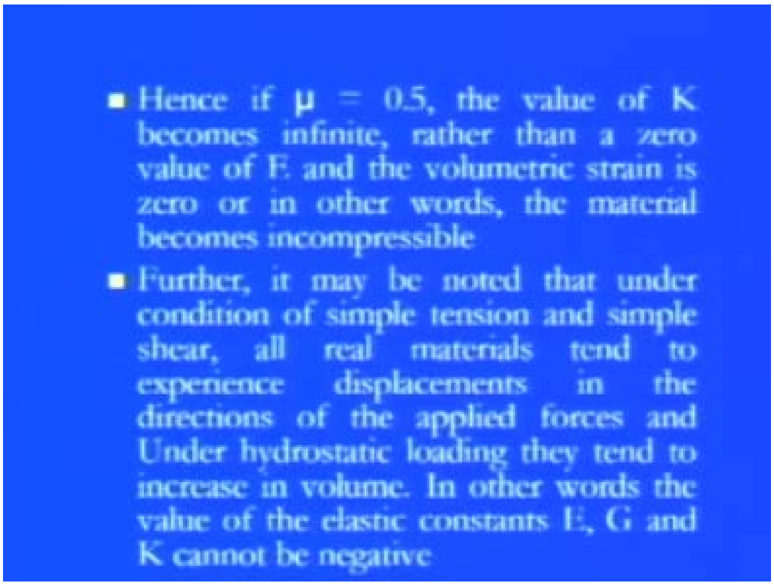
(Refer Slide Time: 35:13)



So, now as we move further we will find, that as we defined, if you want to calculate the Young’s modulus of elasticity, then we can get easily by, you know, like 3 K into 1 minus 2 into shear strain. So, if I am keeping those things, we have the bulk modulus of elasticity that is nothing but equals to Young’s modulus of elasticity divided by 1 minus 2 mu. Or we can say, if I am keeping the value of shear strain as, at, as I told you, that half, then we have the bulk modulus of elasticity is infinite or we can say, that is nothing but the material is incompressible.

And we cannot, you see, you know, like keep all those put, you know, like keep all those conditions applicable. Once we have, you know, like the total, then is half, that is, epsilon v is nothing but equals to, because the K is basically based on the volumetric strain divided by the volumetric stresses. So, if I am keeping those things here, will have, we will have epsilon v, which is the volumetric strain is equals to 1 minus 2 times of gamma divided by E into epsilon. All that means, you see, if we have epsilon x epsilon y and epsilon z, then the summation of those things.

(Refer Slide Time: 36:42)

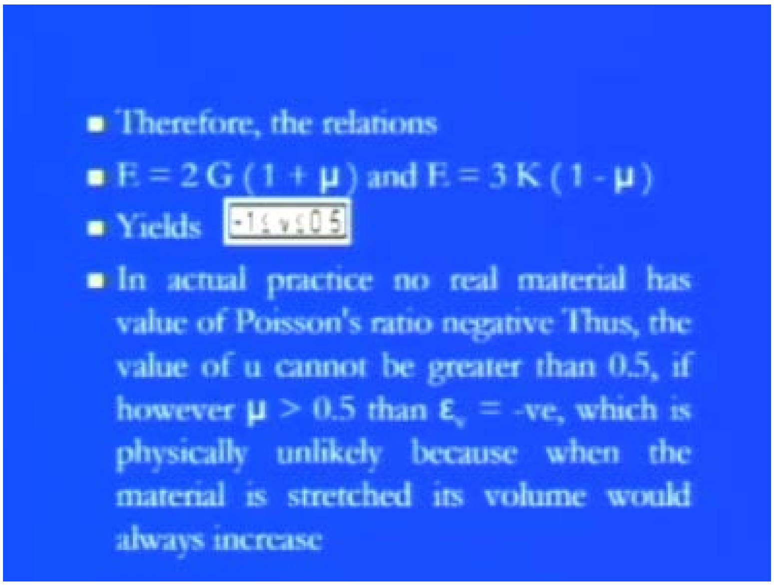



And as I told you, that if we have the hydrostatic stresses, that means, if all the strains are equal in three directions, then we can simply get the 3 times of all, 3 times of 1 minus 2 gamma divided by E into sigma. So, now you see, we can simply replace those features and if I am keeping the value of half, the K becomes infinite, as we discussed.

