Strength of Materials
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Lecture - 10

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee and I am going to deliver my lecture 10 of the subject of the Strength of Materials which is you know like developed under the national program on technological enhanced learning. I just want to refresh those you seen in the previous lecture, which we discussed about that what exactly you know like the strains are. If you want to measure the strain, then you see what the analytical as well as the graphical solution was there. So, in the previous lecture if you just focus, then you see we found that more stress circle as well as more stress circle is pretty similar. You know like on the x axis. Again similar, this normal strain component is there, and the sheer strain components are there on the y axis and how to get you know like that how to extract the information from more circle.

This is pretty similar to you know like the more strain stress circle, where you see the sigma is the normal stress component was there or there is a stress component are there on the y axis. Then we also found that though we can measure the strains, we can you see because we cannot measure the stress, we can measure the strain, but again you see to measure the strain, the angle is very, very important. So, you see you know like we need to put only the single strain gauge, because always strain gauge is the technical, you know like the instrument is there to measure the strain, single strain gauge is not at all comfortable, to measure all the three kinds of strains. So, you see here we need to put the minimum three you know like strain gauges to measure the respective normal as well as the sheer strain components, and then you see you know like the strain rosette, it comes into the picture where the three strain gauges are to be well set up at 45 degree or at 60 degree.

So, at 45 degree you see we establish the relations with using of the three different you know like the algebraic equations are there at the oblique plane strains normal as well as the parallel. So, we found that at still part if we are talking about the 45 degree strain rosette, then at x axis or at y axis, we have you know like the strain component pretty symmetric like it is the straight, you see the direct strain components are there, but you see at the middle portion where exactly you see the 45 degree, you know like the axis is there and at this particular axis, if we have the strain rosette. Then you see this will give you the strain, the sheer strain component, this gamma x y which you see, you know like exactly or as you know like what the direct impacts are there of the other two components.

The meaning is very simple like if you want to measure the strain, you know like the sheer strength and it was nothing butyou see that if you have the middle portion, the two times of the normal strain in the middle portion minus some of the axial. This x and y in the strain axis a epsilon x plus epsilon y. Meaning is pretty simple that if we want to measure the three different components of the strains, we always need the minimum three strain gauges and they need to put at the strain rosette formation is the respective of 45 degree or 60 degree. And then you see we found that you see there are some of the principle strains are there and the principle strains are always exactly you know like analog is there as compared to the principle stresses and then, also we can find it out that actually where the maximum sheer strains are there.

So, this kind of you know like the relations and we also solve for numerical problems to draw the Mohr’s strain circle that actually what exactly the information which we need it to draw the Mohr's strain circle, and you see what we can extract like the remaining information from the strain just by measuring. You do not have to calculate numerically, but all you need to measure the things like what are the coordinates, what are you see the radius as well as you see the angle and all those bla bla things. You can easily find it out by simply measuring those distances as well as the angles.

So, this was you see you know like the kind of discussion which we you know like did in the previous lecture. So, in this lecture now we are, so if you go up to the previous parts, then you use we have now lots of information about the basics of the strength of material like we know the stresses, we know the strains, we know in between the relation between the stresses and strain. If you see the material application is there under the elastic, but the thing you see you know like we know the properties of these stresses and strain, but still you see if these load applications are there on the different material. Then how these you know like the relations are there in between the stresses and strains we do not know.

So, in this lecture you see we are just trying to you know like first visualize what kind of materials are exhibiting the different kind of properties, or we can say this kind of stresses or strains or this Young's modulus of elasticity or this Poisson's ratio or this kind of you see all this either elastic or plastic deformations are there. So, what types of materials are available and how we can categorize those materials in terms of the properties, in terms of their exhibitions under the load and all that kinds of things. So, in this lecture, you see we have not discussed those things.

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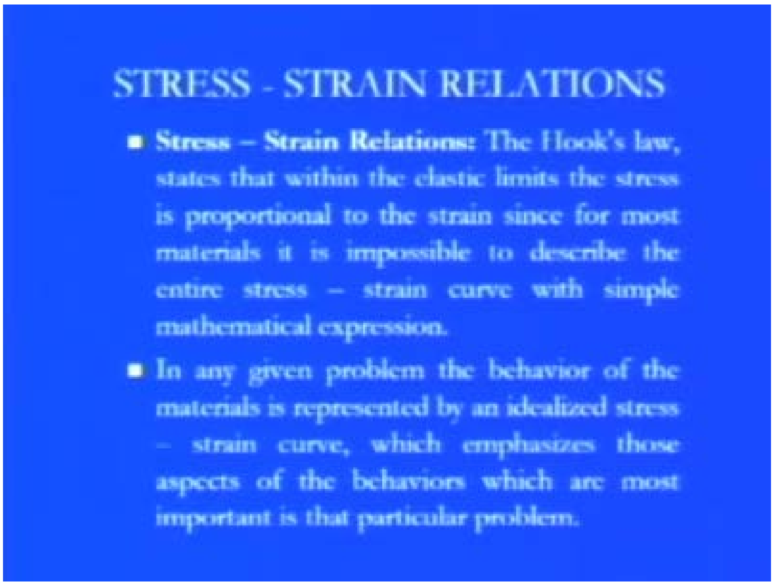

So, you see here first a stress-strain relationship based on the different materials. So, first stress-strain relations, the Hooke's law basic. You see if you are talking about the elastic deformation, then Hooke's law always gives you the stress-strain relationship between the elastic limit, and it says that the stress, the proportional to strain under the elastic limit since for the most of the material is you know like exhibiting this kind of thing. It is impossible to describe the entire stress-strain curve with the simple mathematical expression because you see you know like when you apply the load irrespective of the tensile or compressive, we do not know what will happen beyond that. So, you see you know like we cannot put all the time, the mathematical expressions and get the final. Now, this is up to the rupture point.

So, what we need? We always need that actually like material properties are there and how they are exhibiting on the stress-strain relationship. So, in any given problem, the behavior of material is represented by idealized stress-strain curve. So, we always need the graphical part that how they are exhibiting and that is why you see the universal tensile testing machine. You can say UTM is always giving you whether the tensile test is there, the compression test is there or the bending test is there.

What are the kind of tests are there? You can simply get like you can simply plot those curves with the using of you see these you know, they have props or we can say this is the using of this UTM and these curves are known as the idealized stress-strain curve which emphasize those aspect of the behavior which are most important in a particular problem. Because you see if let us say, if you want to design something and then, the design, the basic feature is that actually what the important part or what the important material properties are. Based on those properties, you can simply find it out that now this material can sustain this much stresses. If you apply the load, it can go up to this extension without any permanent failure or permanent rupture. So, you can go you know like that.

Now, this is good for this kind of application, and based on that you see you need to be very careful that what exactly the material properties are there and which material is really to be chosen to get the desired output. So, here you see we are categorizing those material based on these stress-strain curve properties. So, first material which we are talking about that is the linear elastic material.

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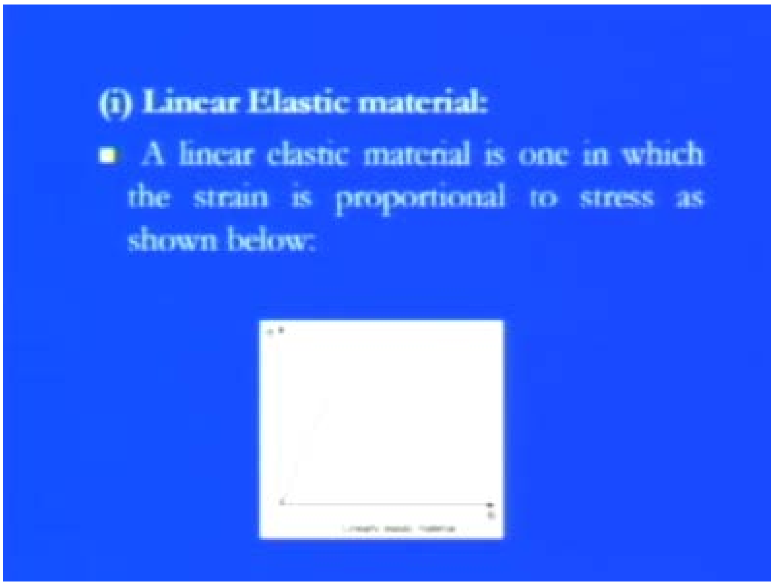

A linear elastic material is one in which the strain is proportional to the stress. That means if you see this particular you know like the curve, you will find that we have the strain epsilon, we have the sigma stress and you see if you apply the load you know like it will simply extend. So, it means you see the stresses are also coming in inducing by the application of load, and at the same time, we have the extension. It means we have the deformation in that particular object. So, we can simply if you want to put the relationship between it mean in between these kinds of in between the stress and strains for this kind of material, you can simply put you know like by the straight line.

So, you see we have the straight line for this kind of material. We have the sigma and epsilon is there. So, this is this kind of material is always known as the linear elastic material and generally, you see the rubbers are there. We can say you know like there are some different kinds of polymers are also there which easily show the kind of elastic. We can see these are the perfect elastic materials in which apply the load; it will extend in straight way. So, you see it will be simply distorted or we can say the kind of deformation is straight way come once we apply the load. So, you see you know like kind of materials are there which is known as the linear elastic material in which the stress is proportional to strain up to the limit. Second is there that is the rigid material. Rigid means like the stone is there. Apply the load. Applied load, it will sustain up to you know like means there is no deformation like you see even if you apply the load, and there suddenly it ruptured.

