

**Vibration Control**  
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**Module - 2**  
**Basic of Vibration Control**  
**Lecture - 5**  
**Shunt Damping**

This is Dr. S.P Harsha from Mechanical and Industrial Department IIT, Roorkee. In the course of this Vibration Control, we are in the mode of active feedback control, and in the previous lecture we discussed about that how we can make the closed loop algorithm for that. And in this you see whatever the error which is coming out from the differences of the desired, and the real outcome can be feed as the actuated value, which can be even magnified by the controller.

And can be you see the system can be perfectly operated as the feedback controller, the active feedback controller, we also discussed about that it all depends on the system parameters. Like if we have the mass tamper and the spring, then how you see the combinedly, all three devices can be acted as the transfer function, and through that you see here the output can be generated from the defined input. And then it can be rather corrected from you see here what are the difference features are coming, in terms of the restoring forces.

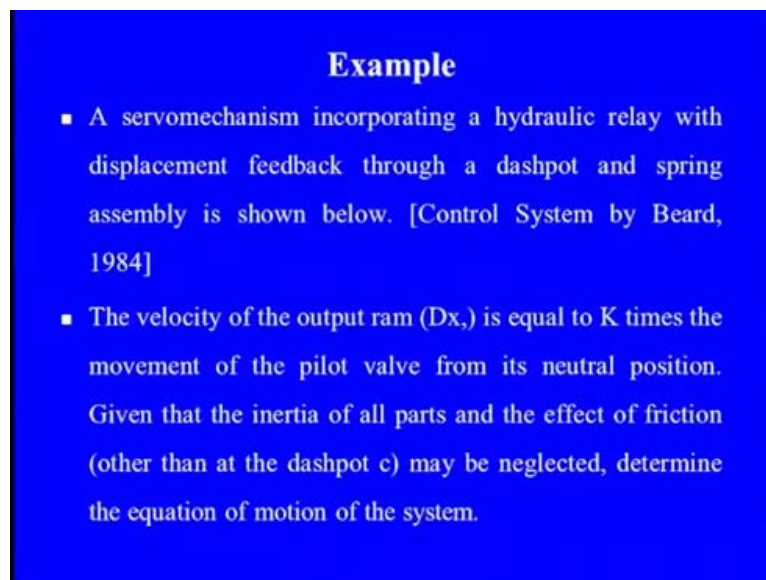
And then we discussed about the stability theorem, that how we see the stabilities are being operated for any system, the single degree to multi degree system, from the Lyapunov to eigen values of the system. If we have the system of the single degree of freedom system, it can be easily predicted whether the system is going in a stable or unstable manner for certain values. If we are just proceeding towards that, just from the eigen value of the theorems, because they are the characteristic roots of those equations and they can simply show, based on the nature of their roots that whether the system is stable or unstable with that.

But, if we have the multi degree of freedom system, say more than 2 degree, then certainly we need to go with the state space model, we need to frame the matrices of all the mass matrix, the damping matrix and the stiffness matrix. And then even you see whatever the gain matrices are there, based on you see whatever the symmetric and skew

symmetric features. And then we can check it out, using the Lyapunov exponent that what will happen with the bounded solution, if we know the initial position and the velocity vectors.

So, these things you see here which we discussed in the previous lecture, in this lecture first we will apply the same concept of the active feedback control to the real mechanical systems, say some kind of the actuation feature. And then we will see that how the shunt damping is coming into the picture and what is its effect in the vibration control theory.

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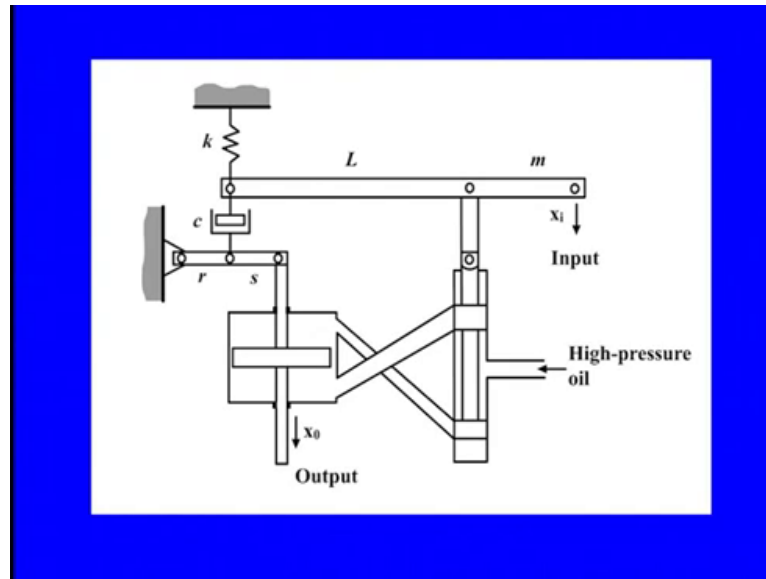
**Example**

- A servomechanism incorporating a hydraulic relay with displacement feedback through a dashpot and spring assembly is shown below. [Control System by Beard, 1984]
- The velocity of the output ram ( $\dot{x}_2$ ) is equal to  $K$  times the movement of the pilot valve from its neutral position. Given that the inertia of all parts and the effect of friction (other than at the dashpot  $c$ ) may be neglected, determine the equation of motion of the system.

So, you see here as we move further, first the example is there we have a servomechanism, which is incorporating a hydraulic relay with displacement feedback through a dashpot, and the spring assembly as I am just going to show that. This example is been taken from the beard, control system by beard and in that you see here, the velocity output of the ram is simply shown by  $\dot{x}_2$ , which is equals to the  $k$  times of the movement.

Whatever you see the positional vector is there of the pilot valve of it is neutral position, given that the inertia of all parts and the effect of friction can be easily neglected. Other than whatever you see the dashpot features are there, now our theme is to see the equation of motion of for that, so this is what the system is.

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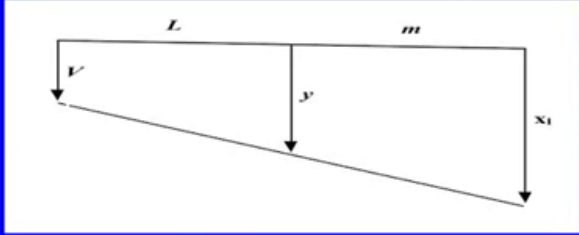


It is a simple servomechanism device, in which we have the lever  $L$  arm, here we are giving the input feature  $x_1$ , the lever has the mass and mass  $m$  and the length  $L$  is there. And in between you see here, we have hydraulic actuated feature with the piston and this you see here when they are being acted the hydraulic pressure oil is being filled there, and according to this actuation here we have the output that. So, this output is basically controlled by the two main points, one we have the spring from the control point that is being connected to the lever.