And again, you see, you know, like the volume strain is 0 in that particular part or the material is incompressible, as we discussed. Further, it may be, that you know, like again depend on that actually, whether the condition is simple tension or simple, simple shear, all the real materials tend to experience the displacement in the direction of the applied forces.

And under the hydrostatic loading means, you see, you know, like it is equal components are there in all three mutual directions. They tend to increase in the volume and in the other words we can say, that actually if the elastic constants are there, they cannot, you know, like either E, G or K, they cannot be negative. Meaning is pretty simple, that if any material, which is under the state of the hydrostatic loading, always it tend to be positive irrespective of whether you see, you know, like the Young's modulus of elasticity is there, shear modulus of rigidity is there or bulk modulus of elasticity is there.

(Refer Slide Time: 37:30)




So, therefore, you see, now based on again the previous relations, which is set up, the Young’s modulus of elasticity is nothing but equals to 2 times of G into 1 plus mu. Mu is the Poisson ratio and you see, E equals to 3 times of K into 1 minus mu. So, you know, like if I am keeping those things, then the key feature in both, you know, like in the, both the formula is the mu value, that is the thing. But you see, the Poisson ratio, which is always you see, you know, like coming from minus 1, which is equal and less to, you know, like mu, which is less than equals to 0.5. In actual practice, no real material has the value of Poisson ratio negative.

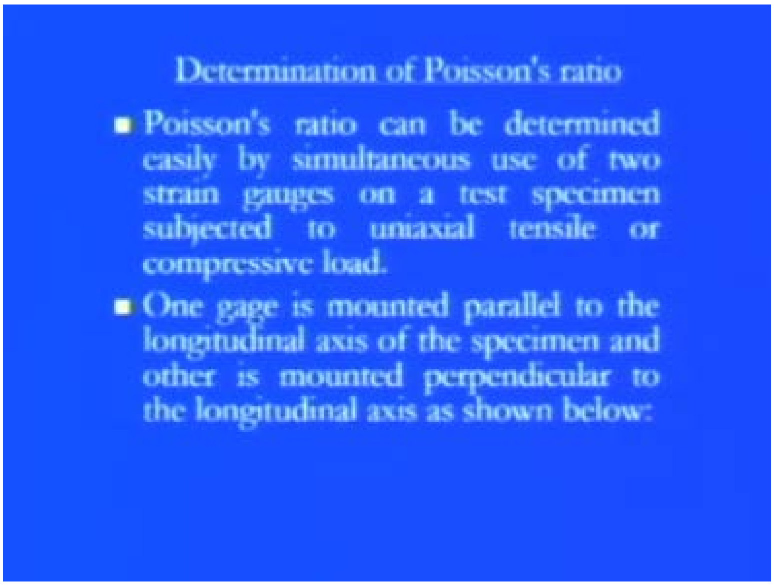
So, this is only a key feature is there actually, that we cannot put the mu value as negative because you see, always if you see, that if you are, you know, like the mu is always, if you go for the physical significance of the Poisson ration. Then you find, that it is when you are stretching a material, it is extended in one direction, but in the other direction always there is a contraction.

So, if you want to measure the mu, then always you will find, that it is nothing but the lateral strain. It is always lateral strain is negative and we are keeping negative outside. So, those negative negative will give you the positive. So, it never be negative. But in, in the real sense, but as far as the mathematical calculations are concerned we always use that, ok. Now, the negative means, the contraction is there; the positive means, the extension is there.

Thus the value of mu, you know, like cannot be greater than 0.5. Again, this is the limiting value. We cannot go in the, you know, like negative side we cannot go beyond 5 because if beyond 5 is there, then material is incompressible when there is no compressibility is there in the metal. As we have shown, that actually if you are keeping mu as 0.5, always this K is going infinite. Means, you see, the material is incompressible. So, this is the limiting value. Mu is always less than 0.5, volumetric strain is negative, which is physically, you know like, unlike, just it is unlikely because when the material is stretched, its volume would be always increased. And when it is increasing, we have a K value, which is volumetric strain; volume stress divided by volumetric strain is always positive side.

So, you see this is the limiting conditions for any Poisson ratio and if you are calculating in any of the numerical and you found, that there is a mu is going in negative side or if it is going beyond 0.5, that means, there is an inaccuracy in the calculation. So, this is, you know, like for the check point to see what the exact relations are there in between E, G, mu and K.

