Vibration Control

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Module - 8
Vibration Measurement Techniques
Lecture - 4
Filters

This is Dr. S. P. Harsha from mechanical and industrial department, IIT Roorkee, in the course of Vibration Control, now we are in the last module and the last lecture. And in this module where we are discussing about the vibration measurement technique, straightaway we discussed about the basics of measurement technique. Then we discussed about what are the dynamic parameters which we can measure and through that, we can characterise the basic vibration signatures.

And when we are measuring these, the dynamic parameters we know that this entire measurement with these sensors or the transducers which we are applying there, at the vibrating body they are in there, they are basically there in the time domain feature. So, the information which is coming in the time domain sometimes, we cannot analyze accurately, so we need to transfer this time domain information into the frequency domain part, using this Fourier series.

And in the Fourier series decomposition, we know that there are various features which we need to because it is pretty easy now, because the mat lab functions are there in, and we can straightaway convert this time domain to frequency domain. But, the match is saying that we need to check it out that, what exactly the waveform feature, whether we are talking about sine wave or the triangular wave or the rectangular wave. Or we whether we are talking about the square wave, what kind of waveforms are there and then what is the corresponding mathematical functions in the Fourier series decomposition.

And in the last lecture, we discussed about the Fourier transforms, when you have the non periodic features in that that means, when you have any kind of abrupt change, means through the shock or the pulse, the impact form. Or when we have the turbulence kind of signature, then instead of just going with the Fourier series decomposition, we can use this straight the short term Fourier transforms.

And for that with the using of in this f of t, which we are saying the Fourier signal, in this the alpha which was the coefficient by changing the decreasing the alpha or by increasing the total time or by decreasing the time step. We can rather characterise these whatever the impulse features are there accurately, so here we discussed and all the part. And then we discussed about the numerical in the numerical problem, that when we are just taking these the sine wave or whether we are just talking about the square, or the rectangular wave, triangular wave.

Then how by simply increasing the time total means, the time span or by decreasing the a alpha, how we can get accurately the variation features in that. So, in this lecture now, again we are going to discuss about the vibration measurement techniques, and in that the filters part which we are always being there. Like the filters are nothing but just it is a mathematical presentation, when there is a noise disturbance or when we just want that these frequency peaks are the important frequency peaks just to featured out.

That what exactly the kind of vibrating these masses or the characterisations are there, then we can allow them and we can restrict the remaining exciting features in the spectra. So, the filters are basically designed accordingly to the desired, what is the decidedness and what exactly the application parts are there in that.

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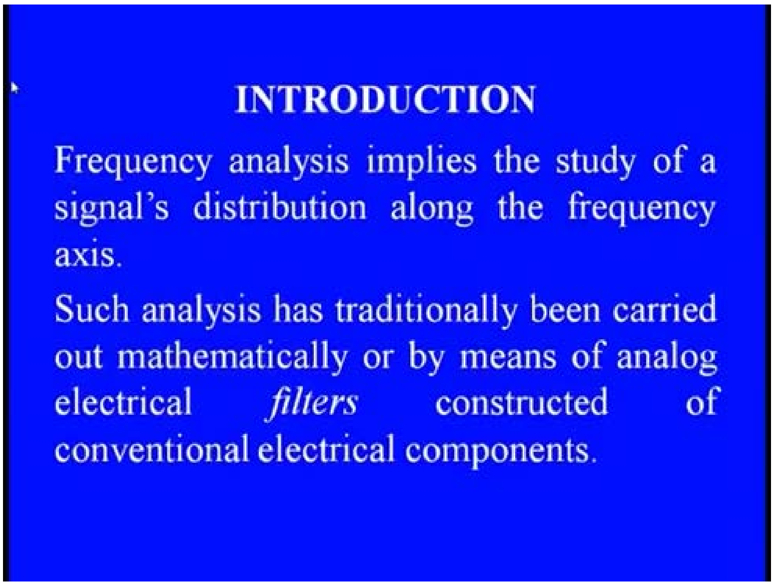

So, frequency analysis implies the study of the signal distribution along the frequency axis, and on one side it is just showing the amplitude, that what exactly the amplitude of the vibration excitations are… So, these frequency analysis has traditionally being carried out, mathematically or by means of analogue electrical filters, which are being constructed on the conventional electrical components in that.

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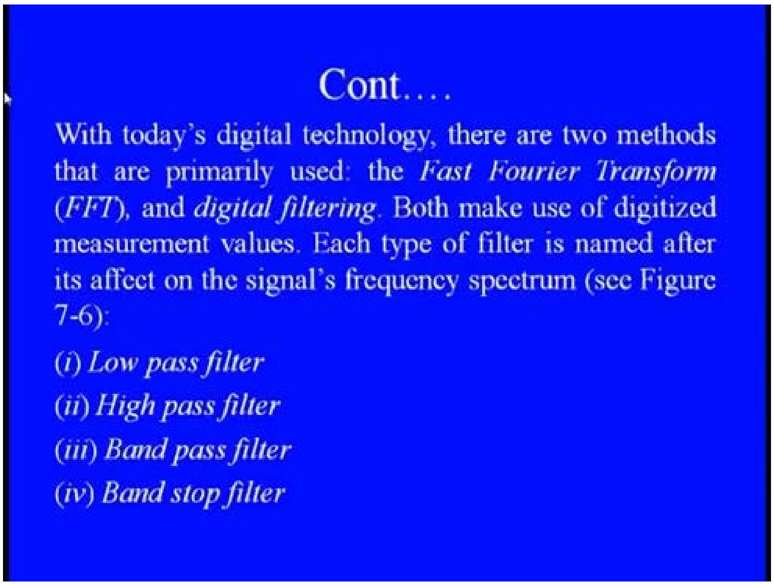

