Vibration Control

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Module - 2
Basic of Vibration Control
Lecture - 4
Feedback Control System – I

Hi, this is Dr S P Harsha from mechanical and industrial department IIT, Roorkee. In the course of vibration control mainly we discussed till now about the various basic controls, basic control theory in that in which we discussed about that when the vibration is transmitting right from the source to the receiver. Then, how we can effectively put some kind of isolators or the damping towards that, and in the previous class we discussed about that you see, you know, like when the flow induce vibration is there.

Then, first of all what exactly the mechanism is there in that and then how effectively we can reduce those flow induced vibrations not only from the amplitude of the frequency side, but also when you see you have such huge sound then what the is effective way to control that. We also discussed about you see when unbalance rotor is there then certainly, we have a huge amount of vibration amplitude and the same time you see it creates huge sound as well.

So, in the previous class you see we discussed all that basic theory about this and then you see what is the effective solution for this. In the coming lecture you see here, we are going to discuss about the another control technique that is the feedback means you see here. If, the system is of such kind of some kind and you see here we have the output. We know, you see here the outcome is basically based on the systems performance and there is a desired output if there is difference in that we can straightway put that difference, which is called the error.

Then, we can get you know like something that you see how we can reduce error to get in a effective control parts on that based on the system performance. So, in this part you see here again we are starting from the feedback control. The feedback control of mainly to types it is a closed loop or a open loop, so again you see here if you are really going towards their effective control system. We can certainly go for the closed loop system and which we can put the error and then we can simply give the kind of feedback.

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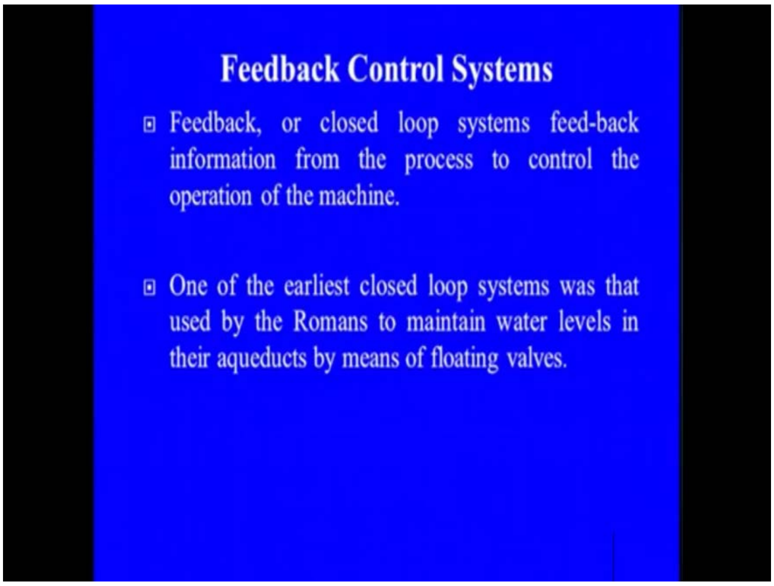

So, feedback or closed loop which is you see the same similar name you see here, systems the feedback information from the process to control operation of the machine. So, feedback is just like you see here the corrected input through which you see here we can effectively increase the performance from the earliest closed loop system was that used by the romans. Right from you see the roman age to maintain the water levels in their adequate by means of the floating valves, so floating valves where acted as the feedback controller in that way.

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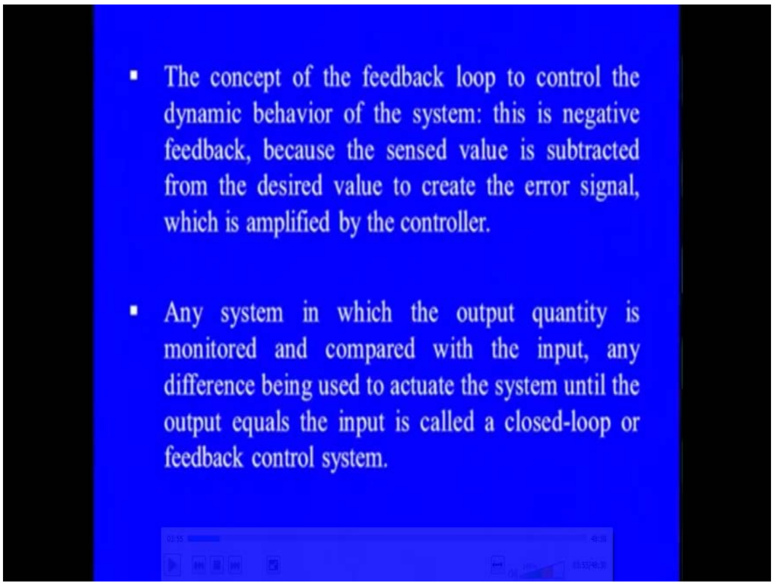

The concept of feedback loop or the closed loop to control the dynamic behavior of system. This is negative feedback because the sensed value means the outcome is subtracted from the desired value as you see based on the system designed or something.

You see here, the desired value to create the error signal which is amplified by the controller and then again you know, like feedback to just giving you know like input towards the system input part. Any system in which the output quantity is monitored and compared with the input any difference being used as the actuator just to actuate the system until output is almost equal to the input. That is what we are saying the closed loop or feedback control system.

So, our actuator in the second iteration is basically the error in between the desired and the designed value and you see here, it absolutely depending on the system performance that how much error can be reduced in that iterations. So, if you are going towards the general system there are 3 models.

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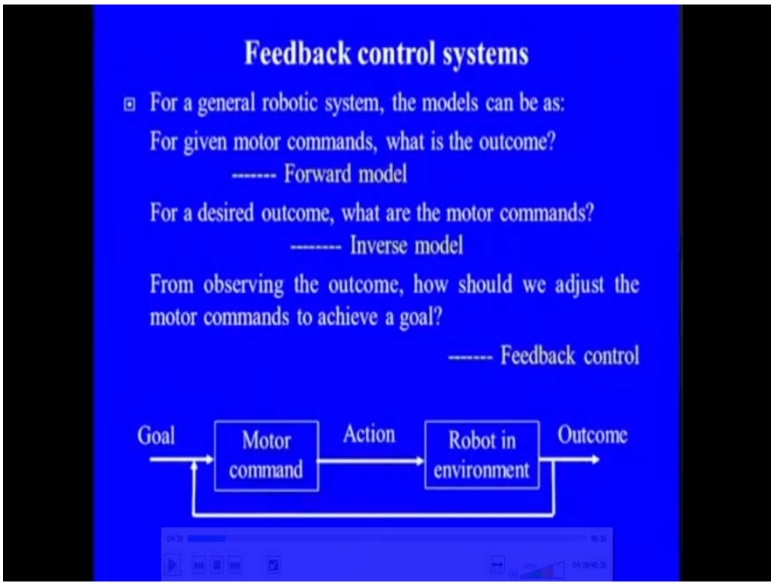

Which can be used as a feedback controller, first if the given part is for a motor command then certainly we are looking for what is the outcome? That is called the forward model for desired outcome, means the outcome is already somewhat the desired value. What are the motor commands? That is the inverse model the reverse model and the third one from the observing for observing the outcome. How should we adjust the motor command to achieve a goal? That is what your feedback control is, so right from the forward and inverse here. The things are coming that how we can achieve our goal.

So, based on that you see here, we can simply frame a basic model about the feedback control system like we have the goal which we are simply giving to our motor command. There is you see here you know, like the process which is happening with motor and the action will be taken place towards the robot in the environment.

So, you see here the command is given to robot right from the action part and the robot is now giving some kind of activity whatever the desired activities are, so it in the outcome. So, if there is a difference between the outcome and the goal, again you see here this is somewhat we are saying that the error and again this error will be. You know, like given as a input to actuate the motor. So, that it can be amplified and again given the action to the robotic feature, so just to achieve the real goal.

So, you see here in this the essential feature in the feedback control system of an any automatic control part. In this is the existence of feedback loop to give good performance and if the measured output is not compared with the input system the loop is open. So, you see here, if you are you know like not looking for an accuracy or something like you see, we just want that the action should be happened.

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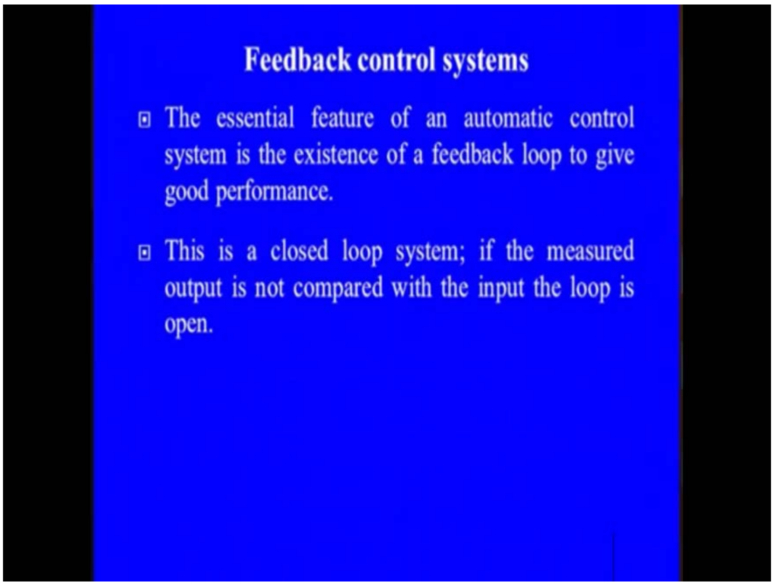

This is a kind of open system whatever the outcome is. Then, you see here, you know like we can just go with the error and all the actuation feature in the second iteration will come into the picture.

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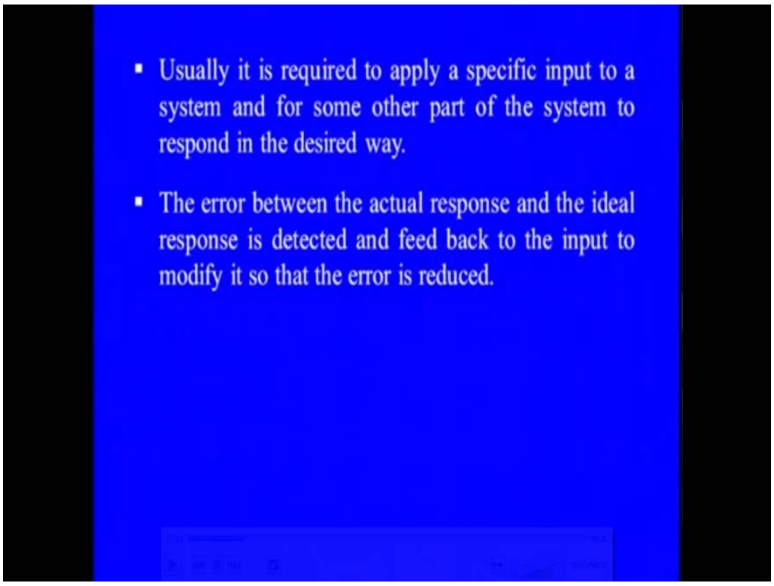

Usually it is required to apply a specific input to a system and for some other part of the system to respond in the desired way. So, we just want that system should respond in a proper way as our desired value should be there as an outcome the error between the actual response and the ideal response is detected.

We need to measure out you see here and we need to get the value accurately and then again feedback to the input to modify it. So, that the error can be reduced and again you see here. It all depends on as I told you, it all depends on the systems characteristics or the system featured that how be iterations does, it require to come closer to the real value your goal.

