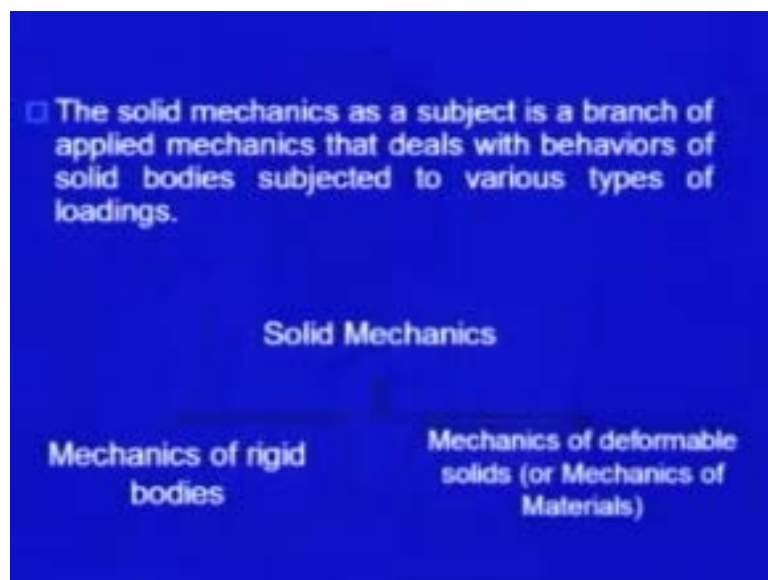


**Strength of Materials**  
**Prof. Dr. Suraj Prakash Harsha**  
**Mechanical and Industrial Engineering Department**  
**Indian Institute of Technology, Roorkee**

**Lecture - 1**  
**Solid Mechanics**

Good morning. This is Dr Suraj Prakash Harsha, faculty of mechanical and industrial engineering department at IIT Roorkee. I am going to teach the basic course of the engineering branch like the solid mechanics in which we are mainly dealing with the deformation or whatever the mechanical forces are there or mechanics of the rigid bodies and the deformable bodies.

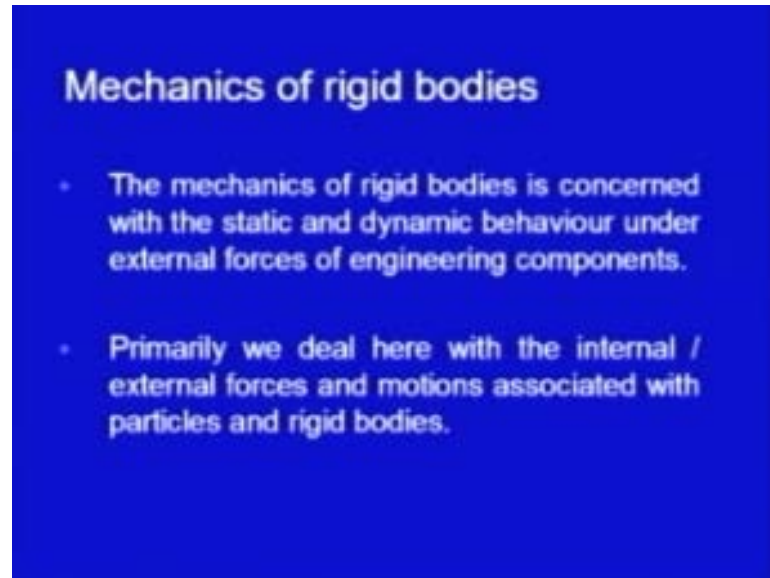
(Refer Slide Time: 00:57)



As you know, the solid mechanics as a subject is a branch of applied mechanics; in which mainly dealing with the solid bodies subjected to the various types of forces or loadings. So, if we want to categorize this solid mechanics, there are two broad categories, because here there is dynamics or static forces, are there in the two solids when they are interacting or they are not interacting from the external excitation forces. So, solid mechanics if we are dividing into two main categories, then these are mainly with the mechanics of rigid bodies. If we have rigid bodies, then what kind of mechanics is there within the rigid bodies or outside with the environment of rigid bodies or the second is the mechanics of deformable solids or we can say rather it is a mechanics of

materials or we can also say that it is strength of materials. So, these are two main categories under the solid mechanics.

(Refer Slide Time: 01:53)



First of all, as we are categorizing this, the mechanics of rigid bodies; in which the mechanics of rigid bodies is mainly concerned with the static and dynamic behaviour under external forces of engineering components. Now, if we are dealing with any engineering problem, we know that there are lots of machineries are there and these machineries are always under the operations of let us say the high speed operations are there or heavy load operations are there; in which we know that actually individual components of these machinery are just bearing the static or dynamic loads.

So, we need to deal that, actually what kind of deformations are there under the static and dynamic forces. So, primarily, if we just concern with the surface level, we are mainly dealing with the internal or external forces. And, the motion associated with these particles or kind of bodies – rigid bodies due to these internal or external forces, because as we apply the forces, definitely there is a kind of motions are there of these particles of the component or we can say the rigid bodies.

(Refer Slide Time: 03:08)

**Forces:** Tension forces present in towing ropes or lifting hoists, where compression occurs in the legs of chair / bench as one sit on it. There are number of different ways in which load can be applied to a member.

- Static or dead loads i.e. non fluctuating loads, generally caused by gravity effects.
- Live loads, as lorries crossing a bridge.
- Impact or shock loads caused by sudden blows.
- Fatigue, fluctuating or alternating loads, the magnitude and sign of the load changing with time.

First of all, as we are discussing in this, the force is the main thing. The variety of forces are there within the rigid bodies or with the environmental of the rigid bodies. First of all, like the tension forces or we can say rather the axial pulling forces, which are always presenting any kind of machinery or the general applications just like here – the towing ropes or lifting hoist. If we are just lifting any object through the ropes or hoists, we are always facing the tension forces, because this is – the axial pulling is there.

While the second kind of the forces are also be existing within the object, because of the load application is there through the compression. So, if we compress the things, the compression occurs in the legs of chairs or benches as you are sitting on it. So, these are all we can say the kind of axial forces are there, because the forces are acting along the one axis; either you are just pulling the object through ropes or hoist, or if you are just sitting on the chair and this legs of chairs are just compressing because of the axial forces. So, these are the number of different ways, are there through which we can apply the load on an object.

So, again there are variety of categories are there under which one is the static or dead loads. These are... The static means actually the magnitude and the direction of the loads are not changing with the time. So, they are just constant. Or, we can say if you apply let us say the 10 newton of force or we have an object of let us say some 10 kilogram; so if you are just pulling or if you are just sitting on a chair, then this kind of force is a static

or dead loads are there. And, that is also known as non-fluctuating loads generally caused by the gravity effects, because if you are pulling or if you are sitting on the chair, the gravitation acceleration is always associated with the mass of the object. So, that is why the gravitational effects are always there under the static and the dynamic loads. Here... So, because... As we have discussed that, actually these live loads are there; so as we are just watching, as the bridge is there under or over the bridge, there are variety of these vehicles are just moving. So, these are just the live loads are there. These loads can be the tensile or the compressive loads. But, the magnitude and the direction of the loads is continuously changing with the time as the variety of trucks or the buses or the different vehicles are moving on as a bridge. So, the real loads – generally we can term as the live loads.

The third way – the type of load in the third way of loading, is the impact or the shock loading. These are always caused due to the sudden blows. So, actually this is also one of the basic load. And, if we want to design any component or any object, we need to focus that, actually whether the load is the steady or it is the impact load or it is a shocking load is there. And, the fourth way is the fatigue or fluctuating. As we discussed the first one, that was the static and the dead loads; that means the magnitude of load is not changing; and it is not changing with the time; while the fatigue or fluctuating – it is always changing; the magnitude is changing or the way of application of the load is always changing with the time. So, fatigue or you can say the fluctuating or the alternate loads; the magnitude and the sign of the load changing with time. This is the main load through which the failure of machine always occurs.

(Refer Slide Time: 06:33)

The externally applied forces are termed as loads. These externally applied forces may be due to any one of the reason, as.

- Due to service conditions
- Due to environment in which the component works
- Through contact with other members
- Due to fluid pressures
- Due to gravity or inertia forces.

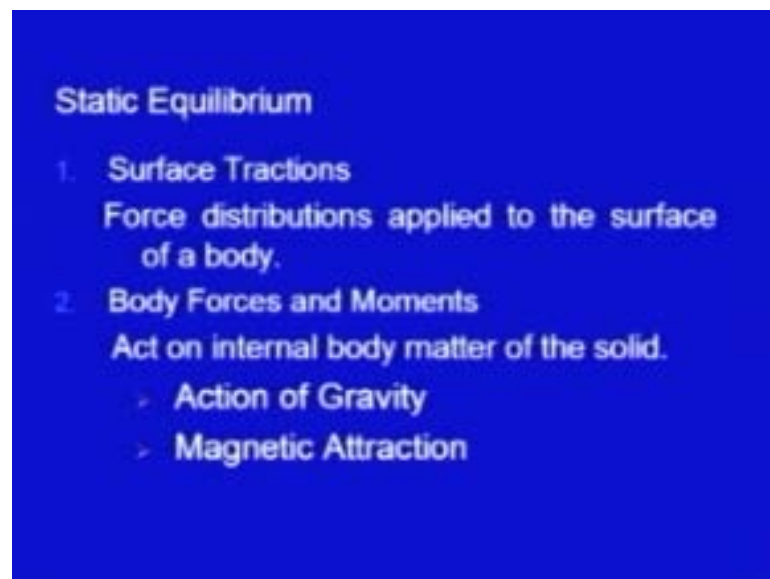
So, we are discussing about the loads. So, again you see as we have discussed that, if we have a rigid body or the deformable bodies, which are subjected to the external applied forces, we need to assume that, under these applied forces – externally applied forces, the system or the object will be in equilibrium position. So, these externally applied forces may be due to the variety of reasons due to the service conditions. If I have an object and it is... or rather I can say that, if I have a machine element, always there are different service conditions are there through which the external forces are to be applied on the body.

Second – due to environment in which the component works. Just like I have an object, which is just there in the open atmosphere; always there is a kind of environmental force is there or we can say the gravitational force is there on that body. Or, if I have an object, which is there within the machine; so what kind of the environment is there; that means if we have any lubricant is there; so definitely, there is a less force is there. So, the external applied forces are subjected to the condition, where – what kind of environment is there in which the object is existing. The third is there through contact with the other members; through which actually this is the basic thing in the machinery. All machine components are just connected with each other; and whatever the force or the torque or we can say the speed is transforming through this connectivity. So, we can say that, all components within the machine or the external, what I can say the objects – they are

always be associated... If they are always be associated with the contact, they always bear the different forces through the different contact members.

The fourth one is that, due to the fluid pressure. Fluid always... Either we can say the buoyancy forces or the hydrodynamic forces are there. These forces – always due to the fluid pressures, because if the fluid is always having some sort of strength through which it always provides the impact or the steady state forces. So, that is why these are also under the category of external applied forces. Then, we have due to gravity or due to inertia forces. These are always presented, because if we have the solid mechanics, then due to friction, always some sort of the frictional forces are there. Or, if we have the fluid, then always the viscosity or viscous forces are there. So, these kind of forces always presented if the situation is like that or the environment is like that. So, these are certain situations through which we can say that, the external applied forces are there within the object.