So, whether we are talking about the fast Fourier transformation or digital filtering, these are the key components nowadays in the digital technology. And primarily we need to apply these two, especially either FFT or the digital filtering, according to our desiredness. So, both make use of digitized measurement values, and each type of filter is absolutely named after it is affect on the signal frequency spectrum, like if we are talking about the low pass filter, low pass filter itself means we need to allow the low frequency component.

And then whatever high frequency components are there which has to be restricted, and correspondingly if we are saying the high pass filter, band pass filter, band stop filter we need to define the ranges. That, what is the frequency the range which is supposed to be allow or which frequency range is supposed to be stopped with these electrical filter design. So, if you look at that in this diagram, where it is clearly showing that we have a machine, which is vibrating.

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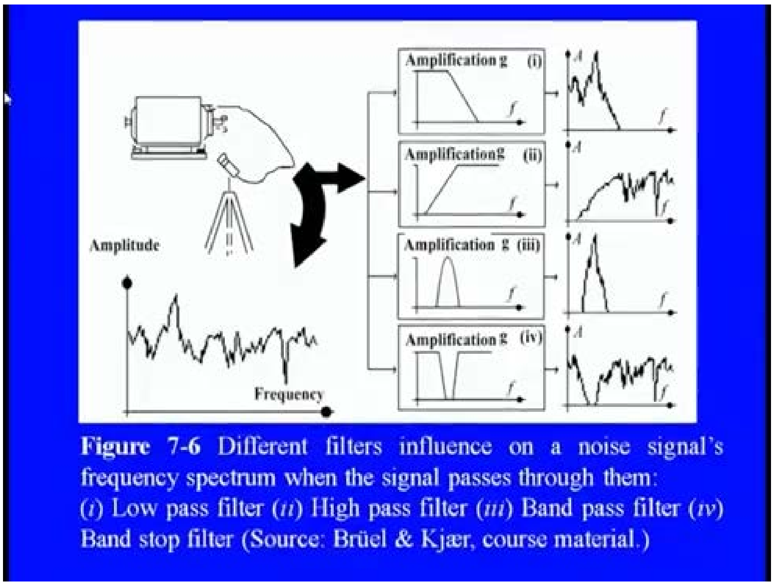

And right now we are saying that we have the accelerometer, an accelerometer is being capturing whatever the frequency excitations are or this whatever the vibration excitations are there, in terms of you see the amplitude and the frequency. So, right now if I am saying that you see, this is what my raw signal in which it is a clear variation is there in the amplitude with the exciting frequencies. So, this is the total we can say the frequency spectra's are there, now the first case when we are saying that the low pass filter.

As per the name itself, we can say we are basically looking in terms of amplitude with the frequency means the initial pass, so if you want to design this that means, this is what the width which is supposed to be provided by the electrical signal to allow the initial part. And the remaining part, means I mean to say that if I am just looking towards this part, this total frequency spectra, so this initial feature of the frequency with the amplitude is being allowed and the entire remaining part is being restricted by that.

So, if I am just looking to the actual situation, when this signal is being passed though this, we can say low pass filter, then I can say that I have this is what my frequency spectra with low pass filter. And this is clearly showing this at the low frequencies, how the variations are there in the exciting frequency with the amplitude. Second when we are saying that the high pass filters, the high pass filters means now, now basically our the analysis range is in the high frequency zone.

So, when we are talking about high frequency zone, then you can see that the second figure which is clearly showing that, this part the initial part is absolutely restricted. And then the frequency component which is to be allowed to analyse, this is what which is falling in the high frequency part. So, this is the high frequency and if I am looking to the actual raw signal, then we will find that the initial feature means the initial peaks and the variation is restricted and the final part is being allowed now.

So, this is if I am looking to the main part, this final part is being allowed and which can be shown here. Now, when we have such kind of say the pulse, in the impulsive form or such kind of in any defect feature is there in, and we are getting some kind of the special we can say frequency components during the running part. So, now we need to band that part that now, this is what I want to analyse that, why it is showing a different nature as compared to all other my signature.

So, then we need to capture the middle part, we need to band the frequency ranges and that frequency ranges is being allowed, so that part we are saying that the band pass filter. So, band pass filter that means, the initial feature is being allowed, say if I am just talking about this part in the main structure, somewhere this exciting peak is high, it is the normal part. But, this peak excitation is at it is highest level, may be because of some defect, may be because of some certain excitation is coming.

If we want to analyse this; that means, we need to allow this range that within, it is being falling at the frequency and with the corresponding amplitude. So, that means, if you look at this amplification part here, you will find that this is what being allowed and the remaining means the lower and higher frequency regions are being straightaway stopped there. So, in this raw signal this is what my now the signature, vibration signature which clearly shows that yes, in this particular frequency region you have this kind of frequency spectra.

