Vibration Control
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Module - 2
Basic of Vibration Control
Lecture - 2
Reduction at Source – II

Hi, this is Dr. S.P Harsha from Mechanical and Industrial Department IIT Roorkee. In today's lecture again we are going to discuss about the basic principles of Vibration Control, in which you see our main intension is to be on reduction at source. In our previous lecture we have seen that, the vibration can be controlled straight way at the source from where the vibration generations are there, by changing some kind of structural changes.

Because, we know that if we want to reduce the amplitude of vibrations, there are two main mechanism which are effectively working other than the viscous damping that is the structural features. Because, we know that the vibrations are transmitting at very fast rate, and even more amplitude with the solid media. So, even we can put instead of the fully material plate or something you see the base.

We can put the perforated plates or some kind of you see you know like the can be provided, to deviate the path of that or else in other way we can provide. The intermolecular you know like the mechanism we can use that, and we can provide some kind of you see the isolators, in you know like at the time of you see the sources just like. You see when the foundations are there, we can provide such kind of materials to prevent the transmission of the vibration from the source to the ground or towards the structural features.

So, many cases which we discussed in the real mechanical practices, and we simply found that there is a great amount of reduction in the vibration amplitude, when we are using these basic principles. In today's lecture also we are going to discuss something about a similar nature that when the vibrations are being generated from the source of various mechanical machineries. Then what is the practical applications or adaptability is there through which, we can reduce the vibration amplitude straight way at the source itself.

So, in that again today we are going to take one of the good example of the cutter, and we know that when there is a you know like cutting operations are being happening, there is a straight impact forces are there from the cutter to the metallic surfaces. And due to that the huge amount of vibrations at the same time the noise is also generated.

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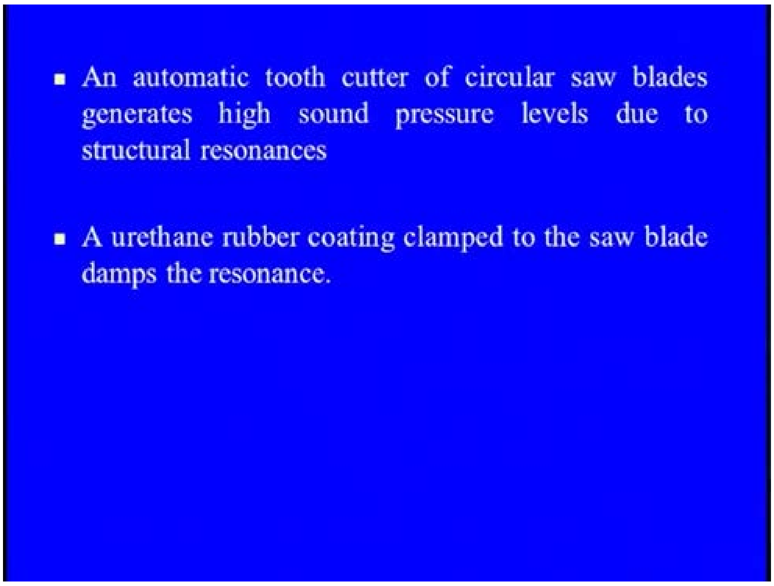

So, an automatic tooth cutter of any circular saw blade generates high amount of pressure level due to structural resonances. And you see here with this, there is a huge amount of vibrations at the higher frequency reasons are generated, and it straight way effect the precision of the cutting operations, and the same time you see here we cannot generate a smoother surface even after the cutting operations at the cutting part or cutting object.

So, you see here we can straight away put a urethane rubber coating, which can be clamped to the saw blade and straight way it will damp out the resonant conditions. Because, you see here we know that wherever the resonant conditions are there is a huge amount of energy, which is being emitted from that. So, this rubber which is nothing but you see a isolator can straight way absorb that amount of energy, and that can be you know like extracted from the source or you see here the during that matting surfaces.

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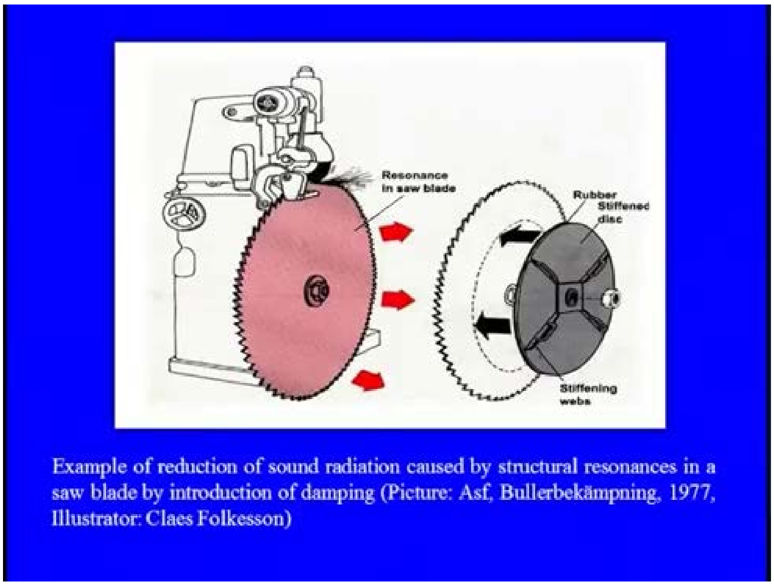

So, you can see straight way you know like on the figure that we have the two main types, one we have a saw blade you see the circular saw you see here and the blade. And then it is being under the cutting operation there is a huge amount of you see, you know like the vibrations are being generated, and these you see here the fluctuations which is absolutely at the resonant conditions, are being just emitted from that. Now, our main intension is to just put this rubber feature here on the blade.

So, you see here this is the portion on that on which we can straight way put the rubber part, and you see here this is you know like the stiffened feature, the stiffened disc is there which can be straight way kept on the rubber feature. And then you see here, this stiffened feature is straight way absorbing the vibrations or the sound or whatever the pressure levels which have been coming out from the interaction of the solid media, it can be straight way absorbed. So, this is one of the effective way and we know that these isolators are really effective in that manner and. through that its really easy to damp out the high frequency vibration from that.

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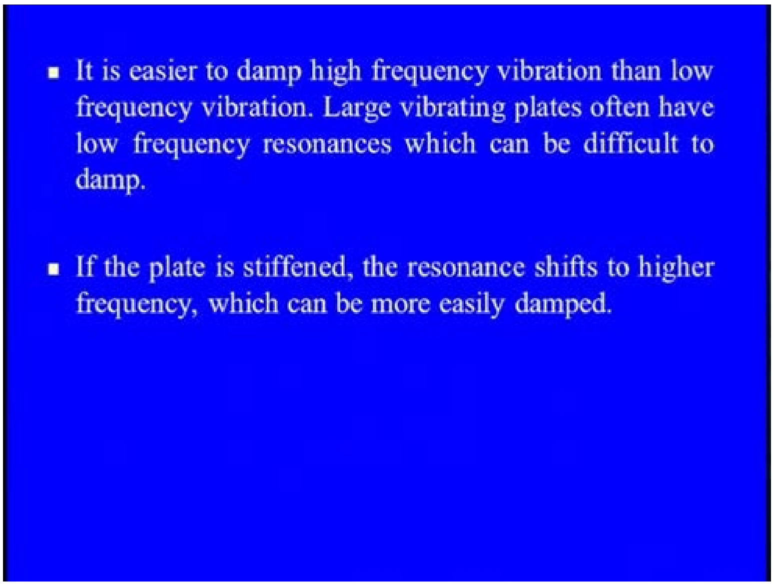

Because, you see one of the theory says that in the vibration specially that you know like it is easy to damp out the high frequency vibration than the low frequency vibration. Large vibrating plates, certainly you see you know like when you have large structure, the number of cycles during the oscillation is pretty low. So, whenever we are just trying to you know like absorb these vibrations, which are coming out from the large plates they have low frequency resonances.