(Refer Slide Time: 09:06)

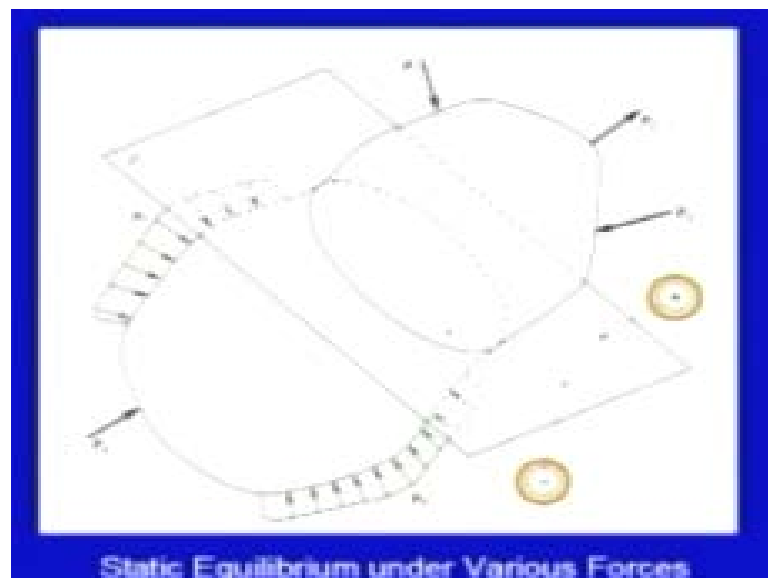


Now, as we discussed previously that, actually we can only analyze the different forces on object if the forces are existing within the object and the static equilibrium. What do you mean by the static equilibrium? Even the force is dynamic or if the force is we can say the impact, we can only analyze the object if the system is under static equilibrium, because if the forces are not set up within the system, we cannot analyze the system perfectly. So, either it is a rigid body or it is a deformable bodies, we need to or we must

apply the condition of the static equilibrium. That is the first and basic equation; only after that, we can analyze a system. So, in this category, first, we have the surface tractions – the force distribution applied to the surface of a body.

Whenever the force is there, the important thing in the force is the application – means where the force applied – means what is the line of action is there of the force, because through which, we can easily find out that, what is the nature of force is there. So, the basic thing in the force analysis is the line of action of force. The second is the body forces. If we apply the force, there is always the internal resistance is there. So, there are internal body forces are there, which always oppose if we apply the external forces. So, there are two categories if we can say the action of gravity, because if you put the object in the open atmosphere, definitely, there is a gravitational force is there. And, if the object is under static equilibrium, then definitely there are some internal body forces; they are resisting this action of gravity. The second is the magnetic attraction. These are all electromagnetic forces. So, these are all the internal body forces, which always comes to the picture when you apply or when object is there within these conditions or the environment.

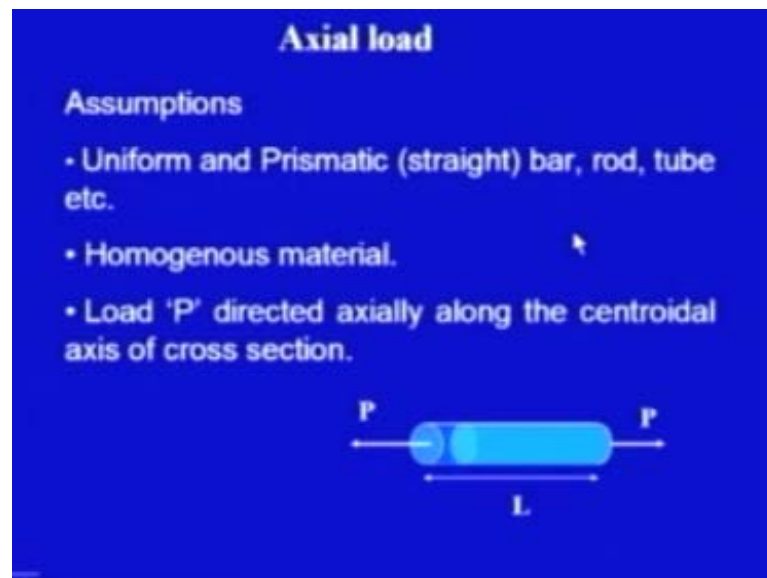
(Refer Slide Time: 11:01)



So, we have an example here. We have an object, which is under the variety of forces. So, we have different forces. And, these forces are applying through the line actions. As we have this  $P_1$ , the line of action is this one; the  $P_2$  – the line of action is outer

towards; and this is there; we have P 3 and P 4. So, we can define the nature of forces through the line of actions; so how they are acting. So, like if we have like P 1, P 3 and P 4, these are all compressive forces though they are – the line of action of these forces are different, but they are all compressing the object. While we have this P 2, the line of action of P 2 is like this, but it is a tensile force, because there is an axial pulling is there. And, if we want to see the internal body forces, then we need to cut through this plane. Let us say I have a q plane; I want to cut this object through this plane, I can easily visualize that, under these forces, this object is under static equilibrium. So, how these internal body forces are set up to resist these all forces. So, we can also visualize these body forces as far as the surface traction these forces.

(Refer Slide Time: 12:16)

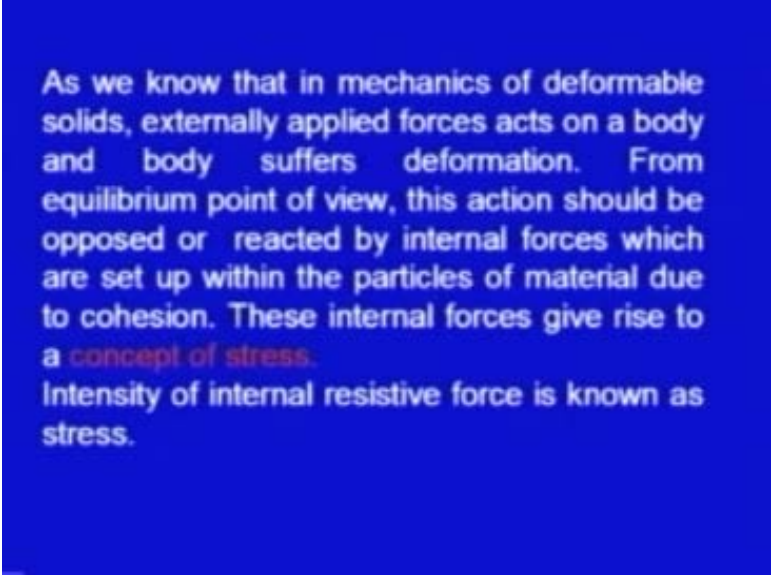


So, as we discussed about the forces, now we have the applied load or axial load. Axial load – the basic nature of axial load is actually that, the load is always applied through the axis or we can say the centroid axis. So, there are certain assumptions through which we can say that, the axial load is applied on an object. First of all, this object – whatever the object we have; rather we can say the bar, rod or tube, it is always having a uniform and prismatic cross section. That means the cross section must be uniform; there is no deviation is there in the uniform; the cross sectional of this area; and it should be straight; or, we can say rather prismatic. The second assumption, which is must that, the material of either the bar, rod or tube must be homogenous and isotropic. The third one, which is the basic condition, because we already termed that, this is the axial load; that



load whatever the P or F is there directed towards the centroidal axis – either the tensile is there or it is either the compressiveness is there. So, we have one object; we can say it is a bar of length L; the axial forces are there towards the outer parts along the centroidal axis of the bar.

(Refer Slide Time: 13:40)



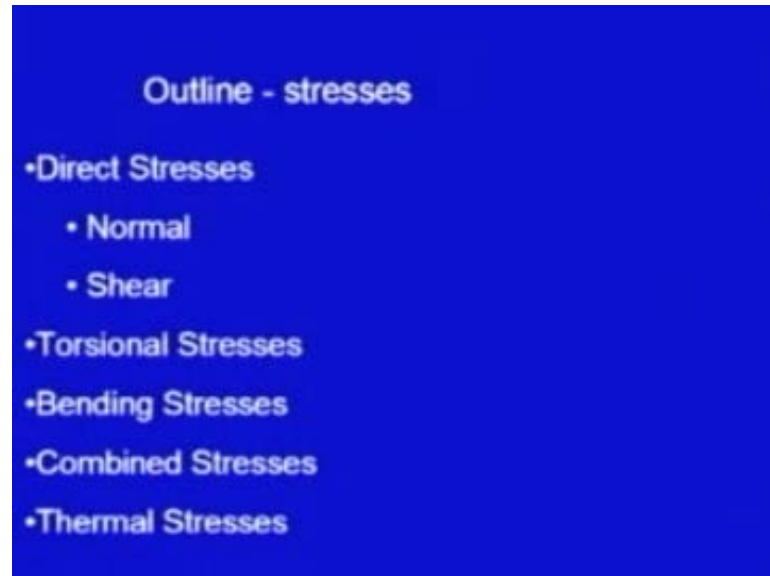
As we know that in mechanics of deformable solids, externally applied forces acts on a body and body suffers deformation. From equilibrium point of view, this action should be opposed or reacted by internal forces which are set up within the particles of material due to cohesion. These internal forces give rise to a concept of stress. Intensity of internal resistive force is known as stress.

Now, as we know that, in mechanics of deformable solids, the externally applied forces acts on a body and body suffers deformation. So, there are two kinds of deformations are there in the object if we apply the force: one is the elastic deformation; one is the plastic deformations. Since the body is deformable; so we can distinguish these two terms as the elastic and the plastic deformations. And, since we know that, under these variety of forces, the body is under equilibrium.

So, from the equilibrium point of view, this action should be opposed or reacted by the internal body forces, which are set up within the particles of material due to cohesion. And, that is why the system is under equilibrium due to the... Even under the variety of these forces are there, the system is in equilibrium part, because of the – there are some certain resistive forces set up within the object. These internal forces give rise to a concept of stress, because we apply the force through different line of actions and the body is under equilibrium; that means there are some internal body forces are being set up. So, we can say that, the intensity of internal resistive forces is known as the stress. So, this is the basic elementary definition of a stress that, it always depends on the

application of load; what kind of load is there; what is the nature of load; and what is the line of action of force is there.

(Refer Slide Time: 15:20)



Now, as we discussed that, the stress is always there – the intensity of the resistive forces. We can categorize these stresses according to the application of force like these – the direct stresses; in which there are two sub categories: one is the normal stress; one is the shear stress. Even in the normal stress, we can subcategorize, because this is nothing the axial stresses, normal stresses. So, either these are the tensile stresses or these are the compressive stresses. The next type of stresses are the torsional stresses. This is again the form of shear stresses, but within the rotational shaft. Then, we have the bending stresses that, what kind of... If we have a loading, which is always creating bending the object; so the bending stresses are being setup within the object. Then, we have the thermal stresses. If the object is there in the different temperature environment, then the thermal stresses are being setup due to the temperature.

Then, we have the combined stresses. If the object is subjected with let us say the thermal and the external applied force is there; that means there is a change of environment due to the temperature. And, the second, the different forces are there, which induced or which created the environment of bending. So, there are sort of we can say that, the combined stresses are always there with the combination of bending thermal or bending torsional or we can say the normal and the shear stresses, which are under the

direct stresses. So, these are the variety of stresses are there, which can be induced in the object due to the application of force – variety of force.