Or if I just reverse that, if I know that because of some irregularities or because of some kind of impactive, impulsive forces I know that I have some kind of non, the non periodic or I have you see some kind of the abruptive changes are there in my signature. And later on you see, once the things are being overcome then I have irregular phenomena, so now if I want to stop this the next filter is the band stop filter. So, band stop filter is now, if you look at the nature of the band stop filter that means, the band which we defined can straightaway stop this frequency component.

And the remaining feature, if you look at the real nature of the signature or this real signature of the vibration, the nature itself says that, this peak is now absolutely omitted. And then we can say that the entire spectrum is being just varying whatever the required features are, so either we are talking about low pass, high pass, band pass or band stop. We can clearly feature our response this, whatever the responses are there in the signature analysis and we can analyse those corresponding, whatever the vibration signatures are there.

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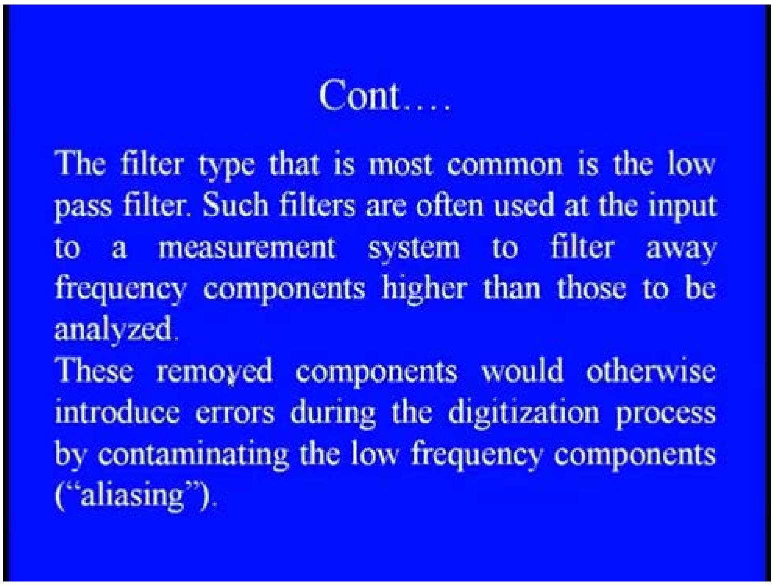

So, as we discussed the filter type, which is more common in, we can say is the low pass filter and such filters are often used as the input to a measurement system to filter, away the frequency components higher than those to be analysed. So, these removed components would be otherwise, introduced the error during the digitization process, and in the digitization process, like this contaminating the low frequency components or we can say the aliasing effect. So, when we are doing some digital these figures in that, so during this entire process, we know that this whatever these low frequencies which are coming in that, the anti aliasing features the features are being incorporated that.

So, we need to remove this, so that is why generally we are saying that, the low frequency components or the low pass filters are pretty common in their application. And during this entire process, when we are removing this we can say that aliasing effect is pretty common in this part, so we are saying that you see the band pass filter.

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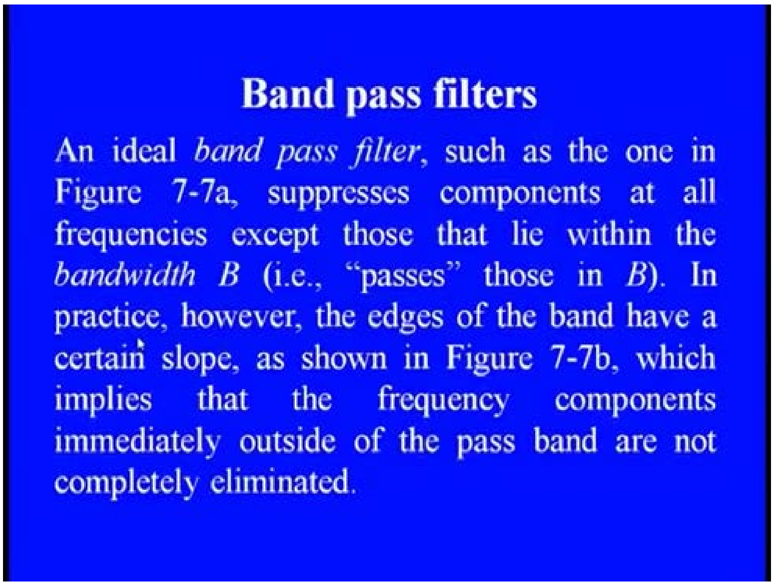

This is what the third one, in that the ideal band pass filter which I am going to show you is just suppressed the components at the all frequency except those, which are being lying within this bandwidth. So, we need to now define the bandwidth, what is the width is there through which the entire frequencies are being passes from these filters. So, in practice the edges of band have certain slope, we know that when they are being passing they have the slope.

And which implies that the frequency components immediately outside the pass band are not completely eliminated, because they are not the squared one, when they are the squared one. Then we can say that whatever the frequency components are there, which are being assed passing through that, they can be completely recovered, they can be completely eliminated. But, since they have the curved shape, so from side these things are being passed through.