(Refer Slide Time: 39:54)

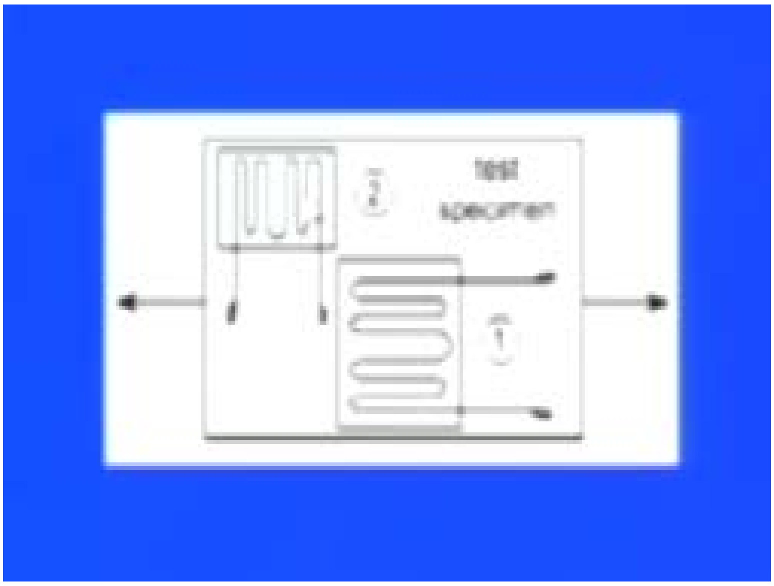



Then, you see, if you move further, how to determine the Poisson ratio? Poisson's ratio can be determined easily by, you know, like simultaneous use of two strain gauges on a test specimen subjected to uniaxial tensile or compressive. Because you see, if you are, if you just go for the physical sense of Poisson ratio, we will find, that if there is an, you know, like if there is, we have a tensile bar and there is an extension is there towards one direction, so there is one strain gauges, which we needed to measure the extension. But there is a contraction is there in the perpendicular direction. So, we need to put the strain gauge in other direction also just to measure, that actually how much contraction is there in the other direction.

So, you know, like for that, at least you know, like we need two strain gauges to measure this extension and contraction in the respective directions and once we know the longitudinal strain, once we know the lateral strain, we can simply compute the Poisson ratio, which is equals to minus lateral strain divided by longitudinal strain.

So, one, you see, gauge is mounted parallel to the longitudinal axis just to measure the longitudinal, as I told you, of the specimen. And other one is mounted perpendicular to the any longitudinal axis, which will give you the lateral strain. So, mu is minus lateral strain divided by longitudinal strain.

(Refer Slide Time: 41:00)




So, like that you see here, in this figure it is clearly shown, that you see here the first one, which is along the length. So, you see, here we have, this is the specimen, test specimen and there is an extension is there towards the x direction. So, this extension is due to the tensile pulling. So, tensile pulling is there towards this direction and along these, you know, like we are keeping the strain gauge parallel, this is the electrical strain gauge in which you see, you know, like we need to put the central line.

So, whatever the changes are there in any of, you know, like the distortion, it is simply measured in terms of the change of, you know, like the voltage and this voltage can be measurable. So, what we can do here? This is parallel to this axis, so this will give you the volumetric strain.

And you see, the other one, just because we just want to measure the lateral strain also, so we need to keep the other electrical strain gauge just perpendicular to this one. So, you see, here we have another strain gauge, which is exactly normal like, you see, this is along this one. So, they are, keeping in this way another one is just perpendicular to this upper one. So, this is like that. So, here in this figure it is showing that. So, whatever the contraction is there in the perpendicular direction that can be easily visible, you know, like in this particular slide.

So, now we have, in one direction we have the longitudinal strain, which can be measured by this strain gauge first. In another direction, whatever the contraction is there due to this loading, it can, it can be measured by the second strain gauge. Once we have both the strains, you can simply put in the formula that Poisson ratio is equals to minus this perpendicular, means, the second one, the lateral strain divided by the first one, whatever the readings are there and this will give you the Poisson ratio at a particular force.

But again, the limiting condition in any of the Poisson ratio is that cannot be negative because always the contraction is there in the perpendicular direction and we cannot go beyond 0.5 because if we go beyond 5, then the respective bulk modulus of elasticity will be infinite. And once it is going beyond certain limit, then you see, if material is incompressible and there is no meaning to measure any kind of strains part. So, that is what you see.