(Refer Slide Time: 16:45)

Stress as internal resistive force per unit area. If the stresses are normal to the areas concerned, these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter ( $\sigma$ ):

Stress Vector:

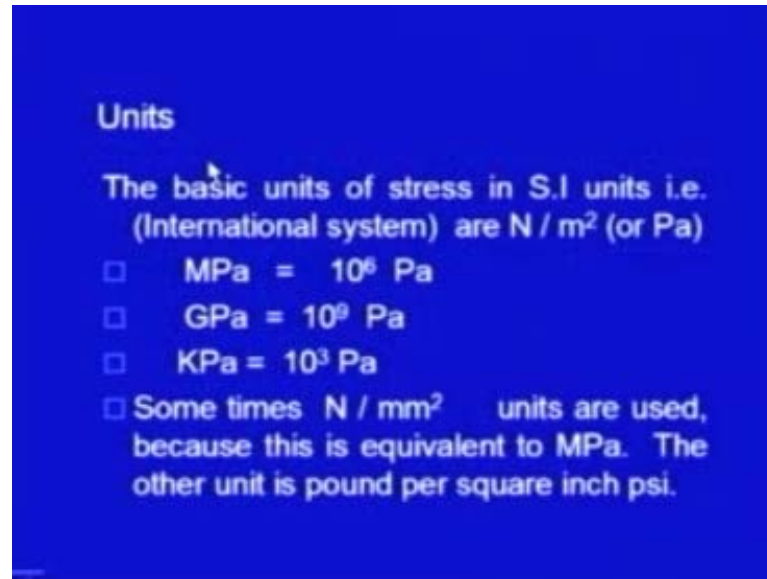
$$\sigma = \lim_{\Delta A \rightarrow 0} \left( \frac{\Delta F}{\Delta A} \right)$$

This is also known as uni – axial state of stress, because the stress acts only in one direction.

Now, as we discussed the stresses as the internal resistive forces per unit area, if a stress... The first case was there – the direct stress – in which the two categories was there: the normal and the shear stress. So, if the stresses are normal to the area concerned, these are termed as the normal stresses, because the stress is nothing but the resistive force per unit area. So, where these resistive forces are there – means what is the effective domain or effective area is there? So, this will decide that, what is the nature of force is there and what kind of stresses are being there in the object.

So, here since the area concerned here is the normal to the object, where the forces are being there. So, that is why this is known as the normal stresses. We can also term this normal stresses as by a Greek letter sigma. So, sigma is nothing but equals to the force applied on the effective area divided by the effective area. Since this is the normal force and we discussed that, these normal forces are the axially forces through axis; so these are also known as the uniaxial state of stress, because the stress acts only in one direction – either the tensile or the compressive. So, these are two categories in the normal stresses.

(Refer Slide Time: 18:03)



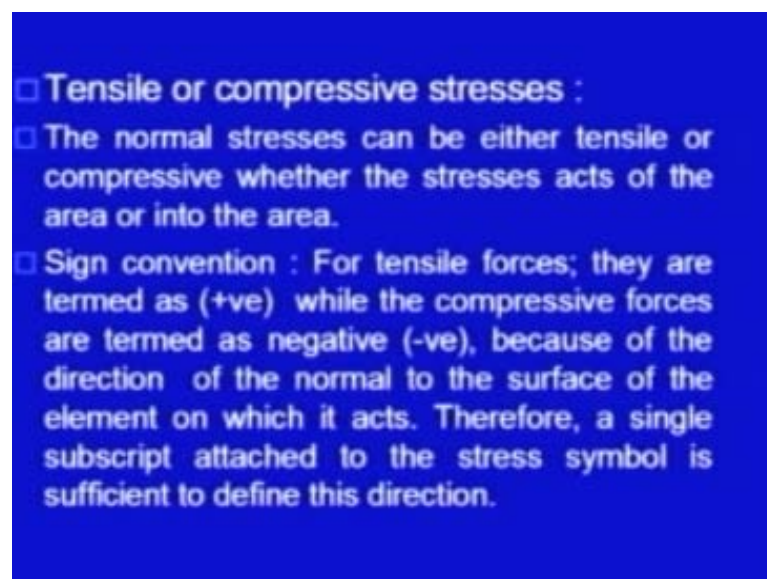
**Units**

The basic units of stress in S.I units i.e. (International system) are  $\text{N} / \text{m}^2$  (or Pa)

- $\text{MPa} = 10^6 \text{ Pa}$
- $\text{GPa} = 10^9 \text{ Pa}$
- $\text{KPa} = 10^3 \text{ Pa}$
- Some times  $\text{N} / \text{mm}^2$  units are used, because this is equivalent to MPa. The other unit is pound per square inch psi.

Since the stress is nothing but the force per unit area; so there are the units like newton per meter square or we can say this is also the Pascal. So, we can even categorize under subcategories that, the mega Pascal is nothing but equals to 10 to the power 6 Pascal; giga Pascal is 10 to the power 9 Pascal; kilo Pascal is 10 to the power 3 Pascal. Sometimes also if we go to the FPS units – not in SI units, FPS units; then the unit of this stress is psi – pound per square... So, there is also one of the basic unit, which earlier we used to analyze the stresses.

(Refer Slide Time: 18:43)

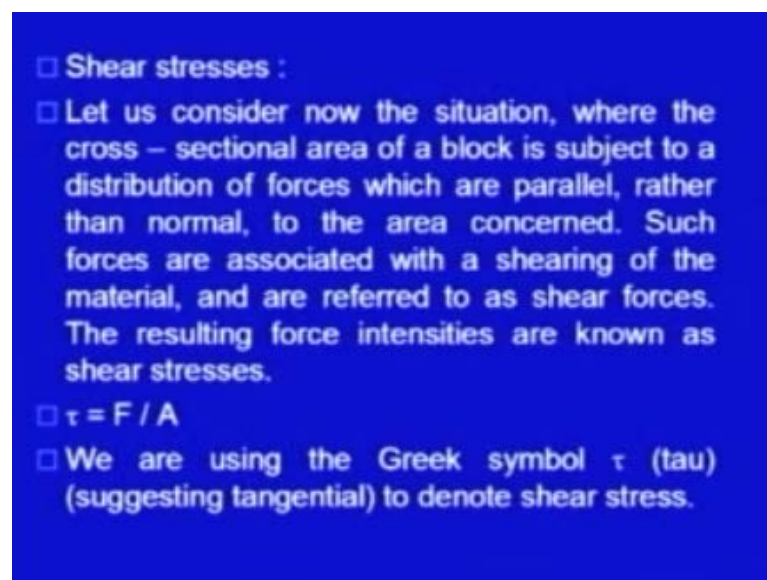


- **Tensile or compressive stresses :**
- The normal stresses can be either tensile or compressive whether the stresses acts of the area or into the area.
- Sign convention : For tensile forces; they are termed as (+ve) while the compressive forces are termed as negative (-ve), because of the direction of the normal to the surface of the element on which it acts. Therefore, a single subscript attached to the stress symbol is sufficient to define this direction.

Now, as we discussed about the normal stresses, there are two categories: the tensile and the compressive stresses. These are all categorized just based on the line of action of force. So, normal stresses can be either tensile or compressive whether the stresses acts of the area or into the area. So, if we are just doing the axial pulling, then these are the tensile stresses. If there is an axial compression is there; the line of action are at both side of the object through one axis and the compression is there, you can say these are the compressive stresses.

The sign convention – if in a normal stress, we are axially... – means if we are doing the axial pulling, these are the tensile forces; they are termed as the positive. Or, if we are applying the compressive forces through the line of action of the centroidal axis, we can say these are the compressive forces and they are termed as the negative. Because of the direction of the normal to the surface of the element on which it acts, we are always considering the same sign – the plus for tensile and minus for compressive forces. Therefore, a single symbol... because only these are all axial – means the line of action of either the tensile and compressive force along the same axis. So, only we need a single subscript to define these type of stresses. That is why we are not going for the different two axis or three axis subscript to define the nature of force.

(Refer Slide Time: 20:10)

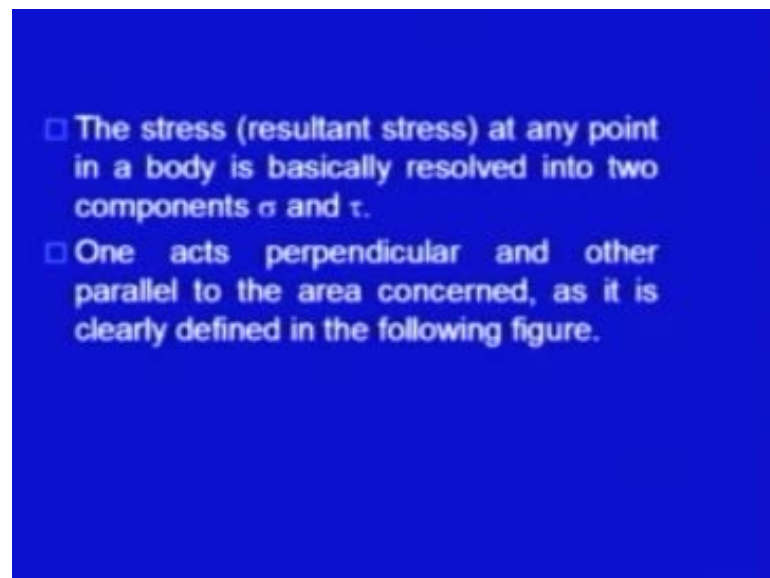


- Shear stresses :
- Let us consider now the situation, where the cross – sectional area of a block is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force intensities are known as shear stresses.
- $\tau = F / A$
- We are using the Greek symbol  $\tau$  (tau) (suggesting tangential) to denote shear stress.

Now, the second types of stresses are the shear stresses under direct stress component. Now, let us consider the situation, where the cross-sectional area of block is subjected to

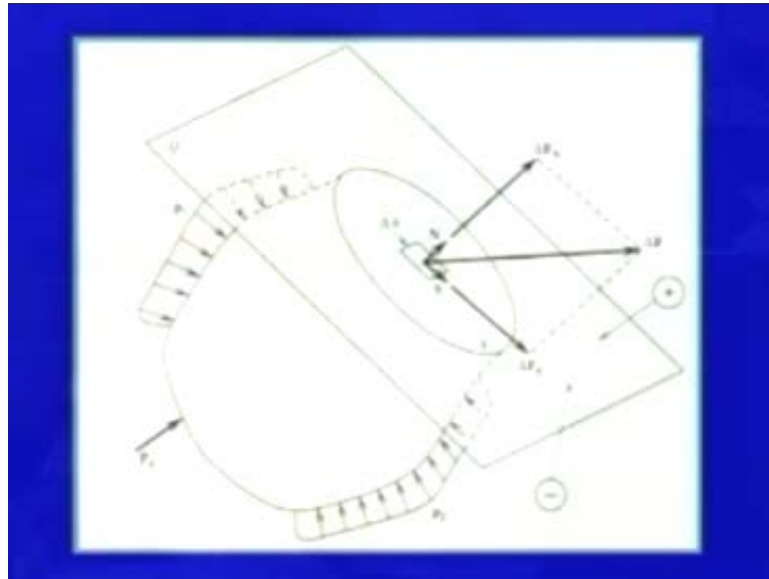
the distribution of force, which are parallel. Now, the force application is not along the same axis. Now, it is the different axis – means there are two parallel axes are there; and at this point and at this point, two different forces are acting. So, definitely, there is a plane, which is just shearing this – the object at the contact points. So, rather than the normal, the area concerned is just parallel. So, the effective area is now parallel; it is not just exactly to the normal to the force. So, such forces are associated with the shearing of the material. Always there is a shearing is there at the effective area and they are referred as the shear forces. So, the intensity of resistive shearing forces are known as the shear stresses, which is also we always define shear stress by Greek letter tau. So, tau is nothing but equals to the shearing forces divided by the effective area. So, the tensile or compressive stresses or the shear stresses are always comes under the category of direct stresses.