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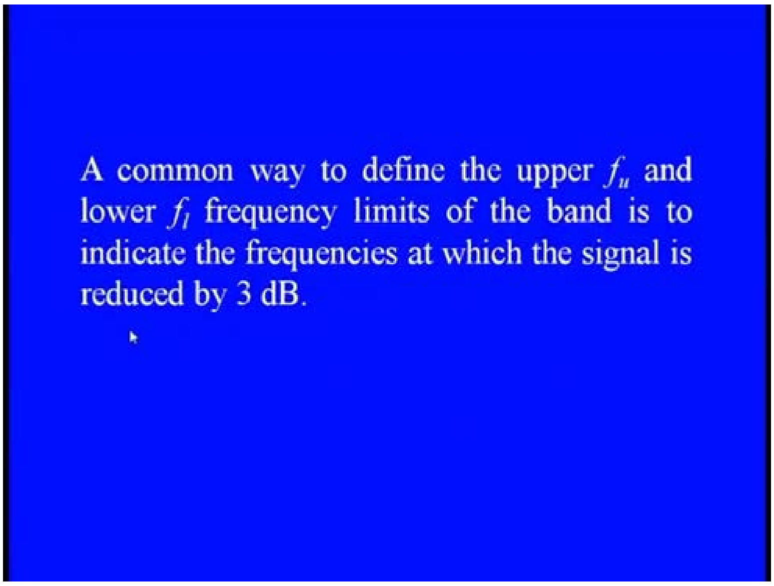

So, common way to define the upper limit, where the curved feature is there and the lower frequency here limit of the band is to indicate the frequency at which the signal can be reduced by three decibel part, 3 d B.

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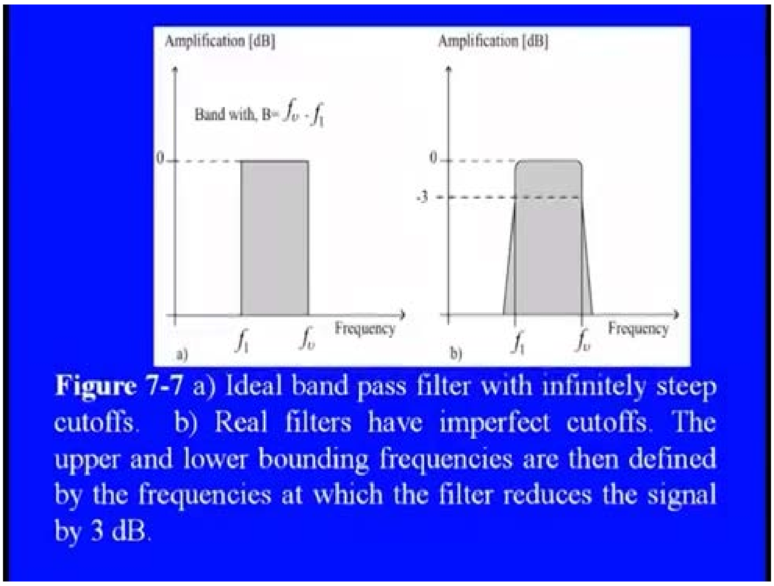

Figure 7-7 a) Ideal band pass filter with infinitely steep cutoffs. b) Real filters have imperfect cutoffs. The upper and lower bounding frequencies are then defined by the frequencies at which the filter reduces the signal by 3 dB.

So, if you are just looking to the exact shape the ideal band pass filter, with these the we can say this indefinitely step cut-offs, so we have either initial frequency, the lower frequency and upper frequency range and if I am saying the bandwidth is this f u minus f l, the upper band and lower band, so this is the square form of this. So, if this is what you see the ideal form, but whenever we are talking about the real waveform of that, then the real filters have to be there with this imperfect cut-offs.

So, we can say that when we are just talking about this, we have the curved feature of our signal part, and when we just want to use this bandwidth there we know that, if you are just going with this part, the slant part which is being there it is a kind of you see the error in that. Because, this is the imperfect cut-offs are there and the upper and the lower bounding frequencies are then defined by those frequencies at which the filter is reduced the signal by three decibel. If you are just looking at that point, if you are reducing the signal by 3 d B that means, up to almost we are in the perfect zone, where we can say that the real filters can pass this much bandwidth is there.

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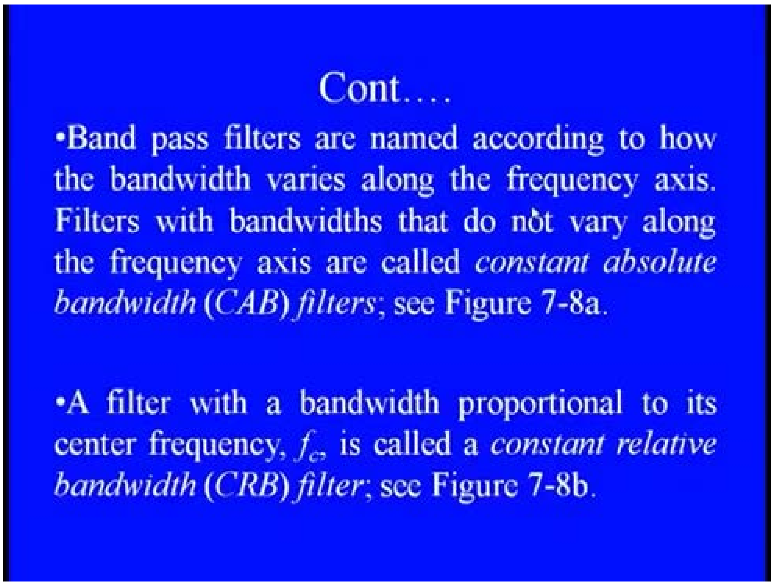