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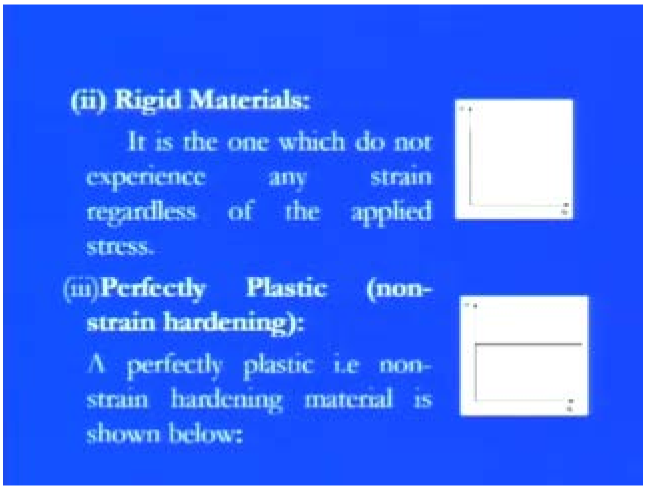

So, it is one of the like the kind of material which do not experience any strain regardless of the applied stress. So, you see we are applying the load, but it is no kind of deformation is there within the microstructure or externally applied kind of that. That means you see whatever the surrounding is there, there is no change of the shape of the material irrespective of what the load application is there, or the stress applications are there. So, you see here this curve will simply blank because nothing will happen. Even if we apply the load in terms of the stresses, in terms of the strain, nothing will come. So, it is a blank kind of that. So, rigid materials are always you know exhibiting. That exhibiting zero result irrespective of the stress and strain. So, we cannot set up any relationship between the stress and strain in this kind of material.

The third is the perfectly plastic. Perfectly plastic is there is no strain hardening like you see you know perfectly plastic straight way you know like go to straight way. When you apply the load, it will permanently sort of deformation is there. That means there is no kind of once you release the load, body comes to its original state. Nothing will happen. Once you apply the load permanently, it will deform like you see if you are taking a thin wire and once we apply the load, it will simply bend and you see even if you apply the load, it will bend and that you see you know like the perfect. It means whatever the load application is there, once the permanent set of deformation exist, this will continue up to the extreme point.

So, you see a perfectly plastic body that is the non-strain hardening material is you know like this kind of material is there, where it will just approach to the maximum. You know like this sheer stress. This normal stress component is there. So, the stresses at maximum and the deformation straight, we will see here this deformation is constant. This means once it is deformed, it will have continuity you know like go even you see that this kind of deformation or we can say the straightway increased keep on these things without any addition of the stress particles. So, that is why it is known as the non-strain hardening. It means there is no hardening is there due to the strain. It is simply you know like showing the straight line towards the strain side like that. Then, you see we have a rigid plastic material.

So, rigid plastic says that you see though it is a plastic material, but some sort of the strain hardening is there. That means, you see a rigid plastic material or the strain hardening is you know like can show some sort of use a variation in the strain part irrespective of the stresses also. So, you see here straight away when you apply the load though it is a plastic material. So, it has a permanent set of deformation right away when the load application is there, but again you see when we increase the load, there is increase in the stress. The simultaneous effect means the simultaneous increase in the strain also. So, you see we can say this kind of strain hardening is there and then, we have which is a pretty common thing. Sometimes perfectly plastic material means you see the materials also having the elastic properties, but as we keep on you know like increasing the load, it shows the perfectly plastic material. So, you see here the elastic perfectly, the elastic perfectly plastic material is having this kind of characteristic.

So, we have the stress-strain curve on the stress and strain up to the elastic limit. You see it will exhibit the proportional limit with the stress and strain. So, up to this point, we can say that the elastic limit and then, it is perfectly plastic. It means you see perfectly plastic means there is no strain hardening after this point. So, we have the constant line in that particular phase, so that is no you see you know like the stresses are coming in that. Only the deformation will go on keeping. So, that is this is known as perfectly plastic with the elastic nature of the material.

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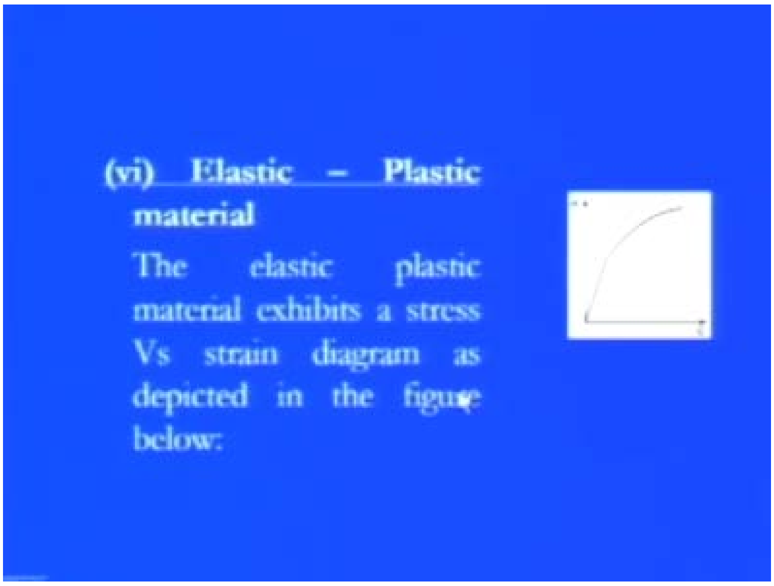

Then we have the last one that is a pretty common. You see the ductile materials are there generally, and they are showing this kind of you know like mild steel is there, high carbon steel is there, high speed steel is there. That is generally we are saying that these are the elastic plastic material. That means you see it is showing you know up to a certain point the elastic nature. It is showing of up to a certain nature of plastic nature in which you see we can say the strain hardening is also available. We have the elastic region and we have the plastic region, and we can easily locate these regions at the two different set of points. So, the elastic plastic material exhibits the stress versus strain diagram which is very common like the diagram, like this is stress and this is strain.

So, we can say this up to a certain limit. There is a straight portion is there. This is the perfectly elastic portion and once you see it goes beyond certain point, and then you see a kind of non-linear relation is there. So, this is the straight relation in case with the linear relation is there between the stress and strain. Now, beyond this point we have a non-linear relation between the stress and strain and that is known as the plastic region. So, you see we have two different regions. If we have a material of nature of elastic and plastic, and we simply you know like by putting the experimentation we can easily you know like separate it out, the different regions of the elastic as well as the plastic.

In general, as I told you that actually irrespective of whether it is a mild steel, high carbon steel, high speed steel, they are generally showing this kind of relationship between stress and strain, and we can simply get with the using of universal testing machine if the tensile is there, compression is there or bending is there. So, this is a very common diagram for showing the relationship between stress and strain for a kind of elastic plastic material. So, you see here whatever we discussed you see right from the perfectly elastic to perfectly plastic to you see you know like the elastic plastic where in the elastic, some perfectly plastics are there or you see we have a general elastic plastic material. All those curves are simply showing a different kind of relationship between the elastic and plastic in between the stress and strain based on what exactly the elasticity is there, plasticity is there or the combination is there.

So, you see how microstructure is playing a key role when the application of load is there in terms of the stresses or in terms of the deformation means the strains this is the very important phenomenon so far. So, now come to the elastic like the stresses and strain. It means if you are talking about 1 is the elastic region and what exactly the relation is there in between the stresses and the strain.

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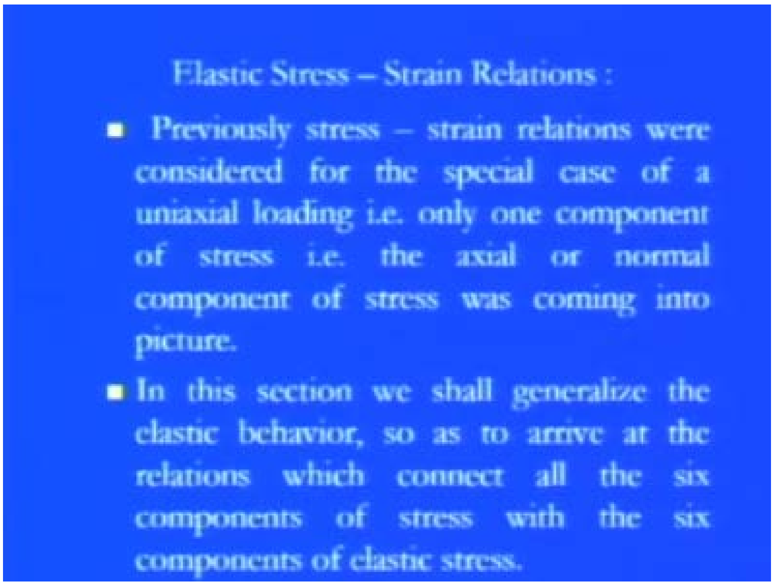

So, previously stress-strain relations were considered for the special case of uniaxial loading. That means you see if we have a kind of uniaxial means the direct stresses are there. Uniaxial loading is there in terms of you see we can say the pulling or compression that is axial or the normal component. The stress was coming into the picture. That means you see we are simply you know like considering the elastic limit first and also, we are considering that the load application is towards the axial. There is no eccentricity or there is no parallel forces are there in that particular component during the load application.

