Industrial Engineering
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Module - 04
Lecture - 09
Network Analysis – II

A very warm welcome to all of you, in the second lecture on Network Analysis, we have already covered the basic concepts of project management. What is a project, what do we understand by project management, how the project can be converted in to a large number of tasks and activities, what are tasks and activities, each and every task would require certain resources for the accomplishment. And all these tasks and resources would be interrelated to one another in the form of a precedence relationship.

We have seen, what are the network representation fundamentals, what is an activity, what is an event and what is a network. We have started the discussion, towards the formation or drawing the network on a piece of paper. So, there are certain rules and regulations that have to be followed, when we represent the network in the form of a diagram or a graphically representation of a network in the form of a diagram on a piece of paper.

So, once the network is ready, then this network is used for project planning and scheduling purposes. So, we can see that how many people would be required on what particular day, which activity is moving at a proper rate, maybe controlling I am not telling, planning like we plan in advance that what will be done at what particular movement of time. Then we have to exercise a certain degree of control also that whether the activities are been being performed at their scheduled rate or are certain activities lagging behind.

And if certain activities are lagging behind, then we have to speed up the process of those activities, so that the project is completed by due date. So, the project completion is dictated by a number of parameters out of which, a late penalty is one of the important performance parameters, if the project is late we have to pay the penalty and that penalty each and every company wants to avoid.

So, we have seen that a project can be represented in the form of a network and that network will further aid the manager, in doing a proper planning and scheduling of the activities, as well as controlling exercise can also be incorporated. So, today we will start our discussion with certain rules that have to be followed in the construction of the diagram or the network diagram.

And then subsequently we would try to see, that what are the other network parameters, other network fundaments, and thereafter we will move on to solve 1 to maybe 3 problems related to the networks. So, let us see now, what is rule number 3, 2 rules all ready we have seen in the last class.

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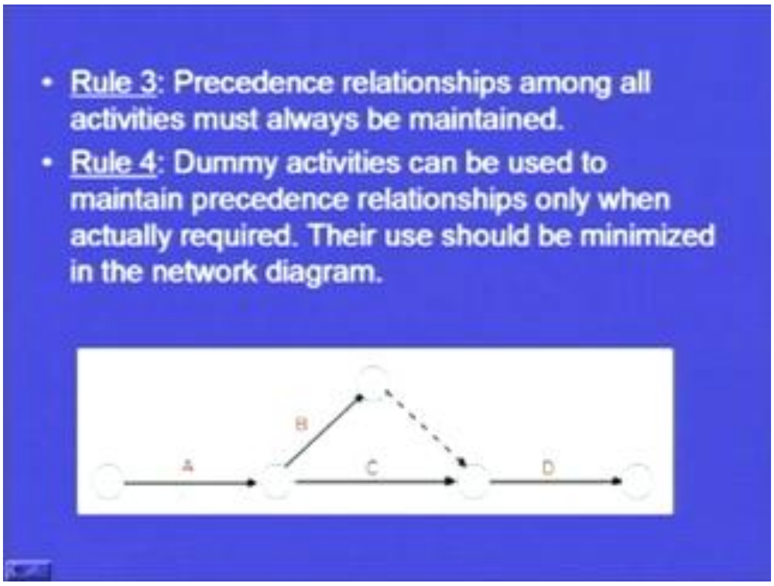

So, rule number 3 says that precedence relationships among all activities must always be maintained. Now, what are the precedence relationships in the network, which is shown on your screen, if you think and if you have understood properly the previous lecture, you would be very easily able to understand that, what are the precedence relationships in this diagram.

Activity A has no predecessor, which means that activity A can start on any given day and it is not dependent on the completion of any other activity, which would have been the predecessor of A. So, A has no predecessor it can start on any day, but activity B as well as activity C, both are having A as there predecessor activity, so activity A is a predecessor of activity B and C. So, B and C cannot start until and unless A is completed.

Similarly, activity D is dependent on B and C, whenever activity B, as well as activity C would be completed activity D can start, now why B is not going directly in to this node or this event. This will result in the violation of rule number 2, which we have all ready covered in the previous class, so activity B is connected to this node with the help of a dummy activity. So, for those of you who were not present or those who have not gone through the previous lecture, this is a dummy activity and this dummy activity is there to satisfy certain logical precedence among the various activates.

Now, rule number 3 says that precedence relationships among the activities must always be maintained. Now, all these activities are bound to one another with certain logical precedence and which has always to be maintained. If that is no maintained properly, then the network would not give us accurate results, maybe by doing all the calculations using say critical path method or program evaluation review technique, which we call as PERT.

Either by CPM or PERT we find out that this is the other project completion time, but if this precedence is not represented properly in the form of a network diagram, then the value that we are getting, maybe incorrect. And that value acts as the basic ingredient for all the project planning and scheduling activities, so that is one of the most important values, that we want to be correct to the best possible manner. So, this precedence and incorporation of this precedence in the network is extremely important.

So, rule number 3 states that precedence relationships among all activities must always be maintained. So, which is one of the most important points to be borne in mind, whenever we are representing the complete project in the form of a graphical network diagram. The fourth point says dummy activities can be used to maintain precedence relationships, only when actually required, there use should be minimized in the network diagram.

So, I mean to say here is that, we should not be too liberal with the use of dummy activities, so wherever a little bit of problem is there we cannot incorporate a dummy activity. So, the total number of dummy activities within the network should be as minimal as possible, we cannot be too liberal with the use of dummy activities, because this will further result in to a lot of calculation.

Because for each and each every dummy activity also the calculations will be done, as we will see in the subsequent slide and this will require a huge amount of memory space allocation, in which this data will be stored as well as the time for doing the calculations will be much more. So, the number of dummy activities has to be limited and these have to be used wherever a kind of logical precedence has to be ascertained.

Now, I have explained in the previous class also just to revise, I would give you an example, D activity is dependent on both activity B and activity C, suppose there is another activity called E, which is only dependent on B. So, if we are not using this dummy activity and B is also going here which although is wrong as compared to rule number 2, but if activity E is starting from at from this point and going in this direction.

This means that activity B and activity C both are immediate predecessors of activity E, which is not correct, only B is the immediate predecessors of activity E. So, use of this dummy activity has given us this point or this event, at which B is completed and E can very easily start from here in this direction. So, in this way we have, we can say and we have incorporated the logical precedence, that E is dependent on B and D is dependent on both B and C.

So, in the way the precedence has been established and dummy activity has been used in order to incorporate this logic or the precedence relationship in to the network. So, use of dummy activity is required wherever logic has to be incorporated in the network, but the liberal use of dummy activities like, we cannot be too liberal with the use of dummy activities. We have to keep a check that, we use the minimal number of dummy activities in the network, otherwise the network will become very, very complex.