So, the band pass filters are named according to how the bandwidth varies along with the frequency axis, the frequencies with the bandwidth that do not vary along with the frequency axis, called the constant absolute bandwidth. That means, we have the CAB filters in which there is a clear absolute the bandwidth are there, and they are just constant all along the frequency axis. But, a filter with the bandwidth which is proportional to this centre frequency FC is called the constant relative bandwidth.

So, if you are juts our basis the reference frequency, means the central frequency and then the filter is being decide based on what is this side, the lower side and upper side we are saying that this is the constant relative bandwidth. And when we are not changing the bandwidth, along with the frequency axis we have constant absolute bandwidth.

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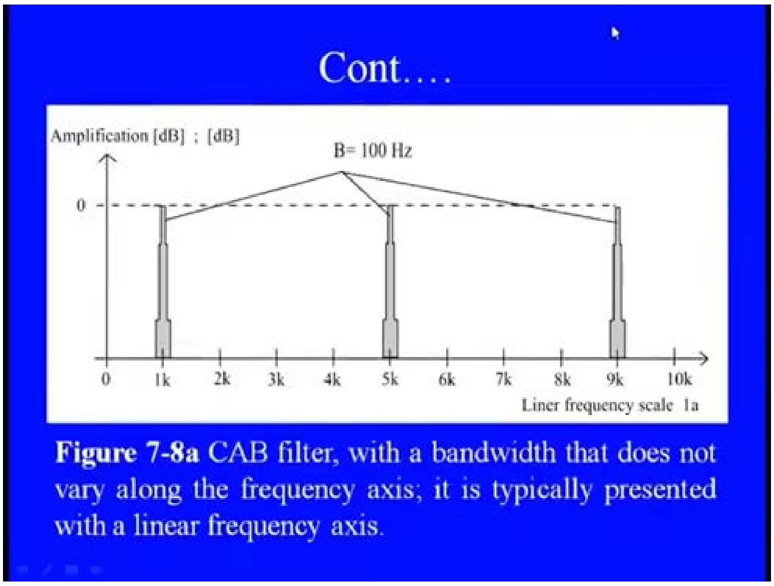

So, that we have constant absolute this, these bandwidths are there in which we have the absolute features are there, so if you look at that, they are absolutely varying with these say if I have the bandwidth of 100 Hertz. So, we can right away start from say this is what the scaling part is there, this is what in between that what exactly the bandwidth gaps are there, and this gap will be constant. So, if I am just going with this say 5 k to 1 k I can simply say that I have the difference the bandwidth is 100 Hertz.

Or even when I am just going from the 9 k to this 9 k to 5 k, then again you say I can say that, I can simply remade you see this part as the constant width this bandwidth. So, here the bandwidth is not varying with the frequency axis, and it is typically presented with the linear frequency axis. So, whatever the frequency axis are there in the corresponding peaks, the peak is repeatedly occurring at the constant frequency and there is no variation in the nature of the excitation peak as well.

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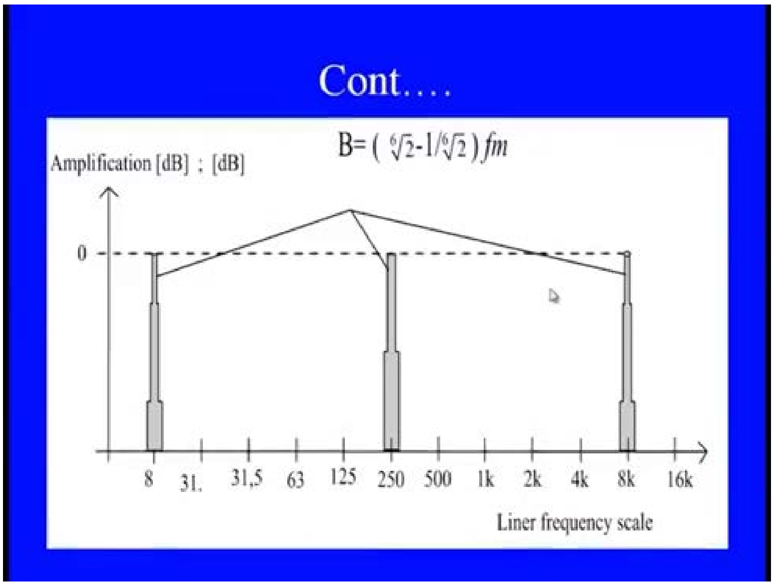

And if you are just talking about the variable means, now central frequency is first being calculated and based on this central frequency now, the other width are being considered there. So, we can say the bandwidth is now nothing but equals to this square root of the 2 minus we can say 1 over square root of 2 whole cube of that into the central frequency. So, here this is what my linear frequency ranges are there, and in between the bandwidth is calculating based on the central frequency and corresponding width accordingly.