And from that it is not an easy to damp out those vibrations, effectively in such a way that as we have done in the previous case. So, if the plate is stiffened; obviously, you see you know like we have, the you know like the stiffness is towards high, so in the stiffness is high, certainly even for the same amount of force the deformation is quite less. But, same time this stiffened plate is you know like creating high amount of we can say exciting frequency, the huge amount of energy at the same time.

So, these resonances with these stiffened plate even the plate is larger, the lower frequencies can be shifted towards the higher frequencies, and that can be easily damped out. So, these are the two practical cases which can be straight way adopted, when we know that the plate is not or plate or saw or any metallic surface is not too large. Certainly straight way at the same amount of exciting force, the high frequency vibrations are generated, and we can put these rubbers or any kind of you see these isolators which can straight way absorb the resonant energies. But, the same time if the plate is quite high the large I mean to say the dimensional feature. Then we need to use the stiffened plate, in which you see the stiffness is quite high, and then these lower frequency vibrations can be shifted to higher frequency and it can be controlled effectively.

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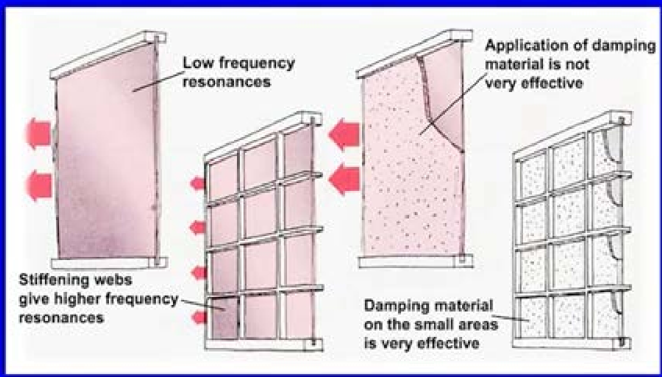

Figure: Principle for reduction of sound radiation by shifting structural resonances to higher frequencies (Picture: Asf, Bullerbekämpning, 1977, Illustrator: Claes Folkesson.)

This is another case you see here, in which the reduction of you know like the sound radiation or the vibration transmission, by simply shifting the structure resonances towards the higher frequencies. You can see that the first figure shows that, we have a huge amount of you know like the thin wall is there, and in this we have the low frequency resonances because of the size and the dimensional things. We can even put the blocks you see here in the stiffening webs we can say, which can even give you see here the high frequency resonances and that can be controlled.

But, again if we are keeping say straight way, the damping material all across the dimension of this wall, you can see that this is the third figure which simply says that we are keeping the damping material. But, even after that since the frequency is quite low because of the dimension we cannot effectively control this, this is the one of the beauty of the vibration theories. But, you see here this is you know like the frequency is quite low, and the damping material by simply blinding we are saying that in blind with that apply the damping material.

We can control no we need to see what exactly the exciting frequency is it is a low or high. And we need to make it first, if it is a low frequency we need to make it into towards the higher side, so you see here what we have done straight way instead of applying a direct material on the wall. We simply put the various webs, by putting the various webs the effective surface area is now reduced towards the lower one, the resonance frequency is now sifted towards the higher side.

And then if we are applying the damping material into these small areas of the webs, there is a effective reduction is there in the vibration, and the sound radiations. Because, of the structural feature of the vibration says that, when you have a clear deviation in the you know like these molecular path. Then certainly you see here it cannot transmitted effectively, instead of having a straight surface of metal and the transmission is very fast it is a molecular phenomena vibration. So, you see here you know like this is the practical case where we are applying these things.

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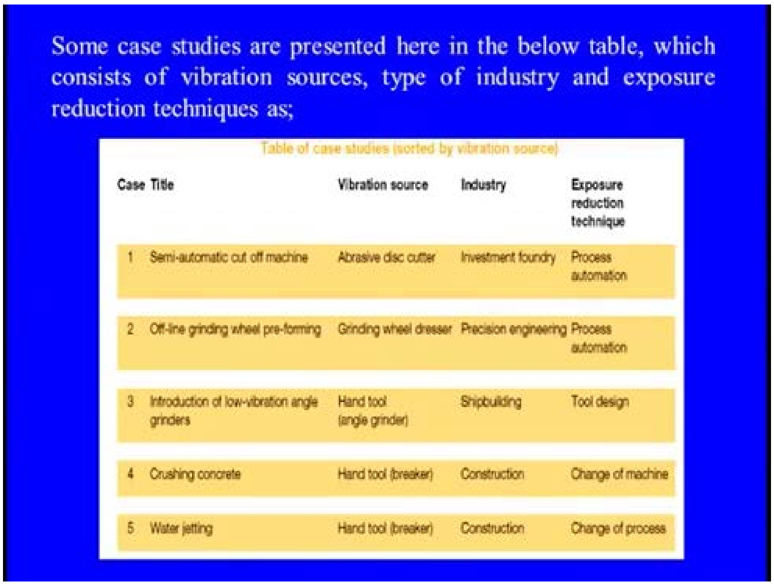
Some case studies are presented here in the below table, which consists of vibration sources, type of industry and exposure reduction techniques as;

Table of case studies (sorted by vibration source)

Case	Title	Vibration source	Industry	Exposure reduction technique
1	Semi-automatic cut off machine	Abrasive disc cutter	Investment foundry	Process automation
2	Off-line grinding wheel pre-forming	Grinding wheel dresser	Precision engineering	Process automation
3	Introduction of low-vibration angle grinders	Hand tool (angle grinder)	Shipbuilding	Tool design
4	Crushing concrete	Hand tool (breaker)	Construction	Change of machine
5	Water jetting	Hand tool (breaker)	Construction	Change of process

But, some of the case studies are there which you know like the we are regularly adopting. That how we can put, the kind of isolators or what kind of you see the vibration sources are there, how we can straight way reduce the vibration exposure by applying the technique. So, you can see on the screen the it is a huge one you see here various components are there, through which you see the vibrations are generated, but few of them are simply explaining here.

One say you see if we have, the semi automatic cut off the machine, means we have a machine in which you see the cutting operations are there it is a semi automatic means some of the manual operations are there. The vibration source in that is straight way the disc cutter of abrasive nature, it is pretty common you see in various industries are there of the foundry and all these things the basic production, we can say level. And then in this the entire process of the automation is straight way affected.

And this is what the exposure feature is, we need to apply the same feature which we have discussed with the saw blade. Even in the other feature, if we have the grinding, wheel with this you see you know like the pre forming feature, the grinding wheel is always. Because, you see you know like it is somewhat, we can say rubbing actions are there the close interaction is there of the two surfaces the huge amount of energy, the sound energy and the vibration is coming out from the sources. And then straight way you see either of any precision engineering or any kind of process engineering, straightway affecting that part or even you see if you are going towards any part say, if we have the crushing concrete you even if we have the water jetting. We know that, there is a clear effect of you see you know like the vibration on the processes which we are adopting in this.