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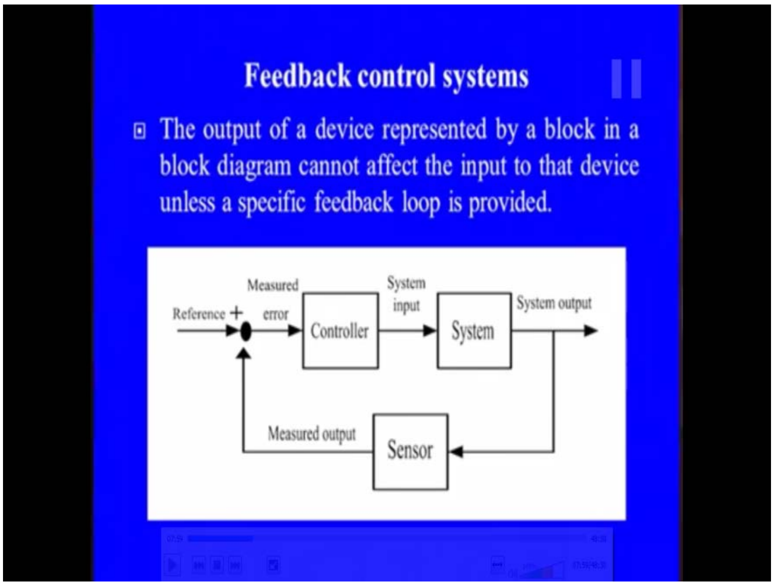

So, the output of a device represented by a block. In a block diagram cannot effect the input that device unless a specific feedback loop is to be provided. So, we can see that we have a reference value as an input then you see here the measured error which is coming out from the systems input and output is to be fielded again to the controller. This is something like the actuator the controller is magnifying that feature and again giving the system input to this.

So, if you are just looking as an simple system, the system is maybe it is linear or non-linear system. If we have a system, there is a input there is a output , but when you see here the difference is there from the output the difference is come out just sending back to the sensor to actuate the measured output again. Simply, you see here, you know the input, you know the output, it will go towards the reference part. The measured error there and then it is going back to the controller. So, if you look at the you know, like the entire system in this you will find that we have the reference, we have the measured error.

There it is just going to the controller system input for the system part. This is what the system performances and then you see here, this entire system right from the system output to the this part the error. It is all you see here based on that how the system is being acted there.

Then, if we are just going towards the next feature then we will find that, that if it is you know like the error is being coming out as the input and output. Then, we need to see that what exactly the nature is there in that.

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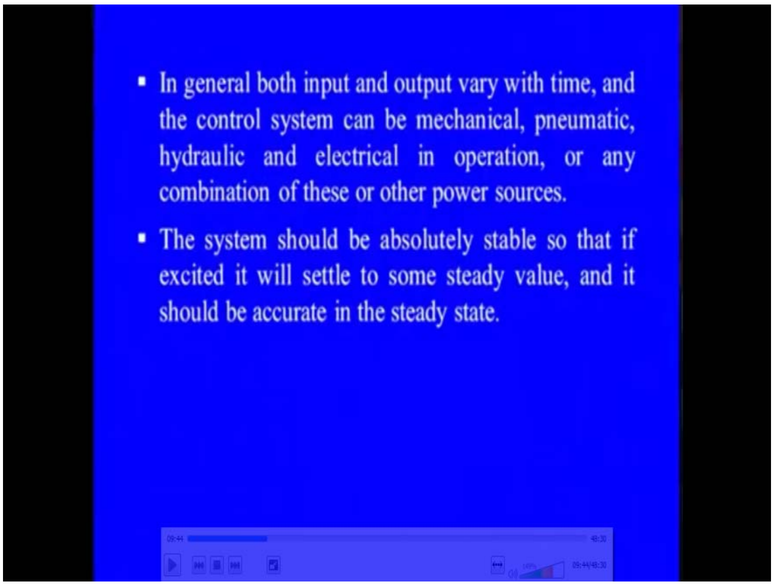

So, in general both input and output vary with time, and the control system which may be you see here the mechanical or pneumatic or hydraulic or electrical in any operation or in combination of anything that can be recorded that can be. We can say, you know like featured out and the differences can be happened and accordingly you see the actuation signal is generated towards the system. So, again you see in that it is not all the time that you have the voltage or you have the current or you have the mechanical force anything can be happened right from mechanical to pneumatic to hydraulic to electrical or even in the combination of these to other power sources.

The system should be absolutely stable. So, that if excited, it will settle to some steady value because if we have the transient nature as an input certainly, it will affect the system performance and whatever the outcome is there at one point of time. If you do the iteration by you know like say the error or something. You see in the actuated part the things will not be accurate during the second iteration or next one, so we need to go up to a certain value of this stable feature just to get the steady state output and it should be accurate in the steady state as well.

So, we can say that you see even whatever the input is there we have to be just check that the whatever the input is going. It is not be in transient mode, it should be in the steady mode, so that we can get in a steady state output and then you see we can straightway check that how much error is there.

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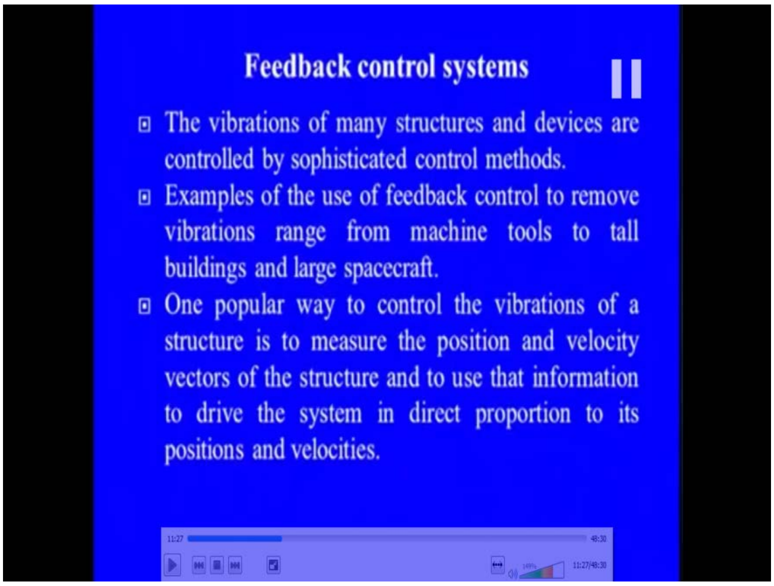

This is very much valid to our vibration problem because vibration of many structure and the devices are simply controlled by the sophisticated control methods. It is not all the time that you see, we need to apply the insulators or material damping or something to effectively control the vibration amplitude some sophisticated methods are also be there just like you see here. The use of feedback control to remove the vibration range from the machine tool to tall buildings and even for the large spacecrafts also, where it is not all the time feasible to apply the isolators one particular way to control the vibration of any structure is to measure the position and the velocity vector of the structure and to use the information to drive the system indirect proportion to its position and velocity.

So, once you get the variation or the range in the displacement and velocity which are the 2 effective dynamic parameters. Then, again you see here, we can simply see that how much variation is there. We can simply feed to control this much in that, so this information is really required to drive the entire system. So, we can use these 2 information effectively how? Now, you see we are taking one of the basic vibration system the spring mass damper of any body in which you see here you can simply see that our desired output is x 0 the input is given as x i.

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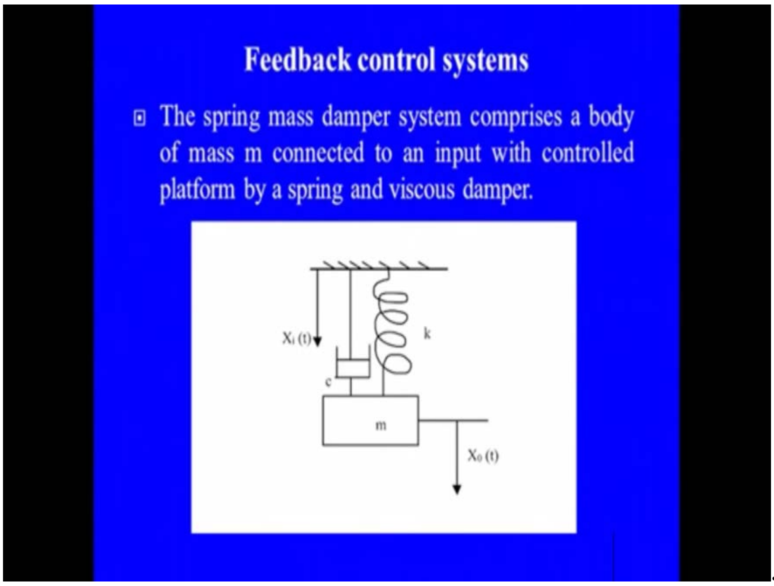

So, in this you see here whether the mass which is just moving with any of the acceleration is introducing the inertia forces. We have the restoring forces and the damping forces and all the forces are being acted in effective way only we need to see that how we can control the platform under which you see here the all excitations are happening.

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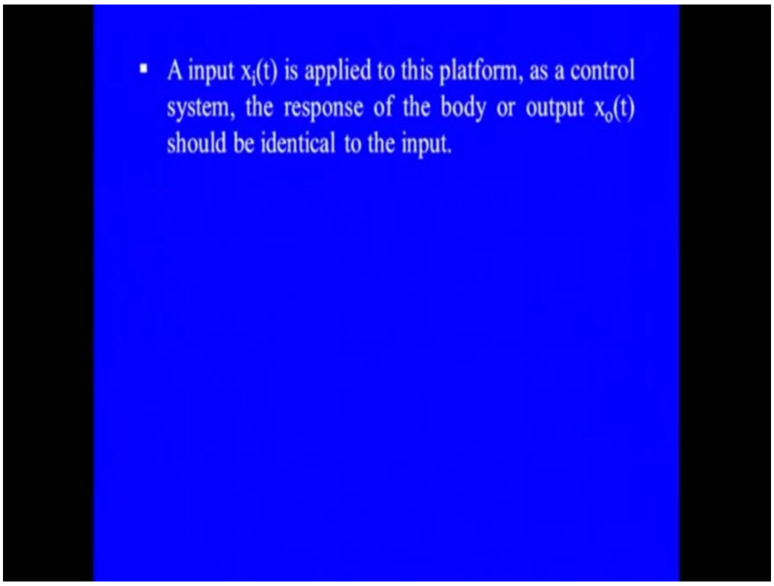

So, as I told you the input is our x i t which is to be applied to the platform through that the excitations are being transmitted and we just want to control this platform. The response of the body or we can the mass is x 0 and we just want that it should be identical to the input system.

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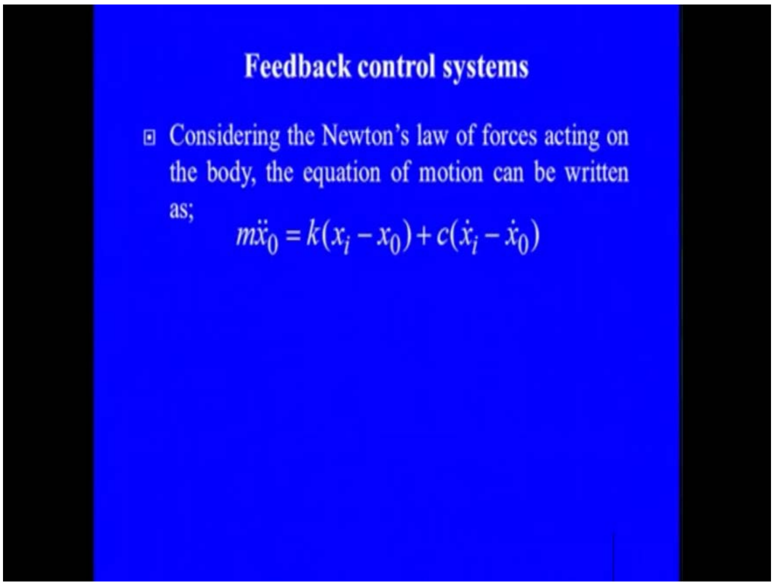

Now, if we apply the Newton's law to all the forces which I told you. We can simply framed the equation of motion m x double dot the inertia force the acceleration since it is there. So, that is why the inertia force is quite dominating restoring force because of the spring k into x i minus x 0 the difference of output and input feature and the damping force c into x dot i x x dot i and x dot o and you see here this equation is clearly giving the relation that this is you know like the kind of motion which has been solved using the harmonic input.

For a general solution again it is a pretty simple a mathematical form of your ordinary differential equation. We can use the D operator the differential operator for our own convenience.