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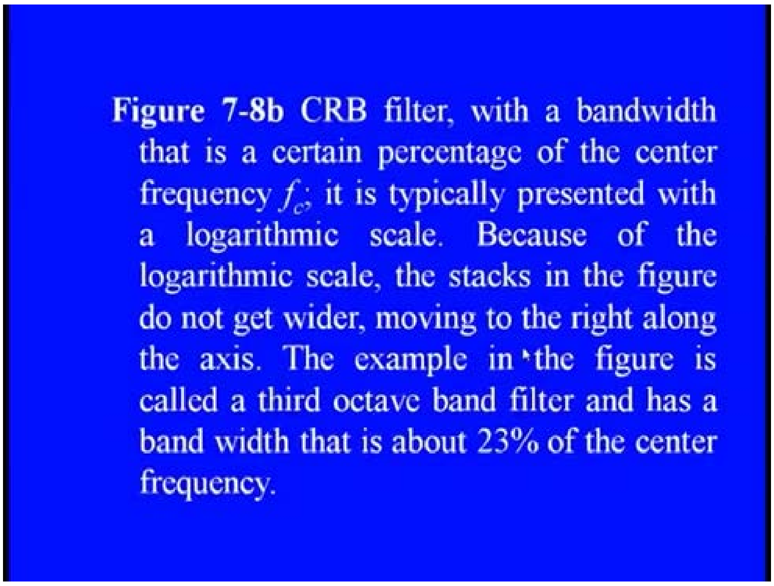

So, now if you are just going to this CRB which is nothing but the constant relative the bandwidth filter, a bandwidth that is a certain percentage of the centralised frequencies. And it is typically presented with the logarithmic scale, because of the logarithmic scale the stack in the figure do not get say the wider, and when we are moving as previously I shown you, just we need to move along the positive axis of x. So, we can say that this figure like is called the third octave band filter, and has the bandwidth is almost we can say the 23 percent of central frequency. So, octave band is to be designed in such a way that, that we are reducing this signal by 3 d B upper and lower, and once you calculate the central frequency, then the variation of this is the 23 percent of either.

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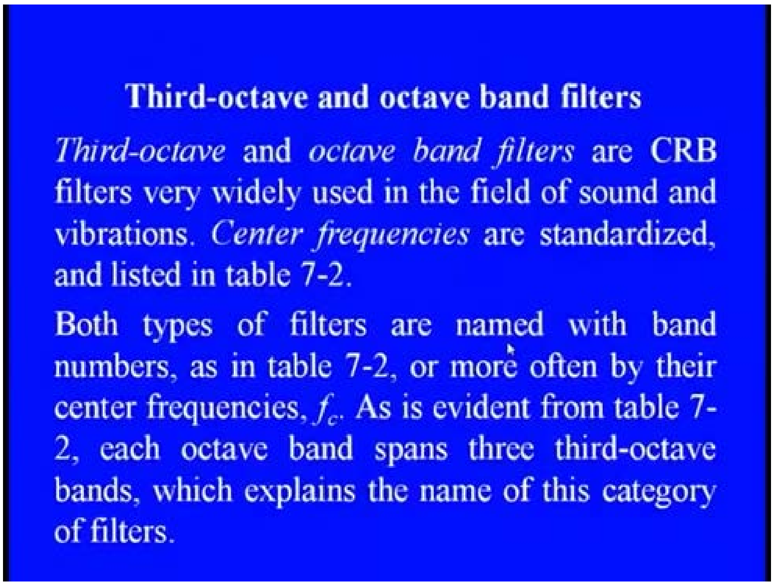

So, when we are talking about the third octave or the octave band filter, the third octave and octave band filter are nothing but the constant relative bandwidth filters and widely they are using in these vibration part. In which the main part is coming that, what is the central frequency which we needs to be standardised there and then we can simply configure the other two part. So, both type of filters are named with the band number, as I am going to show you the table and more often by their central frequencies they can be varied accordingly. So, we can simply see now, that each octave band spans or three octave band in each octave band the span of three octave bands, which explains the name of this category of the filter. Means in each octave band now, there are in the entire span there are three octave bands which are being there within that.

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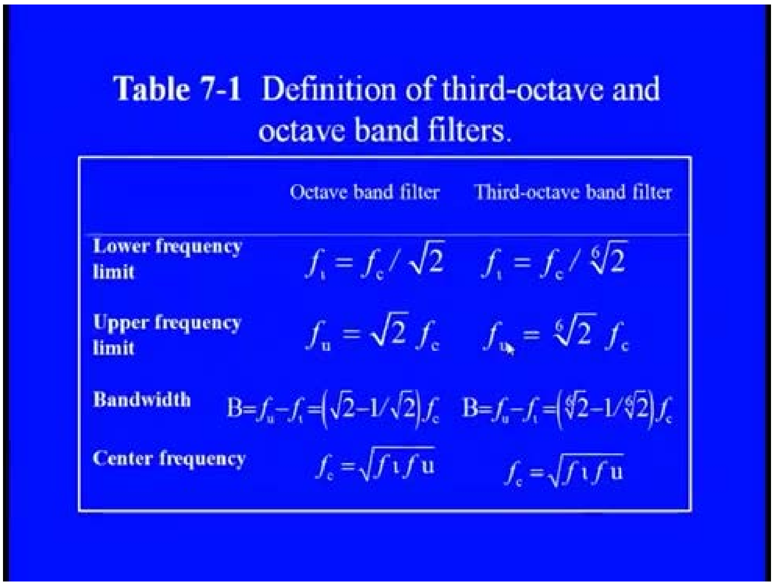