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6	Bursting concrete instead of breaking	Hand tool (breaker)	Construction	Change of process
7	Diamond wire cutting	Hand tool (breaker)	Construction	Change of process
8	Pipeline insertion method avoids trenching	Hand tool (breaker)	Utilities	Change of machine
9	Directional drilling avoids trenching	Hand tool (breaker)	Utilities	Change of process
10	Mounted roadbreaker	Hand tool (breaker)	Utilities	Isolation
11	Reduced-vibration roadbreakers	Hand tool (breaker)	Utilities	Tool design

Even in the next one we can see that, you know like the burst concrete instead of the breaking. If we are simply burst concrete is there are various hard tools are there for especially, and the breaker feature the entire you know like if we do not have you see, the effective control the entire process is changed in that. Even in the diamond cutting, which is you know like even it is a very precise thing.

Since we know that, we need to have the hard tool and with this you see here the high frequency vibrations are being generated. Even in any of these you see here, whether we are using the drilling operations, whether we are using the road breaker or even you see here whether we are using any kind of vibration reduced road breakers are there. We know that there is a straight effect of the vibration on that, this is a clear vibration exposures.

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12	Maintaining chainsaw anti-vibration rubber bushes	Hand tool (chainsaw)	Forestry	Maintenance
13	Chainsaw maintenance and training programme	Hand tool (chainsaw)	Watercourse maintenance	Management
14	Reduced-vibration chipping hammer	Hand tool (chipping hammer)	Foundry	Tool design
15	Sleeve for chipping hammer chisel	Hand tool (chipping hammer)	Steel	Isolation
16	Isolated casting cut off	Hand tool (disc cutter)	Foundry	Isolation

Even in the maintenance processes, even in the tool design, even in the various construction operations. There are you see, the various you know like we can say the operations during which the huge amount of vibrations are being generated, nevertheless you see here we know that since you know like these are the key features of our machining operation. In which the rolling, sliding, oscillating or any kind of operations are being occurred.

Thorough which you see we can perform, the various activities, the huge amount of vibrations are generated. All we need to check it out the three main features in this one, whether the vibration occurred in the low frequency zone or high frequency zone, and third what is the amplitude of vibration? And you see here, even some of the grades are available for that that what exactly the impact of these amplitudes are.

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Preferred and maximum weighted rms values for continuous and impulsive vibration acceleration (m/s^2) 1–80 Hz [BS 6472–1992]

Location	Assessment period[1]	Preferred values		Maximum values	
		z-axis	x- and y-axis	z-axis	x- and y-axis
Continuous vibration					
Critical areas[2]	Day- or night-time	0.0050	0.0036	0.010	0.0072
Residences	Daytime	0.010	0.0071	0.020	0.014
	Night-time	0.007	0.005	0.014	0.010
Offices, schools, educational institutions and places of worship	Day- or night-time	0.020	0.014	0.040	0.028
Workshops	Day- or night-time	0.04	0.029	0.080	0.058
Impulsive vibration					
Critical areas[2]	Day- or night-time	0.0050	0.0036	0.010	0.0072
Residences	Daytime	0.30	0.21	0.60	0.42
	Night-time	0.10	0.071	0.20	0.14
Offices, schools, educational institutions and places of worship	Day- or night-time	0.64	0.46	1.28	0.92
Workshops	Day- or night-time	0.64	0.46	1.28	0.92

The preferred and the maximum weight is RMS value the Route Mean Square value of the continuous, and impulsive vibration acceleration. Simply signify that yes this is the great amount of you know like, the amplitude of vibration is occurred here. And through that; that means, you see here you know like the oscillation feature or you know like we can say, whatever the amount of displacement or acceleration is there it is very significant.

And if you want to control irrespective of whether this much amplitude, in terms of meter per second square right from you see here, 1 to 80 hertz or for low frequency vibration or even in the kilo hertz, we need to check it out that what is the drastic value for that. So, this table you see here which is on the screen is simply shows that, whether you have a continuous vibration or the impulsive vibrations, we can straight way go with you see, the assessment period and what could be the possible preferred value of the vibration, in the vertical and horizontal directions and what could be the maximum value, because these values have a straight impact on the work performance, the accuracy or the precision. And other things you see the human hazard because you see the physiological parameters says that, that if you are bearing the huge amount of acceleration at even the lower frequency, it is a straight effect on the behavior of the human. And the same time you see here, straight it has a effect on the blood circulation as well.

So, even you see when we have simply see that the RMS value, which is you see the weighted RMS, somewhat the reference RMS from this BS code 6472. We know that you see here, even within the low frequency 1 to 80 hertz, these are the only minimum ways. And if you have using that the things have somewhat comfortable for a workman, otherwise the various discomfort levels are there, and there is a various diseases through that.

((Refer Time: 15:54)) And if you want to characterize this, now we just we are just going towards the basic theory of that, one is the frequency response function. Because, you see here the exciting frequencies are coming, so how do we relate with the characterized frequency level. So, frequency response function is nothing but as we know that since it is a function, which is just based on the frequency, so it is the relation between the output signal of a any linear system towards the input signal.

And if we want to relate this, we have a circular frequency CF, you know and in other words we can say that, it is simply a interpretation that how much proportionality. Because, it is a linear system how much proportionality is there in between the complex input and output amplitudes, and it is one of the most important quantity the FRE, we are generally saying FRF, the Frequency Response Function which can be used in the analysis of any kind of vibration and sound problem.

Because, you see whenever we are dealing with these vibration problems straight way, the responses or we can say whatever the feature in terms of signal is coming out it has two domain. One is the time domain, in which you see here we are straight way recording, the variation of displacement, velocity and acceleration, with the time domain, with the time variation rather I should say. And the frequency response, sometimes you see here the time domain signals we are going to discuss by the way in the detail of the signal analysis in this.

But, right now you see the time domain signals does not provide you a clarity of the exciting features. That whether the system is exciting at the one frequency or more than one frequency even, then how many frequency peaks are there. So, we need to convert this into frequency response, the frequency domain through the Fourier series. And that is why you see sometimes we are saying that this is the Fourier transformation, so with this concept now you know like since we are basically discussing here FRF, so in that if the input signal is force.

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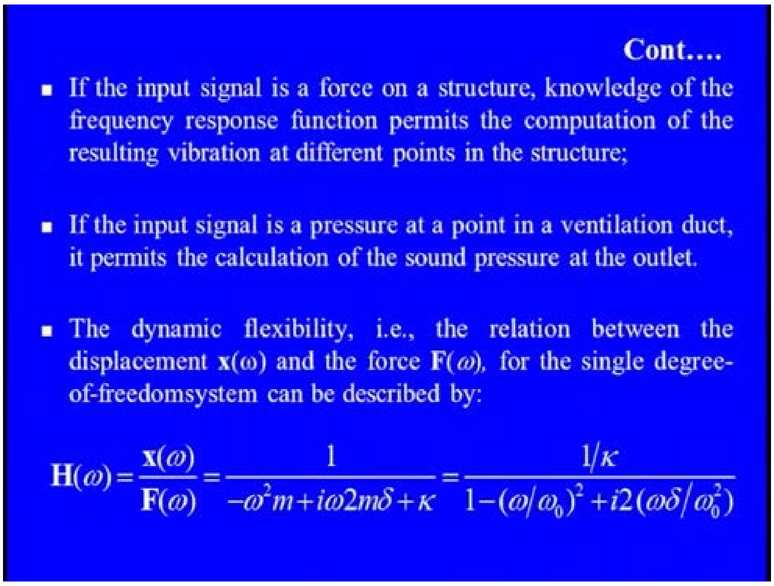

This is one of the important thing that you see what exactly your input signal is, so if for input signal is forced on a structure. The knowledge of frequency response function simply permits you to the computation of resulting vibration at different points in the structure. Because, the force is basically if we define the force we know that, what is a line of action or the point of action of the forces.