And this lever is also connected to one of the other lever, which has you see  $r$  and  $s$  these two points with the damper  $C$ ,  $x_0$  is the output for that. Here we are not considering as already discussed that inertia and the frictional forces in between this, this is one of you see the example just showing the servomechanism in the lever output. Now, you see here if we apply, the basic you see the equilibrium conditions at the lever we know that for this lever there are two different positions, one the input  $x_i$  you see here, on the right hand side the  $m$  is this part and  $L$  is this part. And in between you see here, where the hydraulic actuator of the high pressure oil is being there, we have the  $y$  displacement. And the  $v$  displacement is at the this one you see, where the spring and the dampers are being connected at the beginning feature.

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- For the control rod,



The diagram shows a horizontal line representing a control rod. The left segment is labeled 'L' and the right segment is labeled 'm'. A downward arrow at the left end is labeled 'v'. A downward arrow at the junction of the two segments is labeled 'y'. A downward arrow at the right end is labeled 'xi'. A diagonal line connects the left end to the right end, forming a triangle with the horizontal line.

- where  $v$  is the spring displacement and  $y$  is the movement of the spool valve. Thus;

$$\frac{y-v}{L} = \frac{x_i-v}{L+m} \quad \Rightarrow \quad y = \left(\frac{L}{L+M}\right)x_i - \left(\frac{m}{L+m}\right)v.$$

$v$  is nothing but the spring displacement, because the spring is directly connected to the lever and  $y$  is the movement of the spool valve in between, where you see the high pressure dampers are there. Now, if we apply we know that this is the linear actuations, so whatever the displacement are coming at the different points, they have a linear propagations, as you can see on the diagram. And for this control rod when we apply this linear actuations, we can apply you see the simple equations for that, that is  $y$  minus  $v$  divided by  $L$ .

For the same symmetrical triangle is equals to  $x_i$ , which is the input minus  $v$  divided by  $L$  plus  $m$ , and when we are using this we have now, the spool valve displacement  $y$  is equals to  $L$  over  $L$  plus  $M$  into  $x_i$  minus  $m$  over  $L$  plus  $m$ ,  $L$  and  $m$  these are the clear form of our dimensional features of the rod, from the control positions,  $m$  over  $L$  plus  $m$  into  $v$ . When we are just applying the forced balance, for the spring and the dashpot which are being there within the system we can say that.

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- Force balance for the spring and dashpot gives
 
$$kv = cD \left( \left( \frac{r}{r+s} \right) x_0 - v \right),$$
- So that
 
$$(k+cD)v = \left( \frac{r}{r+s} \right) cDx_0$$
- The flow equation gives  $Dx_0 = Ky$ . Substituting for  $y$  and  $v$  gives
 
$$Dx_0 = K \left[ \left( \frac{L}{L+m} \right) x_1 - \left( \frac{m}{L+m} \right) \left( \frac{r}{r+s} \right) \left( \frac{cD}{k+cD} \right) x_0 \right]$$

It is  $k$  into  $v$ , the  $v$  is the displacement in the spring,  $k$  is the stiffness,  $k$  into  $v$  that is nothing but equals to my restoring force is equals to  $C d$ ,  $r$  over  $r$  plus  $s$  the dimensional feature where you see the dashpot is connected into  $x_0$ , this is my outcome of the output displacement minus  $v$ . Now, you see here from this, we can configured that the  $k$  plus  $cD$  into  $v$  is equals to  $r$  over  $r$  plus  $s$   $cD$   $x_0$ , the flow equations for such system is nothing but equals to  $D$  into  $x_0$  is equals to  $ky$ .

And if you substitute these things we have  $Dx_0$  is nothing but equals to  $K$  and  $y$  the displacement vector, which we discussed previously as  $L$  by  $L$  over  $m$  into  $x_1$  minus  $m$  over  $L$  over  $m$   $L$  plus  $m$  into this  $v$  and this  $v$  is nothing but equals to  $r$  over  $r$  plus  $s$   $cD$  divided by  $k$  plus  $cD$   $x_0$ , when we incorporate everything. So, now I have the flow equations which simply incorporate, the dimensional feature of the rod right from the  $L$ ,  $m$  and you see here the other features are of which we have is the spring coefficient  $k$ , the viscosity of the damping coefficient.

And the same time we have all the dimensional feature of the smaller rod where the damper and the springs are being connected directly. So, you see here from this flow equations, now we can simply rearrange the equation and we can find that, what exactly the equation of motion for this is.

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- which can be rearranged to give the equation of motion as;

$$\left[ D^2 + D \left( \frac{k}{c} + \frac{Knr}{(L+m)(r+s)} \right) \right] x_o = \left( \frac{KL}{L+m} \right) \left( \frac{k+cD}{c} \right) x_i$$

So, the equation of motion for this is  $D^2$  plus  $D$  into these dimensional and the characterized feature of the system  $k$  by  $c$  that is stiffness and damper, plus  $K m r$  divided by  $L$  plus  $m$ , the length of rod plus  $r$  plus  $s$  the length of lower one into  $x_0$  equals to  $\frac{KL}{L+m} \left( \frac{k+cD}{c} \right) x_i$  that is my input feature. So, this is you see the equation of motion, which simply shows that you see how the arrangement can be done for a control of system, and this is you see here can be feeded as the transfer function by  $x_0$  over  $x_i$ .

So, the total transfer function which simply shows the relation between output and input are nothing but equals to this  $\frac{KL}{L+m} \left( \frac{k+cD}{c} \right)$  divided by this  $D^2$  plus  $D$  into  $\left( \frac{k}{c} + \frac{Knr}{(L+m)(r+s)} \right)$ . So, we can straight way feed those things and we can see that, what is the difference in the outcome and how we can do the feedback control part. In the next example, now we would like to use some of the numerical data to find out these things.

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### Example-2

A linear remote position control system with negative output feedback consists of a potentiometer giving 8 V/rad error to an amplifier, and a motor which applies a torque of 3 N m/V to the load. The load has an inertia of 6 kg m<sup>2</sup> and viscous friction of 12 N m s/rad.

(i) Draw a block diagram for the system and derive its equation of motion.

(ii) Calculate the maximum overshoot in the output response to a step input of 2 rad, and

(iii) Given that a tachogenerator is employed to provide negative output velocity feedback, derive the new equation of motion and calculate the velocity feedback coefficient needed to give critical damping.