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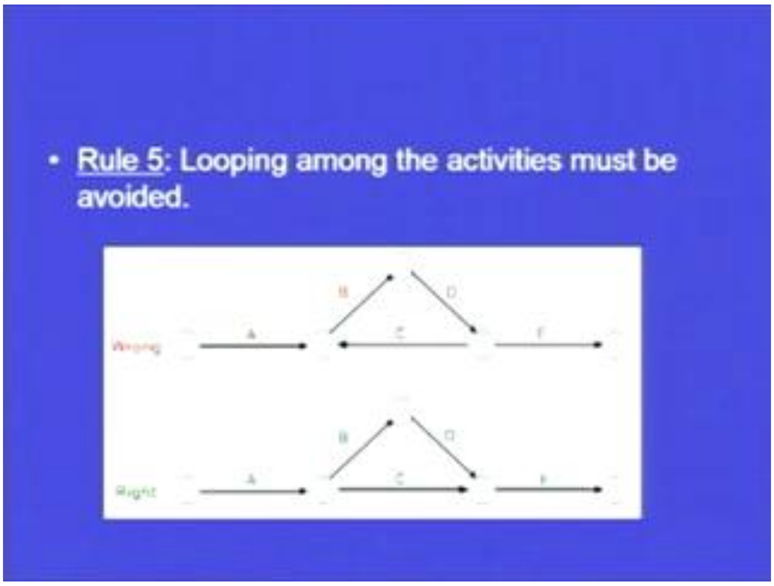

Now coming on to rule number 5 looping among the activities must be avoided, now here we see a loop is being found, this is a wrong network, activity A B C D and activity E. Now, you see B is getting completed here, D is getting completed here and C is getting completed here, so a loop is forming like this. So, whenever in a network there will be a formation of the loop, we will not be able to get the results, so the formation of the loop has always to be avoided and if loop formation takes place, then we will not be able to get the accurate results.

So, how the loop formation can be avoided, this is the right network A B C D and E, just to revise here again related to precedence relationships or the predecessors activities and the successor activities, I would like you to just see that, activity A has no predecessor, activity B and activity C have a predecessor in A. So, B and C cannot start until and unless A is completed, similarly D cannot start until and unless B is completed, so when B activity will get completed, then only activity D will start.

And E activity which is the last activity of this small project is dependent on D and C, so whenever activity C and activity D will be completed, thereafter only activity E can start. So, now there is no loop formation there is no continuous loop like this and this is the correct network.

From this network very easily, we can see that how much time would be required for the completion of this project, so if in place of writing only A, which is the name of this activity, we could have provided the time, in days or months or year required to complete this activity. We would very easily would have been able to find out that how much total time or total duration would be required for the completion of this project, that we would be seeing with the help of an example.

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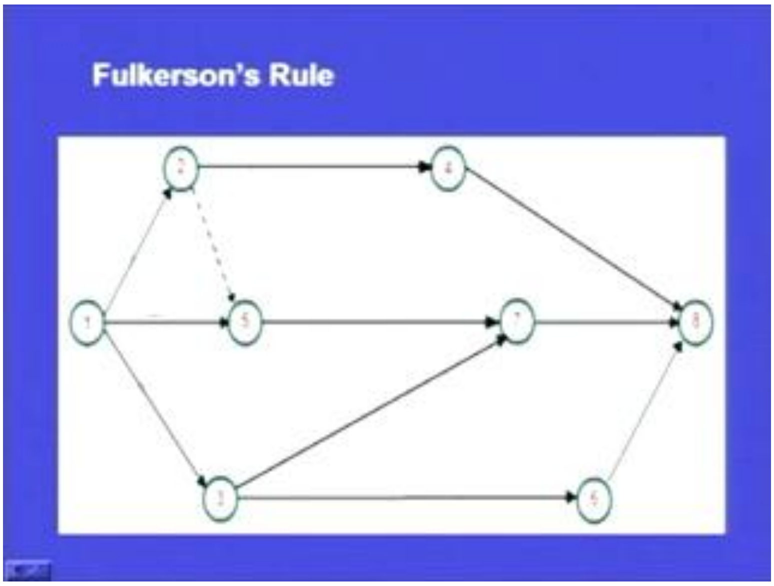

Now, we come on to another point, which is the Fulkerson's rule, now Fulkerson's rule basically is used for naming the nodes, so this is a events of the nodes, that we are going this like 1 2 3 4 5 6 7 and 8. So, all these are called the nodes or the events, now we want to name them, initially we have named the activities in terms of A B C and so on and so forth.

Now, we want to name the nodes also, so node numbering is done with the help of the Fulkerson's rule, now let us understand that, how this node numbering is done. Initially we try to find out a node from which, there are arrows emerging and there is no arrow entering in to that node. So, in your diagram on your screen, you can see that, this is the node, from which there are arrows coming out. And there is no activity or no arrow going in, so this becomes our node number 1, because only the arrows are coming out, there is no arrow going in, this means this becomes our node number 1.

Now, we will strike off all the arrows coming out of node number 1, this we will strike off, this we will strike off and this we will strike off, now we will see that which is the node, in which there is no arrow entering. But, there are certain arrows which are going out, now we see that this is the node, in which there is no arrow entering and there are only arrows going out.

Similarly, this is also a node, because this has been struck off, so this is node out of which, only the arrows are going out and there is no arrow getting in. So, now we have two nodes 2 and 3, in which there is no arrow coming in, but there are only arrows going out, so we start from the top, so this is the top, so we number it as 2 and this is numbered as 3. And all the arrows going out of these two nodes are then struck off, like this is also struck off, struck off, struck off, struck off.

Then we see, that which is the node, in which there is no arrow coming in and there are only arrows going out. Now, suppose then we get this node, in which there is no arrow coming in and there is only arrows going out, so that node then becomes node number 4 and subsequently, this becomes as node number 5 6 7 and 8.

So, we start from the node at which, there are no arrows coming in and only arrows going out, we strike off, we name it as number 1 and we strike off all the arrows going out. And try to search a node, in which there is no arrow entering in and no there are only arrows going out, that we number as the subsequent number, suppose we had numbered node number 1, initially then we will call it as node number 2.

If there is a conflict there are two such nodes at which, there are arrows only going out there is no arrow entering in then we will name the top node as the lower most and the bottom node as the higher node. So, this that was seen when we have seen node number 2 and node number 3 and subsequently, we will move on and there may be certain times certain conflicts maybe there, but this would help us to identify, that which node number should be assigned to which particular node.

So, this would help us where this is just a rule in order to name the nodes or in order to number the nodes, certain conflicts may happen and then, we have to just understand it that this is the way, the activities are flowing. Like in this diagram if you see this is a start point and all the activities are moving all the arrows are moving in this direction and this becomes the end point.

So, the flow of the activities with time is represented, so it is not on the time scale please remember that, this network is not on the time scale, but we can generally see that all the activities are moving from the start towards the end point. Now, we will just review that what type of relationships, may exists between the various activities.