So, straight way we can deviate the different features of the force exactly at the structure point at the different points. But, in other case the input signal is our pressure at a point, in any of the you see this we can say ventilation duct or anything you see here, it permits the calculation of sound pressure, at the outlet. It is not that you see at the structure, we can go at the different points and find out the output feature.

So, the dynamic flexibility in this because you see this is all the dynamic you know like the phenomena is there of this excitation feature. That is the relation between the displacement, and the force in the first feature for a single degree of freedom can straight way find out the H of w, which simply you know like on the screen. H of w is nothing but your dynamic flexibility coefficient is simply the output, in terms of the displacement by input in terms of the force.

And when it is being occurred for any you see the mass damper, and the spring system we can say that. The dynamic flexibility is equals to one over minus omega square m plus iota omega square m delta plus k, where m is the mass distribution of the offset.

Omega is your natural frequency at which the excitation is occurred, delta is your damping coefficient, and k is the spring coefficient or else even you see here, we can rather divide this into the ratios.

And you see we have the two ratios, one is the damping ratio which is nothing but equals to the available damping divided by critical damping another is the frequency ratio the exciting frequency divided by the natural frequency. So, when we are doing this it also you see, the dynamic flexibility is equals to 1 by k divided by 1 minus frequency ratio square omega by omega n plus iota 2 omega by omega n square into delta square.

So, when we are doing this you see we can straight way find out that the dynamic flexibility of any system, in which you see we are basically looking for the output in terms of displacement. When you have the input in the force, it is absolutely depending on the system parameters, one what is the inherent characteristic of the system is means the mass distribution, the material property. And the metal property in terms of the deformation in terms of the damping, so these are all you see you know like the system inbuilt properties are there, through which the excitations are coming out from the system.

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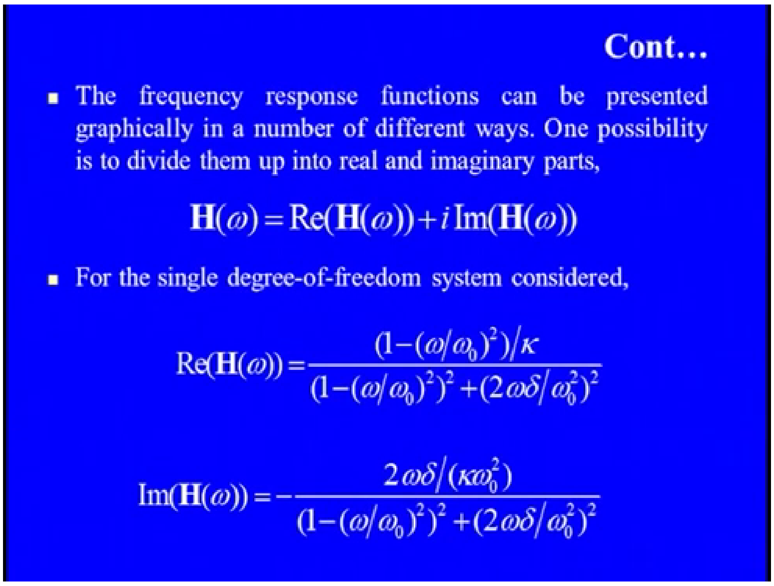

And this FRF the Frequency Response Function can be presented graphically in the various ways. Like you see here, we can simply divide the outcome of this FRF in two terms, one is the real and one is the imaginary one, so when you are just trying to divide.

Because, we know that when the system is under free vibration the complementary function, it is a transient phenomena when the system is under you see the forced vibration means the particular integral, the system is showing the steady state phenomena.

And the roots of these equations can be divided into two main features, one is the real and one is the imaginary one. So, the real part of this dynamic flexibility or in we can say this FRF function is nothing but equals to 1 minus omega by omega 0 that is a frequency ratio square into a divided by the stiffness divided by 1 minus omega by omega square this we can say r square omega this square plus 2 omega zeta by omega square whole square.

So; that means, you see here in the real feature there is a straight impact of the natural frequency, and the exciting frequency ratios are, but in the imaginary one we have a clear interaction of the damping as well on the top of that. So, it is 2 omega delta divided by k omega 0 square divided by say 1 minus omega by omega 0 square to the power whole square plus 2 omega delta omega square whole square. So, this real and imaginary features are clearly showing that there is a variation in these responses, due to the complex or the real nature of the roots are.

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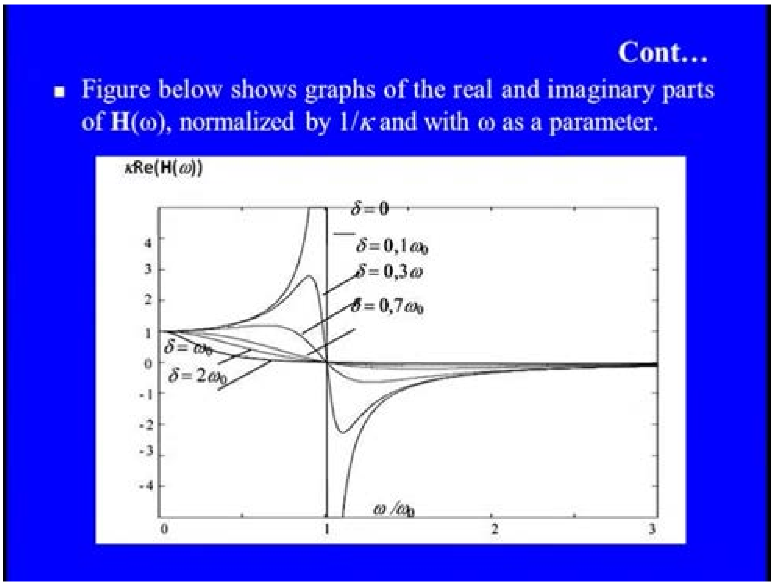

And then you see in this figure we can simply show, the real and imaginary part of our frequency response function, which can be normalized by putting 1 by delta with omega.

So, you can see that we have the real part here in the first figure it is a real feature of this, when you multiplied the stiffness into real figure, and you have the frequency ratio you can see that if the damping is not available in the system. The real root which is simply showing you see here, the free vibration condition is having the maximum excitation at the resonant condition you can see the straight line where the delta is 0.

But, as you see the damping is coming into the you know like the vibration signature, the effect of this feature. You can see there is a straight reduction or we can say, the vibration amplitude is now in the control feature, so we can straight way play that with what is the optimum value of the damping is to damp out, the vibration amplitude critically. So, you see here when we are just going from 0.1 omega o ((Refer Time: 24:19)) or even exactly at this omega 0.

You can see that, when the delta is you see the omega 0 straight way you have a critical damped and you see here, the entire feature is damped out of the vibration or even you see, when we are applying you see the more value of damping that is the over damping case. Certainly you see here it will delayed in the steady state response, so that you see here we need to optimize the thing in that way. So, this is you see the real part in this, and if you are going towards the imaginary part of that.