(Refer Slide Time: 21:24)



The stress – whatever the resulting stress is there on an object at any point in body, is basically resolved into two components. Under the direct stresses, one is the normal stress, which is designed and designated by the sigma; and the shear stress, which is to be defined with the symbol of tau. One acts, because the normal stress component is there. So, it always acts the perpendicular or we can say the normal to the area concerned. And, other one since it is a shearing part is there; so it is always acts on the parallel to the area concerned and it is clearly defined with this particular figure.

(Refer Slide Time: 22:06)



As we see the previous figure, now, this is the cutting plane. At this, the different internal resistive forces are there. As I told you, the different internal body forces are there; they are being setup due to the load application. So, there are two components. If we see this particular element of  $\Delta A$  area; this one is the normal component, which is always inducing towards the normal force. So, this is  $\Delta F_N$  is nothing but the normal resistive force;  $\Delta F_S$  is the force, which is parallel to the plane. So, it is termed as the shear forces;  $\Delta F_S$  is the shear force. So, these two forces are there exactly according to the load application. So, that is why they are termed as the normal force and the shear force. This can be either tensile or the compressive. So, this  $\Delta F$  is the resultant force because of the presence of shear force and the normal force.

(Refer Slide Time: 23:00)

Normal Stress Vector :

$$\sigma_N = \lim_{\Delta A \rightarrow 0} \left( \frac{\Delta F_N}{\Delta A} \right)$$

Shear Stress Vector :

$$\tau = \lim_{\Delta A \rightarrow 0} \left( \frac{\Delta F_S}{\Delta A} \right)$$

Now, we can define again this – the normal stress vector – either the tensile or the compressive is nothing but equals to delta F N – the normal force divided by the effective area. Or, the shear stress is nothing but equals to tau, which is equals to delta F S – this is the shear force. As I discussed, this is the shear force for this particular area along the parallel to the axis of this shearing plane. So, delta F S divided by the effective area. It always acts... The normal stress always acts perpendicular to the effective area concerned; and the shear stress always acts parallel to the area concerned.

(Refer Slide Time: 23:38)

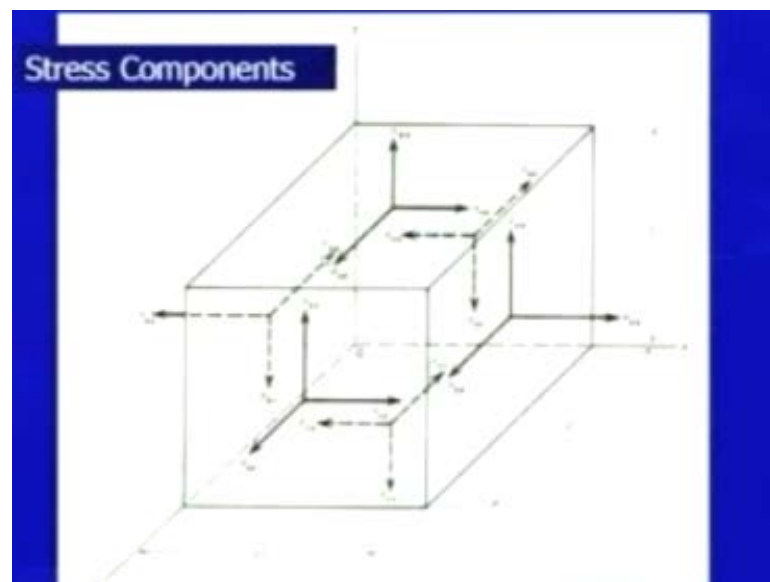
**General State of stress at a points**

- Stress at a point in a material body has been defined as a resistive force per unit area. But this definition is somewhat ambiguous since it depends upon what area we consider at that point.



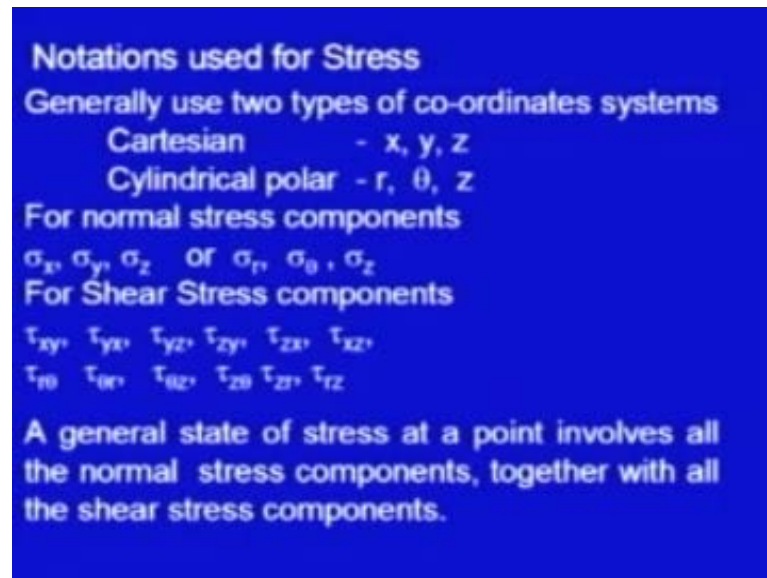
So, we have now direct stress in which there are two main components: one is the normal stress; one is the shear stress. Now, if we need to define the general state of stress at a particular point, then we need to take the small elemental area. So, stress at a point in material body has been defined as the resistive force; as we discussed earlier, internal resistive force per unit area. But, this is not the exact definition of the stress. So, we need to check, because the area concerned is very small in that case; only if you see the previous diagram, you can say that, only these two – means variety of forces are there; but, we are only concerned with the two directions. So, we need to check that, whether it is a fulfilling the exact definition of stress or not; or, whatever the conditions are there with the stress associated or not. So, first of all, we need to check with the variety of directions.

(Refer Slide Time: 24:37)



So, let us say we have in a cube with the dimension of delta x in x component, delta y in y component, delta z in z direction. So, if I have a unit cube with these distances: delta x, delta y and delta z in respective directions; then we can say that, now, we have total 18 elements to describe the stress. That means these two components – that means the normal stress and the shear stress is not at all fulfilling our requirement to define the stress at a point.

(Refer Slide Time: 25:06)

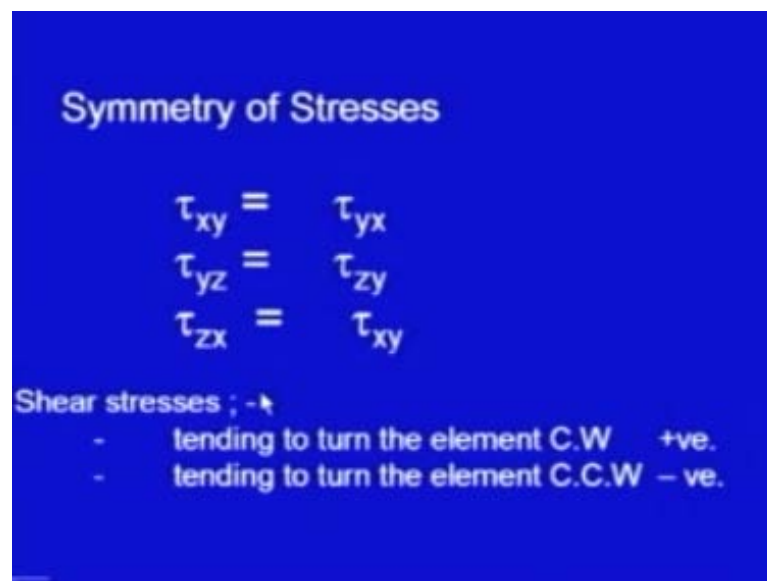


So, now, before starting or before analyzing the cube, first of all, we need to define certain things. Generally, there are certain notations are there to define the stresses. There are two types of co-ordinates systems: one is the Cartesians just like  $x, y, z$ ; and these polar coordinates are there –  $r, \theta, z$ . So, as far as the normal stress components are concerned, they are always there in the single direction. As we discussed, it is a uniaxial stress component is there. So, they are termed as the sigma  $x$ , sigma  $y$  and sigma  $z$  in the  $x, y, z$  directions. Or we can say if we are taking the polar coordinates, then they are sigma  $r$ , sigma  $\theta$ , sigma  $z$ . But, generally, in this subject, we are going to concern of the stress component in the Cartesian coordinate only. And, as far as the shear plane is concerned, which is parallel to the effective area; so they are termed as tau  $xy$ , tau  $yz$ , tau  $yx$ , tau  $zy$ , tau  $zx$  and tau  $xx$ . So, these are the six components in different directions in the Cartesian coordinate we can define. Or, we can say also the stress component is there in the polar coordinate as they are showing here.

So, the general state of stress at a point involves all normal stress components together with all shear components. So, there are total 18 elements which we need to describe the full stress for a single point. As we have seen, here these are in the  $x$  direction; these are the axial stress component; these are again axial component; these are axial stress components are there in the  $x, y$  and  $z$  directions. So, these are all total. This 3, 3, 3, 3; and these are all total 6 faces are there. And, at individual face, there are 3 individual components are there in the respective directions. So, there are total 18 stress

components are there to describe stress at a point. But, since I have taken the cube element, which has a symmetry; so we need to impose this condition that, whatever the stress components are there in xy direction – this stress components in xy direction, which is exactly equals to this direction; that means we have the stress component. Let us say this is in xx direction; this is xy direction. And, whatever this is there in yx direction, they are identical.

(Refer Slide Time: 27:33)



**Symmetry of Stresses**

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{zx} = \tau_{xz}$$

Shear stresses ; - ↻

- tending to turn the element C.W +ve.
- tending to turn the element C.C.W - ve.

That means tau xy, which is equals to tau yx; or, we can say tau yz is equals to tau zy; or, we can say tau zx is equal to tau xz; this is xz. So, these are all due to the symmetry of the elemental area. Now, as we already have termed that, if we have in a tensile stress, we always considered as a positive stress; if it is a compressive stress, then we always termed as the negative stress. So, the same thing is there for the shear stresses. If due to this shear stresses presence, the object is tending to turn in the clockwise direction; we always considered the positive direction. And, because of the presence of the shear stresses, the object is tending to turn in the counter clockwise direction; we always referred as the negative. So, these are the directions of the shear stresses, which are being setup in the object. We can consider simply whether it is a positive effect or it is a negative effect.