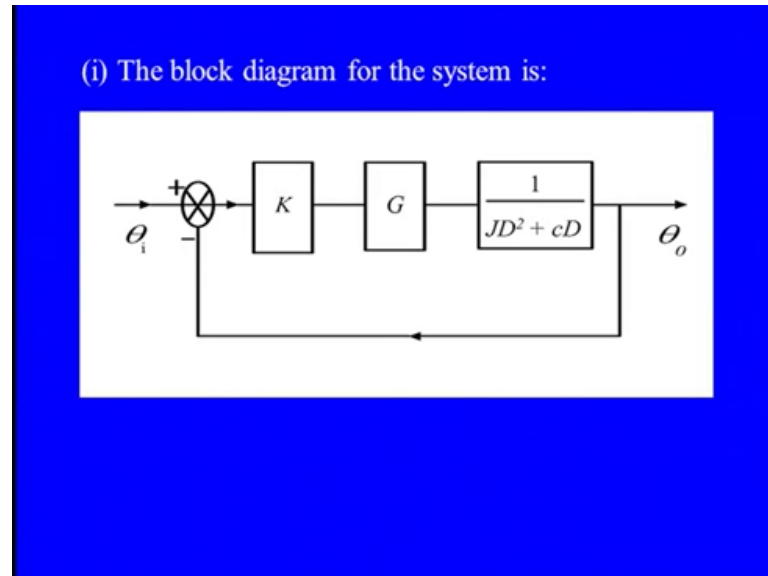
So, we have a linear remote position control with the negative output feedback, which consist of the potentiometer, which gives the 8 volt per radian error. And this one is we are feeding to an amplifier and a motor is there, which applies the torque of almost 3 Newton meter per volt to the load cell. And this load has an inertia of 6 kilograms meter square with the viscous friction of 12 Newton meter per second per radian, this is what you see the viscous frictions are.

So, the system which is not only consisting of the potentiometer just giving the negative feedback, but also you see we have a torque, which is being applied to the motor, the load which is being having the inertia of that and the viscous frictions are there. Now, we would like to draw the block diagram of the system, we would like to calculate the maximum overshoot in the output just to step input, there is a step input just the maximum input just at the beginning is two radian. And then with this given a tachogenerator, which is being applied to provide the negative output velocity feedback, we need to derive the equation of motion and we need to calculate the coefficients for this to be a, which is given to be a negative feedback.

First of all the block diagram, as we discussed you see here not only we have you see, first of all the  $\theta_i$  and  $\theta_o$  are output and input for the system, and there is a error you see here,  $\theta_i - \theta_o$ , which is being considering by the potentiometer. Then it is to be applied to K, G and we have  $J D^2 + c D$ , so this block diagram is

simply showing that we have first the forced features, we have whatever you see the inertias are there in between that.

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And then you see here, when the inertia the force which is being supplied to 1 by J d square plus c D, we have the damping feature including this part. So, the spring, the damper and the mass all three combining together, and along with we have the gain feature G, so we can straightway apply to the equations towards that.

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Equation is  $(\theta_i - \theta_o)KG = (JD^2 + cD)\theta_o$ ,

So the final equation of motion is

where,  $(JD^2 + cD + KG)\theta_o = KG\theta_i$

$K = 8 \text{ V/rad error}$ ,  $G = 3 \text{ Nm/V}$ ,  $J = 6 \text{ kg m}^2$ , and  
 $c = 12 \text{ N m s/rad}$ ,



As I told you  $\theta_i$  minus  $\theta_0$  is the error part is there, which we need to apply to the  $K$  into  $G$  the gain feature, which is equals to the stiffness in gain which is equals to  $J D$  square plus  $c D$  into  $\theta_0$ . So, the final equations if we just want to get it done, then we have  $J D$  square plus  $c D$  plus  $K G$  into  $\theta_0$ , the output is equals to  $K G \theta_i$ . So, overall if you are looking, then the transfer function is nothing but equals to  $K G$  divided by  $J D$  square plus  $c D$  plus  $K G$ .

Now, if you are applying the numerical values, which is being given to us we have  $K$  with the potentiometer reading is 8 volt per radian error, the  $G$  is already given to us, 3 Newton meter per volt, the  $J$  is there the inertia with the mass you see here, 6 kilogram meter square. And the damper is given to us 12 Newton meter second per radian, when we apply these things and along with that, it is given that we need to put the step input.

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▪ With a step input, overshoot is  $\theta_i e^{-\xi\pi/\sqrt{1-\xi^2}}$

$$\zeta = \frac{c}{2\sqrt{(KGJ)}} = \frac{12}{2\sqrt{8 \times 3 \times 6}} = \frac{1}{2}$$

$$e^{-1/2\pi/\sqrt{1-(1/2)^2}} = e^{-1.813} = 0.163 \text{ rad}$$

and overshoot

$$= 2 \times 0.163 \text{ rad}$$

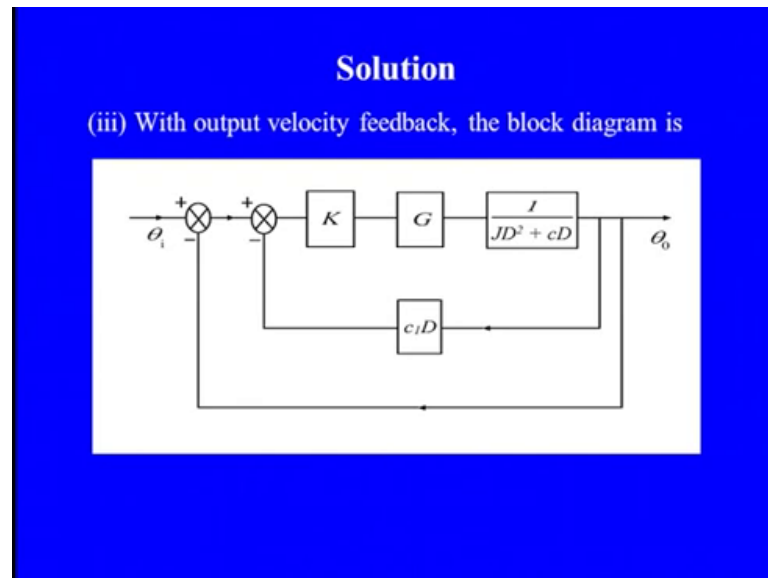
$$= 0.326 \text{ rad, ..... for a 2 rad input}$$

So, the overshoot part in this is  $\theta_i$  exponential  $e$  to the power minus zeta by divided by  $1$  minus zeta square, where the zeta is nothing but equals to the damping ratio. We have all the values of damping  $c$  by  $c$ , so we have  $c$  over square root of  $2 K J$  into  $G$ , so you see here we can apply all the features there and we can get the zeta is half. When we apply to this overshoot, then we have exponential minus zeta pi divided by  $1$  minus zeta square which is nothing but equals to 0.163 radian.

So, we can say that the overshoot is nothing but equals to the twice of this, as the step input is this, so this is nothing but equals to  $2$  into  $0.163$ , that is  $0.326$  for the two radian

input. So, when you have a two input radians and the step input is there, you can get the overshoot which is nothing but equals to 0.326 radian.