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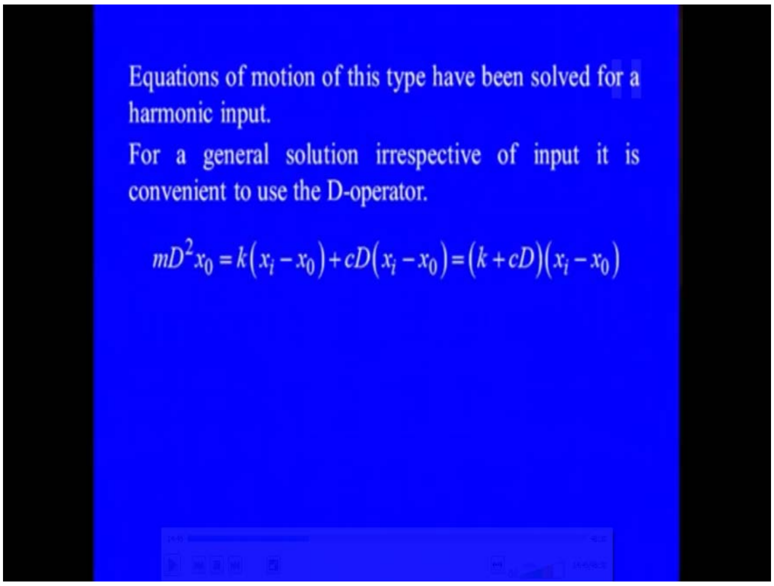

So, you see here what we have we have the basic equation in the differential operator is m D square x 0 in terms of output you see here equals to k x i minus x 0 plus c D x i minus x 0 or else we can say that it is nothing , but equals to k plus c D x i minus x 0. So, this is my force. Now, the inertia force which has to be there you see which we need to feed in that, so you can now design the control loop for that and in that you see it should be noted that although we are using the differential operator which is a neat and compact form of writing the equation , but sometimes you see here it is not that it is pretty easy to solve those things. The solving method is again a simple original differential ordinary differential equation in the originally way.

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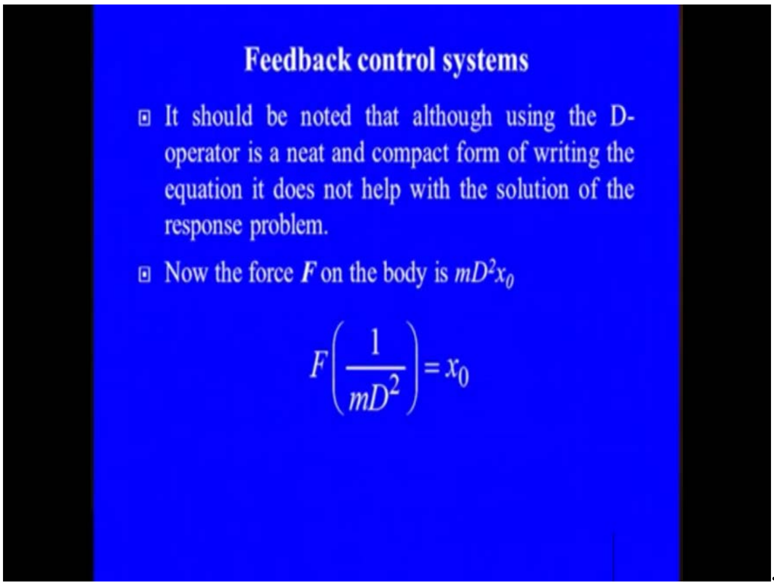

So, say that if force F is acted on the body is the inertia force and in that you see here m D square x 0. Now, we have F 1 by m D square, that is nothing but equals to you see the total output which is coming out from the these. So, with this particular feature with this force now we can draw the transfer function for this.

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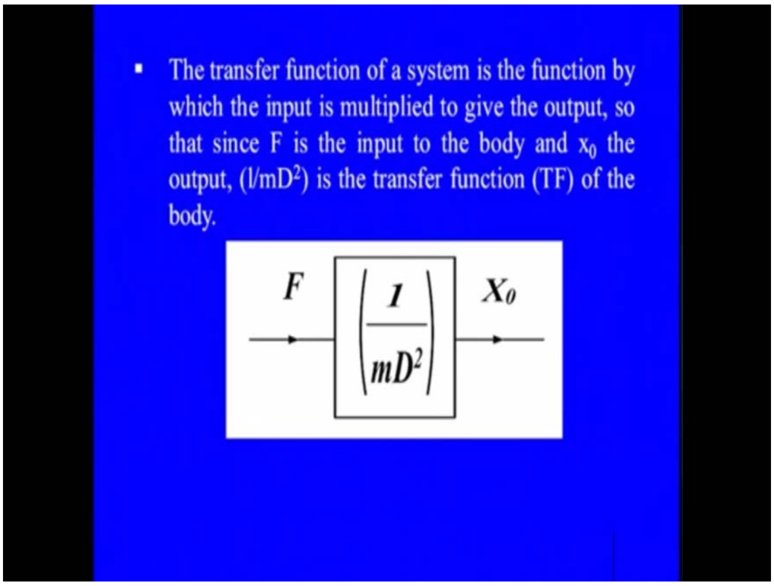

So, the transfer function of the system is a function which is nothing but you see the input is multiplied to the output. So, that the F you see here which is simply a input in the system on the body and x 0 which is the output. If you multiply by the transfer function it is clearly giving that what exactly the relation between the output and input and how much transfer of the vibration can be happened when you have input and output and the media.

So, based on this theory we can say that F is my input x 0 is my output, so in between my transfer function which is a part of my block diagram is 1 by m D square. Now, you see here this is what on your screen we have F and x 0 by 1 m D square is this and 1 by m D square is my transfer function. Now, if I am going to towards the spring damper unit.

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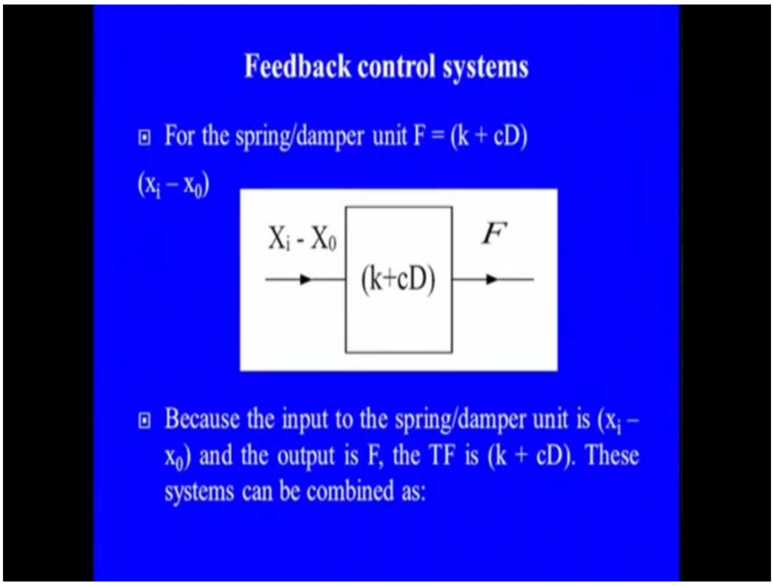

We know that the F which is you see my inertia force is nothing but equals to k plus c D x i minus x 0. So, my x i minus x 0 is the real because x 0 is output x i is input. So, you see what is the difference in that. This is my error and again you see I want to actuate this thing, so x i minus x 0 is now feed and it has to transmit through damping and the spring. So, we have k plus c D and then you see whatever the F is there because the input because we are giving input to the spring or damper unit is x o minus x o and the output is F the transfer function here is k plus c D through which the entire transmission is happening and these system.

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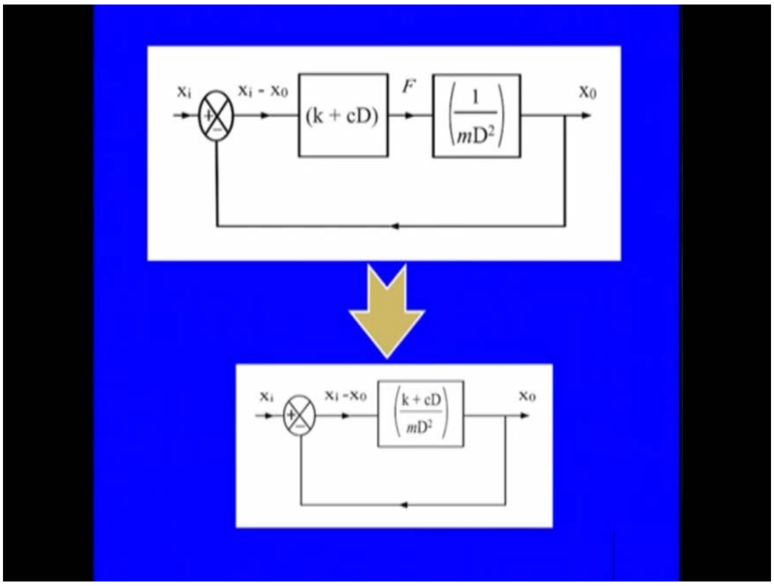

Now, if we just want to combined those things here in the mass as well as in the damping and stiffener then we can say that the overall picture of the system is like that our input is x i the output is x 0. This entire transmission right from inlet displacement to outlet displacement is happening through these devices. The spring and the damper is parallel and they are simply in the connected in between the foundation to your mass. So, k plus c D is my 1 transfer function through which the transmission happen and m by D square m into D square 1 by m into D square is my another transfer function which is simply relating the mass towards the inertia part inertia forces.

So, now in all we have x i minus x 0 is my error function which can be feeded in the iteration feature towards the actuation part. It is going to k plus c D and giving the output F and this F is now going towards the 1 by m D square. So, we can get the x 0 output or else we can combined this because this is the total system a spring mass damper system. So, in all we can say that this is what my control loop system in which you see here the error can be even magnified or this is perfect display of my feedback control. It is x i and x 0 the error is x i minus x 0 which is feed to the transfer function k plus c D divided by m D square. So, the transient or any intermittent feature is simply eliminated by effectively put all the features together. So, we have this x i and x 0 and in between this is what you see the perfect show of the feedback, the feedback control system in the control loop.

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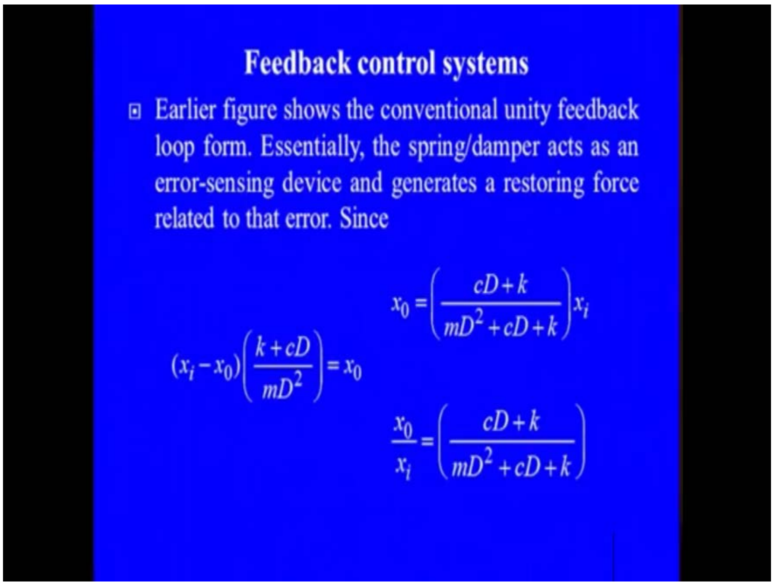

So, in this figure you see here conventionally. We can say that the unit feedback loop is always being formed essentially the spring and damper acts as an error sensing device as we discussed and generates restoring oblique damping force relate to the error and then you see these error is in between that.

So, we can say that since the x i minus x 0 into k plus c D our m D square giving us our x 0. We can say that x 0 can be simply get by c into D plus k divided by this m D square c D plus k where D is the differential operator equals to x i or else or else we can say that x 0 by x i.