(Refer Slide Time: 28:42)

**Stress Components**

*9 components of stress*

$(\sigma_{xx}, \sigma_{yy}, \sigma_{zz})$
$(\sigma_{xy}, \sigma_{yx}, \sigma_{xz})$
$(\sigma_{xz}, \sigma_{zx}, \sigma_{yz})$

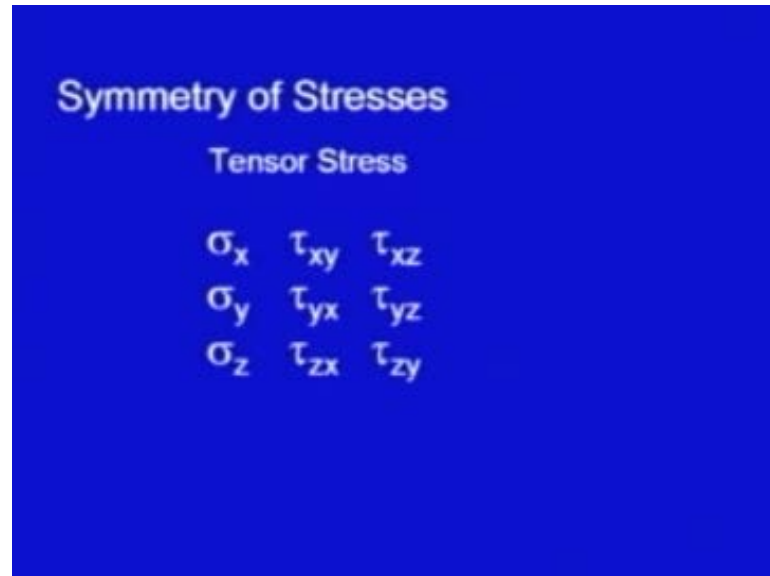
*First subscript refers to the direction of the normal.  
Second subscript refers to the direction of the stress.*

Now, as we discussed, since these are the identical terms are there; so now, we can reduce this 18 elements into the 9 elements. So, now, we have the total 9 components, which we needed to describe the stress at a single point. So, now, we have sigma xx, sigma yy, sigma zz, sigma xy, sigma xz, sigma yx, sigma yz, sigma zx, sigma zy. So, these are the total 9 components. In these, there are two subscripts are there. The first subscript refers to the direction of the normal like this x; so that means now, it is clearly showing that, actually what is the nature; that means the force is there in the direction of the normal or the parallel. The second subscript refers to the direction of the stress; that means the direction of the stress is exactly to the direction of the normal; that means I have the same stress – means there is a domain and the stress is being exactly in the direction of force.

We can say these are the normal stresses, because we are tending that, actually if this stress is acting in the normal – area to the normal, we can say these are the normal stresses. So, that is why sigma xx, sigma yy, sigma zz are always termed as the normal stress component. So, only we need a single subscript to define these things. So, we can redefine this that, sigma xx is nothing but equals to we can replace by sigma x; sigma yy we can replace by sigma y; sigma zz we can replace by sigma z. And the other 6 components – the nature of stress and the area domain is different. Now, here actually the direction of normal is in x direction, while the direction of stress is in y direction; that means they are not matching. So, these stresses are termed as the shear stresses. So, we

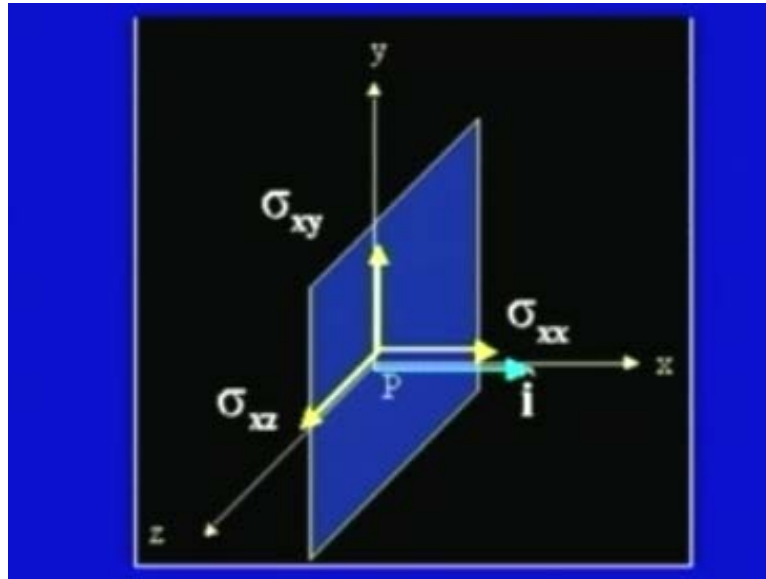
can replace sigma xy by tau xy, sigma xz as tau xz, sigma yx as tau yx, and sigma yz as tau yz, sigma zx as tau zx, and sigma zy is tau zy.

(Refer Slide Time: 30:44)



So, now, actually we have 3 normal stress components and 6 shear stress components are there, which are being designated as sigma x, sigma y, sigma z in three respective directions; while tau xy, tau xz, tau yx, tau yz, tau zx, tau zy are in different planes of shear stress. Since you know that, we have 9 different components including normal and shear stress components to describe a stress at a point; so that is why it is termed as the tensile stress; because in a vector, only we need the magnitude and directions; while here actually it is clearly showing that, what is the nature of the stress is there at a single point of the variety of stresses. So, that is why it is also termed as the tensile stress. And, there are different 9 components are there on a single face.

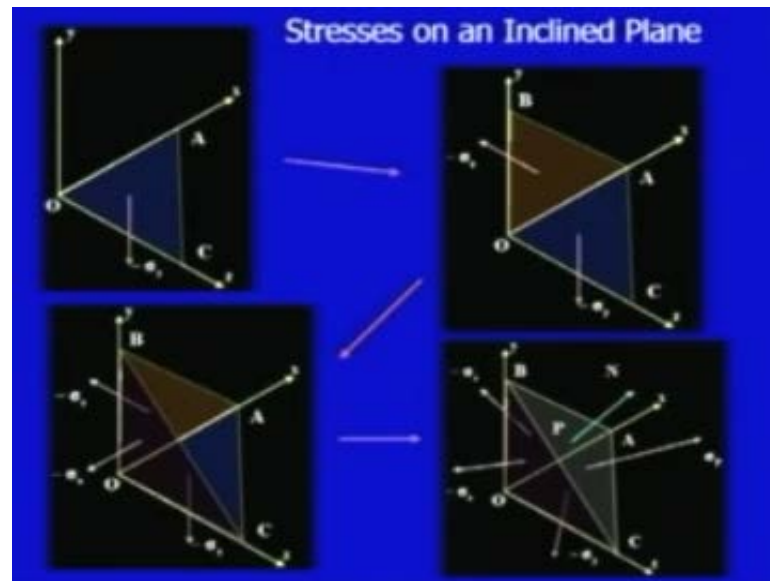
(Refer Slide Time: 31:34)



So, if you are taking the individual faces of the cube, we will find out that, let us say in  $xyz$ , there is a single face in which  $\sigma_{xx}$ ,  $\sigma_{xy}$  and  $\sigma_{xz}$  or we can say it is  $\sigma_x$ ,  $\tau_{xy}$ ,  $\tau_{xz}$  is there. So, at a particular point  $P$ , we have three different notations. So, you can say that, actually since as we discussed in the previous slide that, there are different 9 components and if we are taking the individual faces, we will find that, how these stress are being setup within the individual face of the cube.

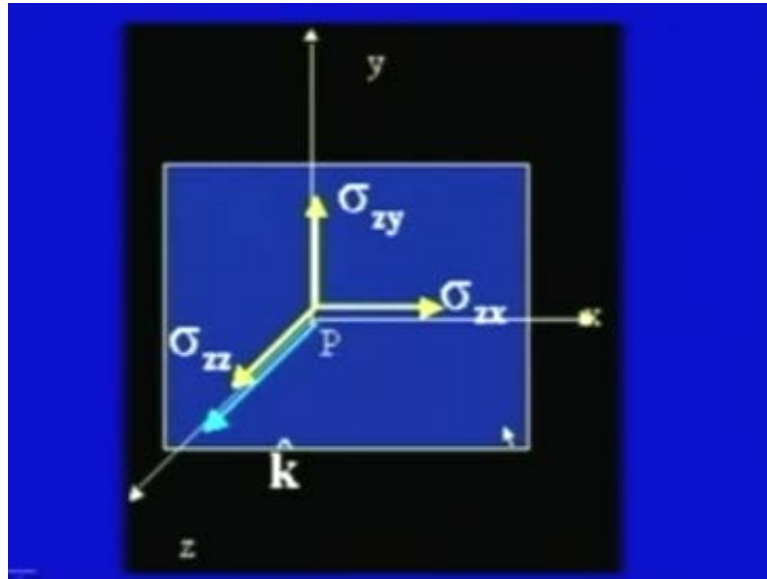
So, this is the first face. The second face – if we are taking, which is just parallel to the upper side, we will have  $\sigma_{yx}$ ,  $\sigma_{yy}$  and  $\sigma_{yz}$ . Or, you can say it can be termed as  $\sigma_y$ ,  $\tau_{yx}$ ,  $\tau_{yz}$ . The similar thing is there in the other directions, where the  $\sigma_z$  is the normal stress; and the  $\tau_{zx}$  and  $\tau_{zy}$  are two different shear stress components are there in the respective directions. So, these three slides will clearly showing that, how the shear stresses are being setup and how the normal stresses are being setup in the system. So, that is why we need exact 9 stress components to define the stress at a point  $P$ . So, this  $P$  point is there. And, these 3; these 9 components are there of the stress.

(Refer Slide Time: 32:56)



So, now, if we are just taking the inclined plane, then how these stresses are being setup. So, if we start from the first slide, we can see here we have the xz plane, where this compressive normal stress is there along the lower direction. So, that is why it is termed as minus sigma y. Or we can say this is another; if we are taking another plane, which is also the xy plane, we have the sigma z. Since it is in the compressive Way; so it is termed as... Since as I told you that, actually if it is a compressive stress, then it is termed as a negative. So, this is minus sigma z and this is minus sigma y. And if we are just going with another direction or we are taking the another slot of this stress component, we can find out we have all these – sigma x, sigma z, sigma y with the normal stress components. So, these are all we can say the different parts are there, where these stresses are being setup in the different directions. And also, there are some other this – what we can say the stresses are there along the directions, which is also we can say the sigma P, which is the resultant stress component due to all stress components. So, these stresses are there.

(Refer Slide Time: 34:11)



If we have the parallel, then we can show the stress by these directions. Or, if we inclined plane, then we can show the directions with these notations. So, these are the different stress notations and the directions on the straight and the inclined plane. So, this was all about the stress; the normal stress, the shear stress component under the direct stress way.

(Refer Slide Time: 34:28)

### Strain

□ If a bar is subjected to a direct load, due to stress in the bar, there will be a change in length. If the bar has an original length  $L$  and changes by an amount  $\delta L$ , strain ( $\epsilon$ ) is,

$$\epsilon = \frac{\delta L}{L}$$

Now, we have the strain. Whenever the load application is there, there is always the deformation is there in the object. That is why we are always analyzing the deformable



bodies under the static or dynamic forces in the solid mechanics. So, the another component, which is very important to analyze the object in the mechanics part is the strain. If a bar or tube, which is having a homogeneous material and all those assumptions are to be applied here also, is subjected to a direct load. So, due to when the load application is there – direct load application is there, the stresses are always being setup, because these are all internal resistive forces intensity in the bar. So, there will be a change in length – means always if it is a tensile; so axial pulling is there. So, definitely, there is a changing in length. If the compressive forces are there, there is a compression is there or deformation is there in the compression side So, there is a change in length or in the diameter with the corresponding type of force – means what kind of a line of action of force is there So, if the bar has original length  $L$ , or and the due to the load application, the change in the bar length is  $\Delta L$ , the strain is always defined as the change in length divided by original length. Since it is the length by length definition; so it has – there is no unit; it is the dimensionless and unit less component.