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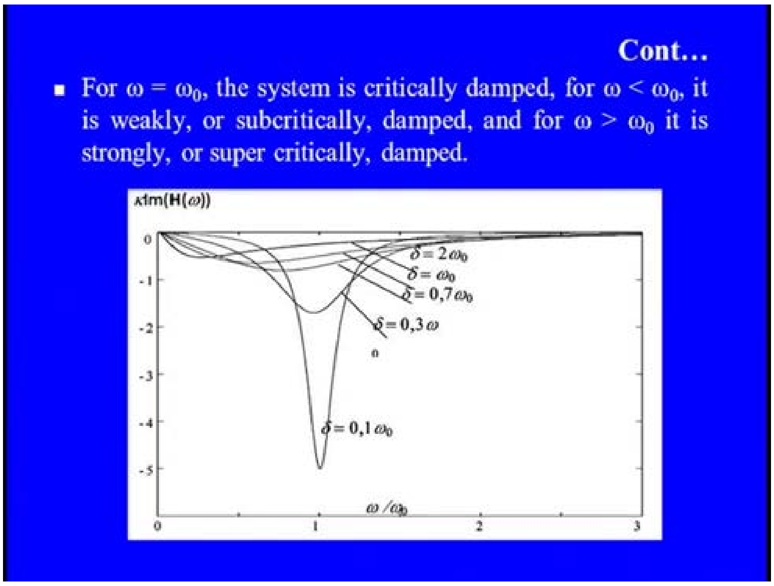

Then again you see here, if we have the damping you know like this is what the k into imaginary part of this, and we have you see omega by omega 0. At the system is we know that it is a critical damped system for omega which is less than omega 0 it is weakly or sub critical damped feature. And when the omega is greater than omega 0 it is a strong one or super critically damped one, but again you see here we need effective damping feature.

So, the same features are being there in this imaginary part which is straight way you see you know like the forcing factor device. So, we can see that even if we are increasing, the damping value, the amplitude variation in straightway drastically reduced, and we can get effective way this is what you see our damped feature is effective utilization of damping in reducing the vibration amplitude. So, this is you see you know like the way through, which you see we can damped out the vibration absolutely at the source only. Even if we are looking towards the other possibility of representation of the frequency response function, we can even divide this into not into the you see the imaginary and the real, we can divide into amplitude and the phase.

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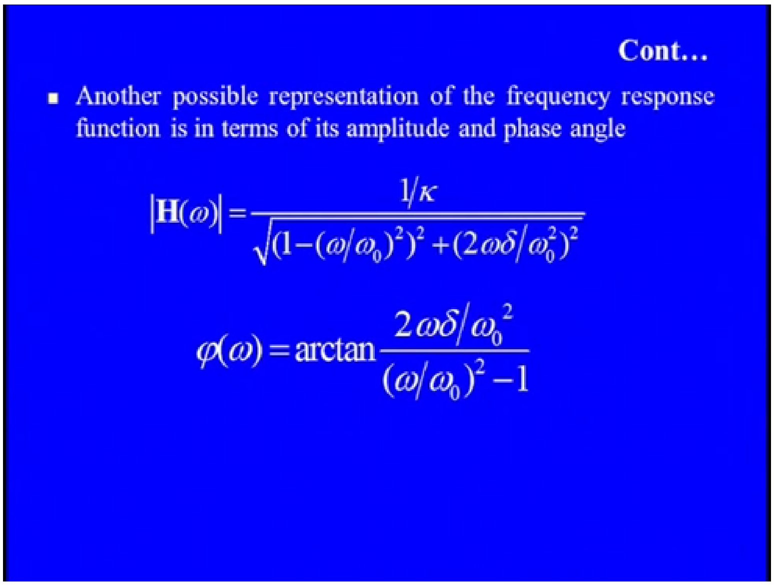

So, over all amplitude of this frequency response function as we already discussed is 1 by k square root of 1 minus omega or omega 0 square to the whole square plus 2 omega delta omega square whole square. And if you are going with the phase then it is nothing but equals to tan inverse of 2 omega delta by omega 0 square divided by omega by omega 0 square minus 1. So, now you see here when we are trying to change the parametric variation with again the frequency response function and the variation of this

H w with the overall value of the amplitude of H w with omega by omega 0 the frequency ratio.

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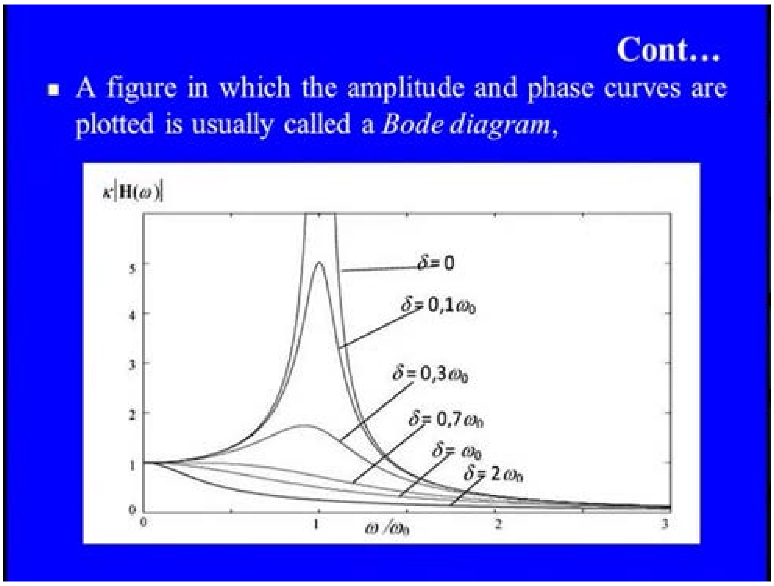

The diagram is known as the bode diagram, which simply gives you the amplitude and the phase curves with that. So, in that you see in the figure it is clearly mentioned that when you are absolutely at the resonant condition, the amount of energy or in other terms the amplitude of this frequency response function is enormous, when there is no damping. So, you see here the huge amount of energy generally we are saying it is a non technical feature, but generally we are saying that it is infinite energy.

So, you see here in other way the huge amount of amplitude is there, we just want to reduce this vibration amplitude. So, when we are adopting you see the damping factor it may be you see the material damping, it may be you see the structural damping, it may be of viscous damping or it is of the mixed nature the complex nature itself. Then it can be straight way brought down the vibration excitation, through adopting the different values of damper.

So, we have delta 0.1 omega 0 same 0.3, 0.7 or straight way if we have, the same value means the critically damped one. The effective utilization is there or else you see even if you are going towards the higher order, we know that the over damping though you see like provides some kind of damping effective damping feature, but the same time you see here, it will delay it your steady state outcome you see here. So, we just want to quickly damped out this which simply featured out from as we already discussed in the critical damping part.

So, this is one and second you see if we are using the phase differences, so if the phase difference you can see that, when we are absolutely at the critical damped feature. That means, you see here you know like the omega equals to delta 1 and it is a resonant condition you can see it is a drastic variation when there is no damping. So, you can see that the phase difference is how the things have been varied, and when you see we are absolutely coming down to the effective damping, you look at that this is the you know like clear variation of damped and first un damped.

If it is a un damped one no damping is there, the phase difference is with the frequency responses are straight you see this is what critical bode diagram is. But, when you see we are adopting the damping, and we are almost approaching towards the critical damped part there is a drastic you see, control in the phase shift or we can say the phase differences. And then you see here there is a clear variation, the smooth variation of the phases.