We are keeping this mu value in between 0 to 0.5 and mostly, you see, we are using the mu value, that is, the Poisson ratio 0.35 to 0.45. So, this is you see the limiting condition for any of the Poisson ratio. So, these you see, you know, like we discussed about many of the elastic constants, like you see, the mainly based on the four, that if we have the Young's modulus. If we have the shear modulus, then how to correlate with the using of Poisson ratio, then if we have the bulk modulus, if we have the shear modulus, then how to relate with the using of shear strain. Because you see, the shearing part is there and if we have the Young's modulus of elasticity and bulk modulus of elasticity, then how to relate those things with the using of Poisson ratio.

So, you see, if you know, out of these four elastic constants if you know the two, then other two can be easily computed because they are dependent to each other. So, if, if you go for independent thing, then you see, they are based on the material side, but it cannot be, you know, like directly related to, to other one. If, if you know the two parameters, two parameters can be easily founded.

So, these, you see, you know, like all the relations, which we set up and these are the important relations if you want to, you know, like, calculate all those things together. But the key feature in all those, you know, like elastic constants are, they are only valid within the elastic regions. Because you see, you know, like once you go beyond certain limit in the extension means, once you put in the plastic region, all these parameters are invalid and we cannot put, you know, like those relations to calculate the different parameters.

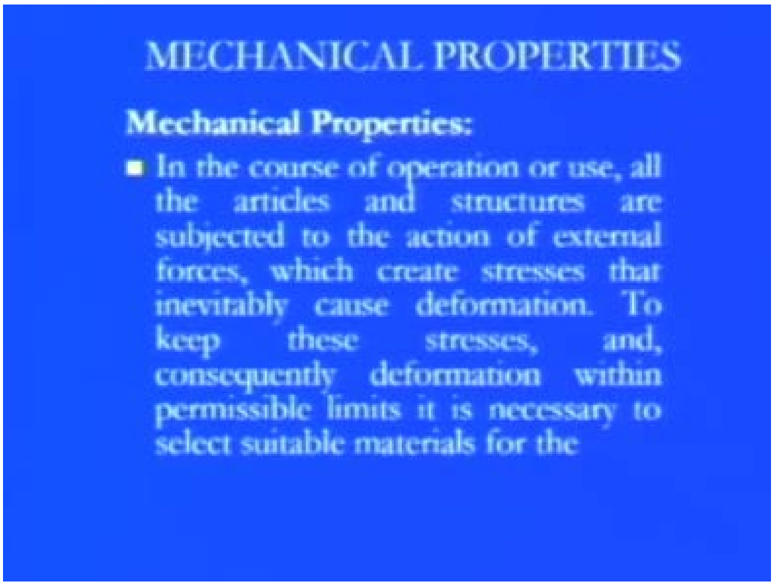
So, this is, you see, this was the one of the key feature was there and that is why, you see, all the Hooke's law or all those parameters are applicable within the elastic region if material is applied and that is why, you see, you know, like we even define the characteristic curves within the elastic region irrespective of the Young's modulus, shear modulus, bulk modulus, Poisson ratio or other isentropic and homogeneous properties. So, this was, you see, you know, like about the key issues of the elastic constant.

Now, you see, we are coming towards the mechanical properties, that what exactly the mechanical property of the materials are there through which we can exhibit the elastic, you know, coefficients or the plastic coefficients. And how you can differentiate it between the plastic region and the elastic region of a material because you see, in the previous lecture I discussed about that.

If you are talking about the elastic region, that means, it is a linear region and always we are defining, that Hooke's law is valid for linear, elastic isentropic material. So, you see, these all are the assumptions.

So, you see, if you want to see the clear microstructure of a material that what exactly is going on when we are applying the load and you see, we are saying, that there is an elastic or plastic regions, You see, these kind of characteristics, only we can, you know, like analyze once, you know, the once we know the mechanical property of a material.