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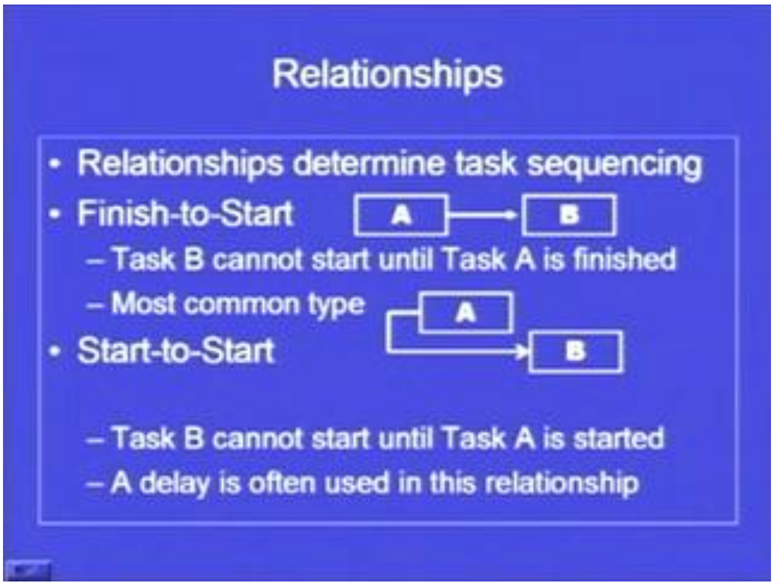

Relationships determine the task sequencing that how the tasks have been sequenced one after the other, so these relationships will help us to understand that, this activity will take place only after the completion this activity. So, there are certain relationships that we follow, whenever we construct a network diagram. So, first type of relationship on your screen is the finish to start relationship, so what is this finish to start relationship, if you say, there is activity A and activity B.

Now, B cannot start until and unless A has been finished, so finish of A will result in the start of activity B, so task B cannot start until task A is finished, which all ready I have told you now, there are two tasks A and B and that relationship among A and B is that finish to start. So, when A will finish then only B will start B cannot start before the finish of A, so there may be a precedence like, we cannot erect the walls until and unless the foundation is ready.

So, A maybe representing the laying of the foundation and B maybe representing the erection of the walls, so as soon as the foundation is ready, erection of walls can start, so A representing the foundation laying and B representing the erection of wall. So, finish of one will automatically indicate the start of the other, so that is finish to start type of relationship. This is the most common type of relationship, which is used in networks, then there is another start to start relationship.

So, B will start when A has started, so both will move simultaneously start to start relationship, as soon as A starts on your screen, you can see B it will trigger of a message and B will also start. So, task B cannot start until task A is started, so this is also a type of a relationship, which has to be sometimes incorporated, when we draw the network or when we construct the network. So, start to start type of relationship, which indicates that task B cannot start until task has started.

In the previous case task B cannot start until A has finished, but here with the start of A task B will also start, there are other type of relationships as well. But, the most common type of relationship, which is used in the networks is the first type of relationship on your screen that is finish to start, which means that B cannot start until and unless has been completed accomplished or finished. Then a delay is often used in this type of relationship, so a little bit of delay will be there as soon as, A has been started, there will be slight delay a trigger message will be sent to B and then, B will also start. Then let us now see the precedence relationships.

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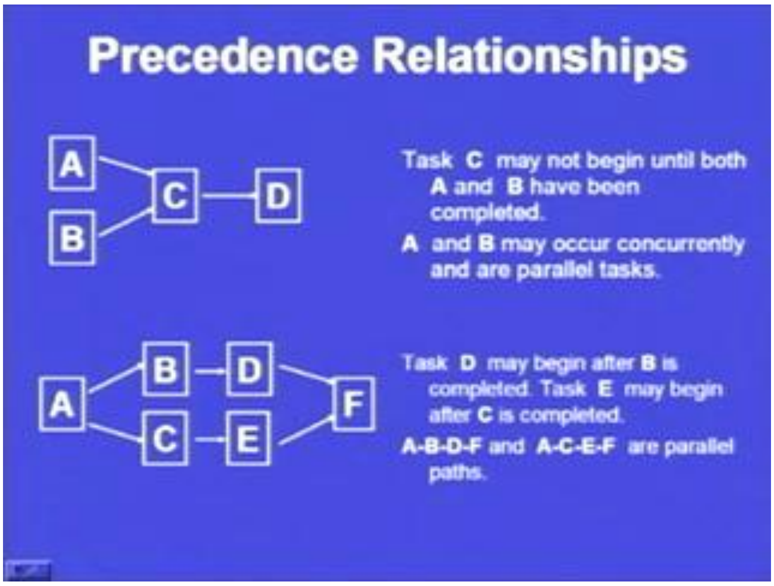

On your screen you can see, C may not begin until both A and B have been completed, on your network you can see, that C cannot start, until and unless both A and B have been completed. A and B may occur concurrently and are parallel tasks, there is no precedence between A and B, A and B can run parallel and as soon as they culminate or as soon as they finish at a particular point from that point activity C will start.

So, I mean to say is that A and B are independent of one another, they can just at the same time. They both of the tasks maybe getting accomplished at one station, one task is getting accomplished at another station another task is getting accomplished, they may run parallel, the tasks or the activities may run parallel, which one A and B.

But, C will only start until and unless A and B have been completed, so as soon as activity A and B gets completed, C will start, so A and B may occur concurrently and are parallel tasks. Now you are see on your screen an another very simple diagram, in which we have six different tasks or activities, now in this you can see task D, may begin after B is completed. So, you can see D may begin after B is completed task E, may begin after C is completed, so you can see on your screen D can begin when B is completed, so it is not dependent on C, so D is not dependent on C.

Similarly, E is not dependent on B, so B is an immediate predecessor of D, also we can see that A B D F and A C E F are parallel paths, so on your screen, you can see that A B D F and A C E F, both these are parallel paths. So, we will see in any project or in any network there will be, so many parallel paths and we will see, which is the longest path that would give us the time required for the completion of that project. So, that basically is the critical path method, that has been devised to do, all these type of project planning and project scheduling activities, so let us see now what is critical path method.

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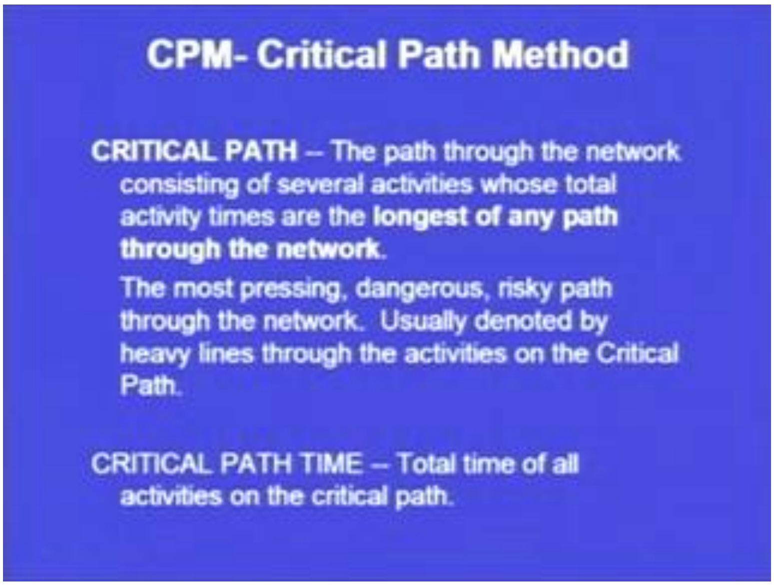

Critical path, the path through the network considering the several activities, whose total activity times are the longest of any path through the network. So, this is the critical path, as all ready I have told in the previous slide you have seen that, there will be multiple paths within the network. So, there were two paths in the previous slide, which we have seen the network consisted of six different activities and there were, two parallel paths running from the start till the end of the project.