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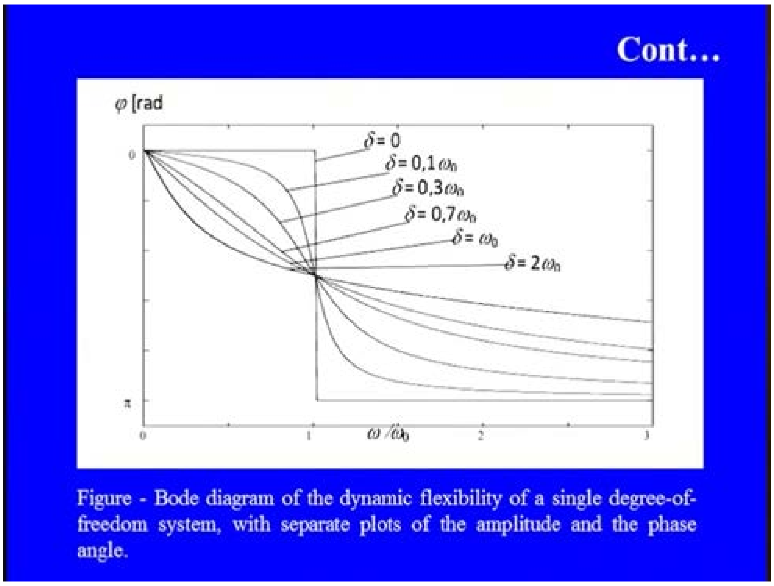

So, you see the bode diagram for dynamic flexibility of any single degree or multi degree of freedom system. They are clearly showing the variation of the amplitude and the frequency or the phases rather I should say of frequency response function that how you see the changes are being there in that. So, now you see here till now we discussed about that how effectively, we can control the vibration in the you know like the structural feature of the sources.

And when they are straight way you know like inducing the vibration by various ways by cutting the operations or by you know like doing various other operations, you know like the grinding operations or when we are doing any manufacturing or this maintenance operation. The huge amount of vibrations generations are there of any kind of industries, and which surfaces are being more exposed, how we can effectively apply. And we know that, if we want to apply say the isolator or any other technique in the basically this passive kind of vibration control, we need to check it out the three main part as we discussed the frequency of lower higher or amplitude.

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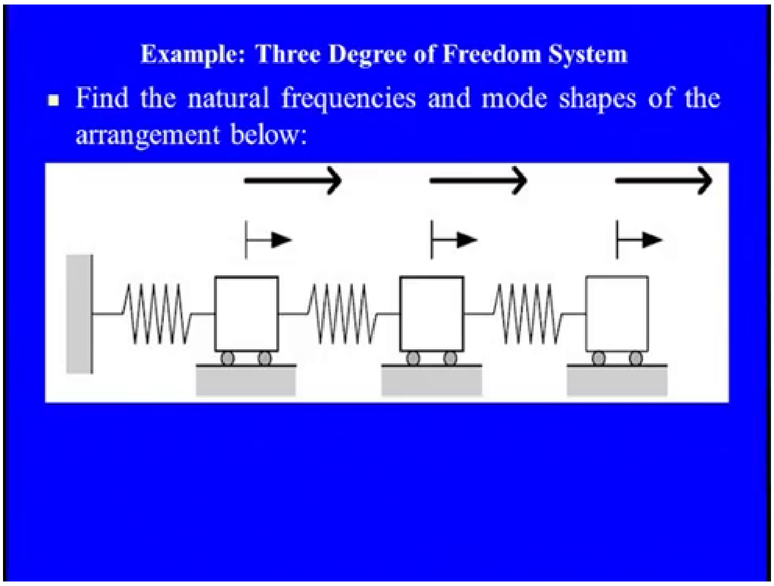

Now, let us solve a numerical problem of higher order say 3 degree of freedom system because this is something you see here through which at least we can we already discussed about the 2 degree of freedom system. Now, since we are in the domain of multi degree of freedom system, let us see that how we can find out the natural frequency and the mode shapes of that.

So, when we are talking about the 3 degrees of freedom, we know that there are 3 masses which are being displaced at three different displacement x 1, x 2, x 3. But, since you see this is the well equilibrium system, so the entire you see mass configuration is controlled by the spring itself. So, we have you see the various stiffness's.

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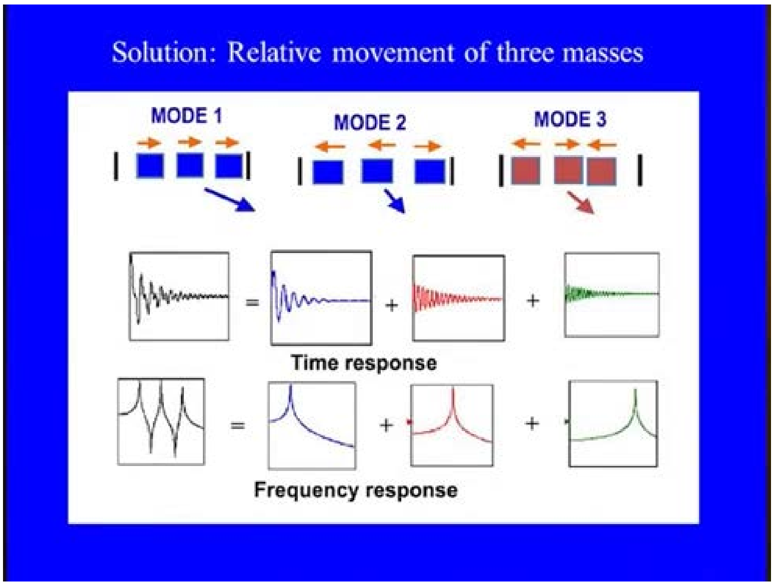

Now, let us look that what the variations are there in these masses, so we know that it is a 3 degree of freedom system, and with this 3 degree of freedom system. We know that there are 3 natural frequencies and corresponding mode shapes or else if we want to simply find out, here you see the things are as we discussed. We know that this is a 3 degrees of freedom system, we can capture the frequency response or the time response from these 3 masses.

And the time response if we are saying that, we want to show that then it is nothing but equals to the combination of all like the acceleration variation with the time in this way or the displacement variation. So, if we want to see the relative displacement of the mass for or the masses for 3 degree of freedom system, you can see that the mode 1 is showing that all the 3 masses are absolutely along one line in the plane, and they are simply showing one phases.

So, this these are in phased vibration even the mode 2, they are also showing that you see the 2 masses are absolutely which are the closest the adjacent masses, the closer one they are in the same phase, and other mass is just going to towards the other side. And in the mode 3, you can see that the adjacent masses are absolutely in the repulsive action, either if we are taking m 1 and m 2 or m 2 and m 3. So, you see here this is the basic we can say the model feature of these masses and due to that, we can get the excitation of different level in the time and the frequency domain.

So, when we are talking about the time domain you can see, it is the overall picture the first one is overall picture of time or time with the displacement variation or time with the acceleration variation anything you see here. And it is equal to the first displacement you see feature of this, second is this and third is this it is a combination of that, but as I told you that one of the drawback of the time response is we cannot find out that though these masses are in the relative displacement position, but what is the exciting feature exciting frequency for this.