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So, for that now if we have the velocity feedback as an output, so we can see that, how we can put that, so the block diagram in this we need to apply one more additional feature, in terms of the velocity output. So,  $\theta_n$ ,  $\theta_o$  are my displacement variation, we can put the  $c_1 D$  as a feedback controller, so that what are the variations are there in the velocity, it has to go through from the damper, and whatever you see the damping matrices are. And in this block diagram it is clear that, when you have such kind of things, in which you see effectively we just want to monitor, the velocity differences it can be easily computed using this.

How you see this, now the equations  $\theta_i$  minus  $\theta_o$  is the error in between the displacement vectors, and minus  $c_1 D \theta_o$  is being acted whatever the error is there in the velocity output into  $K G$  that is the vector with the  $K$  into gain part equals to  $J$  the moment inertia  $D^2$  plus  $c D$  equals to  $\theta_0$ . This equation can be reframed with the velocity outlet as  $J D^2$  plus  $c$  plus  $c_1 K G$  equals to  $D$  into  $D$  plus  $K G \theta_0$  equals to  $K G \theta_i$ . Here  $c_1$  is nothing but the outcome, once you see it has to pass through from the  $c$  and the  $c$  is already being there as the system parameter tamper.

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So that  $[\theta_i - \theta_o - c_1 D\theta_o] KG = [JD^2 + cD]\theta_o$

and the equation of motion is

$$[JD^2 + [c + c_1 KG]D + KG]\theta_o = KG\theta_i$$
$$c + c_1 KG = 2\sqrt{KGJ},$$

For critical damping

$$12 + (c_1 \times 8 \times 3) = 2\sqrt{(8 \times 3 \times 6)},$$

that is

$$c_1 = 0.5 \text{ (N m s/rad) / (Nm/rad)}, \quad c_1 = 0.5 \text{ s.}$$

Hence,

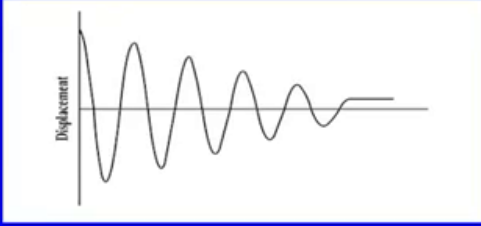
So,  $c + c_1 KG$  is nothing but equals to your critical damping, because you see here ultimately that is what our requirement is that how much damping should be there, for an effective control of the vibrations. So,  $c + c_1 KG$  that is why you see the output error, and  $c$  which is being there, when we add those things it is one of my system design parameter critical damping  $2$  into square root of  $K G J$ . And for this critical damping now we can simply find out what could be the value of  $c_1$ , so we have the value of  $c$  that is  $12$ .

$c_1$  should be calculated the  $K$  value is given as  $8$  from the potentiometer, the  $G$  is already given as  $3$  Newton per meter square equals to  $2$  square root of  $K$  is  $8$   $G$  and  $J$   $3$  and  $6$ , so from that we can calculate the  $c_1$  which is nothing but equals to point  $5$  s. So, now, you see here, when we are simply calculating this, we could easily figure out that you see here this much difference is there as the velocity output is coming, we need to add to, check it out you see that how much error is there. So, this is you see here the two example, which are clearly showing that whether we are looking for the displacement output or velocity output how the feedback controller is being acted there.

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**Vibration damping**

- Damping is a phenomenon by which mechanical energy is dissipated (usually converted as thermal energy) in dynamic systems.



■ Three primary mechanisms involved in damping

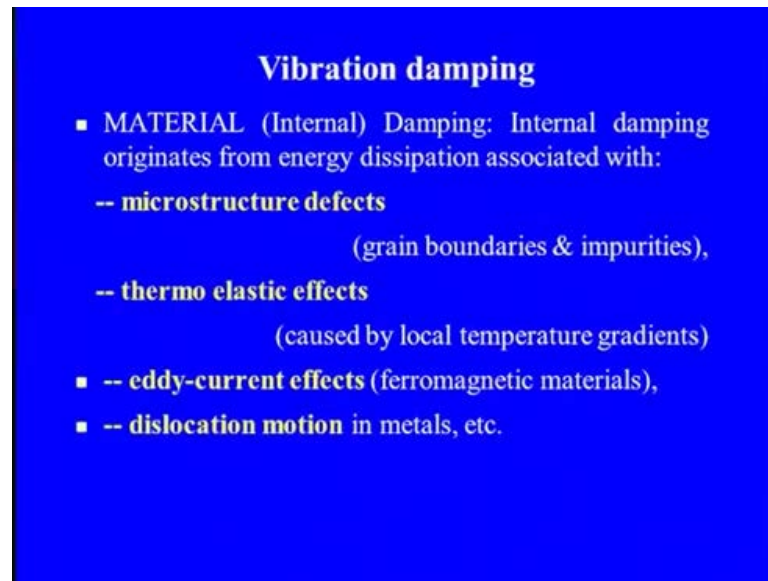
- Internal damping** – of material
- Structural damping** – at joints and interface
- Fluid damping** – through fluid-structure interactions

Now, you see here as the damping is one of the critical feature and in general, if you are looking as a damper which we already discussed, damping is phenomena by which the mechanical energy is dissipated, or it is being converted as a thermal energy in the dynamic system. So, we can see that, the displacement is simply dying out, after a certain amount of time when the damping is playing a critical role. And you see we discussed that there are three mechanism of that, the internal damping, which is related to the microstructure of the material.

Generally we are referred as the material damping, the structural damping which is being there according to the joints or the interfacing of that, it is also known as the coulomb damping or the structural damping. The fluid damping generally you see, this is what the fluid-structure interactions is the viscous damping is there.

So, with these things, if you see we are looking towards the material damping or the internal damping, which is basically you see originated from the energy dissipation associated with microstructure defect. Or the thermo elastic effect, or eddy current effect or any dislocation motion in the metals, it is absolutely giving complex feature of the damping from the metal itself. So, metal damping has various norms in this, as we have just going with any kind of grain boundary or grain structures, with any kind of microstructure defects, it always coming out as the microstructure defective phenomena, even within the surface or the layers of the fibers itself.

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**Vibration damping**

- **MATERIAL (Internal) Damping:** Internal damping originates from energy dissipation associated with:
  - **microstructure defects**  
(grain boundaries & impurities),
  - **thermo elastic effects**  
(caused by local temperature gradients)
- -- **eddy-current effects** (ferromagnetic materials),
- -- **dislocation motion** in metals, etc.