So, critical path once again I will read if for you, because this is the most important point to be kept in mind, so critical path, the path through the network consisting of several activities, whose total activity times are the longest of any path through the network. So, here I would like to address another point, which is not there on the screen, but you need to understand that, that is the basic difference between the critical path method and the program evaluation and review technique.

So, these are two types of network solving prop methods or we can say network solving programs, now critical path method as you all ready this takes care of the critical path and program evaluation and review technique is also based on the same principle. Now, what is the difference between critical path method and program evaluation and review technique. Critical path method is used, wherever the time estimates for each and every individual activity are know with a certain degree of certainty.

We are certain, that if this project is consisting of ten different activities, each for each and every activity, we know that activity A will require 5 days, B will require 7 days, C may require 8 days. So, for each and every activity, we know that the time estimates are known with certainty, so this is more deterministic in nature, why because for each and every activity, we are knowing that this is the time required for completion of this particular activity.

So, how do we come to know that this would be the time required for this activity, this we come to know with our past experience. And that is why critical path method is used in those types of projects, in which we have certain amount of historical data all ready available with us, so this is all related to critical path method. Now, coming on to the program evaluation and review technique, here for the completion of each and every activity, we are not fully sure that how much time would be required for the completion of this particular activity.

So, we have three different time estimates, so what are these three different time estimates, these are optimistic time, most likely time and the pessimistic time. The optimistic time is the time, in which we are very, very optimistic that, if everything goes well the activity should be completed in this much amount of time. Most likely is in between the optimistic and the pessimistic time, and pessimistic time is the time, in which we feel that if everything goes wrong, the activity will be completed in this much amount of time.

How this times are distributed that is not under the preview of this particular lecture, but we need to understand is that in case of PERT, there is a certain degree of probability associated with the time estimates. That is optimistic time, most likely time and the pessimistic time, so this type where this is used, PERT basically is used those type of projects, in which we do not have a substantially amount of historical data available with us, for example, in research and development type of projects.

So, RND we do not know that, we say if with a certain degree of probability, that if this work is completed by such and such date, then this project can be completed by such and such date, so probability is there an element of probability is there. Therefore, PERT is usually probabilistic in nature, whereas CPM is CPM method or the critical path method is deterministic in nature. So, there range of applications is different, the time estimates are different and the application areas are also different, so both of these have their own domains, where each one of them fits in to.

So, in today’s class, we will solve we will see one problem related to CPM and another problem related to PERT. So, now we need to understand is in critical path method, what is the critical path, I think by now all of you might have got an idea that, what is CPM, what is PERT and how these two are different, we will see this with the help of an example also. So, critical path basically is the path through the network consisting of several activities, whose total activity times are the longest of any path through the network.

So, there may be a large number of paths, but for the path for which the summation of the times of individual activities, which are lying on that particular path gives us the longest or the highest value or the summation becomes the highest, that path is called as the a critical path. The most pressing dangerous, risky path through the network usually denoted by heavy lines through the activities on the critical path. Now, why this is most pressing dangerous and risky path through the network, why is it?

This is so, because we know that all the activities, that are lying on the critical path are having certain time, they would require certain time for their completion and this time, we are adding for the activities, that are there on the critical path and we are getting a value of time, that this project will be completed in this much time. Now, suppose that one of the activities gets delayed, so as soon as instead of taking say 5 days, now it is taking 7 days, now 7 will be added to the total path and the overall completion of the project will get delayed by 2 days and that we do not want.

There may be a penalty clause in the contract and that has to be avoided. So, delay in any activity on the critical path will result in the delay of the complete project or the project completion day will get further extended may be by another 2 days or another 5 days and this will result in to a loss to the company. So, that is why this is the most pressing, most dangerous and most risky path, because on certain other paths, we have certain levy available with us, we have certain slack available with us.

So, what way what may happen on the non critical path, even if a activity gets delayed, it is not going to have a direct bearing on the extension of the project schedule. But, if an activity on the critical path gets delayed, then it will directly have a bearing on the projection completion date. So, what is done as a by a project manager, as the precautionary measure is that he will always keep a track on the activities that are falling on the critical path.

If by chance any of these activity gets delayed, then the project manager will keep certain resources will release certain resources, from the non-critical jobs to these jobs, so that to the critical jobs. So, that there length is shortened suppose an activity required 5 days under normal circumstances for completion by certain problems may be, because of certain risk or certain uncertainties or certain unforeseen circumstances the activity got delayed.

So, instead of taking 5 days, now it has taken 7 days and this activity is lying on the critical path, so the critical path will get delayed and the project completion day will be extended by 2 days. Because, if this activity is delayed the whole project length gets delayed, now what we have to do is we have to, see that which activities are following this activity and if this is completed in 7 days. We have to make up these 2 days in the subsequent activities, we may be required to complete the subsequent critical activities by shortage of 2 days.

So, we have to reduce their duration by 2 days and when we know, this is the critical path, this is the schedule by which we are running behind, we will try to increase the speed of performing the subsequent activities or we will thrust some additional resources for those activity. So, that those activities are complete, well in time and by the completion of the project completion date.

So, as soon as the project completion day comes, our all the activities should have been completed, so critical path is a very important path, which helps each and every manager, each and every planner, to take the decisions, so that the project completion day is not delayed. So, we need to understand the basic integrity of the critical path, that if we are able to keep a check on the critical path, our project will always be completed on time.

But, certainly there might be certain circumstances in which, certain unforeseen things may happen and the project may get delayed. But, as a manager we have always to keep a track or of the progress of the work or the schedule of the activities on the critical path, so that the activities do not get delayed beyond certain point. Critical path time, total time of all the activities on the critical path, this all ready I have explained, so we have seen, what is a critical path, it is the most risky dangerous and pressing path and what is a critical path time. Now, let us try to understand that, how easily we can find out the critical path, so there are certain terms associated with the critical path, that we are now going to see, now the terms used in critical path method.

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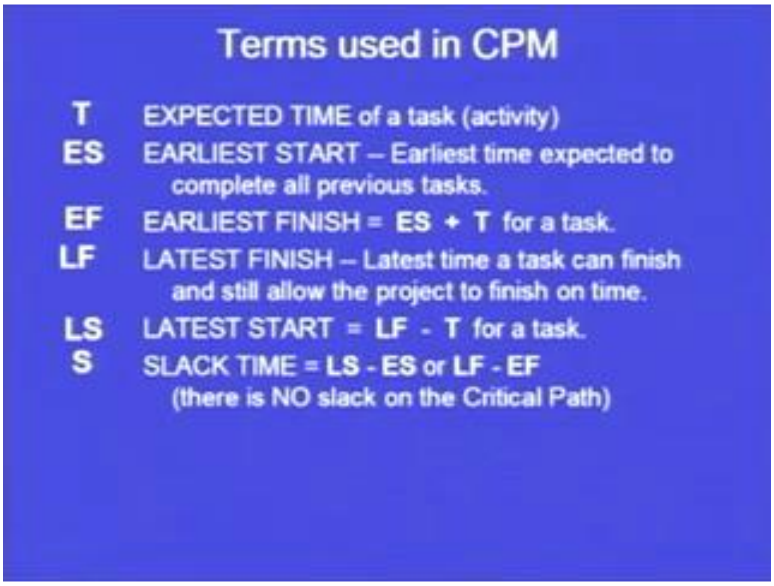

T is the expected time of task or activity, which is the time required for the completion of a particular activity, then we have earliest start. So, earliest start is the earliest time expected to complete all previous task, now here you can understand that when an activity can start at it is earliest. An activity will start at it is earliest, when all it is predecessor activities have been completed, so that is the only thing that is represented on your screen.