So, if you look at this, these basically on the definition of these third octave and the octave band filter, now we are absolutely say in octave band filter, my lowest frequency is absolutely that the f l is nothing but equals to the 1 by square root to 2 of my central frequency. And when I am now categorising this frequency band with the three this part, so certainly my lower band under the third octave band filter, f l is nothing but equals to 1 of square root of 1 by 2 square root of to the power 1 cube, means 1 by 2 to the power we can say the cubic features are there into f c.

So, f c divided by square root of 2 to the power 6, and similarly, you see if I am going for the higher frequency, again I need to go right from the central frequency towards that. So, again this is nothing but equals to the f u is square is square root of 2 into this, because now I need to go on the upper side, so it is into central frequency and similarly, if I am going towards the third octave band filter with this upper limit. So, my upper limit exciting frequencies are nothing but equals to the f c central frequency into square root of 2 to the power 6, means 1 by 6 I can say.

And then based on this upper and lower limit in both octave band and third octave band, we can calculate the bandwidth, so the bandwidth in the octave band filter is nothing but equals to f u minus f l as we know that. So, we can say it is nothing but equals to as we discussed already square root of 2 minus 1 by square root of 2 into f c. And similarly, you can calculate the bandwidth there in the third octave band filter, so again you see it is nothing but equals to f u minus f l which is nothing but equals to the square root of to the power 6 minus 1 by square root of 2 to the power 6 into f c.

And even we can calculate the central frequency in both the cases, so the central frequency in both the cases is nothing but equals to f c is equals to square root of f l f u means, this lower and upper frequency into square root or in for third octave band also, it is nothing but equals to square root of f l and f u. So, this is what you see the clear definition, where the first once we defined the third octave band, and once we define the octave band we can make a clear relation. Because, in the octave band, now we can separate the three different segments and again same every segment is showing the octave band features only in the three forms.

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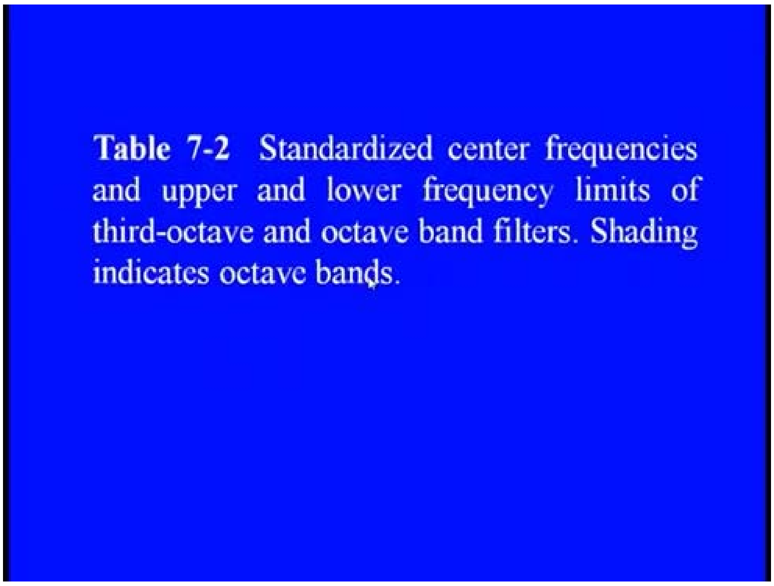

So, now you see here we can standardise the central frequencies upper and lower frequency limit, for third octave band and octave band filters with these the octave band. So, these shading features are there, so if I am just talking about the band number say. I have the band number 1, I can see that the central frequency can be of 1.25, if this third octave band is almost 1.12 to 1.24 and then here when we are increasing the band number say 2, 3, 4 or whatever you see. Say if we are at the band number 3, our central frequency is 2 and the third octave band means you see, f l minus f u is 1.78 to 2.22 and we can get the octave band filter for this is 1.41 to 2.82.

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Band no.	Center Frequency y f_c[Hz]	3rd- Octave Band filter $f_l - f_u$ [Hz]	Octave Band filtert $f_l - f_u$ [Hz]	Band no.	Center Frequency y f_c[Hz]	3rd- Octave Band filter $f_l - f_u$ [Hz]	Octave Band filtert $f_l - f_u$ [Hz]
1	1.25	1.12-1.41		23	200	178-224	
2	1.6	1.41-1.78		24	250	224-282	178-355
3	2	1.78-2.24	1.41-2.82	25	315	282-355	
4	2.5	2.24-2.82		26	400	355-447	
5	3.15	2.82-3.55		27	500	447-562	355-708

That means, we can straightaway start from the minimum for these right from band 1, we can say straightaway, we can say from band 2 to band 4 for this the third octave band filter is being, it is being designed. And the frequency for this is 1.41 as the lower and 2.82 as the upper frequency, so similarly you can go for 5 to the 5 or even 5, 6, 7 for this and then for 8, 9, 10. And then we can go for the for this 11, 12, 13, so for these part specially we can all for three parts, we can go and define that what could be the central frequency, what is the we can say the octave band filter part especially, where the third octave band is there and octave band filters are there. So, for individual part means for individual band number, the octave band filters and third octave band filters are to be designed and in these tables particular.