So, for that we need to go to the frequency response and this the frequency domain feature. So, the second figure is showing that you see, we have 3 frequency peaks because of the 3 masses are exciting at different levels, so certainly you see here when in the first mode, when these you see the masses are in the same phase in phased are there this is my first frequency, the lowest frequency when they are absolutely the 2 masses are in the repulsive feature, the another feature. And all the masses are just absolutely you see you know like in the repulsive nature, the higher order frequencies are there. But, overall picture is showing that you know like the total frequency responses are cumulative effect of all 3 modes of this, so this is the featured part of this.

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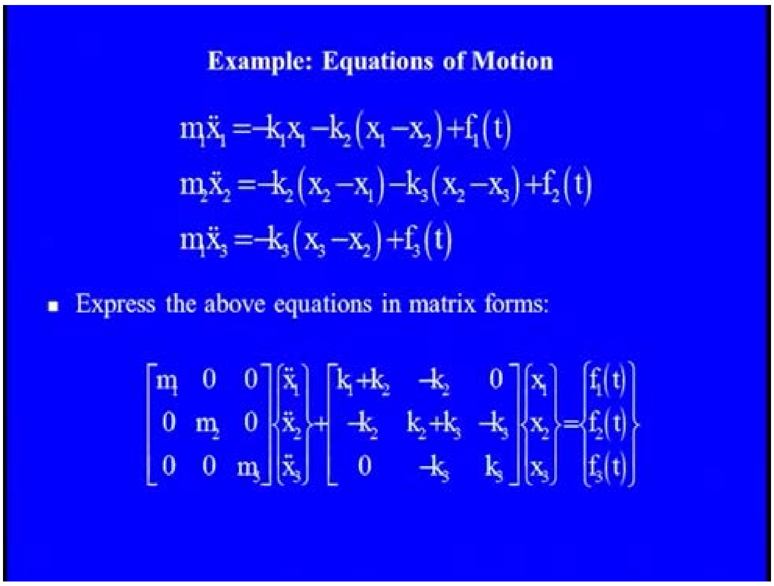

But, now we would like to calculate mathematically this for that first we need to go to the equation of motion. And since it is a 3 degrees of freedom system, and you see all 3 masses are being constrained by the springs, we can go with the equations of motion like that the for first mass, you see m 1 is being constrained by k 1 and k 2. And you see you know like the force is being exerted on that say f 1 t you know we can say that the inertia force m 1 x 1 double dot equals to minus k 1 x 1 minus k 2 x 1 minus x 2 plus this one.

Similarly, for mass m 2 which is being there in between the k 2 and k 3, and even this k 2 has a straight impact of k 1 this x 2 and x 1 you see here. Because, of the stiffness variation of k 1 k 2, so certainly we can say that if we are just going with inertia force this is absolutely. But, with the restoring forces minus k 2 the average or we can say the difference of x 2 minus x 1 and k 3 which is nothing but equals to x 2 minus x 3 the relative displacement of that.

Because, in these in the middle one as we have already discussed that it is a clear impact of both x 1, x 2 and x 3 on the k 2 and k 3. And if we are going towards the m 3 the inertia force is m 3 x 3 double dot, but the stiffness variation is coming only due to the k 3, in which you see the relative displacements are there of x 2 and x 3 and then the force. So, in all you see now we can straight way put into the space form in other way into the matrix form.

And in that we can find that the matrix as we already discussed that in mass matrix all the elements will come along the diagonal one. So, this is what it is m 1, m 2, m 3 into this. And the same time you see the stiffness matrix size is always being positive along the diagonal. So, we can see that in the first part we have a clear interaction of these stiffness's k 1 and k 2, and then you see here we have minus k 2 and this minus k 2 right from the second part, when the m 2 is being you know like under oscillating feature.

So, we have minus k 2 k 2 plus k 3 and minus k 3 and the last one is straightway constraint by only 2 part k 2 and k 3. So, we have minus k 3 and k 2 you see here like this, and this x 1, x 2, x 3 are nothing but the relative displacement of the individual masses m 1 m 2 m 3 and f 1, f 2, f 3 are nothing but the forcing feature.

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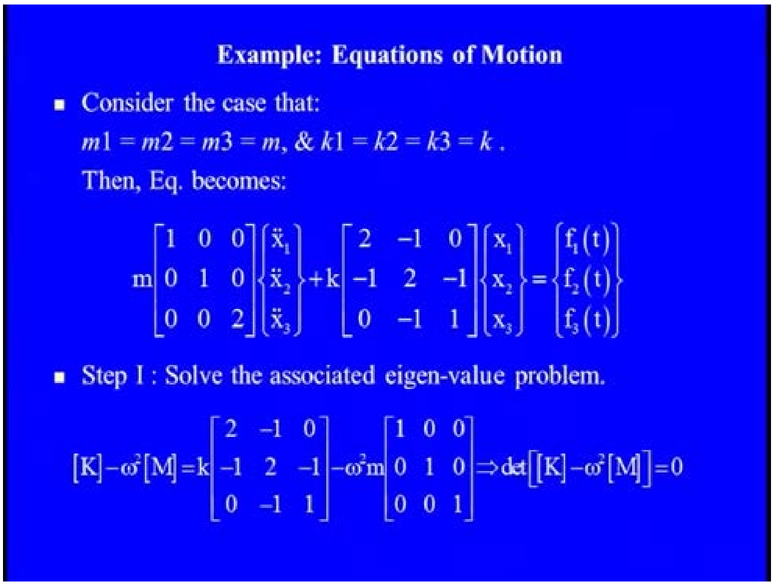

So, now you see if we are saying that say the mass is same it is just a general case m 1, m 2, m 3 is just one mass m. And the same spring is being used, no change in property, the dimensional and the material properties both, and the same time you see here no other forces have been acted on the springs. As just you seen the linear forces along one direction only you can say that the stiffness is same k 1, k 2 and k 3 equals to k. So, when we apply these conditions the simply wide conditions into the equations we will find that our mass matrix since the mass is same. So, we have m 1, m 2, m 3's like that.

And the stiffness matrixes are like this because the stiffness is same and the forcing factor is like that. Now, the first thing is that to calculate the Eigen value or the natural frequency, certainly we know that we need to go with this minus omega square m plus k equals to 0, and we need to find out the determinant of this. So, for that what we have first the matrix formation we have k that is stiffness matrix minus omega square m, so that is m is 1 0 0 0 1 0 0 0 1 in this we have only one here.

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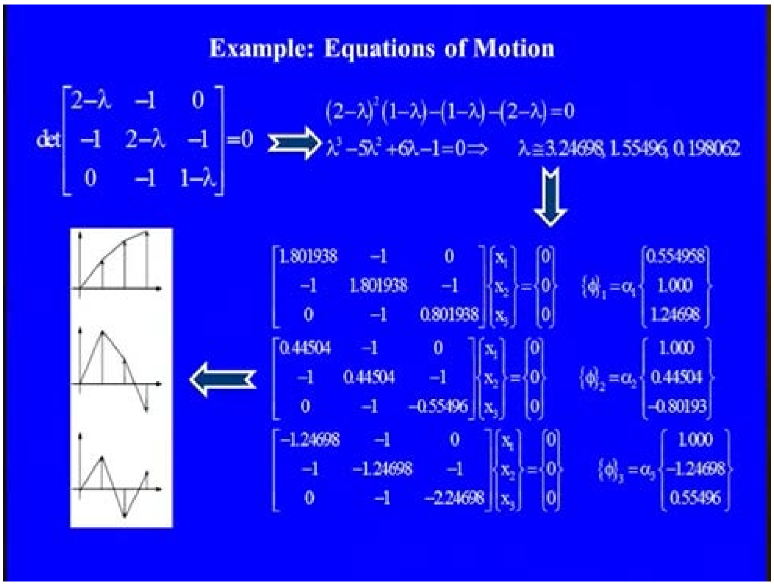

So, now, if we want to find out the natural frequency we need to go to the determinant of this, so you see here in the determinant now you can see that we have the lambda, which is say that the characteristic root for this. And you see, if you are just adopting these things we have 2 minus lambda minus 1 minus 1 and 2 minus lambda and 1 minus lambda with this minus 1 minus 1. So, this is what the determinant is and if you have just to applying basic theory of our determinant feature we have this one.