(Refer Slide Time: 36:09)

- Sign convention for strain: -
- Tensile strains are positive compressive strains are negative. The strain defined earlier was known as linear strain or normal strain or the longitudinal strain.
- An element which is subjected to shear stress experiences deformation. The tangent of the angle through which two adjacent sides rotate relative to their initial position is termed shear strain. The angle of deformation  $\gamma$  is then termed as the shear strain

Now, the sign convention for the strain; since the strain is also termed as what kind of load application is there; because if we have the axial pulling or axial compression is there, always there are the tensile and the compressive strains are there. So, we can similarly as the stress, we can also define the sign convention exactly as the stress component. So, tensile strain is there because of the axial pulling; compressive strains are there because of the compression of the object. So, tensile strains are always termed

as the positive, while the compressive strains are always termed as the negative since the strain is nothing but the defined as the linear strain, normal strain or the longitudinal strain; because if we are pulling or if we are compression, this is nothing but the normal strain or the longitudinal strain, because the direction of that load is in the longitudinal way. The another type of strains are the shear strains, because the shear stresses are always existing within the object. So, there is a deformation is there across the shear plane. These strains are termed as the shear strain. So, an element, which is subjected to the shear stress experiences the deformation – the tangent of the angle through which two additional slides rotate relative to the initial position is termed as the shear strain; and which is always denoted by the Gamma – the angle of deformation.

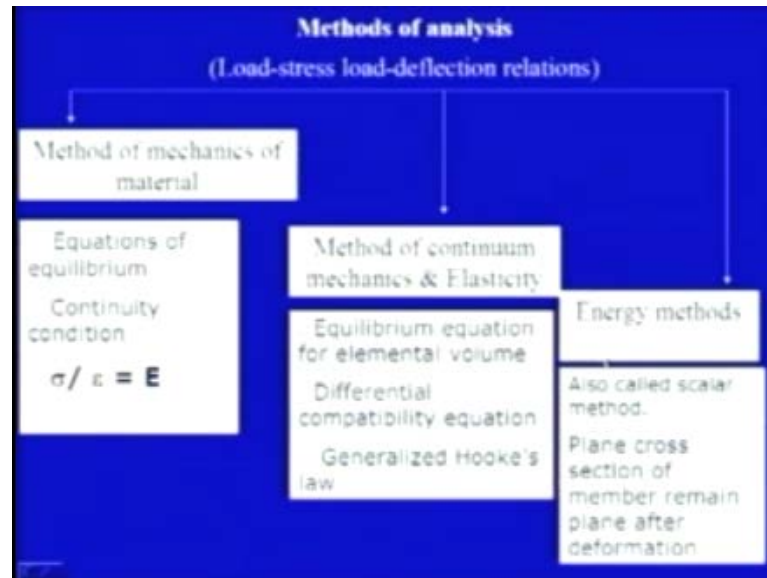
(Refer Slide Time: 37:44)



We have discussed the two important properties under the solid mechanics. These are: one as the load application is there under the equilibrium conditions; these are the load application under equilibrium conditions. We are having too many stresses like normal stresses, shear stresses, torsional stresses, bending stresses. Another thing is – since the load application is there, there is a displacement or rather we can say there is a deformation is there. And, to measure the deformation, we always need the component that is known as the strain. So, now, we have the stress and a strain. Now, we need to... Since these are two – the consecutive components are there; wherever the stresses are there, the strains are always associated with the strains. So, what is the exact relation? So, now, we need to define the stress-strain relations that, what kind of load applications

are there; what kind of deformation is there; then what is the relation is there within the stress and strains. Then, it is also termed as the load-deflection or the load-stress relationship.

(Refer Slide Time: 38:36)

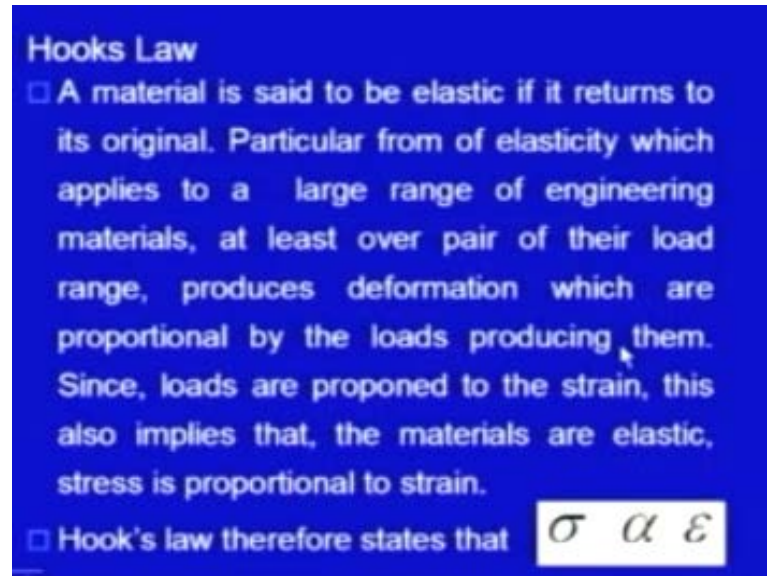


So, now, the method of analysis under the load-stress or the load-deflection relations; now, if you go with the method of mechanics – like the mechanics of material or the solid mechanics, we always start with the equilibrium conditions. If you want to start any problem, first of all, we need to check that, whether this object is under static equilibrium is there or not. After checking those things, we need to go that, what kind of stress and what kind of strain components are there; that means on the continuity equations, whether the stresses – means the tensile or the compressive stresses are there; or, we have the shear stresses. Then, what kind of strains are there? And, what is the relation is there? So, we will discuss this part.

The second thing is there, which is also termed as the method of continuum mechanics and the elasticity, which is beyond our course, because actually we need to go with the elemental area and the volume of the component. And then we need to apply the compatibility equations; then the different generalized Hooke's law is there under the condition of continuum mechanics and the elasticity. The third part is there; that is also known as the energy methods, because – since here the deformation is there and the load applications are there; so definitely, there are different energy components are associated

with the different particles and the object. So, we can also apply the energy methods, which is also known as the scalar methods. But, since it will come in the later part of our course, we will discuss in detail.

(Refer Slide Time: 40:05)



**Hooks Law**

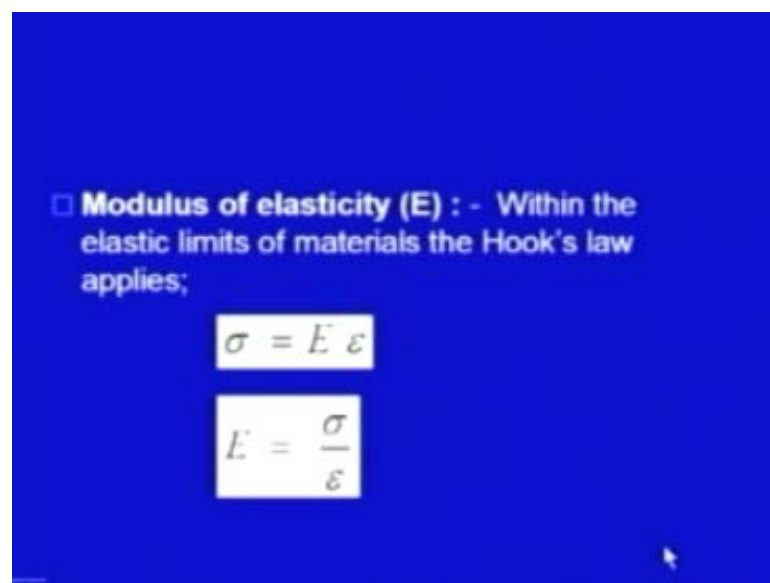
- A material is said to be elastic if it returns to its original. Particular form of elasticity which applies to a large range of engineering materials, at least over a pair of their load range, produces deformation which are proportional by the loads producing them. Since, loads are proportioned to the strain, this also implies that, the materials are elastic, stress is proportional to strain.
- Hook's law therefore states that  $\sigma \propto \epsilon$

Now, we want to relate the stress and the strain component under the different deformation conditions. So, first of all, the Hooke's give a law for the elastic deformations. As we discussed earlier that, since if the load application is there, either the elastic deformation is there or the plastic deformation is there. So, under the elastic deformation, there is a relation with the stress and strain. So, Hooke's gave the law under the elastic deformation. A material is said to be elastic if it returns to its original position – means if you remove the load and body comes to its original state, we can say there is a elastic deformation. And, because of the load application, if a permanent set of deformation exists in the object, we can say this is the permanent deformation or we can say this is the plastic deformation. So, these two elastic and plastic deformations are there.

Particular form of elasticity, which applies to the large range of engineering materials at least over a pair of load range produces the deformation, which are proportional by the loads producing them. Always they are proportional to the load application that, how much load you apply. The corresponding elastic deformation is there in the object. Since the loads are proportion to the strain, this also implies that, the materials are elastic or we

can say the stress is... because the load application is there; the internal resistive forces are there; the stress is there. So, we can say the stress is proportional to strain; or, intensity of resistive forces, the body force is proportional to the corresponding deformation, because the strain is the measure of deformation. So, the Hooke's law, which is valid only for the elastic deformation always gives the relation between the stress and the strain as termed as the stress is proportional to strain; or, we can say the internal intensity of resistive force is proportional to the measure of deformation. So, this is there. The tau is proportional to the xi. So, this is the Hooke's law.

(Refer Slide Time: 42:13)



Now, actually, if we equate this since there is a proportional constant is there; so if we want to equate this, we have the new term, that is E. That is also known as the modulus of elasticity that, what is the magnitude is there of this elastic constant. But, again this modulus of elasticity is again valid within the elastic region. So, within the elastic limit of material, Hooke's law is like that; or, we can say the modulus elasticity is nothing but equals to the stress by strain.

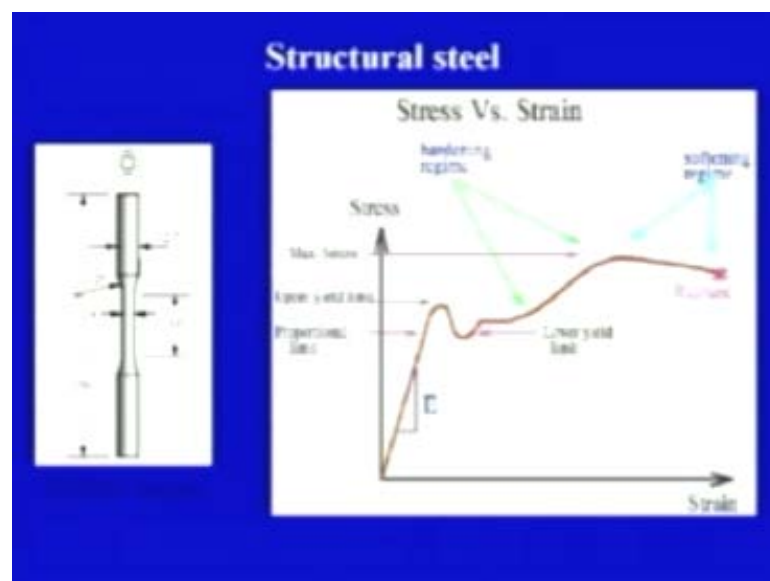
(Refer Slide Time: 42:44)

### Engineering Stress and Strain

$$\sigma = \frac{P}{A_0}$$
$$\varepsilon = \frac{\Delta L}{L} = \frac{e}{L}$$

Or, we can say – again, we can redefine this – the engineering stress, which is also termed as the simple stress or strain, which is equals to the normal stress. If it is normal stress is there, the load applied divided by the area concerned or the original area; and the strain is nothing but equals to the change of length divided by original length. Or, we can say since the change of length is there due to the load application, which is also termed as the deformation; so the strain is nothing but equals to the deformation divided by the original length.