(Refer Slide Time: 45:44)

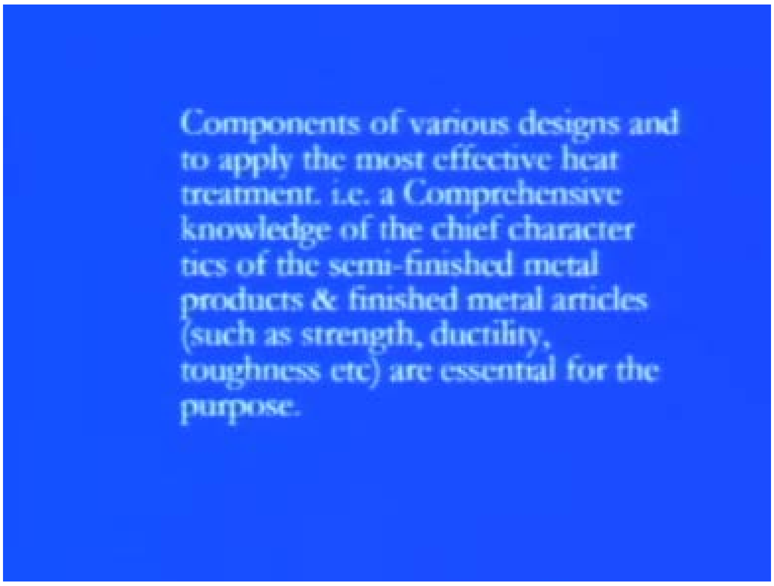



So, in the course of operation or any use, all the articles, you know, like and you see, the structures are subjected to the action of external forces and you know, like, which creates the stresses, that in, inevitably you see, cause the deformation, so whenever you see, we apply the load. And as I told, you see, the load can be defined by two ways, one is the mass into acceleration and other one, which is the important property in the stresses in the point of application of load, that how this load is acting. And due to that you see, we can easily find it out, that whether it is, you see, towards outward, outward, direction, towards the compressive side or towards, you see, the shearing, means, the parallel forces are there. So, what exactly the point of application of this force is.

So, based on that, you see, we can easily find it out, that what deformations are there within those object. And to keep those, you know, the stresses and the consequently, that deformation within the permissible limit, it is necessary to, you know, like we need to select suitable material for that.

Because you see, that let us say, if you are talking about the, you know, like the RCC rod and if we keep, you see, the other material, like brittle material, like the brass is there, the bronze is there, then it is simply rupture because with it cannot bear out the tensile loading. So, we have to be very careful about the chosen of material that what kind of loading is there and which type of material can bear that kind of loading. So, that is what you see, you know, like this kind of properties is very, very important to read it out, try to apply the engineering design.

(Refer Slide Time: 47:08)



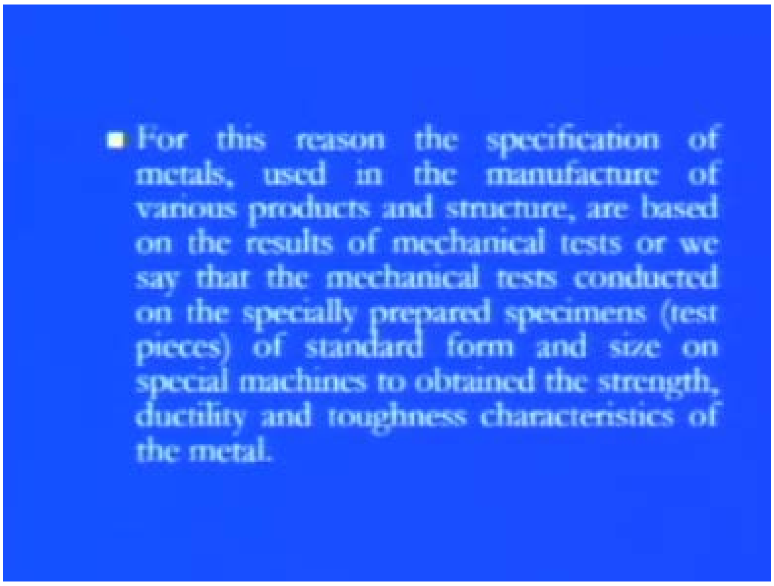

Then, you see, you know, like the components of the various designs and to apply most effective heat treatment, you know, like the heat treating is there, annealing, you know, like, and other, other this heat treatments are there just to refine the structure, that is the, you know, like this comprehensiveness knowledge of the chief character, characteristic of semi-finished material product and the finished metal article, such as the strength and ductility, toughness are really very essential for the kind of engineering design purpose.