The thermo elastic is when we are simply adding any kind of temperature gradient, and the temperature intensity is quite more at certain point, the clear deviation is there at these point, because of the high intensity of temperature gradient. The eddy current defects always we have a ferromagnetic materials in that, these eddy currents are creating, so much deviation at the microstructure level, in the materials. The dislocation because of any force action or the movement there are dislocations of the fibers. And because of that, there are the energy dissipations are there at the internal level through this, so this is the damping phenomena which we already discussed. Now, you see in this chapter, our main intention is to discuss about the shunt damping.

As we discussed that structural damping is one of the important means of damper to reduce the vibration, and the fatigue as well. And vibration can also be suppressed by adding the mass to the system by introducing the spring the system, as we know that when the system is exciting at the lower frequencies, the spring is a effective way to reduce that. When the system is exciting at the higher exciting frequencies, the mass is the effective way, and when the system is exciting absolutely at the resonant conditions, the damper is one of the effective media to absorb the energy at the time of excitation.

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### Vibration damping

- **Shunt Damping:**
- Structural damping is an important means of reducing vibration, noise and fatigue.
- Vibration can be suppressed by adding mass to a system, introducing a mechanical vibration absorber or a variety of other techniques.
- Piezoelectric shunt damping is a popular technique for vibration suppression in smart structures.

So, all three ways are good enough to absorb the vibration or to absorb the energy, so that the exciting level of vibration can be reduced effectively. But, here in this case the piezoelectric shunt damping is one of the popular technique for vibration separation in the smart structures. So, this piezoelectric, though you see we are going to discuss about the smart materials also in this course, but right now you see here, the piezoelectric feature in the material is a smart actuations or the sensing part.

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- These are characterized by the connection of electrical impedance to a structurally bonded piezoelectric transducer.
- Such methods do not require an external sensor, may guarantee stability of the shunted system and do not require parametric models for design purposes.
- The piezoelectric materials are used in conjunction with passive inductance-resistance-capacitance (RLC) circuits to dampen specific vibration modes.

As its name is there you see here, there is a piezoelectric actuations, and because of that you see here some of the ionized effects are being generated within that. We will go to the internal structure in this, and these are the piezoelectric actuations and these sensors in the shunt damping.

They are characterized by the connection of the electrical impedance to a structurally bonded piezoelectric transducers, and such methods do not require an external sensors. Because, that is why you see we are saying these are the smart structures, and you see they are guaranteed stability to the shunted system, and they do not require any parametric modeling for this design purpose. And the piezoelectric materials are used in conjunction with the passive inductance resistance RLC circuits to dampen the specific vibration modes. So, you see this is one of the specific tool, that can be straightway applied to any of these RLC circuit, the resistance, inductance and the capacitance circuit, to damp out the vibration modes.

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- The piezoelectric materials convert mechanical energy to electrical energy, which is then dissipated in the RLC circuit through joule heating.
- Resonant shunt damping circuits, comprised of inductors, capacitors, and resistors, are simple to design and can significantly augment the damping of lightly-damped flexible structures.
- Piezoelectric materials have the unique ability to convert mechanical energy into electrical energy and vice versa.
- When strained the piezoelectric materials produce a voltage difference across the poled terminals.

And the piezoelectric materials are simply you see, if we are going towards the basic formation of that they are absolutely converting the mechanical energy, which are being coming due to the forced action or you see any displacement action is converting into the electrical energy. And which is then dissipated in the RLC circuit through the joule heating, so whatever you see the energies is coming, it is straightway converting into the electrical featured and these RLC circuits are being used there itself.

The resonant shunt damping circuits, which comprised of you see the inductors, motors and the resistors are simply to design, are very simple to design, according to the type of and the amount of energy is being coming out into the system. And can significantly augmented the damping of lightly damped flexible structures, so piezoelectric materials have the unique ability to convert the mechanical energy into this and vice versa. So, any you see the deformation or the strained, feature is coming to the piezoelectric materials, they are straightway producing the voltage difference across the poled terminals. And you see here, whatever because the deformation is coming due to the mechanical energy, these conversion is there into the electrical one.

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- This characteristic has been exploited in various configurations of mechanical sensors.
- Inversely, piezoelectric materials strain when a voltage is applied.
- This characteristic enables piezoelectric materials to be used as mechanical actuators.
- Active control systems require complex amplifiers and electronic sensors.
- Implementation of simple and robust passive control systems using piezoelectric materials decreases the risk of malfunction and deterioration.

So, this characteristic when they are being converted into the voltage features there, is being exploited in the various configurations of the mechanical sensors. And the recent sensors are simply based on the piezoelectric actions only, inversely the piezoelectric material is strained when a voltage is being applied. So, this is also even feasible that when you see here, the strain is being applied to the piezoelectric materials, the voltage is there or when the voltage is being applied, there is a mechanical strain in the piezoelectric materials.

These characteristics enable piezoelectric materials to be used as a mechanical actuators, so these piezoelectric materials even you see in the small patches, they can be act as a sensor, and the actuators together. In active vibration control which we are going to



discuss, requires the complex amplifiers and the electronic sensors together, so implementation of the simple and the robust passive control system using piezoelectric materials, decreases the risk of malfunction and the deterioration of these materials. So, it can be used in both the way effectively, the active vibration control and the passive vibration control.

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- A passive control system is used to damp a single mode of a simple cantilever beam (1-DOF).
- Analysis was done to predict the optimal position of a piezoelectric tile on the beam.
- A resonant shunt circuit was created using an inductor and resistor in series and parallel.
- The electrical impedance frequency was tuned to equate the modal frequency that was to be damped.

So, in the passive vibration control it is being used to dump, a single mode of simple cantilever beam, even you see the analysis can be done to predict the optimal position of piezoelectric tile to the beam. And you see even these are straightway applying to any of the, even the flat structure, the thin structure, even the circular structure to control in an effective way of the vibration as a piezoelectric patches. And the resonant shunt circuits were created using an inductor and resistor, in the series or parallel according to the configurations. And the electrical impedance frequency was tuned to equate the modal frequency which is to be dumped out.

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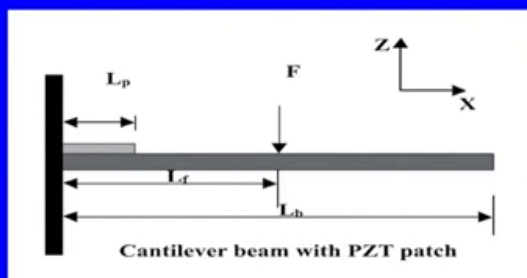
- The efficiency of these shunts depends very much on the ability to
  - transfer strain from the vibrating structure to the transducer material, and
  - transform the strain energy into electrical energy inside the active material.
- The latter is measured by the piezoelectric electromechanical coupling factor  $k$ .