In this section also, we shall generalize the elastic behavior you know like the particular elasticity is there when the application of load is there within the elastic limit. So, as to achieve the relation between you know a relation which convert all the six components of the stresses because it is the plain stress. So, we have you see the components, three normal stresses, three sheer stress components are there with the six components of the elastic stresses.

So, we have all the kind of stresses means all the components are there because we are not, though we are considering the tensile part of the stress, but here we are also assuming at the same point that it is a plain stress, and all the six components are there because of the symmetricity. So, you see we just want to convert those things in generalized way. The state of the stress that what are the six components are there of the stress, and how they are you know like behaving when you know like we are talking about the elastic behavior part. So, first of all you see in this category, the first key feature is there. If we are talking about isotropic property of a material means you see we have a ductile material, but if it is showing the isotropic one.

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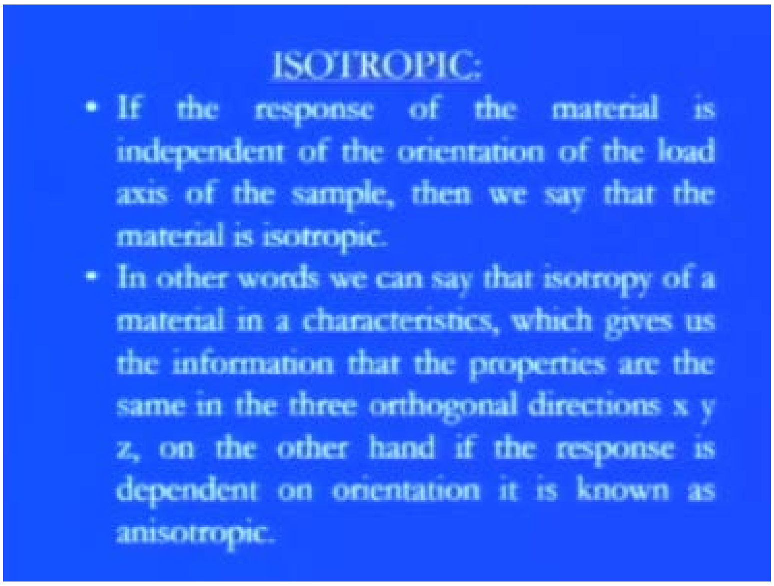

If the response of a material is independent of the orientation of the load axis of the sample, then we can say that the material is isotropic. Isotropic means you see irrespective of what the orientation is there at what angle, you know like it is putting or what kind of load application is there, whether it is the x direction, y direction, z direction, you see the load application is their irrespective of what the load application or in which direction it is there. If this is what are the responses are coming, if they are you see perfectly same in all directions, we can say that this kind of material is known as the isotropic material or in other words, we can say the isotropy of a material in characteristics which gives us information about the properties are same in all three or the one or the mutually perpendicular direction x, y, z or we can say if the response is dependent on the orientation, it is known as anisotropic. That means, it is just a reverse side that an anisotropic.

It means you see isotropic material is that kind of material rather you see like if you put any kind of load in any direction at any angle, even I should say that the material properties are same. It is independent of orientation of the load application on the element. This material is known as the isotropy and if a material is very much sensitive to the load application, at what point of application is there, at what angle, they are simply applying the load. This material property is known as the anisotropic.

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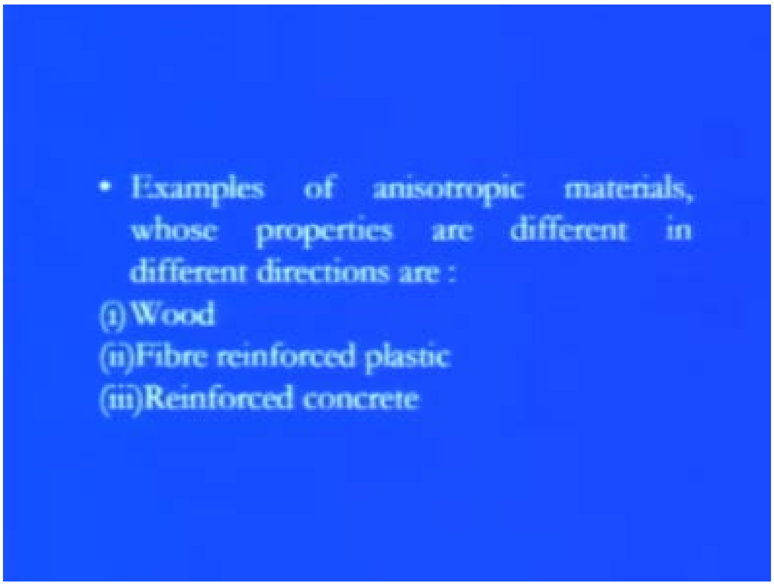

So, this is you see the both kind of features are example of an isotropic material because you see this is a very good property that you see you know like sometimes, it is showing these are you see you know like an isotropic material which are you know like which are not showing this kind of relation which very dependent on that what the load applications are there and they are exhibiting all different properties in the different directions. Examples of anisotropic materials are wood. Wood you can see you know like if we apply the load sometimes, and that is why you see the woods are characterizing. Sometimes woods are you know like always strengthen. Various types of woods are there. Some types of woods are very much strong and towards the axial loading, sometimes they are very much strengthened towards the compressive loading and that is why according to the applications there that which layers are you know like supporting what type of forces.

So, accordingly you know like we are also characterizing that actually since they are exhibiting the different material properties in the different directions. So, always you see we are categorizing this wood and anisotropic material, and then you see fiber reinforced plastic these you know like these fiber operated, these reinforced plastics are always forming in that way that they are supporting some compressive loading, or sometimes it is tensile loading, so that the layer formations are in such a way that these fiber layers are supporting in a different directional forces. So, you see since they are forming in that way only that they are supporting different properties in different directions. So, always you see you know like we can categorize this fiber reinforced plastic is an isotropic material.

Then, you see it is a pretty common example is the reinforced concrete and then, that is why you see the concrete material we are keeping in the railway tracks that you see, they can simply compress or they can absorb the axial vibrations. So, you see they have a very good compressive. So, the axial part is there. So, always this is the perfect example of anisotropic material that they are very much, they are always good in the axial or we can say the compressive part. So, they are you see actually they are showing some good properties in that, and they are not showing the similar kind of trend or similar kind of property in the other directions. So, these are you see some of the three examples which are exhibiting different material property in the different directions. So, this was the first property which is a very common property and which is a very popular property was there that the isotropic property. Then, you see we have the next property is the homogeneous property.

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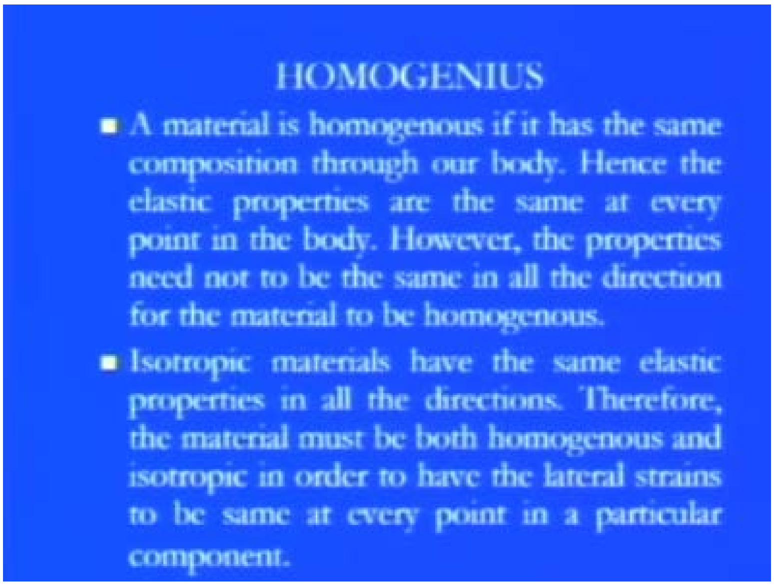

A material is homogeneous means you know like it is a kind of homogeneous mixture kind of thing is there. So, if it has the same composition throughout or body like you see our cell is there, if you take any part of our body and if you just take it off the cell, you will find that the cell characteristics are homogenous for a unit body. Similarly, you see we can say that the material homogeneous because if it has same composition throughout this particular material body, hence the elastic property are same at every point in the body. That means, you see you know like because as you apply the load since they have the same compositions, definitely they will exit the similar kind of properties under the same application of load is. However, the properties need not to be you know like the same directions of the material just like you see in the isotropic.

So, this is the clear difference though you see you know like it will show the property is same at every point, but that does not mean that actually these properties are similar in all three directions. It might be sometimes you know like the properties are different in different directions. So, homogeneous materials are those materials which have the same composition and because of the same composition, the elastic properties are same at every point of body, but that does not mean that they are same in all three directions. So, isotropic materials have the same elastic property in all directions.

So, that is the beauty of these elastic properties. So, they are exhibiting all the properties you know like the similar kind of properties in the different directions also, but here you see the homogeneous materials are showing the different properties in the different directions. Though they have individual points at any point in the body, they are exhibiting the similar kind of you know like the elastic properties because of the homogeneous mixture or the homogenous composition of the material when in those bodies.