Because you see, after you know the ductility, then we can simply say, that yeah, this mild steel is, you know, the ductile material, even we can stretch up to this limit, then it will show the elastic limit, elastic region or plastic region. Or even we can say, that if it is high speed steel or if it is a high cut, this, this high speed steel or high chromium steel is there, then what kind of, you know, like these characteristics, they can exhibit under the application of those loads.

Or if I am saying, that if I am just using the cast iron or this brass or bronze, then what kind of characteristics they are showing and what kind of ductility is there, whether they are showing any ductility property or the brittle material or there is simply fracture is there. Or whether, you see, you know, like they have the, what kind of carbon percentage are there through which we can say, that yeah, this much hardness is there and we can apply this kind of loading there.

So, these, either the strength or the ductility or the hardness or the toughness, all these, you know, like are the key features of any material through which we can say, that yeah, if this kind of loading is there, this material is very much suitable for that kind of application. And that is what, you see, we can even refine through the heat treatment of these materials.

(Refer Slide Time: 48:36)



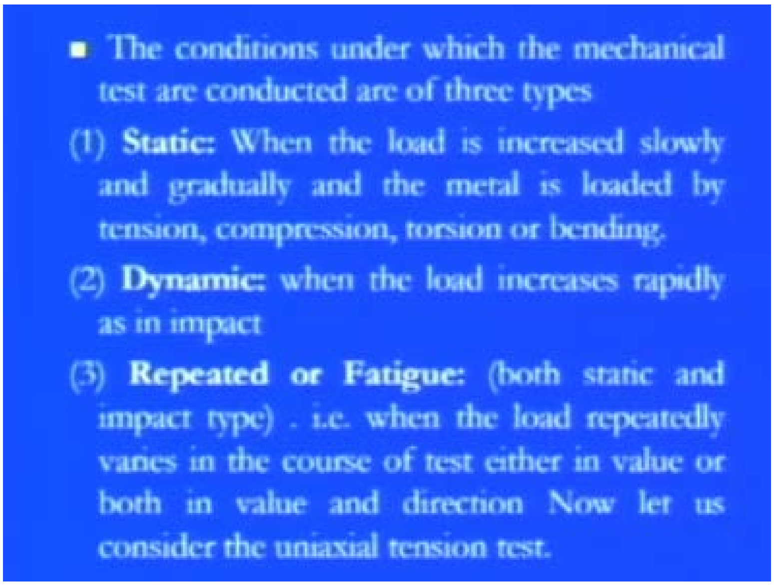

So, for these, for this reason, the specimen, the specimen of a material, the metals used in any manufacturing of, you know, like the various product and the structure are simply based on the result of the mechanical test. So, that is what you see, what we are doing here prior to apply anything or prior to, you know, like go for any application, we would like to check, that actually, what actually the microstructure is there and if we apply the load, like the tensile load or you see, the compressive load or the shear stresses or the shear loading, then how these microstructure is changing.

And you see, whether it can, you know, like, like we are applying up to a certain limit and there is no, and once we release the load, if there is no exact permanent set of change in the microstructure is there, we can say, that yeah, this material is very much sustainable or suitable for this kind of loading. So, you see, prior to go for anything we need to check it out the microstructure.

So, mechanical tests are very, very essential before designing or we can say, that actually the mechanical test conducted on, in a specially prepared specimen, like you know, like the test pieces, always we are first checking on the test pieces only of the standard form and the size on the special machine to be obtained the strength, ductility and the toughness characteristic of a material.

So, you see here, these, these three, four properties are very, very essential prior to go for design of any, you know, like the object and we need to check it out, that actually whether this can be, you know, like sustainable for impact loading, direct loading, shear loading or what. So, you see, here this kind of, you know, like these properties are very, very essential.

(Refer Slide Time: 50:12)




And for that the condition at which this mechanical tests are conducted, we are simply categorizing the three types. One, the static type, because you see, as we know, that if the static loads are there, then how these microstructures are changing. So, when the load is increasing slowly or we can say, the gradually, the material is loaded by tension, compression or torsion or the bending. That means, you see, here if there is a hydraulic press is there, you see, as we, we can see in our universal testing machine, the hydraulic always, you know, loading is always kind of slowly, there is no direct impact is there. So, the microstructure is changing slowly.

And as and when you see it is extended, these all elastic limit, proportional limit, yield limit or this plastic limit can be easily, you know, like clearly defined, that what are the limit is there for these, these regions. And we can say, that actually if you want to show these things on the graph, it can be clearly shown by the graph also.