(Refer Slide Time: 43:14)



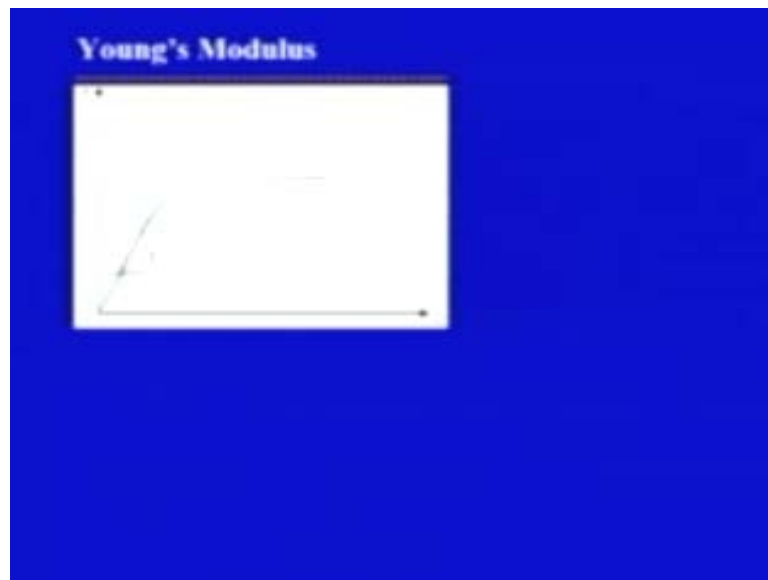
Now, if we take an example of structural steel – means if we have a steel and there is axial pulling is there; we can define the stress-strain relationship with that. So, now, we have one axial bar is there; if it is... the axial pulling is there of this particular bar, which is having the homogenous material and the isotropic material, this is also having the same cross-sectional. This deviation is there because of the load application. So, we will discuss this part. Initially, it is just like as good as the uniform bar. So, as we apply the axial loading, maybe it is compression or it is the tensile pulling. Now, we have the corresponding stress-strain diagram. So, initially, if we apply the load, there is elastic region is there or we can say there is a proportional limit is there, which gives you the stress is proportional to strain or we can say there is straight line is there. So, up to this point, which is termed as the proportional limit; the stress is proportional to strain.

Again, if we apply the load, there is an increase in deformation; there is also increase in stress. And, it will give a point, which is known as the yield point. Yield point also comes under the elastic deformation. This is the last point; where, if you remove the load, body comes to its original state; that means this is the last point or there is a limit, where the elastic property of material remains closed. So, that is this. Then, again if we have a dip, this dip between the upper yield point and lower yield point is known as the yield plateau. So, yield plateau is nothing but it gives you a straight relationship is there that, even if you apply the load, there is a deformation in this region, but there is a reduction in the stress.

First of all, why there is a reduction stress? You are applying the load, but there is a reduction in stress. Why? Because as we apply the stresses, there is a change in the microstructure of the object; there is a short period, which is very short in the structure of steel or any tensile or we can say ductile material, which is very short period. But, there is a period, where the material or the microstructure needs time to being setup. So, this is the time, where there is a chance to reduce the stresses. So, here this is the plateau, where there is a reduction in the stress, but there is increase in the deformation. So, this region or this dome is known as the yield plateau. Again if we apply the forces, again there is now, increasing in the stress with the deformation increasing. There is a point, where we are getting the maximum stress, which is also known as the ultimate tensile stress. So, ultimate tensile stress is nothing but this is the maximal stress being offered by a component before fracture. Then, again if we apply the force, again there is a

deformation. But, after certain point, we are getting the rupture; that means there is a failure. So, now, we have... There are two categories: one – we have the elastic deformation part; one – we have the plastic deformation part. The elastic deformation part is this.

(Refer Slide Time: 46:49)



The elastic deformation is here up to the Young's modulus, because Young's modulus is only applicable where the Hooke's law is valid. So, this is – up to this part, there is we have the elastic deformation. Or, in the previous diagram, up to this point, we have the elastic region; and this remain part is known as the plastic deformation. Then, we have the two regions: one is the hardening region; one is the softening region. These two regions are always there within the plastic region.

First of all, what is the hardening region? And, second – what is the softening region? The hardening region is being as once we are approaching towards the plastic deformation... This is the starting point. As we are just approaching towards the plastic region, the object always offers the maximum resistive forces; because now, the microstructure here is being already setup. So, now, there is a permanent deformation existing. This is the permanent deformation existing state is there. So, here there is a maximum hardening is there. Once we are getting the maximum tensile strain; that means once the object offers the maximum resistance; beyond that, now, it loses its hardness. So, that is why now, this part in which the object is not offering the maximum



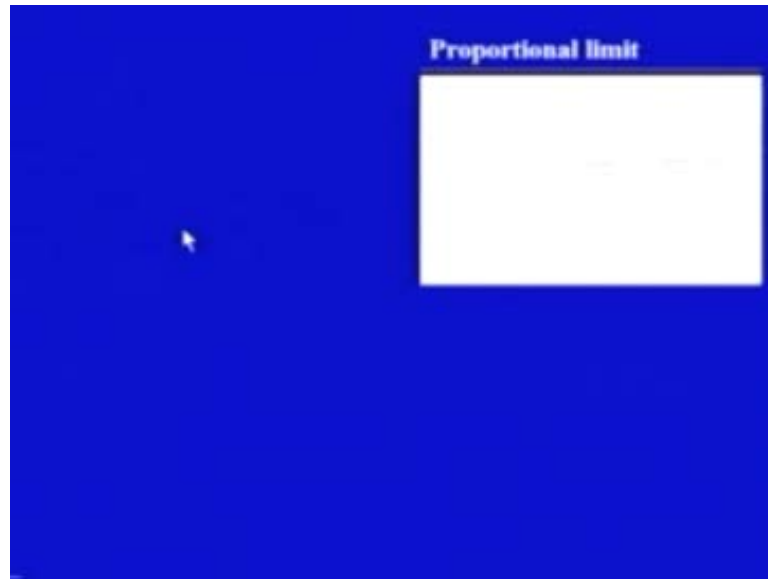
resistance – it is known as the softening region. So, that is why these are the hardening and the softening region – depends on what kind of internal resistance provided by the object.

(Refer Slide Time: 48:32)



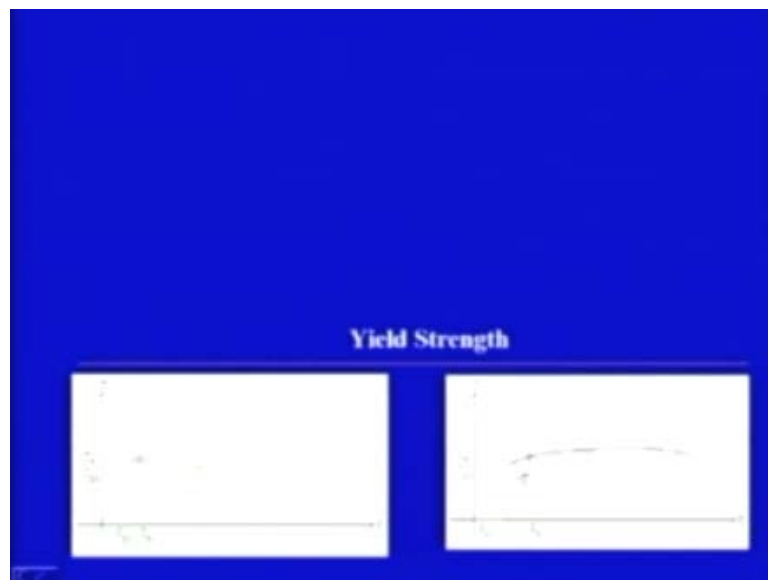
Now, we can even categorize those things with the different graphs – like first, we have the Young's modulus. So, if you want to calculate the Young's modulus, which is only applicable to the elastic deformation or we can say till the proportional limit, then this is the proportional limit. If you want to measure this thing, we need to apply the tangent on this graph; that means this  $E$  is nothing but equals to the  $\sigma$  by  $\epsilon$ ; that means the stress by strain.

(Refer Slide Time: 48:55)



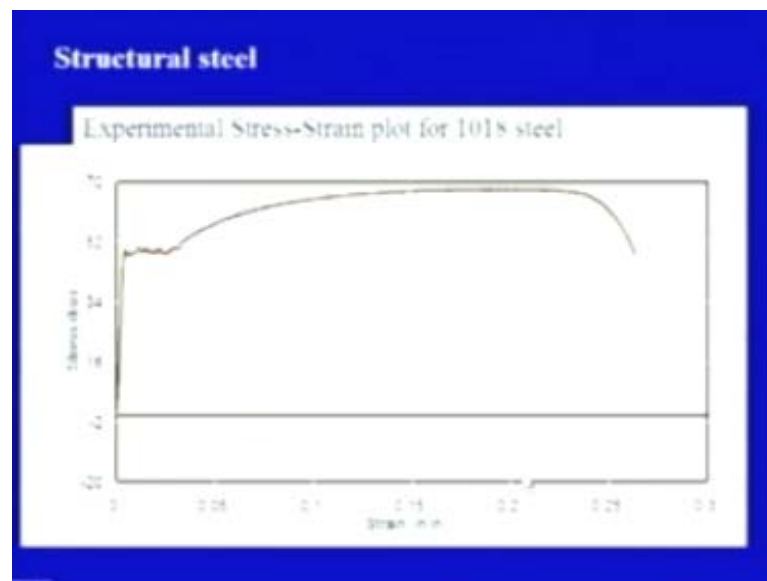
Then, we have the proportional limit. Again, since Hooke's law is valid up to the proportional limit. So, this is the proportional limit. I can get this stress of the proportional limit at this particular point. If you go beyond this, then there is no more the linear relationship is there between the stress and strain. Now, there is a nonlinearity comes in the picture; that means we have now the stress is not exactly proportional to the strain beyond this point. So, that is why it is known as the proportional limit. Here we can measure the stress at the proportional limit and the strain at the proportional limit.

(Refer Slide Time: 49:30)



Now, we have the yield strength. Yield strength is also the part of the elastic deformation. That is the last part; that means we can say this is the limit of the elastic region or we can say this is the starting point – beyond that, there is a... That is a starting point of the elastic region. So, we have now, the elastic.. The yield strength here – upper yield strength or lower yield strength we can say; and this is the corresponding strain is there at both points. And, this is the upper yield strength; this is the lower unit strength; and this is the yield plateau is there.