Therefore, the material must be both homogeneous as well as the isotropic in order to have the lateral strength of the same at every point in a perpendicular or at a particular component. That means, you see if we have the lateral stress and the longitudinal means if we have a longitudinal strain and lateral strain, if you are saying that actually these are same at every point, then it must be first homogeneous just to have you know like the same elastic properties at all points, and also it should be having the similar isotropic material, so that they will have the similar kind of you know like these elastic properties in all directions.

So, the elastic properties must be same in all direction, and it must be same at all points. Then, we can say that the lateral strains are similar at every point of this kind of material and that is why you see generally we are preferred to have a material homogeneous as well as the isotropic properties. Then, you see you know like after all incorporating both the properties like you see the homogeneous as well as the isotropic, we can say that the strains are proportional to stress.

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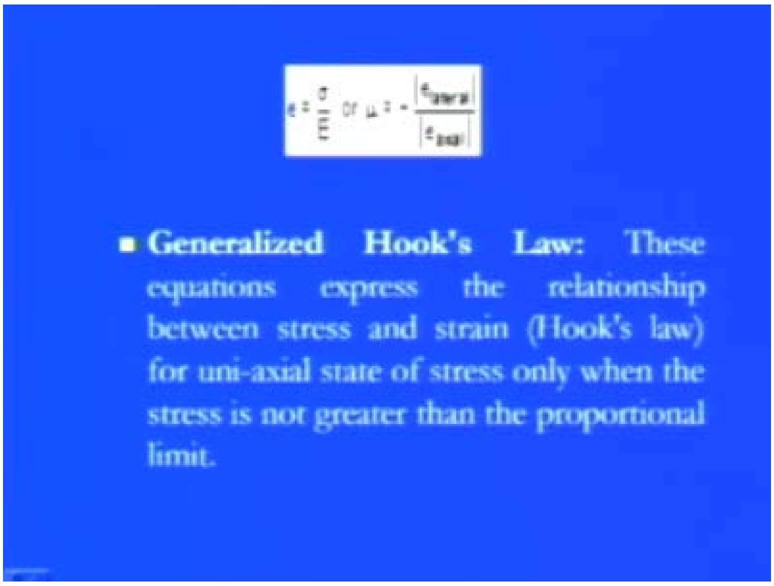

The stress we can easily measure with using of a strain gauges or rosette. Young's modulus as we discussed that it is a property of material, so it has a different value altogether for the different material like you see as we discussed in the previous case that there are six types of materials like it is perfectly rigid or perfectly elastic or perfectly plastic or elastic perfectly plastic or elastic plastic. So, you see you know like based on that Young's modulus of elasticity is having different you know like the values. So, once you have you know the strain which is measured part, once you have the Young's modulus of elasticity, elasticity which is a special properties of material, you can easily get you know like the stress part which is you see you know like the sigma is epsilon into E, or we can say they can also get you see the Young's modulus.

Sorry, this Poisson ratio which is nothing butequals to minus of this epsilon lateral divided by epsilon axial. That means you see what are the kind of deformation is there, absolutely it is based on that what the isotropy and homogeneous things are there because if it is not exactly equal in all the direction, then definitely you see it will exhibit the Poisson ratio which is you see you know like has exactly, which has a different meaning for minus epsilon. And lateral means what the deformation is there in the x direction, and what the deformation is there in the other mutually perpendicular direction irrespective of y or z. So, this is you see you know like this kind of discussion which even you know like it is discussed that this homogeneous or isotropic part is there which is exactly similar to this kind of mathematical expressions. So, now you see since we are talking about the elastic material, so we need to you know like go for a generalized Hooke's law.

So, these are you know like some of the equations which express a relationship between the stress strains within this elastic region. That is why we are saying that the Hooke's law for uniaxial state of stress only when the stress is not greater than the proportional limit because you see once it goes beyond to the proportional limit means when the stress is not proportional to strain, then you see that means, we have a kind of the permanent set of deformations are starting to exhibit. That means you see you know like we have the microstructure which has a kind of dip or which has a kind of you know like the deviation is there in between those microstructures. So, we can say that then these uniaxial states of stress you know like is not exactly proportional things. That means, the Hooke's law is not valid for those kind of you know like the stress and stress relationships.

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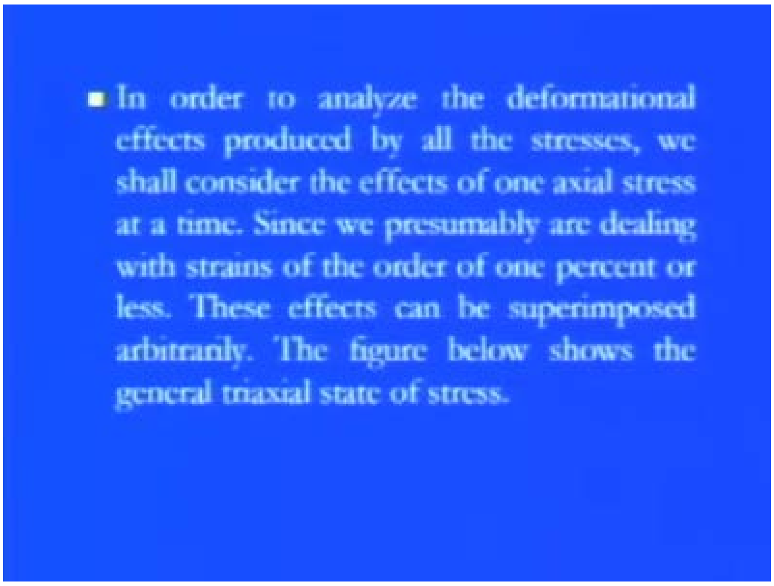

So, in order to analyze the deformed you know like that this deformational effects producing by all the stresses, we shall consider the effects of one axial stress at a time.

Since, you see we you know like presumably are dealing with strains of the order of 1 percent or even less, then that means, you see we have to be very careful that actually rather we are going beyond certain region you know like beyond certain point or not actually. Because if we are you know like dealing with the small limits of these stress and stresses, then that means, you know like whatever the deformable properties are there or I should say actually whatever these elastic limits are there, this is well stabilized and we can say the Hooke’s law is valid for this kind of relationship, and usually we can say that these effect can be also superimposed arbitrarily, because you see this id to locate the regions. Now, this is the proportional limit and then, you see the transition region slots that sometimes it is showing the elastic, sometime plastic and then, you see the plastic region starts.

So, if you see it is very much you know like it is not an easy task to exactly you know like find out those locations. So, you see these effects, whatever the effects are there of the elastic plastic or in between that are really you know like superimposing those things, and it is always taking arbitrary like that. So, now you see we can simply show you know like the tri axial state of stress in generalized way in this particular figure as we discussed.

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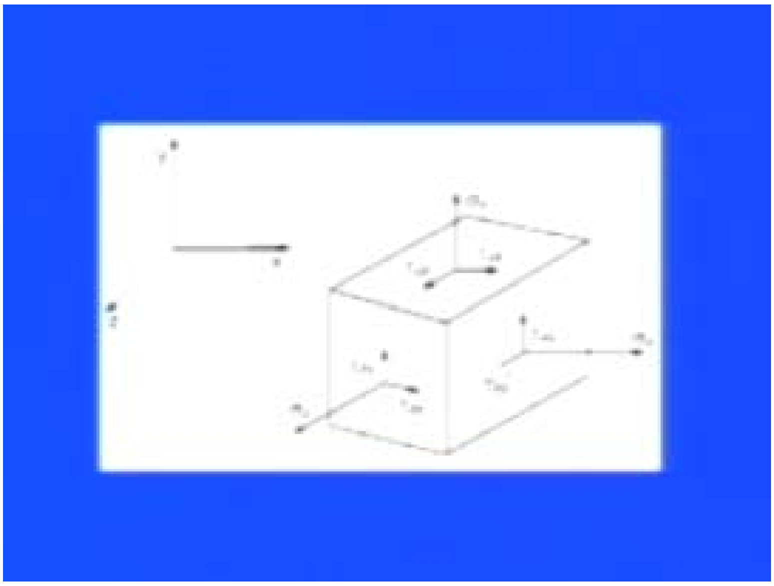

In this we have the three mutually perpendicular stresses x, y and z and you see you know like in those things always your two mutually perpendicular stresses are there. These are the axial stresses sigma x and sigma y, and you see we have the tau x, y. So, if you want to express those things in you know like the tensor way in which there are the nine components, so we can also show all those things in a generalized unit cube ways here. Here we have it is just kind of you see the difference of those things that this is my plane where you see the normal stress components are there. So, this is you see the sigma x because it is normal to this particular plane and then, you see the parallel stresses are there either we cancel the gamma x, z or gamma x, y, it is in the x domain.

Similarly, we can you know like drag this information for other two domains like if we are talking about the y domain, then you see we have the sigma y which is perpendicular to this plane and then, we also have this tau y, z and tau y, x. That means you see these two parallel forces are there which are you know like influencing the sheer stresses in this nature, and this in normal stress component is there which always inducing the normal stress component towards this body. Similarly you see you can drag the information for the z axis, where this is the plane normal to the plane you see this sigma z is there and then, when the forces are set parallely and tau z, x tau z, y and tau z, x will give you the clear cut idea about how this stress formations are and then, how these stresses are being well set up.