So, the efficiency of these shunt circuits is absolutely depending on the ability, to transfer the strain from the vibration structure to the transducer material, this is one of the effective way that how much the strain is being transferred from the structure to the material itself. And then the second thing it also depends on that how much transfer of the strain energy is into the electrical energy inside the active material, and these two things just defines the performance of the shunts. And you see these strained energy can be measured by the piezoelectric electromagnetic couple factor which is to be applied there in the circuit straightway.

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### Application of Shunt Damping for a Cantilever Beam

An analytical model has been developed to determine point displacements, mode shapes and strain energy of a cantilever beam. A beam clamped at one end and free at the other was used.



Now, you see we just want to see, the application of a shunt damping for a cantilever, we have a simple beam and you can see that the PZT patches being applied, on the top of the beam and then you see here the force factor is being applied at the central feature of there. So, we just want to see that, according to this we can straightway determine the point displacement the mode shape, and the strain energy of the cantilever and this beam is clamped, it is a cantilever beam, so it is clamped at one end. Another end is free for that, you can see this one this is what the length of piezoelectric, this is the total length and then we have you see here like this particular  $L_f$  is the length where the point load is being applied to that part.

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**Application of Shunt Damping for a Cantilever Beam:**

- The Euler-Bernoulli method is used to model the cantilever beam.
- The governing undamped equation of motion for the beam for forced motion under zero initial conditions can be written as:

$$\rho A \frac{\partial^2 w(x, t)}{\partial t^2} + E_b I_b \frac{\partial^4 w(x, t)}{\partial x^4} = F(t)$$

where  $w$  is the displacement of the beam,  $\rho$  is the density of the steel beam,  $A$  is the cross-sectional area, and  $F(t)$  is the external force applied to the beam.

So for that we can apply the Euler Bernoulli theorem, to find out the model of the cantilever beam, and the governing equations for that is the undamped equation for a beam, for forced vibrations and with the initial condition 0. So, we can say that it is  $\rho A \frac{\partial^2 w(x, t)}{\partial t^2} + E_b I_b \frac{\partial^4 w(x, t)}{\partial x^4} = F(t)$ . Where the  $w$  is the displacement of the beam the density is  $\rho$ ,  $A$  is the cross-sectional area and  $F$  of  $t$  is the force applied.

So, this is my inertia force which is being there  $\rho A \frac{\partial^2 w}{\partial t^2}$  plus  $E_b I_b \frac{\partial^4 w}{\partial x^4}$  and  $I_b$ , the  $E$  is my young's modulus, because in that I presume that the beam deflection is under the elastic deflection only.  $I$  is my cross-sectional feature the moment of inertia, and  $\frac{\partial^4 w}{\partial x^4}$  is my another feature, which is just saying that how the

deflection is being occurred along with this one E into I this is nothing but equals to the flexible rigidity equals to F of t. So, this theorem just gives the forced balanced condition that you see here, when the forces are being applied, then how the inertia and you see the flexible rigidity part is being coming together in terms of F of t.

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**Cont...**

- The boundary conditions are:
 

$w(0, t) = 0,$	$w_{xx}(L_b, t) = 0,$
$w_x(0, t) = 0,$	$w_{xxx}(L_b, t) = 0$
- Considering a harmonic forcing function applied to a single point on the beam, according to Fig. 1, we can write:
 
$$\frac{\partial^2 w(x, t)}{\partial t^2} + c^2 \frac{\partial^4 w(x, t)}{\partial x^4} = \frac{f_0}{\rho A} = \sin(ax) \delta(x - L_f)$$
- where  $\omega$  is the frequency,  $L_f$  is the position of the applied force and  $c^2$  can be written as:
 
$$C^2 = \frac{E_b I_b}{\rho A}$$

So, now the boundary conditions, if you we are applying we know that the initial displacement is 0, at w equals to this 0, t 0 even the w of x at the localized region it is also becomes 0, the w of x at L b, t is 0 w of x L b, t is also becomes 0. So, you see here it simply says that when initial conditions are 0, that w is 0 at the L th point, even the initial condition is 0 at x point the same w x is 0 at the total length. Considering the harmonic forcing function, whatever the force which we are applying it has a harmonic excitation.

Then if we apply to these two things we know that there is a change on the right hand side, because of the harmonic excitation we can say del w by del t square plus see we have E b I b, so you see here we can say E b I b by rho A is my C square. So, C square delta 4 divided by delta x 4 equals to F 0 by rho A or we can say that it is nothing but equals to the sin of a x delta x minus L f, where omega is the frequency. The L f is nothing but the position where the force is being applied and C square is nothing but the coefficient which is the ratio of E b I b the flexible rigidity divided by the rho A.

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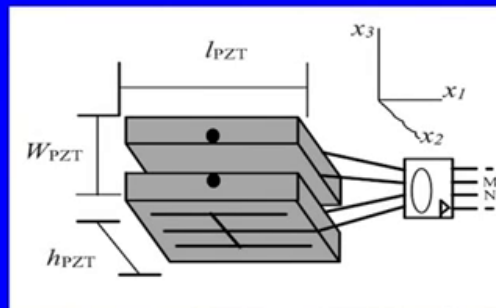
### Application of Shunt Damping for a Cantilever Beam:

- A piezoelectric material is a three-dimensional device poled along one axis.
- Most piezoelectric patches (PZT) are poled across the thickness with electrodes through the top and bottom planes.

So, piezoelectric material is the three dimensional device, which can be poled along any axis and these PZT patches, the piezoelectric patches are poled across the thickness with the electrodes through the top and bottom planes.

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- As a voltage difference is applied across the electrodes ( $x_3$  direction in Figure 2), a strain is produced in the other two directions ( $x_1$  and  $x_2$ ).



PZT patch design on cantilever beam

So, now, you can see that when a voltage difference is applied across the electrode, as you can see in the diagram, a strain is produced in other two directions, so you look at that you see here, the voltages are being applied there. So, this is what the voltage difference is right from in the axial direction, but in the other two direction  $x_2$  and  $x_3$ ,

there is a clear strained feature is there. So, PZT patches can be designed on the cantilever beam in such a way that, we have a clear deformation or the strain as the voltage is being applied to the system.

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**Application of Shunt Damping for a Cantilever Beam:**

- For the given experiment, axis 1 is bonded along the horizontal neutral axis of the cantilever beam.
- When deformed by the cantilever bending, the PZT produces a voltage across axis 3.
- The material transverse coupling constant ( $k_{ij}$ ) is used to describe the relationship of energy transfer from the  $i$ -axis to the  $j$ -axis and is specific to the PZT patch design.
- A simplified description of the PZT damper is to convert mechanical energy into electrical energy and then dissipate the electrical energy in the form of joule heating through a resistor.