Earliest time expected to complete all previous tasks, so all previous or activities, when they are complete then only this activity can start, so ES will give the earliest start for a particular activity. Then earliest finish, so when it has started at it is earliest time and we know that T is the time required for the completion of the that activity, then earliest finish will be, earliest starts plus the T or the time for accomplishing that task. Similarly latest finish, so the latest time a task can finish and still allow the project to finish on time.

So, this latest finish will give us that, if suppose there is a last activity, which requires 5 days for completion and the total project length is 24 days. So, last activity requires 5 days, so what is the latest finish time for the predecessor activity for this, if the project has to be completed on time. Now, the project has to be completed in 24 days, last activity requires 5 days and this activity can only start, when certain other activities have been completed.

So what would be the latest finish time for those activities, that is 19 days, because if they finish later by 19 adding 5 more days will not, sum up to 24 and the total project may get delayed. So, latest finish is the latest time a task can finish and still allow the project to be finishing on time, so the project will finish on 24th or may be after 24 days 5 days for the activity, last activity, so the previous prior activities should finish latest by the 19th day or the end of 19th day, that is a latest finish time, for those activities.

The latest start, so latest start is given by the latest finish all ready, we know what is the latest finish minus the time required for accomplishing that task. Similarly as we calculate is the slack time, which is given by the latest start minus earliest start or latest finish minus the early finish. And this gives us a value or the nodes or the elements, where there will be no slack and the points, at which there will be no slack or the nodes, at which there will be no slack would fall on the critical path, so there in no slack on the critical path.

So, no slack means, that if anything gets shifted on the critical path, any activity is delayed the overall project duration will be delayed. But, may be on certain other paths, there is certain slack available, that the activities can be shifted forward and backward, so that we are able to release the resources from those activities for the activities on the critical path.

So, network evaluation steps, now we have seen what is a critical path method, what are the rules for drawing the network diagram, what are the salient features of the project. We have seen the Fulkerson's rule for making the network or numbering the network, now we will see what are the network evaluation steps?

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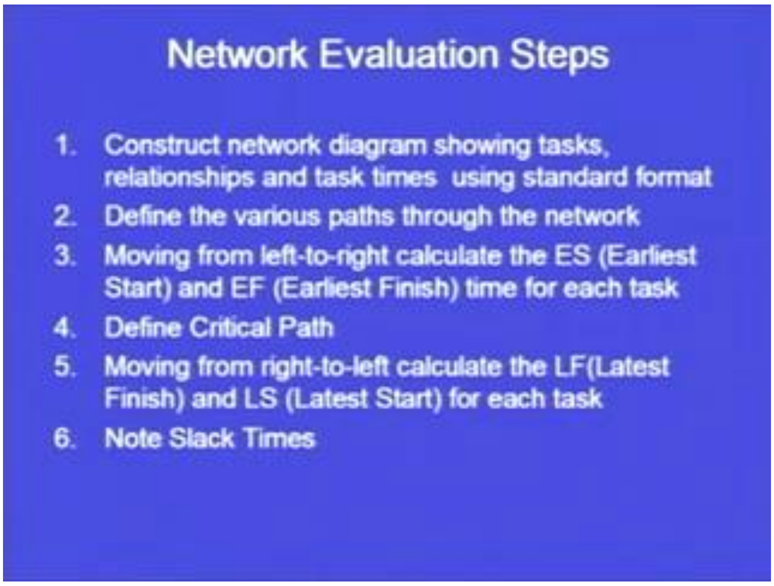

Construct the network diagram showing tasks relationships and task times using the standard format, we have all ready seen certain very simple networks, but mind you, this is these are very simple networks, that we are seeing, here in this particular lecture. But, the networks can be very, very complex, we will see towards the end of this lecture certain very complex networks also.

So, first step is to construct a network, diagram showing tasks interrelationship and the task times for each and every individual activity. Define the various paths through the network, so all ready we have seen that, there will be number of paths in the network. Moving from left to right calculate the early start and the earliest finish time for each task. Now, for each and every task we will see that when can this task start at the earliest and when can this task finish at the earliest.

So, mind you please take in to account, that these calculations have to be done moving from left to right of the network right from start towards the end, we would move and we will see, when this activity can start at the earliest, when this activity can finish at the earliest. For example say activity number 1 or activity A first activity, it has no predecessor, we will say it can say at day 0 and then this will progress maybe, it requires 5 day, it will take 5 days to finish.

Now, the activity B, which is dependent on activity A when can it start at the earliest, it can start at the earliest when A has finished, now A is finishing after 5 days. So, B can start at the earliest on day 5 may be on day 6, when 5 days have been taken by activity A. So, similarly moving from left to right, we would be able to find out the earliest start and the earliest finish time, for each task. Now, we can find what is the critical path, which would be the longest path.

Moving from right to left, we can calculate the latest finish and the latest start for each activity, because now we know this is the critical path, this will be the time or the scheduled completion of the project starting from that point moving behind, we can calculate, what is the latest finish and what can be the latest start, for each task. Now, we can know slack times, that which activity has a slack and which activity has no slack, obviously the activities which do not have a slack or slack or equal to 0 would be falling on the critical path.

Now, let us once again remember on your screen all these points have to be taken in to account, whenever we are solving any problem related to the network evaluation. So, we draw the network, we calculate the latest start, early start, latest finish, early finish, we see, which one is the critical path and then, we calculate the slack for each and every activity. Now, let us come to a very simple practice problem.

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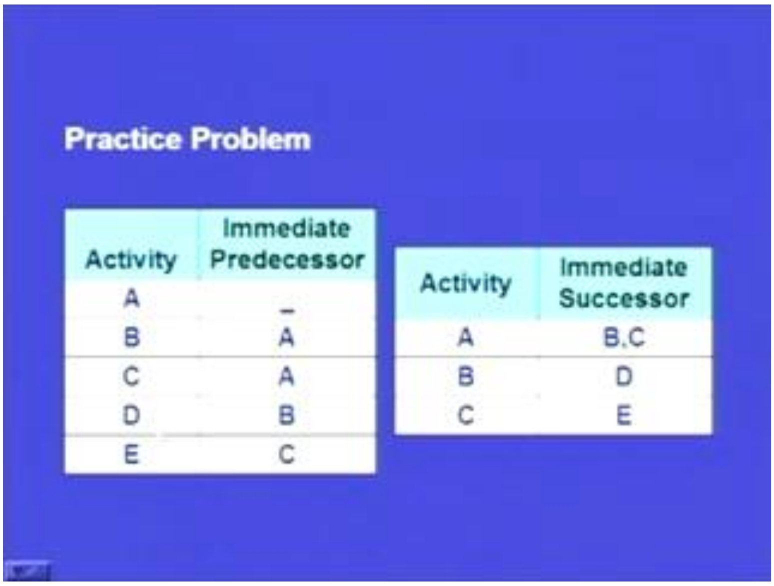

On your screen you have a practice problem, you can see any problem related to a project will be represented in this type of a tabular form. Even we can add another column, in which we can give the description of these activities also, but for simplicity here, we can say here this is giving the names of the various activities, which have to be completed in order to accomplish the complete project.