So, lambda is nothing but equals to 3.2 this plus minus this or we can say it is nothing but equals to see we have the 3 main features because it is a cubic part. So, certainly you see here there are 3 values of the lambda is, we can say that the lambda is 3.2469 whatever it is 1.55 and 0.198. So, first natural frequency when they are in the phase all the masses it is 0.198, the second in which you see here one of the mass combination is in the repulsive nature it is 1.55 and all 3 are just in the repulsive nature means you know like in the reverse order we have maximum frequency 3.274 the omega and 3.

And you see here now, if we want to calculate the mode shape means the relative displacement of that. We can simply see that again we need to go to the basic equation and put you see the alpha 1, alpha 2, and alpha 3 to see the variation of that in x 2 by x 1 and then again x 3 by x 2 and then x 3 by x 1. So, in these things you see here again we have applied those basic features there in the lambda, so for first natural frequency we can simply put that lambda is 0.198062. So, we have the first 1.80 this 1 minus 1 minus 1 and this, this.

So, when we are putting this first lambda value the natural frequency or we can say the Eigen frequency. The corresponding 512 is nothing but equals to alpha 1 and these features, this is showing you see simply the values of x 1, x 2, x 3 in the relative positions. In other way when we are putting the lambda is 1.5549 means for the second natural frequency the 52 is equals to alpha 2 1.4 and minus this one. So, here you see in the in the elements of the matrix we have now 1 minus.

So, now you see one mass is now in the repulsive nature or in other way you see here, we can say that the third when we are keeping 3.24. So, 2 minus 3.24 is 1.2468 and when we are going towards the third natural frequency, again if we look at that it is starting from 1 that is you know like 1 phase, then opposite phase and again in one phase. So, you see here all 3 masses, are in the different directions according to their relative positions, so now, you see we can simply find out that this these are the relative position.

And if we want to plot these featured into the real mode shape diagram, then we can simply find that this is the way, first figure shows that all 3 elements which are simply you see in this equation say 512 is 0.55 51 basically 0.55 1 and 1.2 are this, they are absolutely in the same phase. And you see the masses are arranged in this direction, second we have the 52 which is nothing but equals to alpha 21 0.445 and minus 0.8. So, you can see that they are just going up to 1, then coming down the second mass.

And the third mass relative position is just in the opposite manner and because of this deviation the exciting frequencies are quite high as compared to the first one. While if you are going to the other feature, which is we can say the outer phased, you can see that the first is 1, the first mass is absolutely on the top of that. The second mass is now going down, exactly in the opposite direction minus 1.2, and third mass is again going on top of this. So, this is you see here you know like in 1 and 2 m 1 and m 2 the orthogonal position and even m 2 and m 3 is also along the orthogonal feature.

And because of this you know like the different mode shapes and the critical locations, the stiffness is quite high and the exciting frequencies will be of more nature. So, we can see that starting from the first natural frequency 0.19 and the third mode of this is the natural frequency is 3.2, which is you see if you find out the ratio it is very significant. So, why we have used this numerical problem here because we know that this is you see you know like the vibration is there, the vibration is at the source is there.

But, when we are going towards the higher order of mode shapes, we can simply find out that the frequencies are quite dominating because of the stiffness variation. And if you want to effectively control those things, we need to check it out that which mode shape is there, what is the relative position of the masses are accordingly we can adopt the control strategies. So, in this lecture you see we discussed mainly about the vibration control, at the source when the structural featured are quite dominating there.

Some time you see the metal damping is also playing the key role, and the that is why you see accordingly we are always adopting the foundations from the brittle or any other material. Because, we know that though they are harder, but they are very good damper also, and they can absorb the huge amount of energy even at the resonant conditions. Second thing is there, which is even quite dominating here is when you are at the you know like say multi degree of freedom systems, means you see the multi body systems are there.

And we know that they are absolutely rotating in the various coordinates, we need to check it out for same component of or various component that what is the relative displacement or relative positions are there during the excitations. And we can find out that you see with this deviation there are significant, exciting frequencies are and accordingly we can adopt the control methodology. So, these you see the 2 chapters which we discussed about the basic vibration control.

Where absolutely focused on the structural vibration, when you have a huge structure or when you have a small structure when the cutting operations are there or any impact or continuous vibration is there or you see even when we are just talking about the low frequency or higher frequency. Then how we can adopt according to the machine specific or according to the this operation specific, we can simply adopt the control method.

But, these are all you see in the domain of solid metals or we can say you see you know like any solid media. And then you see we discussed also about, the frequency response function which is the true representation of the exciting featured off that, it is a mathematical representation. And in that you see there were 2 main we can say the characteristic features are there, either we can use the imaginary or the real part of this because it is a dynamic flexibility, so we can use this or else we can go with the amplitude straight way and the phase one.

And both are clearly showing that, when you are adopting you see when we are absolutely at the resonant condition. Then the damping is one of the significant criteria to reduce the vibration level and, but again you see the damping is not like you see you can put any kind of you know like, say damper there or any either the viscous damper or material damper. You need to see that, what exactly the requirement is according to the system inherent property. And corresponding you see the damping is to be added there itself, for an effective control of vibration.

And that you see we are simply adopting according to the damping ratio, so you see this is all about you see the solid media through which the vibration is transmitting or the sound. In the next lecture, which is you see again you see we are absolutely going with the basic vibration control, but this vibration is now induced due to fluid flow. So, flow induced vibration is again in many of the books I did not find much of the material there, so that is why you see I would like to discuss.

Because, you know like sometimes when we are talking, we are only talking about the solid feature in the vibration. But, the fluid featured of the vibration is also one of the significant criteria because when the flow is just you know like flowing through the pipe or the air is passing through the pipe. Certainly there is a huge amount of vibration and the noises are there, and in our industry this is a very common feature of that.

So, since we are talking about the basic vibration control at least I need to dedicate one lecture, on this flow induced vibration in which the fluid either the air or the liquid is playing a key role, in inducing the vibration or we can say the sound. And there are various ways to effectively control these by changing either some of the shape or you see the adrupt you know like we can say what are the changes are there, where the maximum turbulences are creating there. And with these turbulences the huge amount of vibrations are being generated.

So, we can you know like adopt some different passage or there is a change in the adrupt we can say changes are there. We can simply change the shape or the size of those different shapes of the adrupt areas, where like adopting these smooth surfaces, the turbulences can be reduced drastically. And then you see here we can see that, there is a clear reduction in the vibration and the sound generations from these pipes or these any of these you know like the areas. So, in lectures in next lectures we are going to discuss about the flow induced vibrations.

Thank you.