And then you see here when this particular system, in this particular, we can experiment axis 1 is bonded along with the horizontal neutral axis of the cantilever beam. And when you see the deformation is taken place in the cantilever banding, the PZT produces the voltage all across the vertical direction 3. And the material which has a transverse coupling constant  $k_{ij}$ , because of its nature is used to describe the relationship between how the energy is being transferred from one axis  $i$ , to other axis  $j$  and that is very specific according to the PZT patch design is.

So, how the PZT patches are being placed there, from where the voltage is being coming into the coming into the PZT patches or the actuations are there. And then how you see the other two directions are being coming another strain form, straightway depending on how the PZT patches are being applied to the system. And the simplified description of the PZT damper, to convert the mechanical energy into electric energy and then it is dissipated the electric energy in form of the joule heating through the resistors.

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**Application of Shunt Damping for a Cantilever Beam:**

- The equation for power dissipated by the resistor is;

$$P = i^2 R = \frac{V^2}{R}$$

- Maximizing the current through the resistor increases electrical damping.
- A simple resistor shunt circuit can be used as a broadband damper.
- The resistor will effectively dissipate energy from all modes of vibration.

So, you see here, if you are looking towards the basic formation of that, then we will find that there is a clear equation for this as, because you see the ultimately it has to transfer through the resistor, then we have the power P is nothing but equals to i square R or V square by R. So, how much the resistances are being there, and how much current is being passing through, so maximizing the current through the resistor will certainly increase the electrical damping, this is you see the unique feature of that.

We need to maximize the current through the resistor, so that we can increase the electrical damping, a simple resistor in the shunt circuit can also be used as a broadband damper. And that is what you see we are applying this principle to the shunt damping and the resistor will effectively dissipate the energy from all modes of vibration. So, it does not matter, whether we are in the higher harmonic mode or lower harmonic mode, whatever the resistance feature is, the resistor will certainly dissipate the energy in a very effective way, in all the sort of vibration modes.

So, again you see here from this theory, we can simplified that since ultimately the electrical energy has to pass through from the resistor, then we need to check it out that you see, how we can maximizing the current. So, that when it is passing through the resistor, the damping can be increased effectively.

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### Parallel RL circuit

- An inductor-resistor shunt circuit was implemented to create an electrical resonant frequency for single mode damping.
- Placing the inductor and resistor in parallel to the PZT allows for the simplest electrical frequency tuning because the inductor and resistor can be altered independently.
- Optimum resistor and inductor values for a parallel configuration were calculated using the method outlined by Wu and Bicos (1997) as:
  1. Experimentally determine  $\omega_o$  and  $\omega_s$  for the cantilever beam.  $\omega_o$  and  $\omega_s$  are the PZT open-circuit and short-circuit modal frequencies for the cantilever beam.

So, there are two main we can say the circuits are there, when we are going with the parallel RL circuit, the resistance and induction circuits. So, inductor resistance shunt circuit can be straightway employed to create an electric resonant frequency, for any single mode damping as well. And when we are placing this, inductor or resistance in the parallel way, because it is a parallel RLC circuit to the PZT patches, which are allowing for the simplest electrical frequency tuning, because of the inductor and resistor are simply alert, they are being acted independently.

So, we can say that we need to frame both the things, the resistor and inductor in a parallel way just to adjust our PZT feature. And we can simply optimize the resistor or inductor values, R and L values for a parallel configuration, and which was given by Wu's and because in 1997, that how we can simply get those optimum value. So, they said that first you can make open circuit and the short circuit model frequencies, the  $\omega_o$  and  $\omega_s$  for cantilever beam.



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**Parallel RL circuit**

1. Calculate the generalized electromechanical coupling coefficient for mode 31,  
$$K_{31} = \sqrt{\omega_0^2 - \omega_s^2} / \omega_s^2$$
2. Determine the PZT capacitance at constant strain.  $C^T$  is the pre-bonded PZT capacitance.  
$$C^s = (1 - K_{31}^2) C^T$$
3. Calculate the normalized tuning frequency  
$$\alpha = \sqrt{(1 - K_{31}^2) / 2}$$
4. The optimum parallel inductance is:  
$$L_{opt-p} = 1 / [C^s (\omega_s \alpha)^2]$$
5. The optimum parallel shunt resistance is  
$$R_{opt-p} = 1 / [\sqrt{2} * \omega_s C^s K_{31}]$$

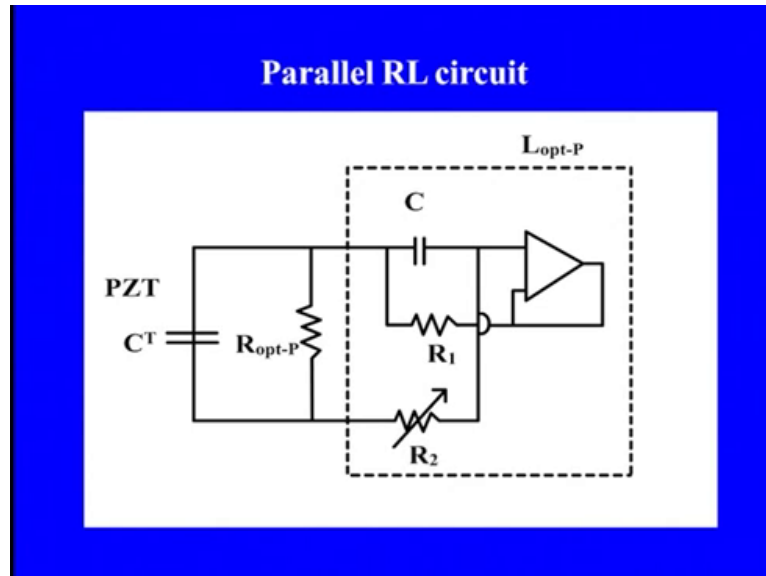
And once you determine these things for a PZT one for open and the short circuit model, then you can calculate the generalized electromechanical coupling coefficient. For any kind of you see the mode that is  $K_{31}$  or for that particular mode which is nothing but equals to square root of  $\omega_0^2 - \omega_s^2$  divided by  $\omega_s^2$ . And then we can simply find out the capacitance of the PZT at the constant strain, so we can say that you see what the  $C^T$ , the capacitance feature is the prebonded PZT capacitance  $C^s$  is nothing but equals to  $1 - K^2$ .

This one the  $K_{31}$ , which we have calculated on top side,  $K_{31}^2$  divided by  $C^T$  and then we can calculate the tuning frequency  $\alpha$  which is nothing but equals to square root of  $1 - K_{31}^2$  divided by 2. Once you get the normalized tuning frequency, then it is pretty easy to find out the optimum parameters for parallel inductance and the resistance. So, for the inductance as well, we can calculate the  $L_{opt}$  is nothing but equals to  $1 / [C^s (\omega_s \alpha)^2]$ .