Now, the complete project here is consisting of five different activities, now what are those five different activities, activity A B C D and E, also we have on our screen the immediate predecessor for all these activity. Now, activity A has no immediate predecessor, which means that A is not dependent on any other activity, A can start on it is own, activity B has A as it is immediate predecessor. Similarly C also has A as it is immediate predecessor, activity D can only start until and unless activity B has been complete and activity E can only start until and unless activity C has been completed.

So, we know the activities, we know the immediate predecessors for the activities, similarly activities having their successors also is shown. Now, successor is the activity following the previous activity, now A will be succeeded by B and C, B will be succeeded by D and C will be succeeded by E. Now, this is a very simple problem, in which only the logical precedence has been provided, although for another set of calculations, in order to make the full use of the network, we can have another column here, in which the time for completing all these activities would be provided.

For example A may take 5 days, B may take 7 days, C may take 2 days, D may take 1 day and E may take 5 days. So, when the time is also provided very easily, we can find out the critical path and we can find out that how much time would be required for completing this project. But, here we are only concerned with the logical precedence among the various activities. Now, let us see that when this data is available to us, when we have five different tasks, which have to be completed and there is a logical precedence among these tasks, how we can represent this type of a problem, in the form of a graphically representation of a network.

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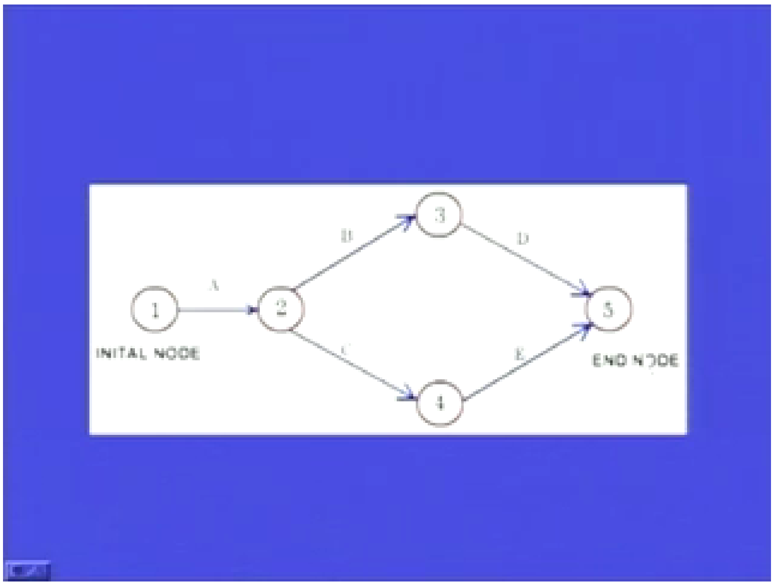

Now this a network on your screen, so we have initial node and we have a final node, so any project from the beginning of the lecture, we have been discussing that, what is a project it should have a definable beginning and a definable ending. So, this is a definable beginning initial node and we have a definable ending also that is the end node. Now, you see from this network, if you remember A has no predecessors. So, it can start from day 1 or day 0, B and C have A as their immediate predecessor, similarly activity D has B as the immediate predecessor.

So, we can see that, this precedence relationships have been shown on your screen and all these precedence relationships have been incorporated in the form of a network diagram, that you see on your screen. So, we had a problem of this type, in which we had the activities, we had the immediate predecessors and also the immediate successors and we will we have been able to represent graphically, that problem in to the simple network, in which we have all the activities and the precedence relationship incorporated in to the network.

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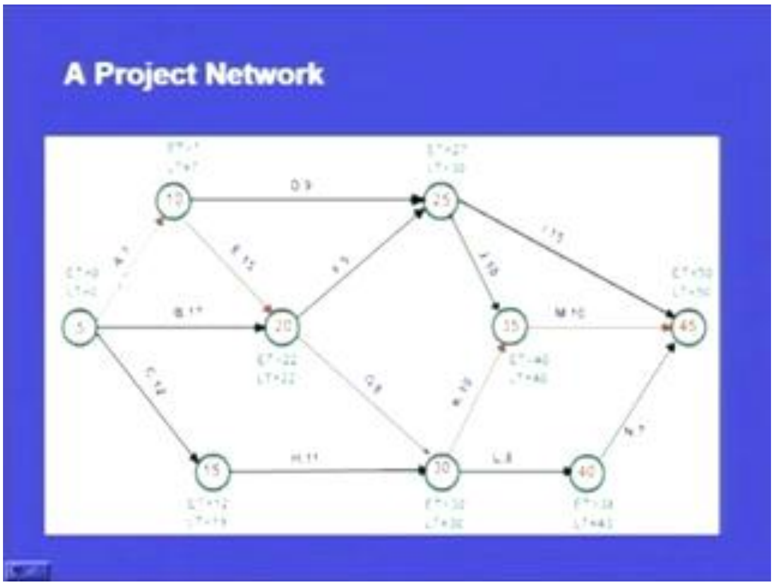

Now, you can see a very complicated network consisting of an array of different activities with their logical precedence also, so you can see right here activity A B C D E F, then we have G, we have H I J and so many K L M, and so many activities are there in this network. And then we have done all the calculations for these activities, we can say that activity D, suppose this is activity D, this requires suppose nine days for completion, when can this activity start at the earliest.

Suppose starting from left activity A B and C can start at the earliest at 0 day, so very first day these activities A B and C can start, now when we go to activity D, when can it start at the earliest, it can start at the earliest at the end of 7 days. So, it can start after 7 days, similarly activity E can also start after 7 days. And activity H can start after you can see H will start, only when C is completed and C is taking 12 days. So, activity H will start only after the completion of 12 days.

So, earliest start for H is 12 and earliest start for E is 7, earliest start for D is also 7, so in this way, we see that, we can construct a network diagram, first thing we need to ascertain is the logical precedence among the activities. For example is your network you see activity J, J can only start until and unless activity D and F have been completed as soon as D and F have been completed J can start.

Similarly I can also start when D and F are completed, so logical precedence has to be incorporated the arrows have to be properly aligned and when this logical precedence is correct, then only we would be able to get the exact idea, that this complete project would require, how much of time for the completion. So, here we can see 50 is the last, so 50 days would be required, if you say all these time domain things are in days, we will say that 50 is the overall project duration.

So, it would require 50 days for the completion of this project, so we have calculated the earliest start, earliest finish, latest start, late finish all those things, all those parameters can be calculated, if we have the time value associated with each and every activities. Now, let us now come to the critical path, let us now try to find out the places or the nodes where the slack is 0. So, you see, 50 minus 50 slack is 0 here, then this is 40 minus 40 slack is 0, here then 30 minus 30, 22 minus 22 and then 7 minus 7 and 0.

So, we can say A E G K and M, these are the activities, that are lying on the critical path, now let us sum up that how much time is required are they giving the longest path, now you see activity A 7 plus 15 22 plus 8 30 plus 10 40 plus 10 50, so this red line indicates that this is the critical path. So, we mean to say that activity A E G K and M represent the critical activities and the summation of the times of these individual activity is going to give us the longest path.