This is again you see here, the outcome by input in income incoming that is nothing but you see the transfer function for overall system is a c D c is the damping coefficient. D is the differential operator c D plus k divided by m D square plus c D plus k. So, this is the total transfer function of a perfect spring mass damper system and we can get the accurate you see here outcome by input.

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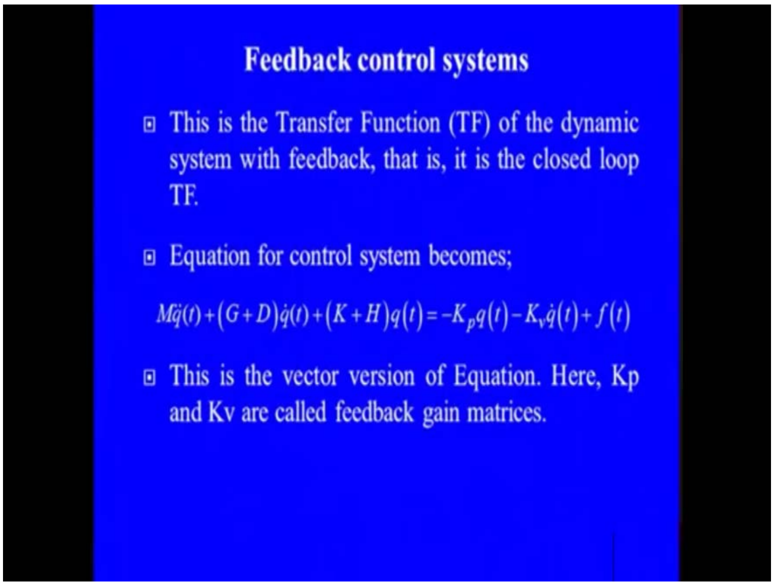

So, this transfer function of any dynamic system with the feedback is also known as the closed loop transfer function because it provides us a clear feedback in terms of the restoring force. For the equation we can generate this is M q double dot q is my coordinate of a generalized feature or state space coordinate. We can say G plus D q dot t K plus H q dot t equals to minus K p q minus K v q dot equals to F.

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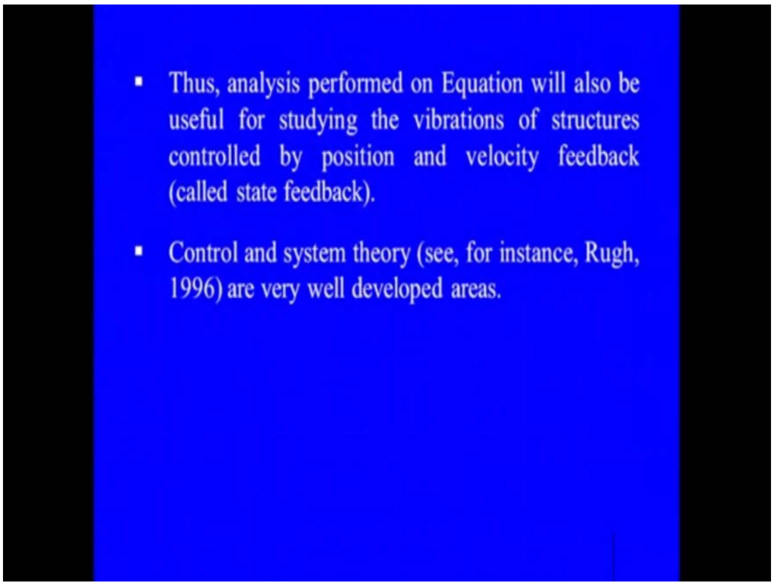

Here, you see we have 2 gained features, so again you see here we can apply a gain matrices because we need to amplify those thing. So, certainly you see here this vector version of the equation which I just show there. In that they are 2 coefficients just showing K p and K v are the feedback gain matrices because ultimately whatever the error is coming.

I need to feed again back to the damper as well as to the spring thus the analysis performed on the equation will also be useful for studying the vibration of a structural feature which can be controlled by the position and the velocity feedback. Generally we are saying that the state feedback in terms of the K p and K v because you see here, we are giving feedback to the q of t and q dot of t which are absolutely related to the spring and the damping facilities and you see here. Now, if we are just going towards the other feature. We know that every system is not a single degree of freedom system. We need to go for multi degree of freedom system as well, so the state space form of the equation is always applicable to that.

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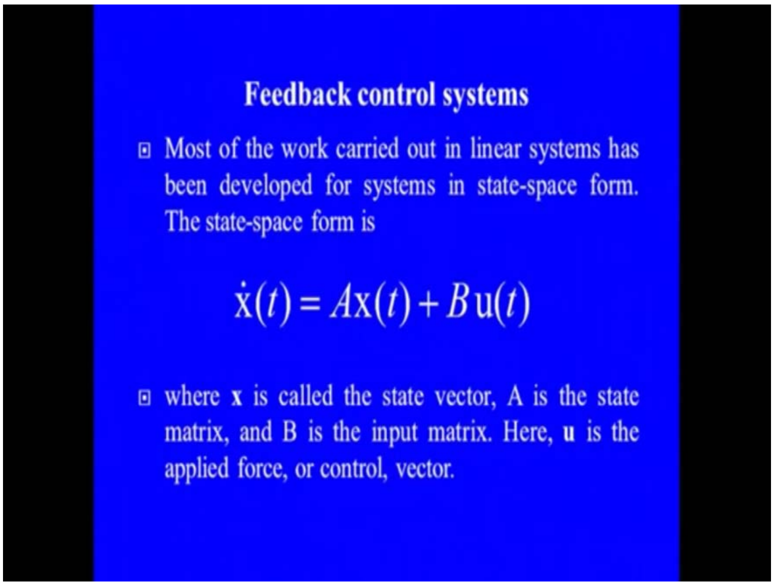

So, most of the work carried out in any linear system is just developed in the state space form and the state space form is always you see here x dot t is always giving you the into x of t is the velocity is nothing but equals to some coefficient a into x of t. The state vector and B u of t where u is the applied force or any you see control vector. We can say, so here the x is the state vector a is the state matrix and B is the input matrix because A is you know like absolutely related to the displacement and B is related to the u of t where the u is the applied force or control vector or anything which we can say.

So, again the state matrix is there the input matrix is there and then we have both the state vector and the control vector in the equation so this is you see the state space form for that.

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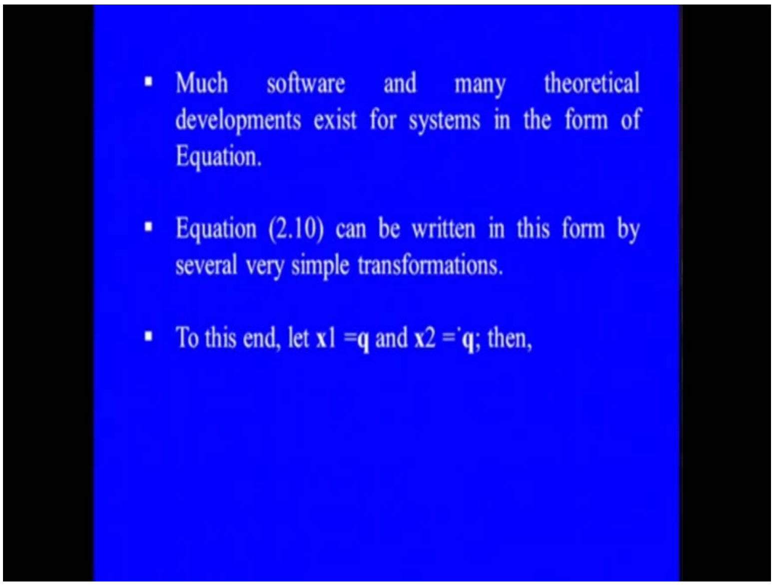

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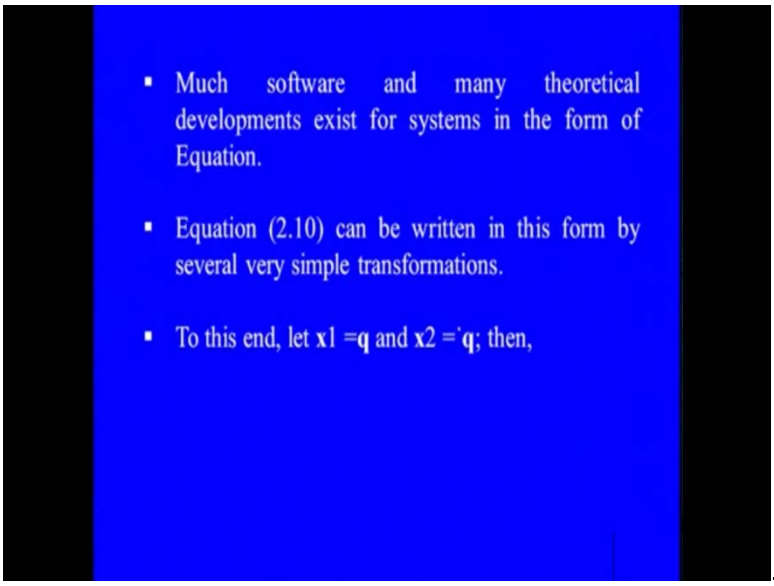

Many of the software you see here which is being you know like applied which is being generated and the theoretical development exist for these kind of equations only. So, the equation can be written in various ways the transformation, so we can say that if I have the state space coordinates say q for displacement and q dot for velocity. I can say that x

1 the initial feature x 1 equals to q and x 2 equals to q dot then I can write the equations for 2 coupled system is x 1 dot t is nothing but is equals to x 2 because we know that x 1 is q x 2 is q dot.

So, x 1 dot is equals to x 2 and x 2 dot which is nothing but the acceleration feature is simply giving you the basic equation of the motion. If, you are just see then we will find that minus D plus G the gain and the differential operator into x 2 of t minus K plus H x 1 t equals plus f of t.

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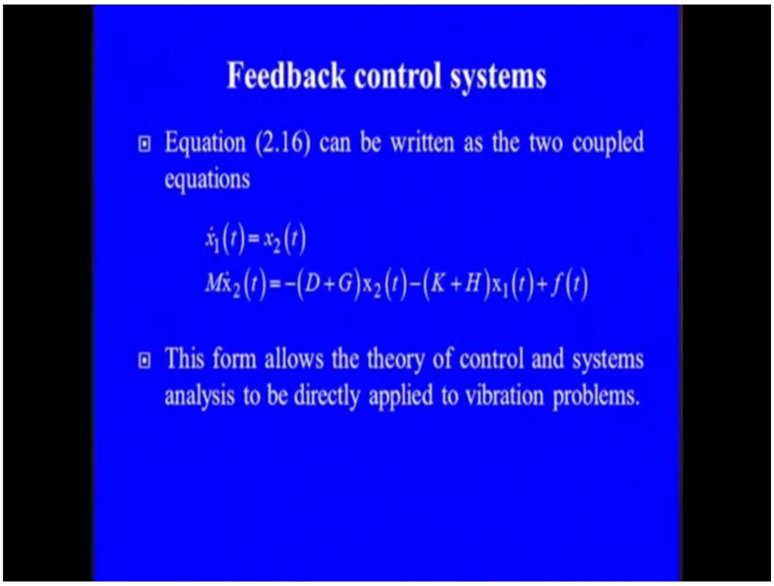

The forcing factor this form allows the theory of control and the system analyses to be directly applied to any vibration problem because we have effective input in terms of the displacement and the velocity. Now, if you saying that there is a matrix which is being there for you know like say mass or say you see here the damping or for stiffness. So, say if we have the mass matrix M inverse is always there in terms of the inverse of mass matrix. We can say that the inverse and the original matrix is always giving the identity matrix that is A 1 form theoretical form of the equation.