Now, you see here it is not exactly equal in all three directions meaning is that if the force point of this application of the force is different you know like direction, then we have all three directional stresses and these you know like the nature of the stress is different as you can see here, and the magnitude also depends on that what the effective area is where and the force application is.

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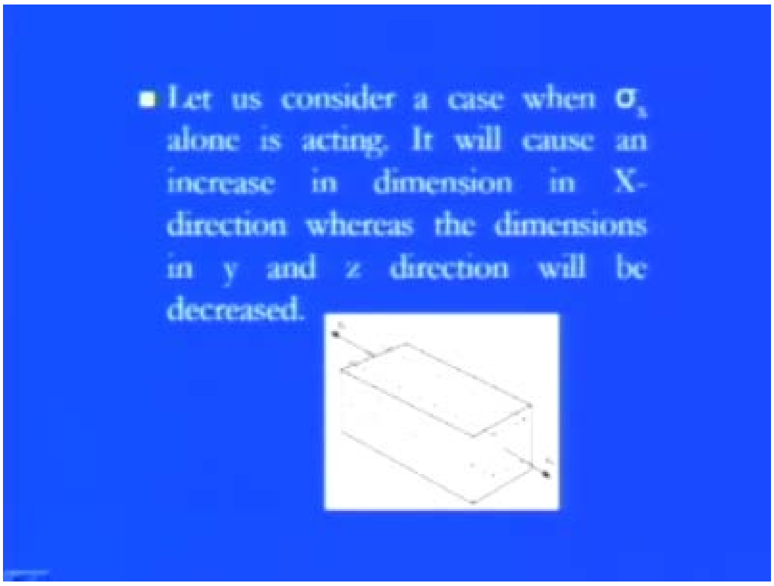

So, now you see you know like if you apply those concept into the main category, then you see we have you know like let us consider a case, where sigma x is simply acting. That means you see we have a simple axial pulling is there. Only the force is dominating in the x direction and then, what will happen means what exactly the deformations are there, that we will see in this particular diagram. So, here it is you see we have a simple cross bar. You see if you are dragging towards the outside, then we have this tensile loading is there towards you know towards the outward direction and due to that, there is a contraction in the lateral part.

So, the longitudinal stress is sigma x, but while you see in the lateral part, there is a kind of contractions are there. So, it will cause an increase in dimension in x you know like direction whereas, the dimension y and z will be decreased. So, the given figure clearly shows that there is an axial loading is there in the x direction, while contraction is there in the other direction because if the axial loading is there in the x direction that is in a pulling side, that is an extension is there in the x direction. So, we can compute you know like the strain component in all three directions because of the axial loading only.

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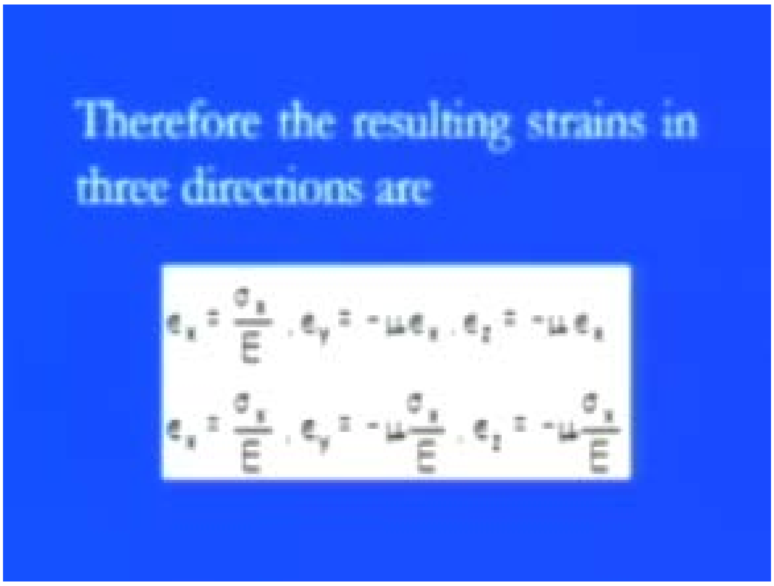

So, we have you see now the things resulting strains in all three directions are because we are pulling in the x direction. So, straight way the strain will come, the normal strain will come or we can say the direction strain will come in the x direction, that is epsilon x is nothing butequals to sigma x by E, because we can straight way measure that part of the extension, but you see there is an contraction in the other two directions. And you see if you want to set the relationship between the lateral strain as well as the longitudinal strength, always we need to consider the Poisson ratio. So, with the consideration of the Poisson ratio, we can take the strain component in the y direction. So, epsilon y is nothing butequals to minus mu times of epsilon x.

So, now you see you know the epsilon x. So, what will happen? What the contraction is there due to the Poisson ratio, this Poisson ratio is computing the total ratio of lateral strain to the longitude, the longitudinal strain to this lateral strain. That means, you see how much contraction is there, you can easily compute with this particular coefficient this Poisson ratio into epsilon x, or we can say epsilon z also in z direction. We can also calculate the strain component here that is equal to minus mu times of epsilon x. The meaning is pretty simple that once you have the epsilon x because this is my axial loading, and it is a pulling kind of that is there we can straight away get the sigma x by E epsilon x. Once you have the epsilon x by using the Poisson ratio, you can get epsilon y as well as the epsilon z also.

Similarly, you can you know like by keeping those things, you can even have those things that epsilon x is nothing butequals to sigma x by v y sigma x by E, or we can say epsilon y is nothing but equals to minus mu times of epsilon x, or we can say sigma x by E or epsilon z is nothing butequals to minus mu times of epsilon x, or we can say minus mu times of sigma x by E by simply manipulating this particular relation. That means you see we can easily get the three components of strains. The direct strains epsilon x, epsilon y, epsilon z even if it is loading in the x direction. So, similarly you see if you are talking about the y direction, you know like that if the normal stress component is there in the vertical direction, means you see now I am pulling from the vertical direction and there is you see the extension is there in the y direction. So, then what will be the impact.

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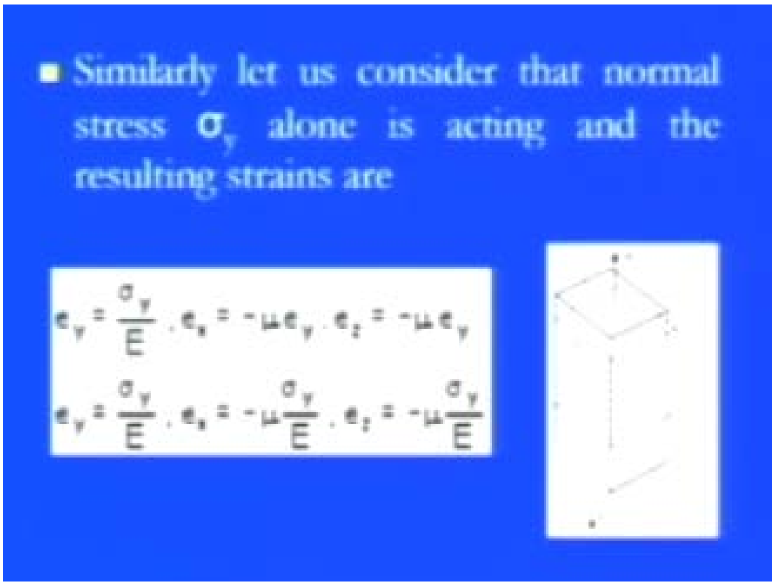

So, you can see the figure here that we have you know like the rectangular bar is there and from the extreme corner, there is an extension. So, sigma y is there and due to that, you can see this you know like the kind of extension here. So, this is the extension 1, this is the extension 2. So, once you have the extension in y direction, straightway you can calculate the strain component to the y direction and that strain component is known as the linear strain or the normal strain. So, it equals to sigma y by E or you can say you see the remaining part which is contracting. So, using the Poisson ratio as we discussed in the previous case, again using the Poisson ratio we can calculate the strain component in the other two directions.

So, epsilon x is nothing butequals to minus mu times of epsilon y. Epsilon z equals to minus mu times of epsilon z or we can say epsilon y which is the main you know like the coordinating, I should say the dimension, because the load application is there from that plane only which is equal to sigma y by E. So, you see here epsilon x also can be calculated with using of the required available information minus mu times of sigma y by E and epsilon z. It can be also calculated by mu times of sigma y by E. So, these you see you know like again just a load application is there in the one direction, but we have you know like the strain component in the three mutually perpendicular direction that what required deformation is going on, or what the required you see you know like the change of dimension is there, where due to the effect of one axial loading. Similarly, you know like we can go for the other third direction that is the z direction.