Now, what is the importance of this longest path already I have explained that, if any activity on this path gets delayed the project cannot be completed in 50 days the project will get extended by a day or 2. So, in this way we can represent, a very big project, which can be divided into a number of small tasks and activities, which are having independence relationship or a precedence relationship with one another can be represented in the form of a network, from which we can find out the critical path.

And then this critical path is the most risky dangerous path on which we have to focus all our attention, so that the project is not extended beyond the due date. Now, the important thing to note here is that the times are known with certainty, A will be completed in 7 days, B will be completed in 17 days, C will be completed in 12 days.

But suppose, this is CPM method, now we want to solve the same problem or maybe a different problem with PERT in that we will have three different time estimate, namely optimistic pessimistic and most likely times. And from these time estimates, we calculate the expected time for the completion of that particular activity.

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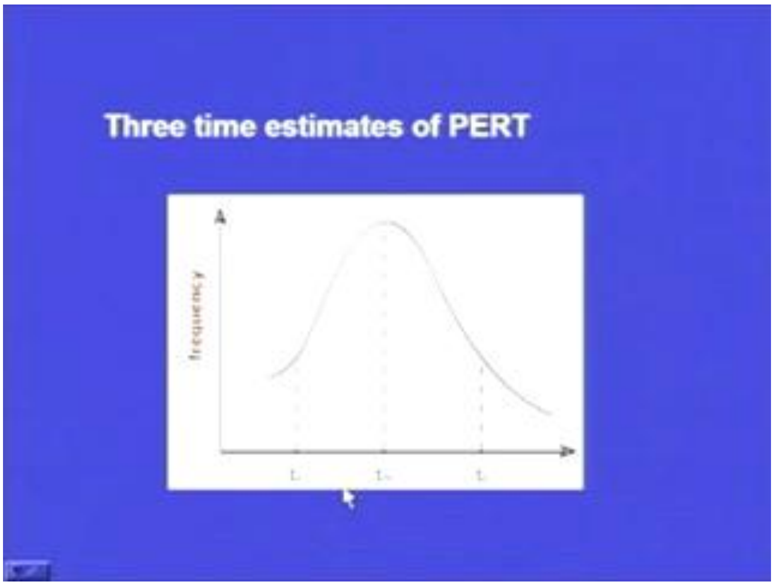

So, now let us go to the PERT, now three time estimates are there in PERT, first one is the most optimistic time, most likely time, this is the probability of most likely time and the pessimistic time, so optimistic time most likely time and pessimistic time. So, there are three different time estimates in case of PERT, as it is probabilistic in nature.

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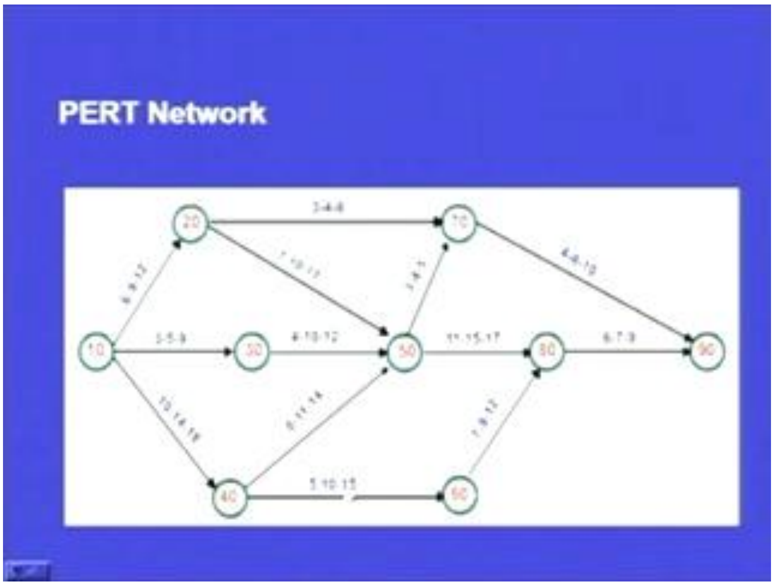

Now, we see that we have a PERT network as well, in this network, if you observe, very clearly it has been seen that 6 9 12 7 10 13. So, 6 9 and 12 means, that 6 is the optimistic time, 9 is the most likely time and 12 is the pessimistic time. So, we are not sure that, this activity 10 20 will be completed in how much amount of time, but most likely we feel, it should be completed in say 10 days.

Similarly, for each and every activity, we have three time estimates, that is the optimistic time, the most likely time and the pessimistic time. Now, suppose we have this problem what type of information can we derive out of this, certainly here also we can calculate which is the critical path, but we see for each and every activity, there are three time estimates.

So, we need to calculate one time estimate out of three these time estimates for each activity and that expected time for each activity would be used for calculating the critical path or finding out the longest path in the network and that expected time is calculated from these three time estimates.

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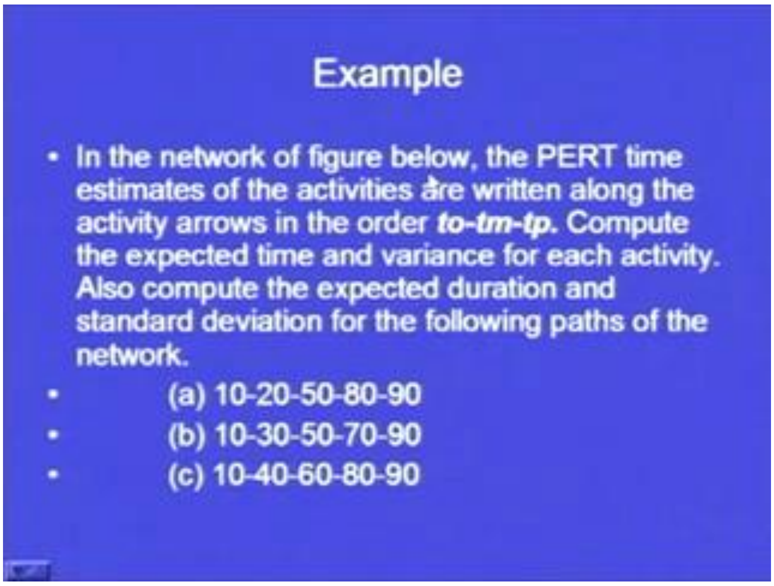

Now, let us take an example, in the network of the figure not below, which you have seen in the previous slide, the PERT time estimates of the activities are written along the activity arrows, in order of to-tm and tp. So, optimistic most likely and pessimistic time, compute the expected time and variance for each activity and also compute the expected duration and standard deviation of the following paths of the network.

Now, there are different paths, so like path is 10 20 50 80 90 10 30 50 70 90 and 10 40 60 80 90, so let us see 10 40 60 80 90 10 20 50 70 90. So, we have three different paths and for each and every path, we can calculate the standard deviation.