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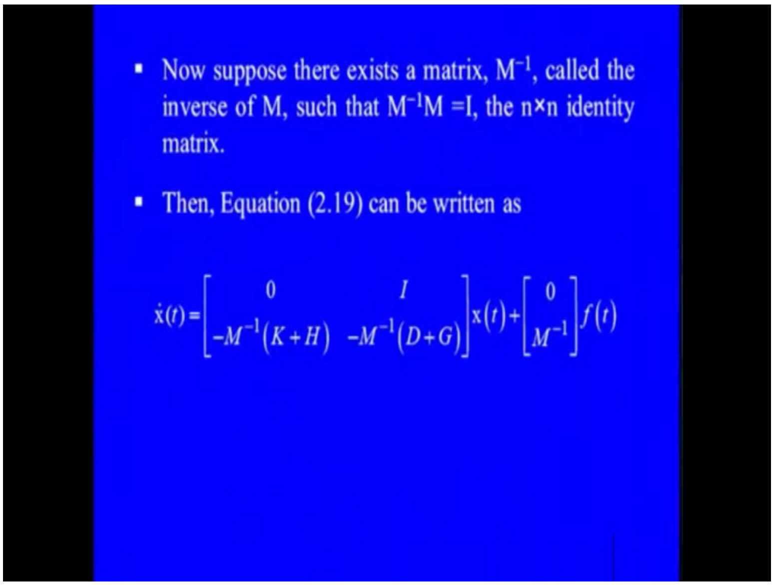

So, now we can write the equation x dot t equals to the matrix formation. In the state space form that is 0 1 minus m inverse K plus H and minus M inverse D plus G because now we are trying to convert this into the real state apace form into x of t plus 0 over inverse f of t. Now, this coupled equation is now in the state space form or we can say in the matrix form where we can say that we can simply put the state matrix just like you see. If, you remember the first case which we said that the x dot t is nothing but equals to a into x of t plus B into u of t.

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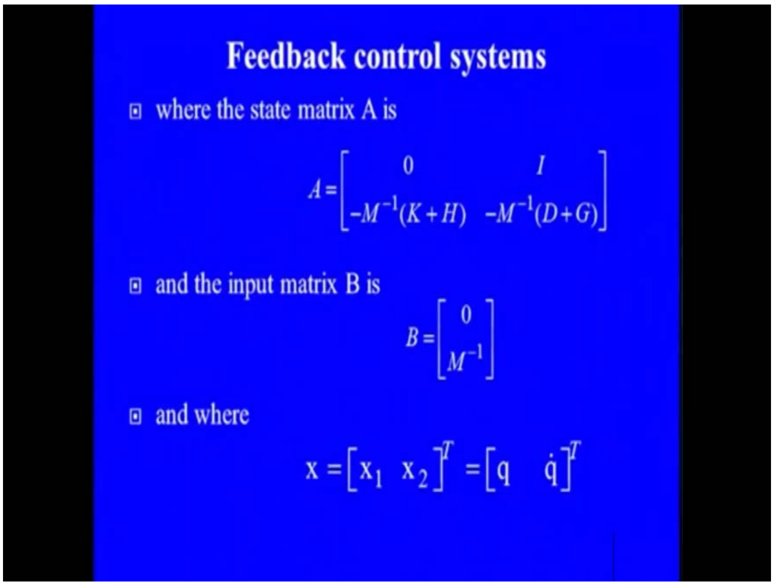

Here, the A is the state matrix which is nothing but equals to the first feature related to x of t that is 0 1 minus M inverse K plus H minus M inverse D plus G and the B which is the input matrix is nothing but equals to 0 over M inverse and you see where we can say that the x which is my input feature, you see here in the state matrix. So, we can say x is x 1 comma x 2 transpose or else we can say that this is q of t and q dot of t in the transpose manner.

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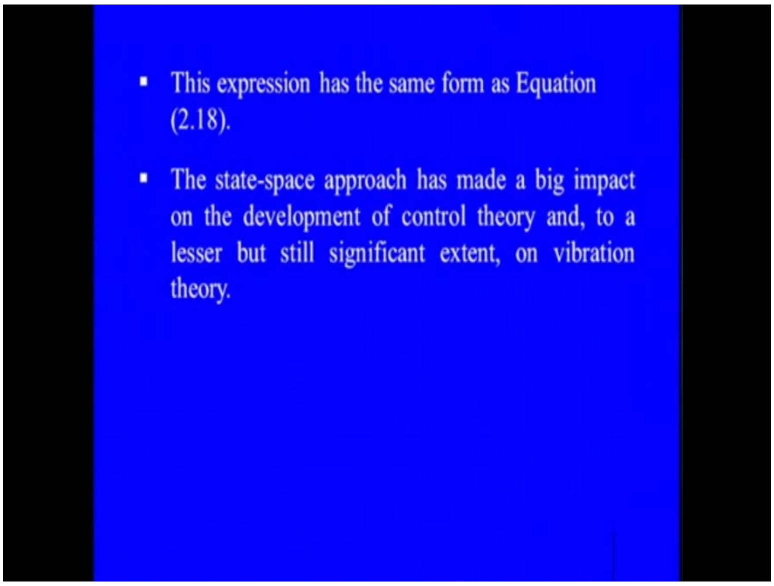

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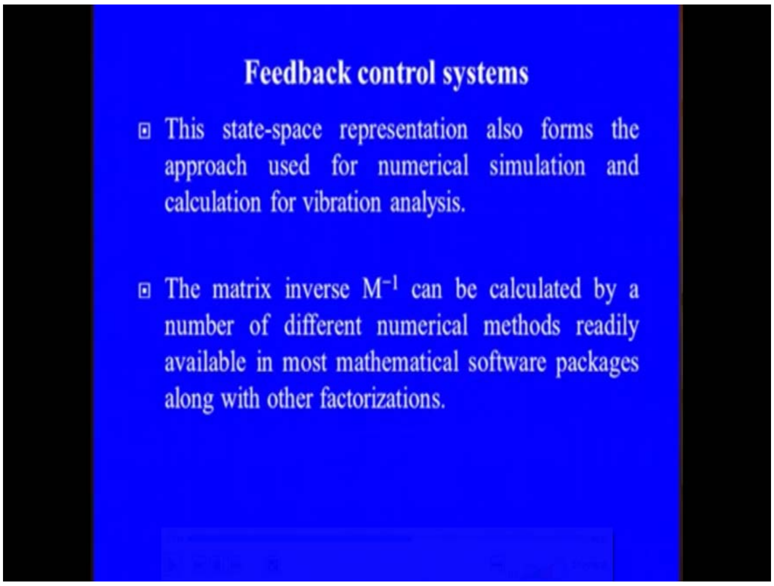

So, the state space approach has made big impact on the development of control theory and to a lesser, but still significant that we are applying to multi body systems and also to vibration theory as well. So, this state space represent an representation also forms the approach used for numerical simulation and calculation of the vibration analysis the matrix inverse M. Inverse can easily we calculated can be also calculated easily by the number of different various numerical methods and you see here we can straightway go to the factorization of this part. The simple calculation can also show that you see the how second order matrices can be formed.

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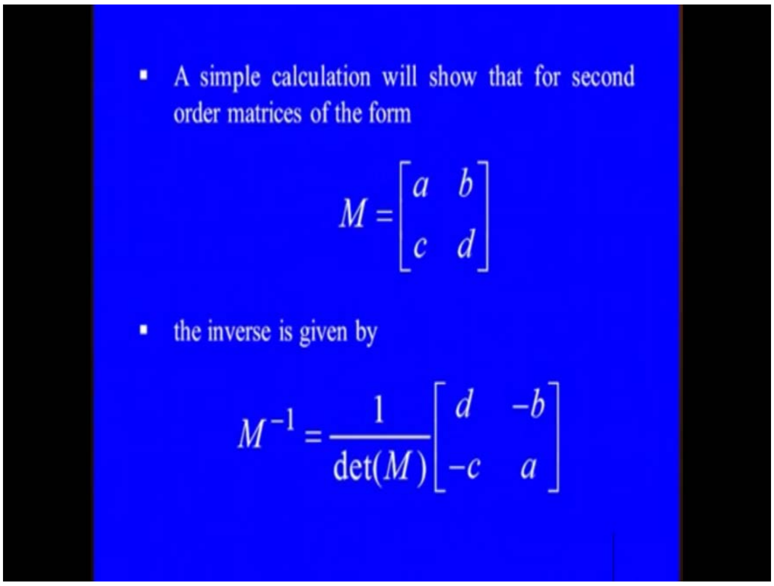

Say, we have M the mass matrix is a b c d. We know that the M inverse is nothing but equals to 1 by determinant of M into to the transpose of this d minus b minus c a. You see here, from this we can calculate the determinant of M is nothing but equals to a a minus c b or else we can say that if it 0 then a d equals to c b where M is singular. If we have both of the determinant is 0 and therefore, the M inverse will not be exist in this because it is a singular matrix. So, in general, you see it should be noted that if matrix inverse exist then only you see here. It is a unique solution for that further inverse of the product of the square matrices, can also be you see according to the matrix theorems A B inverse is nothing but equals to B inverse into A inverse.

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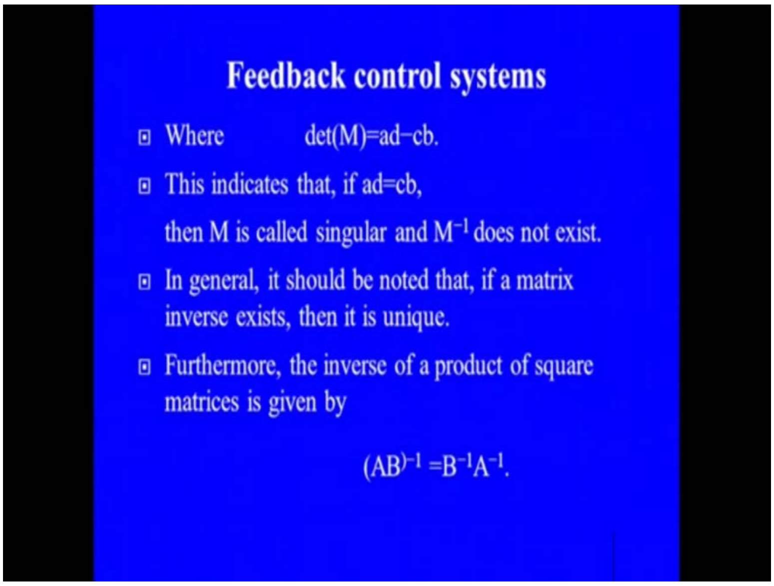

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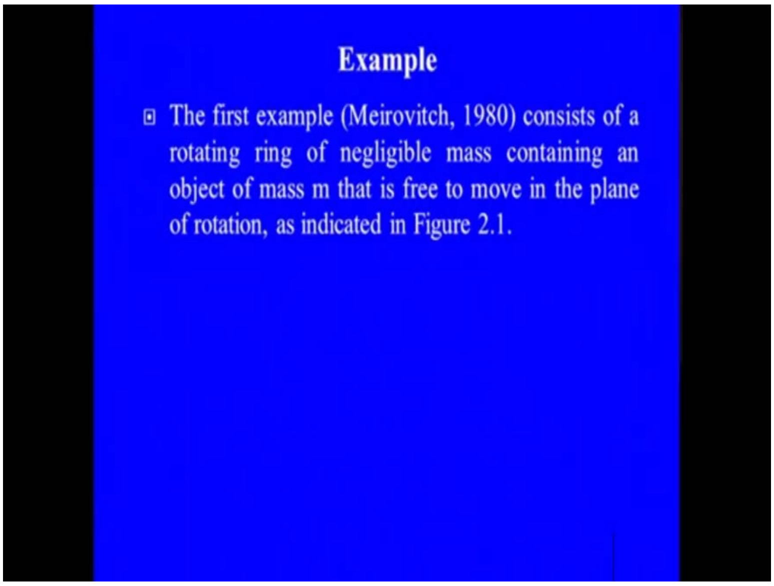

So, now you see here, we are taking one example which is from the meirovitch book that how you see a rotating ring with the negligible mass, can be framed you see in the equation. How we can you know like put the feedback controller for that, so we have a rotating ring. I am going to show you the figurer there itself which has you see the negligible mass containing in a object of any mass.