(Refer Slide Time: 50:03)



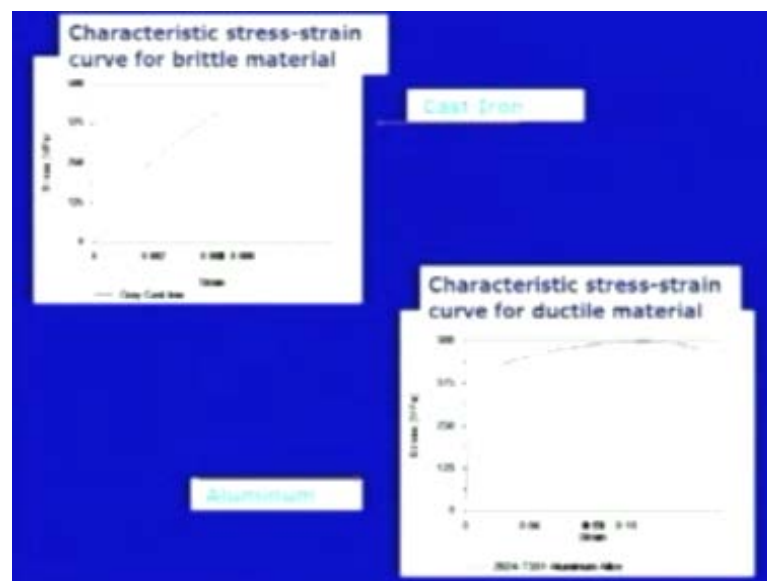
Now, again if we go to the different steel, that is, we have an experimental stress-strain plot for 1018 steel; that means here we have the different carbon structure with that. So, here we can clearly find it out that, actually this yield plateau is no more deviating. Here it is pretty simple is there; that means here the stress is not decreasing; that means if we have the less carbon component, is there in the steel, it always offers more resistant; that means this stress is no more reducing; though there is a deformation, but stress is not reducing. So, again we can clearly define that, the stress and strain.

Here this is the proportional limit up to this point; this is the yield limit. Here this is the lower yield limit and upper yield limit. This yield plateau is there, which is almost remains constant as you apply the load. And then here the hardening part will come; this hardening region is there. Up to this point, this is the ultimate tensile strength. And then

immediately because of less carbon percentage, immediately after maximum tensile stress – this strength, it will go for the failure. So, this is the rupture.

Now, since we are discussing the previous figure – this figure for the structural steel; and this figure for the different... This is also the structural steel, but different carbon composition. This mild steel or the structural steel is always termed as the ductile material. The ductility is nothing but the property of material through which it can extend up to a certain limit without failure. So, we can clearly distinguish the elastic and the plastic regions for the ductile material. And, whenever there is a rupture or the failure is there in the ductile material, we are getting almost the shape of cup and cone structure. So, this is the basic property of ductile material. This will always good in the tensile strength. So, they are having maximum tensile strength. That is why they can be at the tensile stress up to maximum. So, that is why we will find that, the steel – mild steel or the structural steel has an application in the RCC or wherever the tensile load application is there, because they are more comfortable under the tensile loads.

(Refer Slide Time: 52:33)



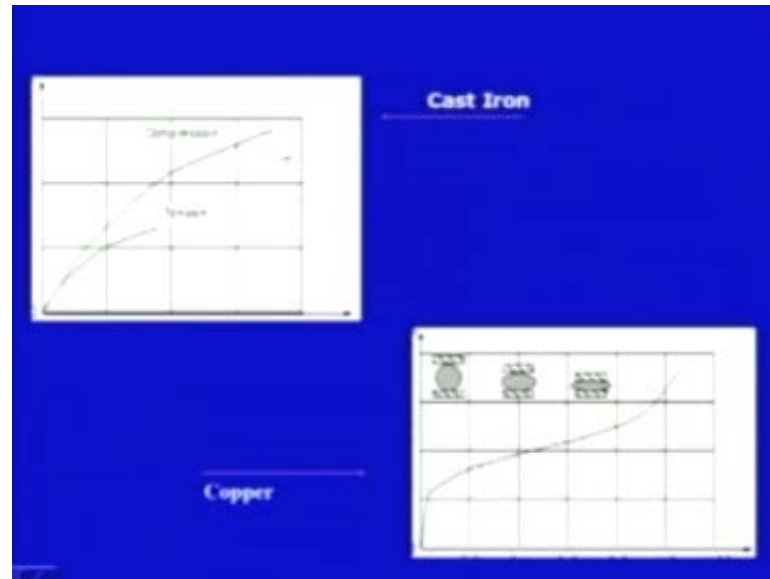
Then, we have the cast iron. The cast iron, which is having more carbon percentage always offers the damping property; that means it is always good in the compression region. That is why the foundations are always being provided by the cast irons. So, what is the important nature is there; they are always good under the compressive strength; while mild steel or we can say the ductile material is always good in the tensile strength.

So, cast iron, which is also termed – which is also under the category of brittle material; they are having brittleness – means they are not good under the tensile load; they are always good under the compressive load. So, again if we are going for the axial pull, here there is a stress-strain relationship is there, but it is not showing any elastic region or plastic region; or, we can say there is no yield – upper yield or lower yield limit is there; or, there is no proportional limit is there.

Straightaway as you apply the load, there is a stress component; there is a strain component; and beyond up to a certain point, it will show some relationship, which is non-linear, which is... There is no linear relationship is there for the stress and strain component. It will show the non-linear relationship. And, up to a certain point, it will show something; then it goes for fracture. And, at the point of fracture or rupture, we are having the zigzag or irregular shape at the fracture point. Like you see in ductile material, we are always having a kind of – not exactly – a kind of cup and cone structure; while for brittle material or the cast iron, we are having the irregular shape.

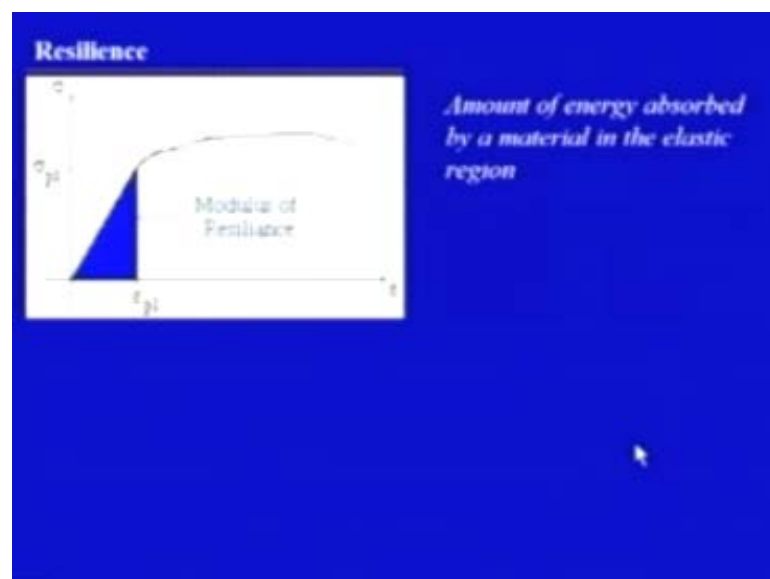
If we take the another example of the aluminum, which is also a kind of ductile material; it is not exactly showing that, where is the proportional limit is there; where is the upper yield point or the lower yield point is there. But, we can... Generally, we can get it as exactly there is a straight line is there; that means not much deformation is there, because this is a different carbon composition is there. So, not much deformation is there. But, there is a stress component; and beyond that, we can easily point it out that, where is the proportional limit is there and where is the yield point is there and where is the maximum tensile strength is there, and then what is the fracture point, because we have the ductile material. So, this is for aluminum.

(Refer Slide Time: 54:50)



Then, we have the cast iron. So, as we discussed earlier, the cast iron is always provide good damping; that means there really good in the compression. So, the stress-strain component for tensile the... Straightaway if you apply the tensile load, there is a straight failure. But, if there is a compression or compressive loads are there, the compressive stresses are there in cast iron; they are showing very good strength over this. Then, we have the copper, which is also some sort of the ductile material. So, it always offers here... If you see here; if you compress those things; then you will find these things here. So, this is nothing but the stress-strain characteristics for the copper.

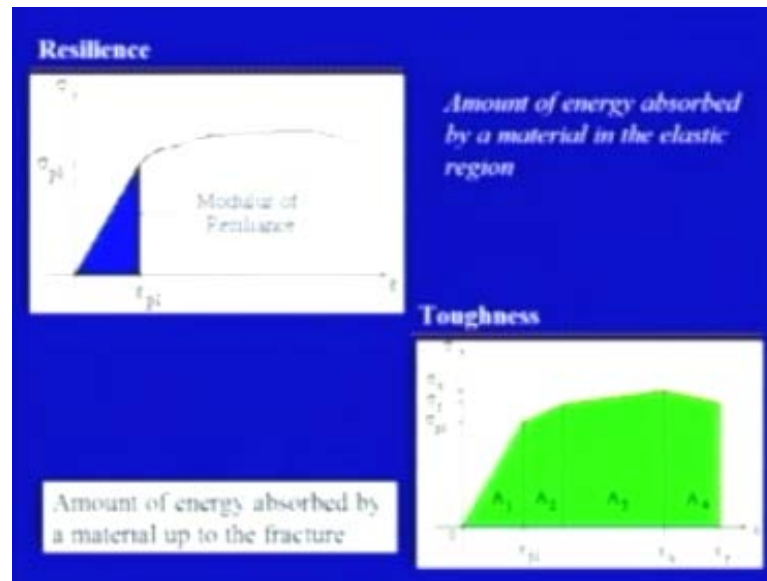
(Refer Slide Time: 55:36)



So, these are all the graphs, which is shown that, actually if we have different carbon composition or if we have different – these microstructures, then you see how these different regions are coming under the stress-strain relationship. So, in the previous graphs, these are all the diagrams, which clearly shown that, actually there is an important part is there that, what kind of load application is there and how the stresses are being setup within the object, and what kind of deformation is there. And how we can correlate the using of Hooke's law or modulus of elasticity or with the different strength like the ultimate tensile strength or the yield strength or the proportional limit or the fracture strength. So, this part we discussed here.

Now, as we see in the previous diagram of the stress-strain diagram for structural steel, we have different regions. So, one region was there under the elastic deformation. So, area under the elastic deformation – whatever the elastic region was there; this area, which is clearly showing here by blue line is also known as the modulus of resilience. Resilience is always coming even if the impact loading is there or the steady state loading is there. So, amount of energy absorbed by a material in the elastic region. So, that is the important term. If the area, which is going beyond this term, then it is not defined as the term of resilience. But, whatever the amount of energy, which is being absorbed while the axial pulling is there – so whatever the energy absorbed by the material since you are applying the load. So, definitely, the particles are absorbing the energy because of the load application So, how much energy is being absorbed in the elastic region is termed as the resilience.

(Refer Slide Time: 57:24)



So, now, the next part is the toughness. If we go beyond the elastic region and whatever the amount of energy is being absorbed by the material under the axial pulling up to the fracture point or up to the rupture point is known as the toughness. So, there are two regions. One is the elastic region. And, whatever the amount of energy is being absorbed by the material till elastic region, is the resilience. And, if you go up to the plastic region; that means up to the last point, that is the toughness. So, these are the two properties of material that, how much energy is being absorbed. So, it is a resilience and the toughness.

So, now, we just want to conclude these things that, actually today, we discussed about the initial part of the solid mechanics; that means how much, what kind of forces are there; what type of the different mechanics is there of the rigid body or the deformable body; how the internal body forces are being setup; how the stress components are acted on an object. And, the stress is always to be defined with the tensile stress; that means we need at least 9 components to describe the stress. And, whenever the force application is there on an object, always there is a sort of deformation, which can be measured with the property of material; that is known as the strain. We can define the relationship between the stress and strain, and the elastic and the plastic region. So, that part we discussed.



Now, in the next class, we are going to discuss more about that, actually what kind of other parts of stresses are there and how we can define the relationship, if the object is not the straight bar; if the object is of a different geometry.

Thank you very much. Have a nice day.