So, this is, as far as the static is concerned because you see, the element is under, you know, like equilibrium when the static loads are there, whatever the forces are there, these forces are simply applying on the outer side and internal intensity of the resistances are exactly equal and opposite, opposite. So that we can say, that there is no, you know, like the randomness or the turbulence is there within the microstructure in the static loading.

Second part, as per its name, the dynamic, when the load increases rapidly because in the dynamic we are saying, that the force is absolutely dependent on the time. So, when the dynamic forces are there, just like, you see, you know, like when a shaft is moving or we can say, the gears are just rotating in that or the cam shaft is moving, the impact loading is always coming. So, the impact means, actually the high amplitude of load is coming, just boom like that and then, just releasing.

So, whatever this kind of loadings are there, we have to be very careful. And for designing this kind of, you know, like the components, always we are taking the dynamic factor of safety. That means, higher factor of safety because it can, you know, like go up to any extension in the stress level or the strain and that you see, it is very, you know, like random part is there. And I should say, it is momentary and due to in this moment, whether the material can sustain or not, that has to be carefully designed.

Then, you see, you know, like the third kind of loading is very, very essential and you can find in, in many of the machines this third loading, that is, the repeated or the fatigue loading, which is very dangerous So, both static and dynamic, this impact type or dynamic types are different. But that is, you see, when the load, you know, like repeatedly varies in the course of the test, either in the value or in the nature value of the direction, both.

Then, you see, we can say, that this is the kind of the fatigue means, the sinusoidal part is there. It is coming in the positive as well as the negative direction and repeatedly, it will come up to certain limit.

So, you see, here if you want to design the component for this kind of loading, then we have to be very, very careful in the chosen of the, the engineering factor of safety. Because you see, once we choose any kind of factor of safety here for impact loading, that is, because the toughness will give you that kind of thing or we can say, the static is there. We can say, that the hardness is, will, hardness will give you the clear picture.

But as far as the repeatedly or the fatigue loading is there, then we have to be very, very careful in choosing the engineering factor of safety because all the time you see the micro, the microns or the microstructure of the material is sometimes compressing, sometime extension, sometime compression, sometime extension. So, that means, you use, the R keep on changing, the dynamic forces are there, as well as some part, you see, they are also facing the static forces, means, you see, you know, like the transient phases are there in between the structure of the, in between the microstructure of the element.

So, you see, here if you want to set up the stresses and the strains within that particular material, you have to be very, very careful and that is why, you see, we have chosen the maximum engineering factor of safety for this kind of loading. And now, you see, you know, like we just want to see, that actually, that what, what will happen in the uniaxial tensile test.

So, you see, in the next chapter we are going to discuss about, that what will happen if there is a uniaxial test is there. If there, you see, you know the combined testing is there and the uniaxial tension test, always the universal testing machine, UTM, is very, very clear, you know, like applicable in which we can apply the tensile test, we can apply the compression test, even we can apply the bending or we can, you know, like the simply, you know, are shifting, just applying the torque at the end. We can simply say, that the torsional or the shearing part is there.

So, you see here, in this particular chapter we have discussed about the elastic constant of the various kind. Mostly, we will find, that the three elastic constant with the other constants are there, like you see the E, G and K with mu 2, mu and gamma.

And also after that you see we simply set up some relations amongst these and the last part actually we discussed about what the, this mechanical properties of the materials are and what the importance of these things. So, all the discussion is simply based on why actually we are you know like going for the microstructure as well as the associated mechanical properties are.

So, in the next lecture we are going to discuss about, that if we apply the tensile loading on a uniformly, you know, like structured bar, then what will happen if we apply the load and keep on increasing the static way. Even then, how we can simply segregate it, the elastic region, the transient region, when you see this elastic region is going to the plastic region.

So, even there is a transient region and how we can define the values, the limiting values for these regions. That is also equally important. And then, you see, what is the plastic region and what are the stresses, stress components are there in the plastic region and the elastic region. And then, you see, generally we are defining the yield limit of that. So, what is that yield limit, what will be the proportional limit?

So, these are some of the limits and these are, you see, you know, like the values through which we can say, that the material is, you know, like simply showing the elastic behavior. And all those constants, which we which we have shown in the previous chapters or in this chapter also can be valid throughout these particular behavior of the material.

Thank you.