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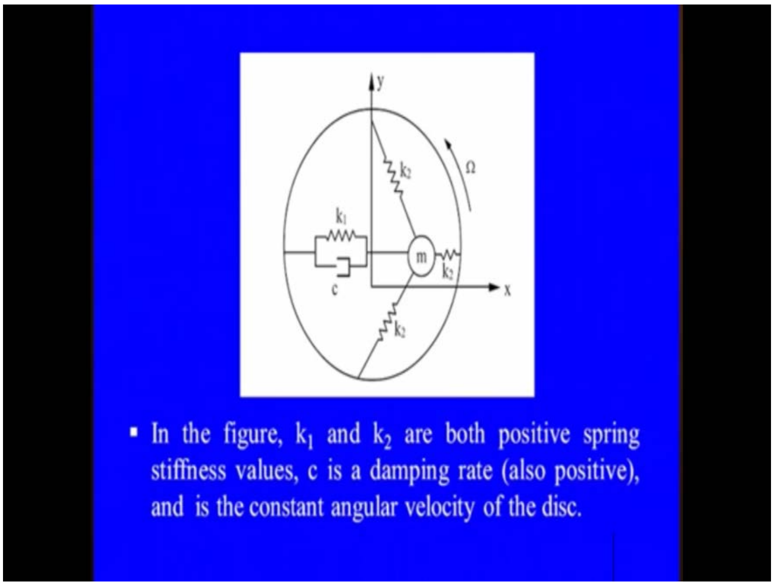

That is free to move in the plane of rotation and you see here. This is what it is we have plane. You see here, in that and it is being rotated here and you can see that we have the springs connected to all the feature. This is spring and damper is k 1 and c and this mass is now connected to all other featured by the same other spring k 2 here.

So, this mass is now constrained. It is on the base plate, you see here, we are not considering the mass of the base plate , but there is mass which is absolutely on the table and it is being rotated and been constrained by that in the figure. There are 2 springs k 1 and k 2 and you see here since it is always being acting both compression and the extension, so the it has a positive spring stiffness. There is a damper see also which just shows the damping rate there itself and is being you see you know like the entire table is moving in the circular feature. So, we have angular velocity of the disc omega that is the omega is given to, given to that one.

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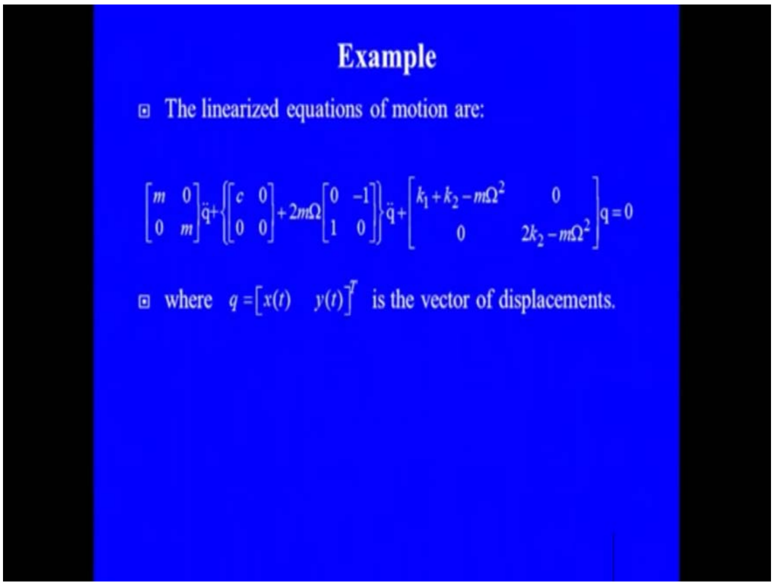

We can frame the equation of motion for this as you can see that and m 0 and 0 m is with the q double dot the inertia force in the state space. Then, we have c 0 0 0 because the c is just connected to one feature and then 2 omega that is q dot q omega 2 m omega 0 minus 1 1 0 q dot is there and then in that another form of the stiffness matrices.

We have k 1 plus k 2 minus m omega square as it a angularly rotating feature 0 0 and 2 k 2 minus because you know like both side the springs are there. So, it is 2 k 2 minus m omega square into q the state space form equals to 0 where q is my coordinate. Simply, shows the displacement vector is x of t and y of t transpose, so I have you see, you know like both the terms together in that as you see the coupled 1 in x and y direction.

Now, you see we can simply framed the matrices for mass for D damper and for stiffness and they must be symmetric while we have the gain matrix which should be skew symmetric. So, that the system can be framed in the gyroscopic system because of the circular risk circular disc.

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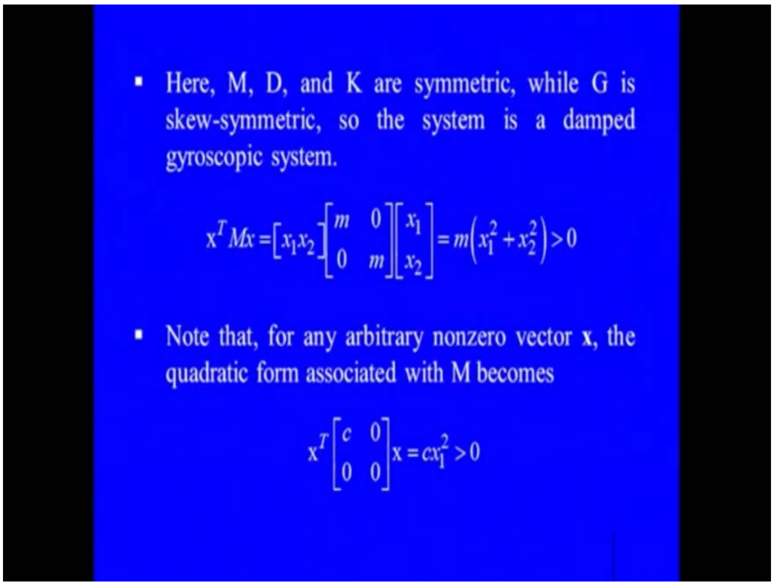

So, now you see if we apply this, we have x transpose mass matrix into x is nothing but equals to x 1 comma x 2. In the linear frame of transpose the mass matrix is m 0 0 m and x 1 and x 2 are just my displacement vector or else we can say that m into x 1 square plus x 2 square and it must be greater than 0. To have an effective control on that note that for any arbitrary non 0 vector x. The quadratic form is absolutely associated with the matrix m and it becomes x transpose c 0 0 x, which just gives that c into x square or else we can see because you see, it is just a transpose feature. So, x square and it should be greater than 0.

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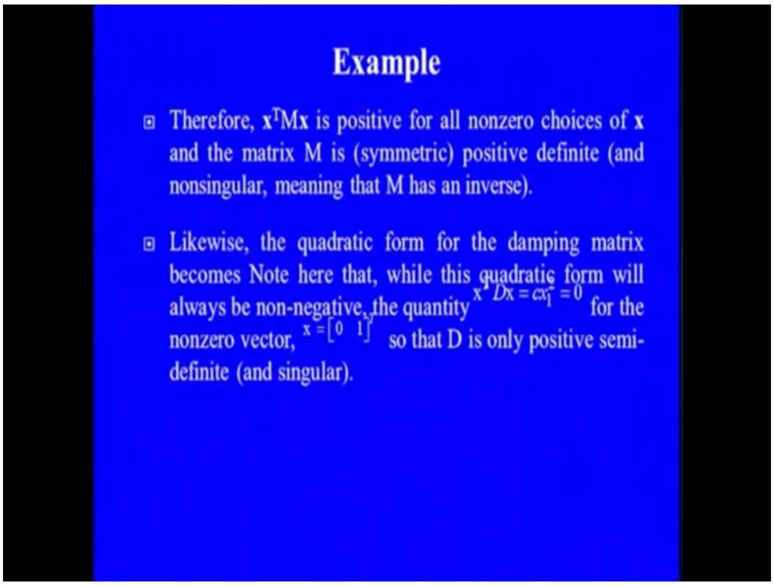

Similarly, you see here the x transpose M x must be positive for all nonzero choices of M for x and matrix M obviously. It must be in the symmetric feature for any nonsingular. That means you see here the M inverse must be exist likewise the quadratic form of the damping matrices c 1 x square becomes you see here, you know, like in the quadratic feature because you see. We know that it is velocity oriented feature and it always be nonnegative.

We can say that x transpose D x or it is equals to c x 0 must be a nonzero vector. So, we can say that ultimately our D is a positive semi definite or we can say a singular A. The damping matrix and you see here. Now, we just want to calculate the quadratic form of the equations and then we just want to see that what is the real value of x, so for that we need to go to our gain matrix.

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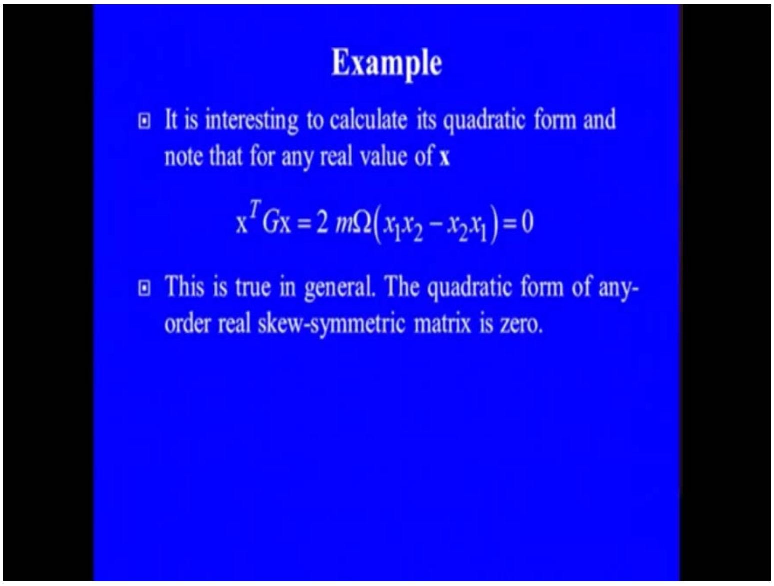

So, it is x transpose g x is equals to 2 m rotational feature tau or the omega into x 1 x 2 minus x 2 x 1 and this is you see, you know, like always true for any general conditions because you see here it just shows the quadratic form of the any order real skew symmetric matrix. You see here in that we can say that in any form of the quadratic it must be equals to 0.

So, you see here, this is one form of the equation which just shows that how we can get the gain matrix displacement matrix. This damping matrix and the mass matrix for that and how we can simply control by simply putting all these values together the another feature in the control system of any vibration control is the Lyapunov stability. We people are all taking about the system is stable unstable. You know, like linear non-linear, but the important thing is that for any effective control, we need to check it out the stability first because you see here ultimately we need to put the parametric ranges to see the stable feature of any system.

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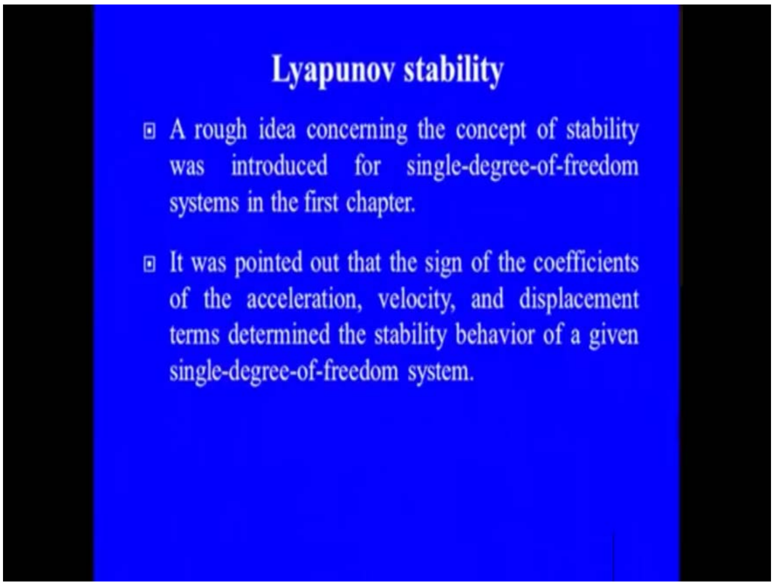

So, rough idea concerning to the concept of stability as we discussed. You see with the characteristics routes of the characteristic equation in any single degree of freedom in our first chapter. As we discussed that you see whether the routes are real or the complex and then you see here how the routes if they are just going positive negative then whether they are showing the system equations or the system is stable or unstable.