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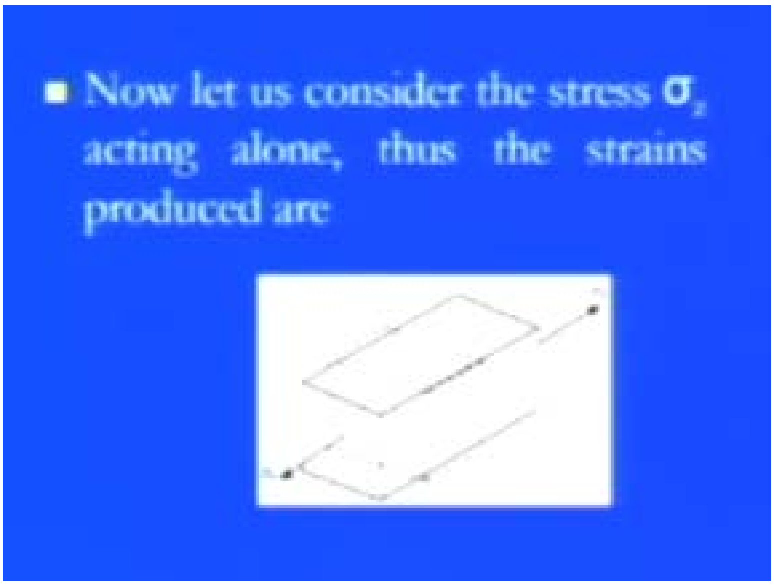

So, you can see the figure here along the z direction, and then again you see the required you know like the strains again will come in all three directions.

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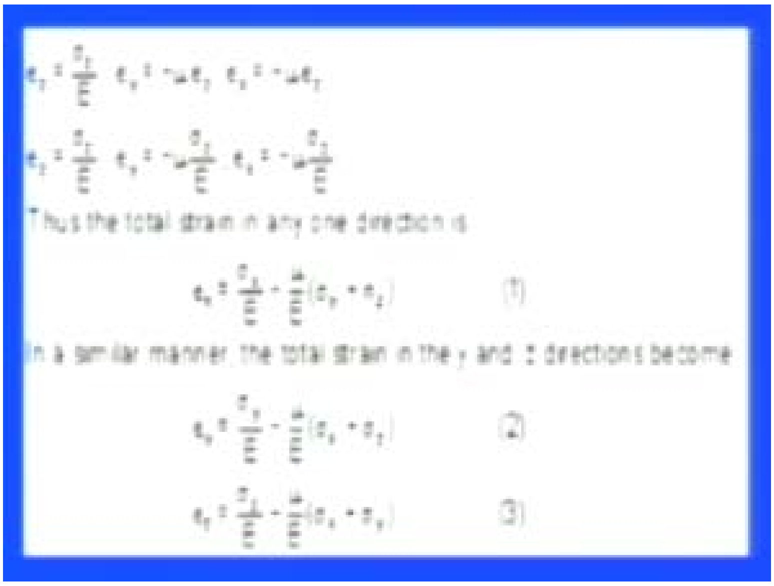

That will be equal to see epsilon z which is equal to sigma z by E because this is the dominating parameter through which this deformation or we can say the deviation is going to go on, and then you see you know like the other parameters will come as sigma E by z minus mu times of sigma z minus mu times of this sigma z for other two directions irrespective of the epsilon y or epsilon x. So, if I compute that part, then you have epsilon z which is equal to sigma y sigma z by E, but epsilon x is minus mu times of sigma z by E and epsilon y is nothing butminus mu times of sigma z by E. The meaning is pretty simple. The load application is there, individual load application is there in all x or the y and z direction, but there is an impact of even if you are taking the individual component, it is an impact of other two directions also which we can compute.

So, if I am saying that actually all three mutually you know like perpendicular directions are actively attaining and the load application is there in the respective directions, so we can say that the total strain in any of the one direction at any point if the load application is there in all the direction. Then you see epsilon x is nothing butequals to sigma x by E minus mu by E because you see you know like now again the very important feature is that this load application is under the elastic limit only. So, minus mu by E, the Young's modulus of elasticity times sigma y plus sigma z. So, you see other two you know like the stress components are always present in the extension of x direction, but they will come with the modulus of elasticity as well as the Poisson ratio.

Similarly, you see in that manner also, we can contain you know like the total strain component in y and z direction as well. So, epsilon y is nothing butequals to the main coordinating parameter sigma y by E minus the combination of other two directions minus mu by E into you know like the sigma z plus sigma x. The meaning is again simple. There is a minus sign there because of the contraction in other two directions because if you pull in one direction, always there is you see the reduction of the dimension is there in other two directions. So, that is what you see minus sign will always be introducing in those things, and if you are counting the epsilon z in the z direction and that load application is in the z direction, then epsilon z is nothing butequals to sigma z by E minus mu over e into sigma x plus sigma y.

So, you see here if you are you know like going for a generalized part, then you will find that we can simply compute these strain component in x, y and z correspondingly with the combination of sigma x, sigma y, sigma z in individual direction also. So, in this following analysis, sheer stresses were not considered. That is the key point here you know like till that only we found that sigma x, sigma y, sigma z because you see these normal stress components are only acting and due to that, you see we have the deformation and it can be complete of using of this Poisson ratio and the direct stress component. So, that part we discussed, but if the amazing thing is still there is no sheer component are there.

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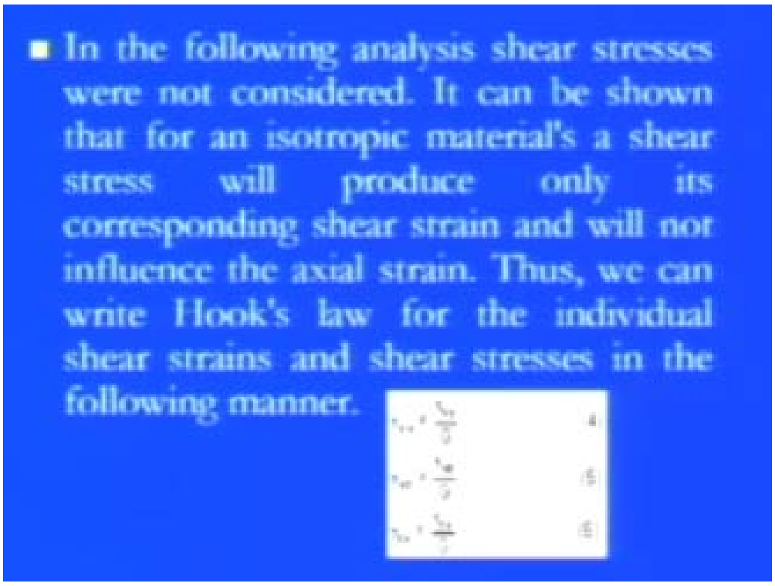

So, it can be shown that for an isotropic material, a sheer strain will produce only its corresponding sheer strain. It means you see there is no sheer stress. The sheer strain component will come in form of deformation until and unless the sheer stress is not there. You see we cannot you know like influence the axial strain and because of the axial strain, even we cannot say that the sheer strains are there because you see the deviation in the angle is. Thus, you see we just want to write the Hooke's law for individual sheer strain and stress in the different manner. You see here if the sheer stress is there, this mean the gamma x, y will induce the sheer strain in this particular you know like due to tau x, y shear stress, we have gamma x, y. So, gamma x y equals to tau x y divided by G.

So, now you see here as far as the axial strains are concerned or the normal strains are concerned, a new modulus of property was there that the Young's modulus of properties, but here you see since we are talking about the sheering of plane, so due to the sheer stress present as we said that the sheer strains are there. That means, you see this Young's modulus of property is not at all valid. Then, we need to define the new modulus, that is the sheer modulus of elasticity and that is denoted by G. So, G is only valid whenever the sheering is there, and E is only valid whenever the axial stress and axial strains are there.

So, you see these two different properties of materials are there, and they are exhibiting the different value in terms of types of load. If the tensile loading is there or the compressive loading is there, then E will come. If the sheer loading is there, then the G will come. So, if we are talking about the x y plane and that the sheer stress is there at this tau x y, so the sheer strain will come down. X y is equal to tau x y divided by G, or we can say similarly in other two planes y z and z x. Similarly, the sheer strains will come and the sheer strain in y x plane means if we have the y and z plane, then this gamma x y is nothing butequals to tau x y divided by G or we can say also if we are talking about the z x plane. Then this gamma x z is nothing butequals to tau x z divided by G, but these relation within the sheer strain and sheer stress is only valid when elastic limit is there.

So, you see here all equations are straight from first three. This epsilon you know like x, epsilon y and epsilon z or we can say this gamma x y, gamma y z and gamma z x, all six equations are known as the generalized Hooke's law and you know like constitute the equations for the linear elastic isotropic material. That means you see here this is only valid when we are taking the material property within the elastic region of the isotropic material. Then, we can say that all six equations which we have shown here, 3 for the normal strain components, 3 for the sheer strain components are only valid. When these equations are used as written, the strain can be completely determined you know from the known values of the stresses always because you see you know like whenever the load application is there, the stresses are first forming and because of the stress formation, the strains are there. If you want to maintain the equilibrium, for that always there is a counter you know like evidences are there in form of the stresses only.

So, the strains are always coming due to the stresses and if you want to compute the strains, the stress, the components of the stresses are always there in that. So, to engineers the plane stress, you know like situation is a much of reference because you see always we are keeping, always we are talking about the plane you know like that what x y plane is there, or y z plane is there, or z x plane is there. That means, you see you know like the third axial stress or third axial strain is always going into the zero. So, either sigma z or tau x z or tau y z means the normal stress component, or you know like this, other sheer stress component, other directions are always keeping zero. So, we can say the above set of equations whatever we have written in the previous six form, it can be again you see reduced to the new form because if we are keeping sigma z is zero, we are keeping tau x z is zero, we are keeping tau y z is 0.