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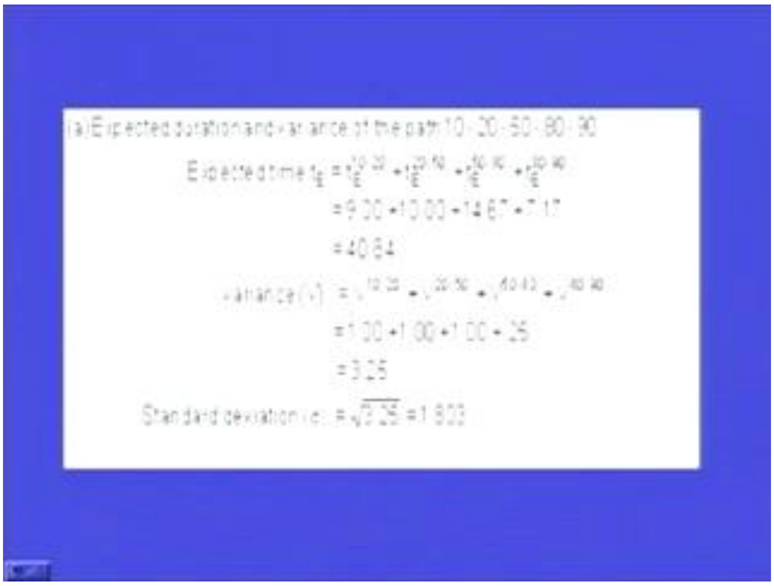

Now, here on your screen, the first path expected duration variance of the path 10 20 50 80 and 90, now expected time, total expected time of this path will be given by the expected time of 10 20, expected time of 20 50, expected time of 50 80 and expected time of 80 90. So, expected time is calculated, this is an exercise for you to go and find out, that how expected time is calculated, what is the formula that is used to calculate the expected time, from the optimistic most likely and pessimistic time given, for each and every activity.

So, we calculate the expected time for each and every activity and then, we calculate total expected time for that particular path. Now, variance is also added up, variance for each and every activity, there are four activities on this path 1 2 3 and 4, so the variance of each and every activity is added to get the complete variance, for that this particular path. And similarly, when the variance is known very easily, we can find out the standard deviation also.

So what is the need of finding the variance and standard deviation is that, the time is distributed over a period of time, may be for each and every activity the time is not deterministic the time is not constant it can take any value within the range, varying from the optimistic time to the pessimistic time. And that is why there can be a large number of critical paths, that maybe there in one particular network.

So, multiple critical paths are there, that is why we need calculate the variance and standard deviation. Now, this is for the expected duration of other path that was 10 30 50 70 90, similar calculations have been done.

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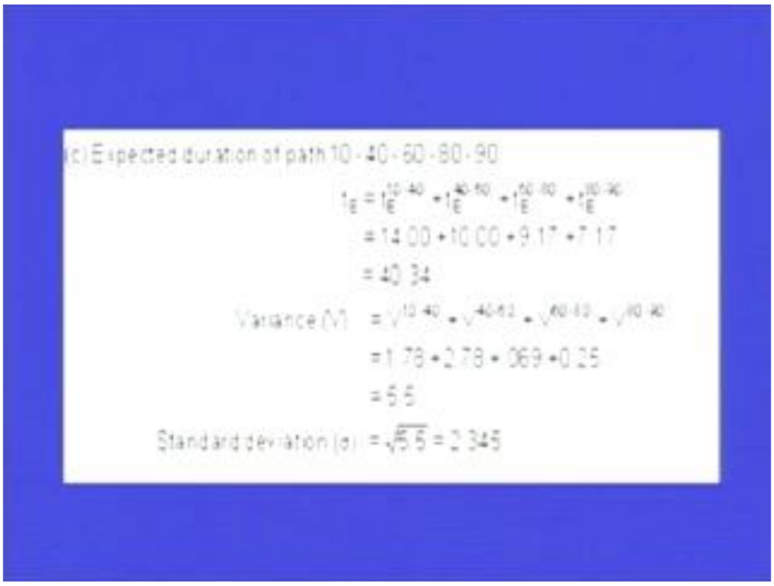

And similarly this one is for the third path 10 40 60 80 and 90, now you may say that, why there is a difference 10 20 30 40, why the names have been or the node numbers have been done as 10 20 30. Because, sometimes this is a such a dynamic scenario, we need to add on certain activities, which may creep in into the complete project. So, in order to keep that kind of flexibility to incorporate certain other activities in to the network we keep, sometimes name we keep the names as 10 20 30 40 like that.

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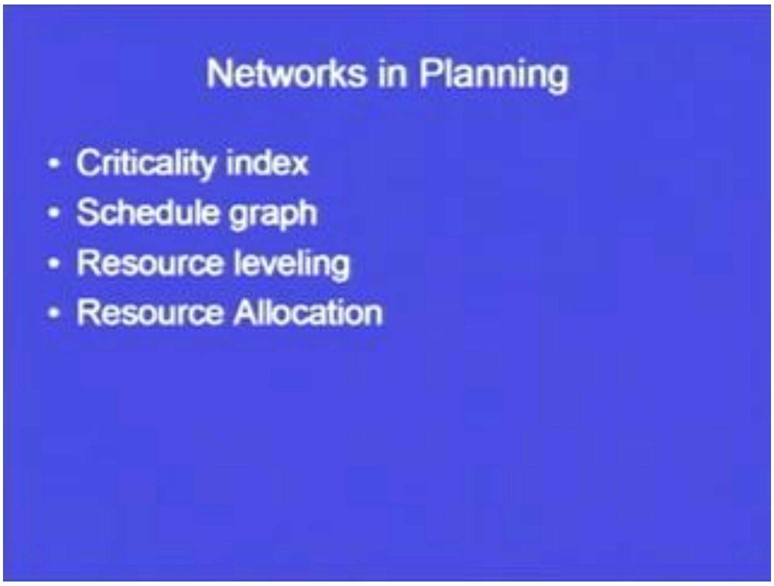

Now, we use the networks in planning, so one thing that is more important is the criticality index. Now, suppose for a PERT network we do the simulations with the time in the range is like, optimistic time is five and pessimistic time is twelve and in between there is a most likely time. So, this activity can take time, anywhere between five and twelve and in between there will be a most likely time.

So, if we do the simulation and find out the critical path again and again, we may find out number of critical paths. Then for each and every activity, suppose we do 1000 simulations, one activity is following on the critical path, 300 times out of the total 1000 simulations. Then we say that the criticality index, for this particular activity is 300 divided by 1000.

So, 1000 simulations done 300 times, this activity has fallen on the critical path, so this will help us to identify those activities, which are prone to fall on the critical path and we can focus our attention on those activities. Now, suppose for a particular activity, the criticality index comes out to very close to one then, it means that this activity is going to fall on the critical path most number of times.

Similarly, these networks will also help us in the planning with the if we draw a schedule graph, schedule graph is time scale graph, in which we will draw the activities with their precedence relationships and allocate the resources and optimize the resources on the daily basis or day by day basis. Similarly, resource leveling and resource allocation can also be very easily carried out with the help of the network.

So, in these two lectures on network analysis, we have seen the basics of project management, we have seen that what are the salient features of a project, we have seen that what are the rules that have to be followed for constructing the networks. We have constructed certain networks, and then we have seen what is a critical path, also I have described, what is the major difference between critical path method and program evaluation and review techniques. And in between the lectures, I have been telling again and again, that these networks are going to help a program manager or a manager in a manufacturing enterprise or any other person, who is related to the projects in one way or the another in project planning and project scheduling.

Thank you.