So, it was pointed out that the sign of the coefficients of even any dynamic parameter may be acceleration velocity or displacement term is simply determining the stability behavior of any single order system. So, just you see, this sign notations are indicating whether the system is conversing or diverging part whether it is stable or unstable part.

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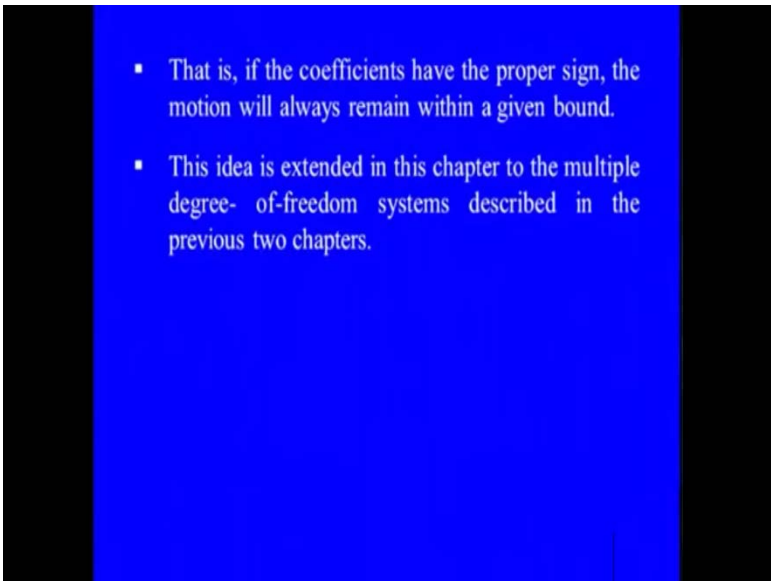

So, you see here, if the coefficients have the proper sign, the motion will always remained in the same when they are just given bounding features and this idea is simply extended here. In this chapter for multi degree of freedom system as we know that you know like when we are going for these things, we need to check it out for the state space form how we can control these with the using of these characteristics routes.

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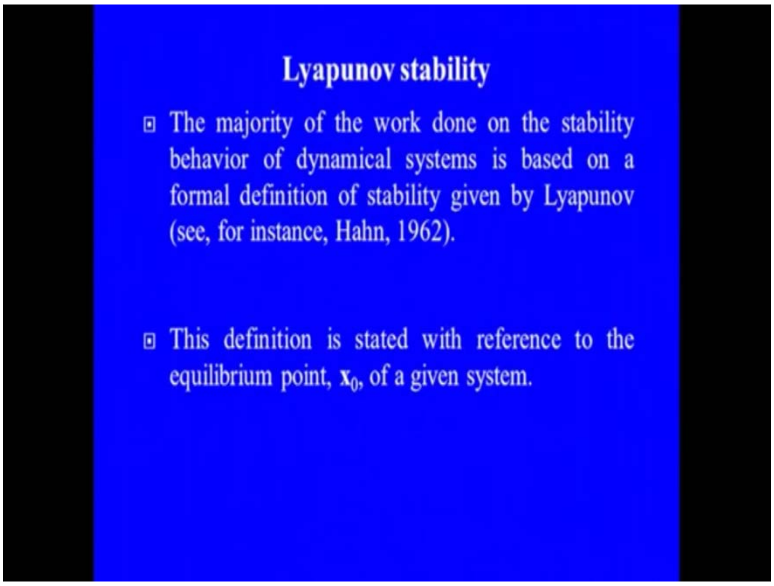

So, the majority of the work on the stability behavior of any dynamical system for multi body system is based on the formal definition of a stability by given by the Lyapunov which was published by Hahn in 1962, and this definition is stated with the reference to the equilibrium point the stable equilibrium point, x 0 for any given system. So, we need to make an reference point and then we need to check it out whether the system is bifurcating with any extension or whether it is you know like going towards diverging or converging feature.

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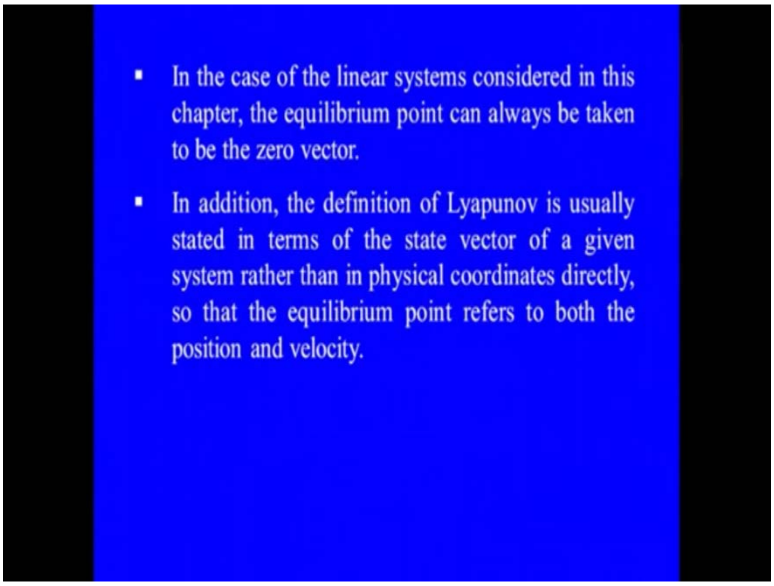

In the case of linear system, which generally considered in this chapter. The equilibrium point can always be taken from the 0 vector and the definition of Lyapunov exponent is usually stated. That the state vector of an given system under the state vector under any you know, like given system rather than the physical coordinate directly. So, that the point which we are referring can be simply just based on the position and the velocity feature.

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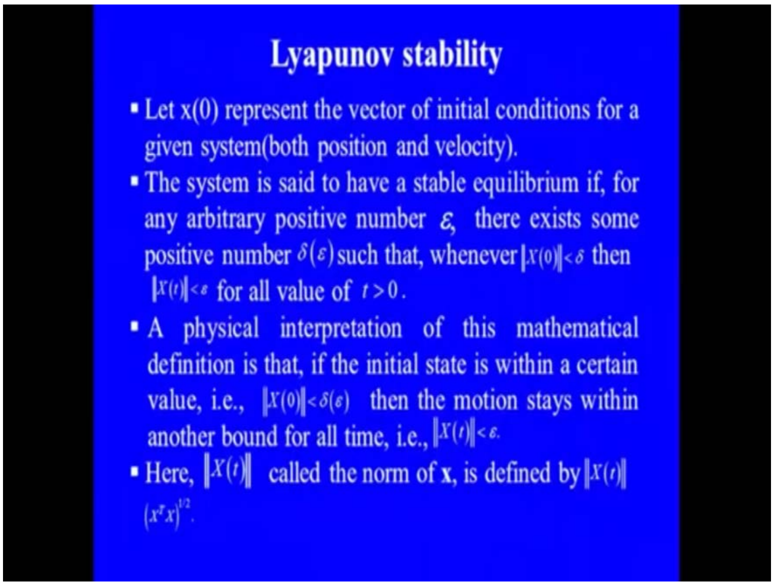

So, say you see, if you are saying that x 0 is the representation of vector of initial a condition for a given system both for velocity and the position itself. The system can be said to a static equilibrium or stable equilibrium, if for any arbitrary positive number which say zeta or any arbitrary positive number is there exist. Some positive number such that whenever x of 0 means the initial representation initial condition is less then delta.

Then, we can say that any increment x of t will also be less then this positive number delta for all values of t for all values of a preceding t where the t greater than 0. The physical interpretation of this mathematical definition says that the if the initial state is within the certain value then the motion will always be stays within the bound of the what whatever the conditions are there for all the time.

That means, you see here, we never cross the barrier of bound even we are proceeding with any iterations of that t 1 plus t 2 plus t 3 plus t 4 like that and here x of t which we are saying that the preceding step is called the normalized factor of x is defined by x of t. Modulus is equals to x transpose x t to the power half as we, as we are moving further, so this is you know, like one of stable condition for equilibrium position that you see here. You know, like always x of t when we are preceding is less than that arbitrary number for any value of t 0 t is greater than 0 and if we are saying that say x of 0 when we are initially assuming is representing the initial condition of vector for both the system is absolutely in the form of the stable vector.

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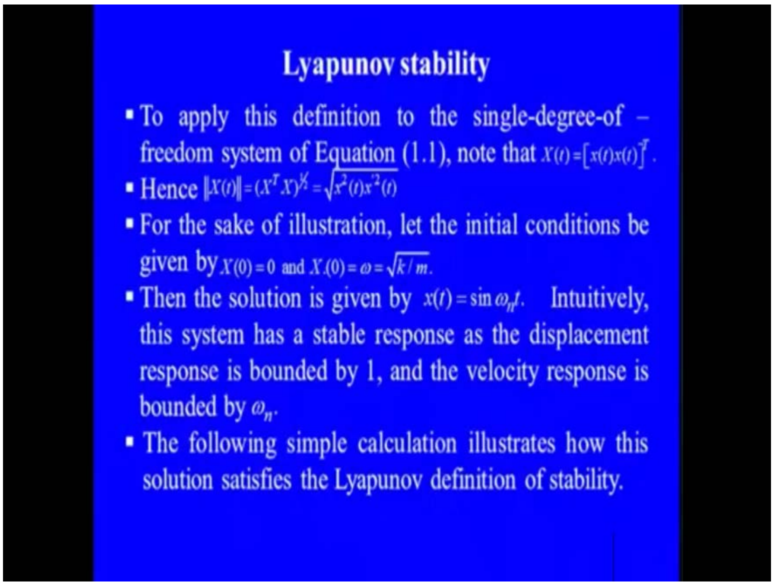

If, we apply this definition to single degree of freedom system. We can say that the x of t is nothing but equals to the x of t and then you x dot t with the transfer's feature or else we can say that when we are applying the amplitude feature of that it is nothing but equals to x of t x square root or we can say that it is nothing but equals to the square root of x square plus x dash square t. Then, you see here, if we are applying the initial condition say that you see x of 0 is equals to 0. We can say that x dot t which is nothing but equals to say because this is the input harmonic excitations. We know that say x is nothing but equals to a sign omega t the x dot is nothing but equals to a omega cos omega t.

So, it is a phasor difference and if you are applying to x dot 0. We can get you see the omega which is nothing but equals to square root of k by m so the solution which is given by x of t as sign omega n t. This system has a stable response as the displacement response is bounded by 1 and the velocity response is bounded by the natural frequency omega n. So, this is the characteristic feature here that if we bound the system solution by unique displacement.

Then, we have the velocity response is absolutely bounded by the natural frequency because the velocity is the linearly dependent on the natural frequency just like the acceleration is a non-linear dependent acceleration is omega square. We can simply go to find out the solution which can satisfy the Lyapunov stability definition. So, that how you see what the steps are there. Now, we are going to check that you see how the things are being there.