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$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\epsilon_z = -\mu\frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} \text{ and } \gamma_{xy} = \frac{\tau_{xy}}{G}$$

Their inverse relations can be also determined and are given as

$$\sigma_x = \frac{E}{(1-\mu^2)}(\epsilon_x + \mu\epsilon_y)$$

$$\sigma_y = \frac{E}{(1-\mu^2)}(\epsilon_y + \mu\epsilon_x)$$

$$\tau_{xy} = G\gamma_{xy}$$

So, the new form of these six equations is first the normal strain component in the x direction. Epsilon x is nothing butequals to sigma x minus E sigma x divided by E minus mu times of sigma y by E because minus mu times of sigma z by E will be gone. Similarly, you see for y direction, this epsilon y is nothing butequals to sigma y by E. This is dominating parameter minus mu times of sigma x by E because minus mu times of sigma z by E will be eliminated here because of the plane strain and plane stress. Third is you see the epsilon z in the z direction which will come due to the load application in the x direction. So, you see here minus mu times of sigma x by E minus mu times of sigma y by E, or you can say minus mu times of sigma x plus sigma y by E.

So, you see it is the induction or we can say the interaction effect of these two stress component x direction as well as the y direction, or also we can say that you see the stress component in other direction, x y direction, it can be easily be that tau x y is nothing butequals to gamma x y by G. So, you see these are the three equations and if you see you know like the inverse relations, it can be also you know like well set up which can be also determined with the following relations that sigma x is nothing butequals to you know like E over 1 minus mu square into epsilon x plus mu times of epsilon y. That means you see now we have the total like if you know the deformation in x direction, if you know the deformation in y direction, you can set the relation with using of Poisson ratio and Young's modulus of elasticity, and you can get the value of the stress in the axial direction.

So, if you want to compute the stress in x direction, we can easily compute with using of E Young's modulus of elasticity divided by 1 minus mu square Poisson ratio square into epsilon x plus mu times of epsilon y. So, you see epsilon x and epsilon y can be miserable, put those value and get the value of sigma. Similarly, you see you can also get the value of sigma y which is again you see computed E over 1 minus mu square epsilon y plus mu times of epsilon x because you see here irrespective of sigma x and sigma y, what kind of load application is there which is more of dominant nature. So, we can easily use these things. You see as far as the sheer stress is concerned, again it is pretty simpler you know like simple to get those sigma x y which is equal to modulus of elasticity G times this gamma x y.

So, you see you know like these either irrespective to a straight relation because now the six, you know like these six relations can be merged into four relations of the generalized

Hooke's law as we can see here these three normal strain components, the sheer strain component. So, these four relations are only applicable whenever we are you know like using for the plane stress and plane strain, or if we are talking about you see you know like the inverse problem, but we can easily compute sigma x, sigma y and this tau x y for our calculation part to observe the stress you know like values if you know the strain values.

So, Hooke's law is probably the most well known and widely used. This constitutive equation which you know like one cannot say that all the engineering materials are linear isotropic ones because you see this is the perfect limitation of the Hooke's law is that we cannot say that actually these all you know like what are the materials which are available in this universe. They can show the perfectly you know like linearly. First linear means the stress is proportional to strain and the elastic also within this elastic region. They will show anisotropic. So, like we can say, this is the perfect limitation for the Hooke's law that these equations are valid within this region only, and that is why you see it is not we can say generally acceptable for all the materials which are you know like applicable in this universe, because now in the present times, new materials are being developed. In everyday you see these fibers, composite polymers, all those materials are available and they are exhibiting the unique features. They are for a specific kind of application.

So, you see many useful metals exhibit the non-linear responses because generally that is why whatever the metal which are you know like or we can say whatever the responses are available in this universe, or we can say the realistic part which is very close to the nature are always non-linear. You can take any example and you will find that any process, any material, they are always you see exhibiting the non-linear responses. So, that is what you see the first when they are exhibiting the non-linear responses between the stress and strain. We can say that you see you know like this Hooke's law is not at all valid for those things.

So, again you see we have to be very careful that actually if they are showing the non-linear relationships between the stress and strain, or if they are not showing any elastic region, straight way if some sort of plasticity is there in between that. Then we can say that this Hooke's is not at all valid, but the beauty of this Hooke's law is you know like if you are applying the load and if it is within the linear elastic region, whatever these relations which we have shown previously, all the six relations for all three dimensions or four relations for two mutually perpendicular directions, they are absolutely valid and it is pretty easy to calculate those required parameter for that.

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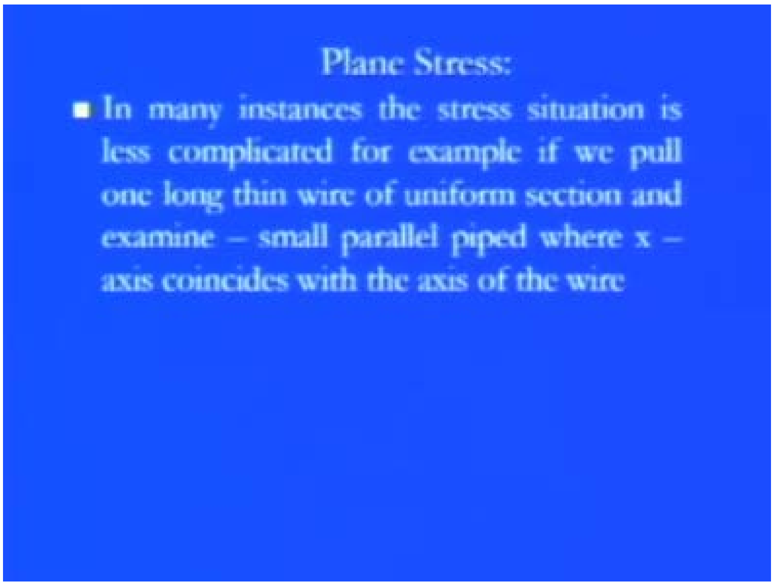

So, now you see as far as the plane stress is concerned, in many of the instance the stress situation is less you know like complicated. For example, if we pull 1 long thin wire of a uniform section means if you have a thin wire of uniform section and if you are pulling, then always we will find that actually it is you know like more of the uniform nature. The load application is pretty uniform and whatever the area reduction is there, this is also you see a sort of symmetric. There is no stress concentration is there at the different points kind of that. So, we can say that actually you know we can simply find the small parallel piped it which is symmetrical you know like in the x direction, where the x is always you see towards that part, towards the main direction where the load application is there, and only we need x as well as the y direction for total description of the stress.

So, we do not have to go for the complicacy of those things. So, generally you see whatever the material like you see we have you know like the RCC rods are there for making the building, so always we found that actually they are having a good tensile loading is there. So, that means only to analyze that kind of thing we know that what kind of loading can come on that way. Axial loading is there or you see whatever the compressive part is there, but it will come in the plane part only. So, we do not have to go for the complicacy to analyze those kinds of things. Only you see a small region and the simplicity of the stress are well you know like stabilized, and it is you know much of the information is already being like described from those kinds of thing. So, this plane stresses are always you see playing a key role for ease of analysis.

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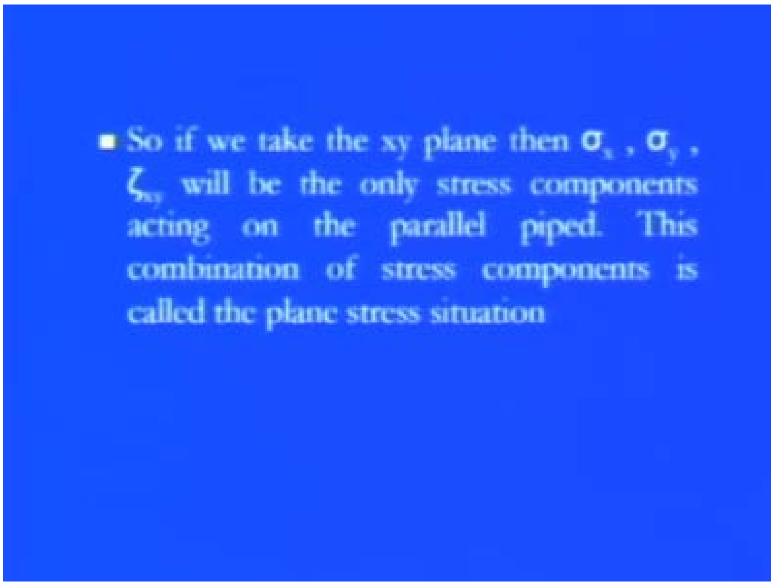

Then, you see if we want to take any x y plane, any of that you see all three you know components or stresses can be easily described. As I told you in the previous lecture you know like slide that sigma x, sigma y and tau x y, easily you know like can be described on the plane when you have a simple parallel piped. Then, you see sigma x is there in the x direction sigma, y is there in the mutually perpendicular direction, and this tau x y is there in this parallel surfaces are there for the clockwise direction or counter clockwise direction as far as the rotation is there. So, we can easily show their nature of that how they are you know like what is the nature tensile compressive clockwise, anti-clockwise as well as the series concerned and also, we can show that actually you know like because of these things, what will be the combine effect on this parallel piped.