And the similarly, we can calculate the optimum parallel shunt resistance, which is nothing but equals to  $R_{opt}$  is  $1 / [\sqrt{2} * \omega_s C^s K_{31}]$ . So, both the values irrespective of you see the inductance and resistance can be easily calculated, once you are if the position of calculation of the electromechanical

coupling coefficient, PZT capacitance and the tuning frequencies  $\alpha$ . So,  $K^2$ ,  $C$  and  $\alpha$  once you calculate, then you see the other things can be easily calculated.

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And then we can arrange the parallel  $RC$  circuits in the accordance of PZT, so we can see that the PZT  $C$  dash is this one, the capacitance and then you see we have the normalized parameter of the parallel circuits. So, we have  $R$ , the  $C$  and  $R$  this is what you see the parallel features are there and then you see when it is just passing through the resistance, the entire this circuit the dotted one, it can be effectively utilized to find out the parallel  $RL$  circuits in that. Even we can go with the other series  $RL$  circuit.

So, when you see you are arranging the resistance and inductor in the series part with the PZT patches, we need to apply the feature Kirchhoff law, where the current is constant throughout the element in the series. And a resistor or inductor in series with the PZT can achieve the maximum current through the resistor itself, so in optimum resistor and the inductor values for the series circuit, can be evaluated again by the same.

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**Series RL circuit**

- By Kirchoff's law, current is constant through elements in series.
- Therefore, a resistor and inductor in series with the PZT can achieve maximum current through the resistor.
- The optimum resistor and inductor values for the series circuit were calculated following the procedures of Hagood and Von Flotow (1991).
- The dissipation tuning parameter was calculated by
$$r = RC^s \omega_o$$
- The optimal circuit damping was determined by
$$r_{opt} = \sqrt{2} * K_{31} / (1 + K_{31}^2)$$

The process are its was given as the Hagood and Von Flotow in 1991, in which first we need to calculate the dissipation tuning parameter, which is R is nothing but equals to resistance. And you see the C s, w o and when we calculate this, we can straightway find out the optimal circuit damping parameter that is R optimum is nothing but equals to the square root of 2 K 3 1, which was calculate earlier divide by 1 plus K 3 1 square.

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**Series RL circuit**

- The optimal series shunt resistance is
$$R_{opt-s} = r_{opt} / C^s \omega_o$$
- The electrical resonance frequency is
$$\omega_e = \frac{1}{\sqrt{LC^s}}$$
- The optimal series configuration inductance value can be calculate by setting the electrical frequency equal to the short-circuit frequency ( $\omega_e = \omega_o$ ).

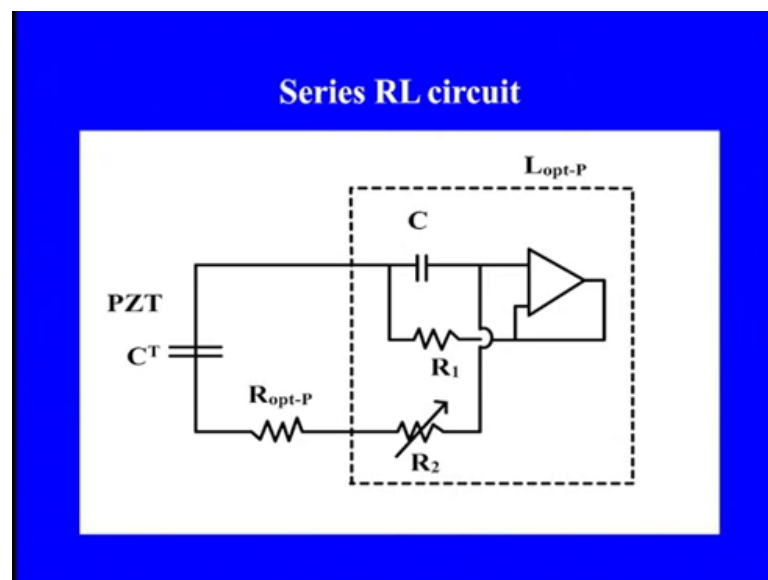
$$L_{opt-s} = \frac{1}{C^s \omega_o^2}$$

Once we calculate this the optimal series shunt resistance can be easily put towards that the R optimum this is nothing but equals to the small r optimum which we calculate

previously divided by  $C s \omega_0$ . And once you calculate this the electrical resonant frequencies  $\omega_e$ , can be easily find out using this  $L_c$  that is you see the inductor and the capacitance, so  $1$  by square root of  $L_c$ . So, the optimum series configuration inductance value can be easily calculated by setting the electrical frequency, that is you see the  $\omega_e$  equal to the short circuit frequency.

That is means you see the  $\omega$  equals to  $\omega_0$ , so we can find out the optimum parameter of the inductor  $L_{opt-P}$  is nothing but equals to  $1$  by  $C \omega^2$ , and we can see that, this is what the arrangement is.

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Now, you see here, we have the parallel the series arrangement for that, and this is you see the  $R_{opt-P}$  here, so both are just in the series solution and then you see here, we have the resistance, and the this dotted part is just giving the  $L_{opt-P}$  part means the inductor optimum, just to see the optimum parameters. So, in the shunt damping, as we know that, the damping is coming out from the material the damping is coming out from the viscous.

But, the shunt damping is always one of the unique feature in the smart structure, where we are just using the capacitor, the inductor and the resistor to find out the effective way to control the vibration from this. So, we know that you see here, how we can opt the parallel this RL circuit or the series RL circuit, according to the requirement of the system. So, this lecture is simply showing that you see, when you have a active vibration

feature and you want to control these things, we are always using the piezoelectric material, which can be used as the actuator or can be used as a sensor towards that.

And then we can straightway calculate that how these PZT patches are being applied to the system to the real mechanical system along with this inductor, resistor and the capacitor. But, in these things the two main features are being coming out, that how the resistor and how the inductor and the capacitors are being straightway coming, coupled together with this our this PZT patch, and then accordingly the resistor will form the real power dissipation.

Because, you see ultimately this mechanical energy which is forming the strain, in the feature is being converted into the electrical energy and here, the resistor is playing a key role. Because, if we want to maximize the current we know that how the resistor is being there just to have an effective control on the energy itself. So, this lecture is unique in it is own lecture, that how the PZT patches are being used in the RLC circuit and then you see here how we can control not by externally, by the internally by simply putting the PZT patches in a proper direction for actuation, or for sensing of those vibration.

So, that the feedback can be effectively you see the feedback control can be effectively used and you see the gain matrix can be effectively we can say design accordingly to our requirement. So, in the next lecture now, we will again discuss about the vibration control feature that you see, when we have a different material choices together than how we can means the design consideration, then how we can apply those vectors for the vibration control.

Thank you very much.