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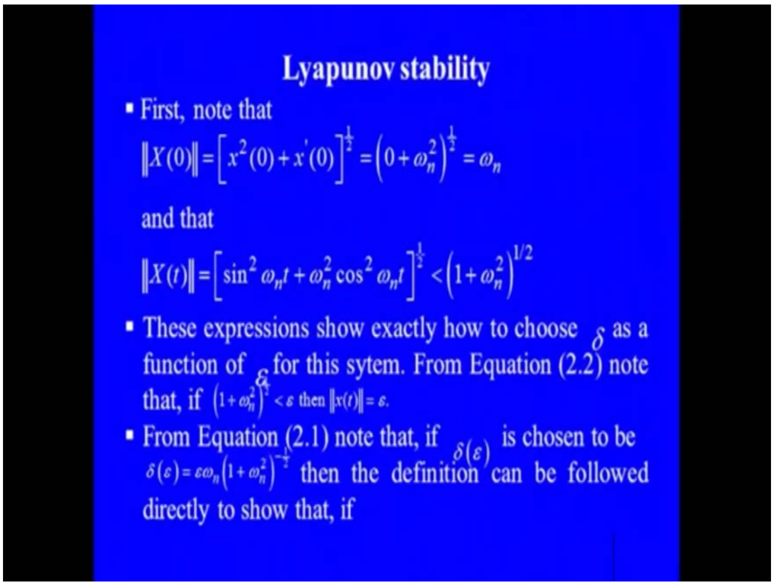

So, first of all we know that the initial condition should be pretty accurate. So, X of 0 when we are applying, it is x 0 square plus x dates x dot 0 square and if we are just applying this is 0 omega n square and square root half of that. So, we have the omega n as in a modulus of x 0 and if you are going with the x of t as an increment of that we have the sign square omega n. The sinusoidal feature of that plus omega square and cos square omega n t square root and which must be less than 1 plus omega n square root of this and this expression is exactly shows that how to choose the delta as the function of the arbitrary value zeta of this system.

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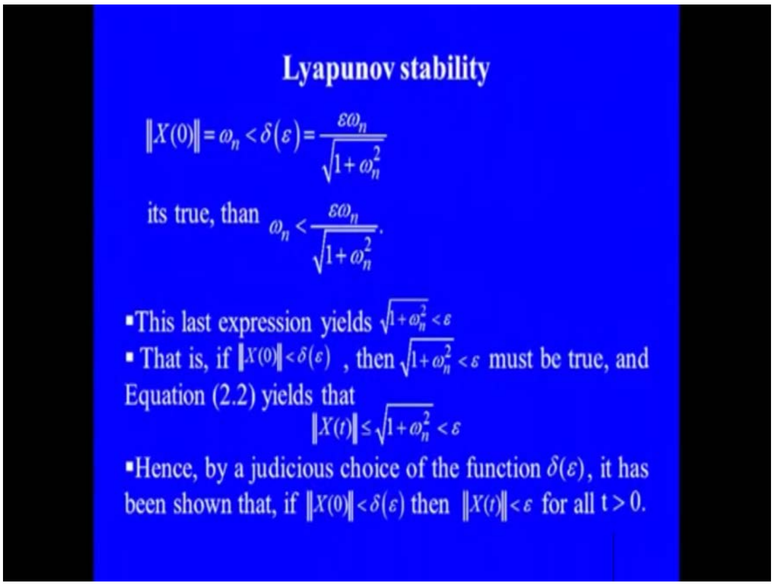

We can say that this is, if 1 plus omega and square of half that is nothing but the modulus of x of t is less than the arbitrary value. We can say that we can simply move further without having any unstable bounded solution x of t equals to zeta. Then, you see here, if the delta which is the function of arbitrary value is chosen to be in such a way that it is zeta omega n 1 plus omega n square root.

Then, certainly we can say that it is straightaway applied to the condition, if we have you see, the omega the x 0 which is equals to omega n minus zeta. This 1 or we can say that the modulus of x of 0 which is nothing but equals to omega n is zeta omega n divided by 1 plus is square root of 1 plus omega n square.

This is absolutely true, then omega and is less than this 1 and this last expression is just giving that 1 plus omega n square root which is the modulus of our x of t must be less than the arbitrary value chosen. If, this is true then we can say that ultimately x of t which should be less than or equals to the preceding step of our displacement which must be less than equals to square root of 1 plus omega n square or else it should be less than the arbitrary chosen value.

Hence, you see here by this judicious choice of this function. We can say that if x of 0 the model of the initial part is less than the any function of the arbitrary value then we can say that the x of t must be less than that arbitrary value chosen for any time increment for this Lyapunov exponent.

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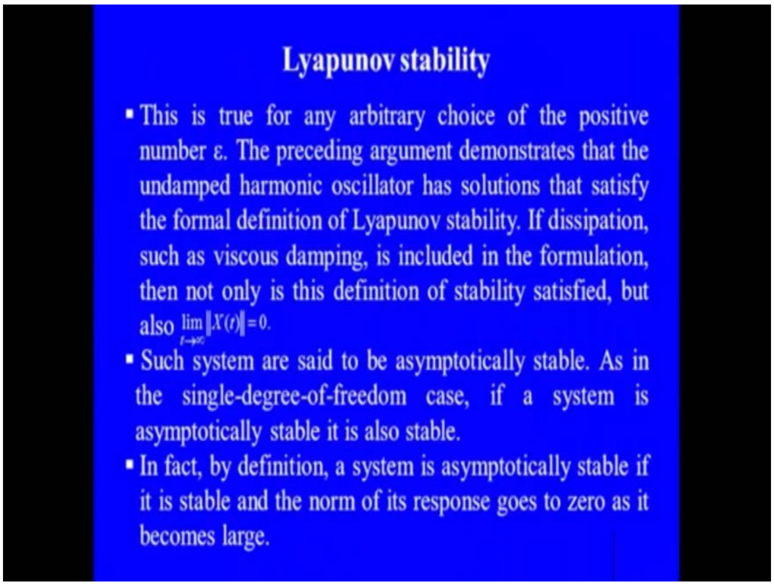

So, this is true for any arbitrary choice of the positive number zeta and the preceding argument demonstrate that the undamped harmonic oscillator has the solution that satisfy the formal definition of Lyapunov exponent and if the dissipation such as the viscous damping or anything is being included in the formulation.

Then, not only this definition stability satisfy, but also because you see here, there is additional source here which is just used to extract the energy from the system back, which simply leads the system towards the more stable form. We can say the limit t tends to infinite x of t must be equals to 0 and such systems are said to be asymptotically stable in case of even the single degree of freedom system. The system is also asymptotically stable even it is also called sometimes the stable features and in fact by definition a system is asymptotically stable.

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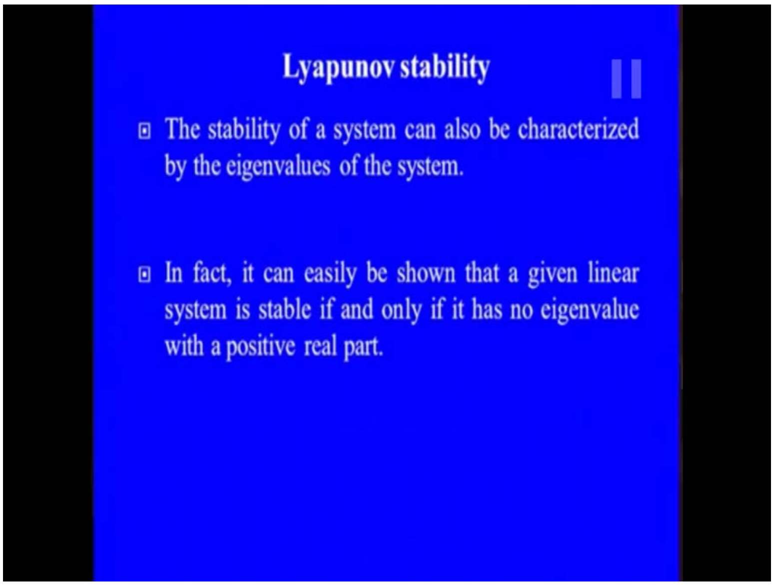

If, it is stable and again that norm of its response goes to 0 as you see t b t becomes large. So, you see here, if we are going up to the infinite time increment the system becomes even more stable form. So, this is you see here one of the mathematical representation of the Lyapunov stability and this the system can also be characterized by as we told as I told you the Eigen values to the sign of these things.

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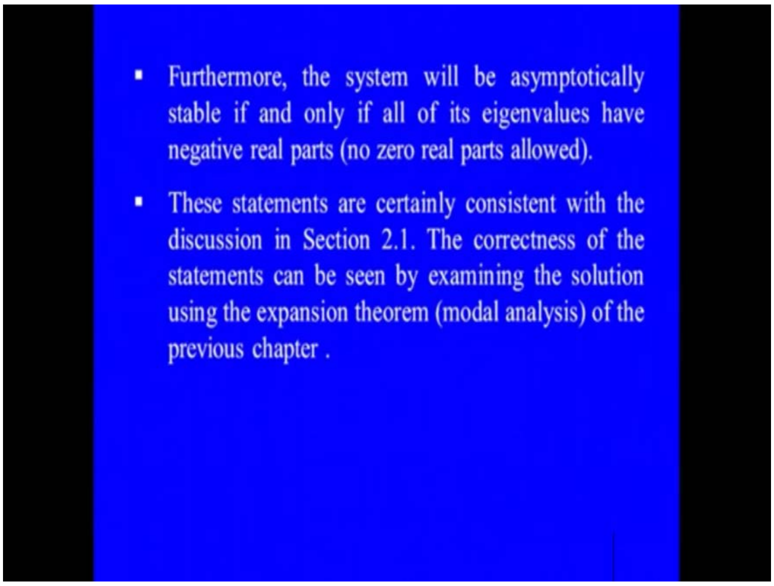

In fact it can easily shown that a given linear system is stable if and only, if it has no Eigen value with the positive real part because if it is, if it has a positive real part that means you see some additional energies to be feed to the system and that makes system unstable furthermore the system will be asymptotically stable if and only if it is all Eigen values have negative real parts nonzero real part allowed and these statements are certainly consistent with all the sections which we discussed previously.

The correctness of the statement can be seen by examining the solution using the expansion modal analysis of that. So, the Eigen value approaches to stability has attracted both the necessary and sufficient condition.

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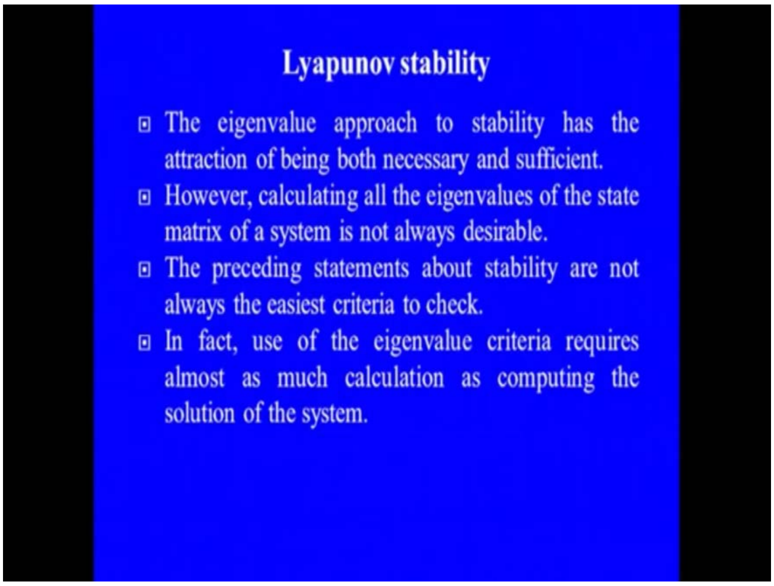

However, the calculating Eigen values of this state matrix of a system is not always desirable because the Lyapunov exponent is giving a clear picture about the bifurcation of that and the preceding statement about the stability are not always the easiest criteria to check. In fact we can use the Eigen value criteria require almost much calculation as compared to the solution of system. So, when we are talking about this. We know that the system is stable either conditions whether we are applying the Lyapunov theory in which you see here the initial displacement and the velocity vectors are being defined and then further in the preceding steps.

We can see that whether the solution is bounded or not, and we can check it out that whether it cross the barrier of that certainly of any chosen value that means you see here, the system is going towards the converging part some additional energy is to be supplied to the system. So, system makes the unstable or else you see here even the Eigen value that is nothing but the characteristic routes. The nature itself speaks whether the system is going towards the stable or unstable manner, so you see this is all about this chapter. The next chapter we are going to discuss something about even the feedback control and then you see here, how the theories can be applied for the vibration control.

Thank you.