So, this combination of the stress components are always called the plane stress situation that actually true. Mutually perpendicular stresses x y and you see the sheering is there on the top of surface. So, that is what you know like the plane stresses are you know like well stabilized. The stress formation is there and if you are going for the complication like in that the tensile form of the stresses always, it gives you a complicacy and you know like for calculating all those parameters and all the different kind of loadings are there, it will be very difficult for you know like to analyze things. So, that is what you see we always prefer to be analyzed based on the plane stress as we discussed in this slide.

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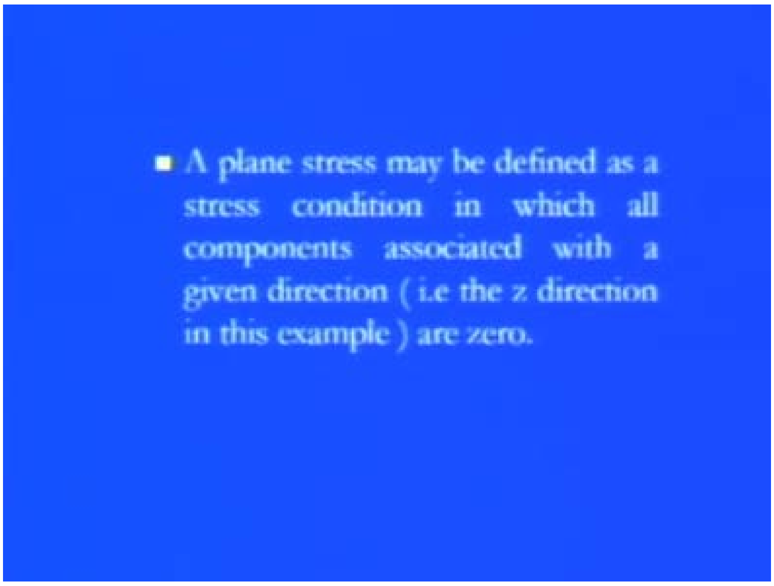

So, now you see a plane stress may be defined as the stress condition in which all components associated with the given direction like you see if you are saying that the z given directions are supposed to be zero like you see if I am saying that the x y is there. So, always sigma x, sigma y, tau x y will come. So, other components, the z component will be gone out. So, sigma z, tau x z, tau y z will be gone out or if I am talking about the y z plane, then x component will be gone, sigma x will be gone out or tau x z, tau y z will be gone or if I am talking about the y z plane, then only the respective parameters will come like sigma z, sigma y. If I am talking about z x, then sigma z and sigma x will be there and tau z x will be the dominating parameters.

The meaning is very simple. Whatever the plane which you are considering the dominating parameter in terms of the normal stress and the sheer stress will easily you know like located, and we can simply see the observation of that how they are you know like putting their effort and how we can (( )). So, what is the combined effect is there of those stress component. So, now you see the last part of our lecture is the plane strain because as you know that the plane stress is playing a key role for the formation of the stresses in these. Then, what about the plane strain?

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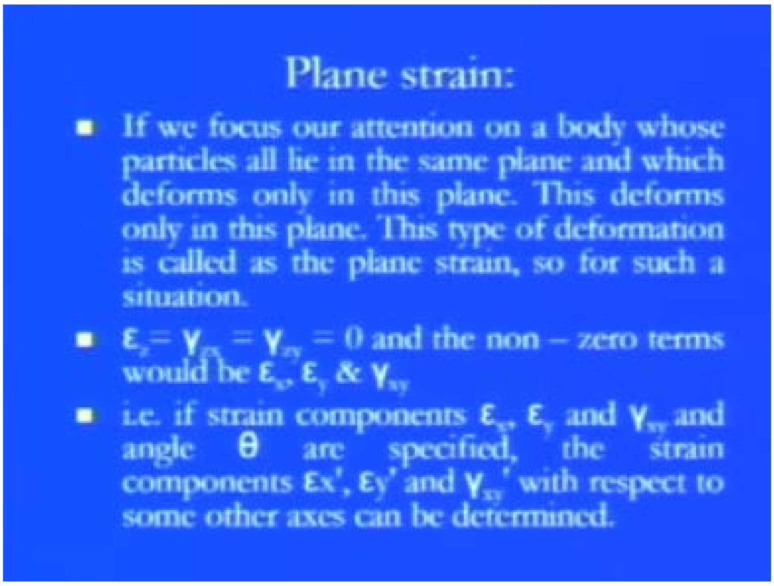

So, if we focus our attention of a body whose you know like the particles are lying in the same plane, there is no change in the plane. They are exactly well set up within the plane which deform only in the plane only. Then, you see we do not have to go for all the other parts of the deformation and complicatedly you see analyze those things. So, this deform only in the one plane. This type of deformation is called as the plane strain, and you see you know like this situation is generalized situation.

So, if I am saying that actually if I am taking x y plane, where the plane stresses are there in x y, and sigma x and sigma y, and tau x y are the dominating stresses are there in these things, the third stress component and the normal as well as the other two sheer stress component, these are zero. Then, similarly you see the strain components are also you know like deriving from those kinds of relations and we can say that this epsilon z will be also zero because you see the stress component is there. Sigma z is zero. So, obviously, you see epsilon z will also be zero and similarly, you see the other two components of the sheer strain, the tau z x and tau z y will be also zero because you see in the plane stress also we compute that the tau z x and tau y z will be zero. So, gamma x z and gamma z y, this will be zero because you see we are not considering any kind of deformation in the z direction, irrespective whether it is normal or sheer strain is.

So, we can say the non-zero terms are only four terms, only three terms. If we are taking this x y plane, one is the epsilon x and epsilon y, these are the two normal strain component and third one is the gamma x y which is you see that is the sheer strain component. So, if strain components epsilon x, epsilon y and gamma x y are always presenting at the end of theta is specified, and it is pretty simple you see you know like it will give you all the kind of required information which you want to analyze for a plane strain. If you want to cut you know like the inclined plane, and if you want to measure, then what will be the strain component as there epsilon theta and this gamma theta and you can easily get with you know using these three components epsilon x, epsilon y and gamma x y.

So, we can say that actually these components are well in you know like it will give the whole information about any kind of body. So, that is what you see. You do not have to go for all you know like the six parameters within this generalized Hooke’s law. Only these three parameters are well enough to you know like give all kind of information with respect to some other axis can easily find it out. So, this is you see the plane strain. So, if the plane strain and plane stress of that, that is why you see generally if we go further, our analysis is mainly based on the plane stress and plane strain. That is why you see we discussed them thoroughly here that y plane stress and plane strains are necessary and good enough to analyze any kind of structure in which these three types of loadings are there. Two mutually perpendicular you know like the normal loading and the sheer loading is there.

So, you see in this lecture, we discussed about that you know like if we have the different variety of the material and how we can express the stress-strain curve, the idealized stress-strain curve for you know like it is exhibiting the elastic as well as the plastic region. If we have you see you know like if you are talking about only the elastic region, then there are two important properties which we discussed that isotropic property that you see all the material property which if the material is showing all its property equal in all directions, and it is isotropic property. And if you see material showing at every point, the equal property not in all directions, then it is homogeneous.

So, these two properties are very important and if you are saying that if you have a material which is linear following the elastic and isotropic material, then there are six you know like equations, three for the normal stress component and normal strain component and three for sheer stress and sheer strain component in which you see we defined the Young's modulus of elasticity for the linear you know like this normal strain component linearly,. And if the sheer modulus of elasticity for the linearly sheer properties for you see where the sheer stress and sheer strains are there and we set up the relations.

In the last part we discussed about that if we have you know like the plane stress and plane strain, then we do not have to you know like consider the third direction. We can simply put the zero you know like instead of going to the complicacy of the analysis. So, you see sigma y or sigma z, sigma x and sigma y is well enough in the normal stress condition, and if you want to go for the sheer stress and tau, yes the tau x y is well enough for the required you know calculation. So, see this part which we discussed in the lecture and in the next lecture, you see we are going to discuss that actually how you see you know like we can put if you see we have you know like go beyond the elastic limit. That means you see if you have general, let us say the tensile bar is there and if you are pulling that part, then how the material you are just talking about a general mild steel metal which I told you that it is general elastic and plastic material. If you are applying the tensile loading and how it will exhibit not only the elastic region which we have discussed right now for Hooke's law.

If you go beyond those things that what are the different stresses, stress components are there like you see the yield stress ultimate tensile strain, because the tensile loading is there and then, you see if you are going up to the fracture, then what will happen, what kind of specific shapes are there at the time of rupture or fracture. If we are taking about the mild steel and if it is a different material altogether like you see if we have a material of you know like elastic, but plasticity is there, then you see what kind of a shape is there. So, this kind of you know like the relation which we want to set up and not only these homogeneous and this isotropic isentropic properties, they are well enough.

We also you know like need to define that actually what will be the strain hardening is there if you want to compute a straight way from the graph itself that what will be the you know like this rigidity or the hardness, or any other you know like the property of the material. And how we can get and what are the impacts of these properties on the behavior of the stress and strain, or the behavior of the material to get the responses in between the stress and strain we can easily set up. So, you see this kind of relations which we are going to discuss about you know like the stress strain for the different material in the next lecture.

Thank you.