We can go up to the band numbers almost up to 43, and there for even when we are talking about say 41, 42, 43 for 41 we can say the central frequencies are 12500, and then we can simply define the third octave band. Say for first 41 band number it is 11200 to 14100, for 42 it is 14100 to 17800 and for 43 it is 17800 to 22400, and when we are talking about the octave band filter, in which you see a lower and upper limits are there for band number 41 to 43, then we can say that it is for 11200 to 22400.

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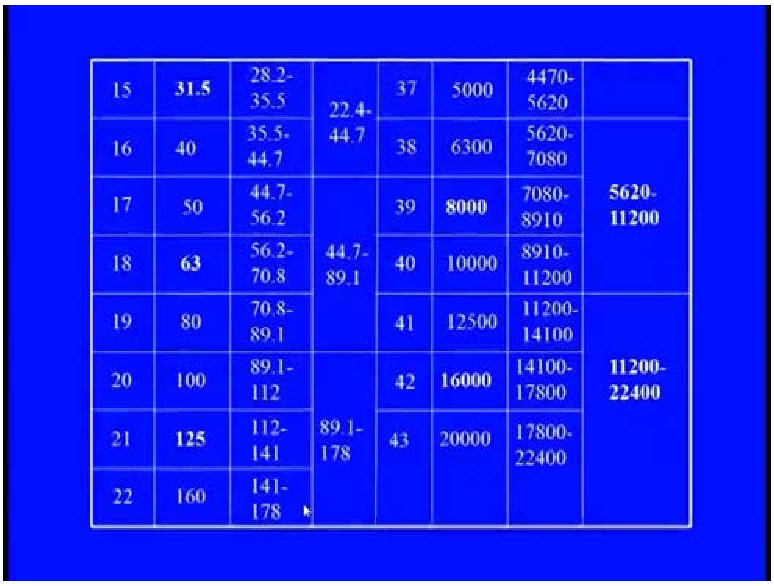

15	**31.5**	28.2-35.5	22.4-44.7	37	5000	4470-5620	
16	40	35.5-44.7		38	6300	5620-7080	
17	50	44.7-56.2		39	**8000**	7080-8910	**5620-11200**
18	**63**	56.2-70.8	44.7-89.1	40	10000	8910-11200	
19	80	70.8-89.1		41	12500	11200-14100	
20	100	89.1-112		42	**16000**	14100-17800	**11200-22400**
21	**125**	112-141	89.1-178	43	20000	17800-22400	
22	160	141-178					

So, this is almost we can say, once you define these central frequencies in between you see here the both the bandwidth can decide and then accordingly the filter can be designed for many number of the band numbers. So, for 43 we have shown in here in this that, how we can design those features together, now in the another section, we are now going to discuss about the addition of the frequency components.

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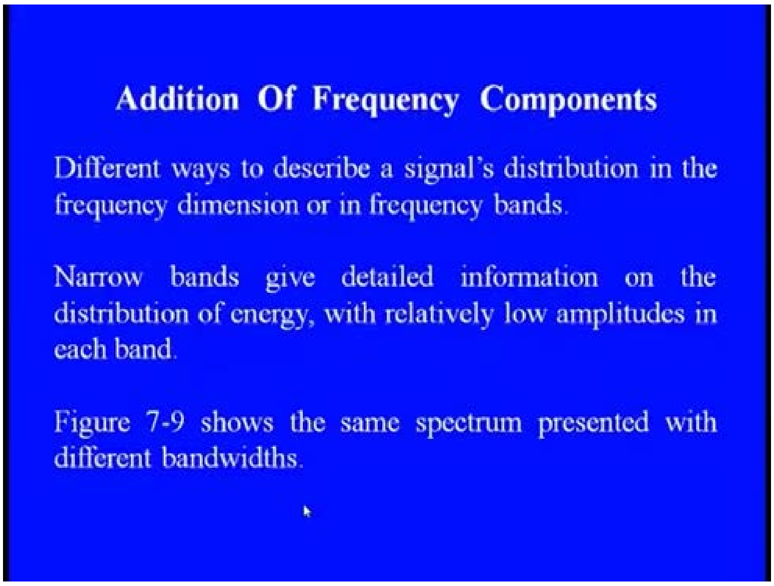

So, there are you see various ways through which we can describe the signal distribution in frequency dimension, or we can say in the frequency bands. And when we are talking about the narrow bands, they are simply giving the detailed information on the distribution of energy and in that with the relatively low amplitude in each band. So, the narrow bands are always just discretized those entire informations, within that how the distributions are there at those the lower amplitudes.

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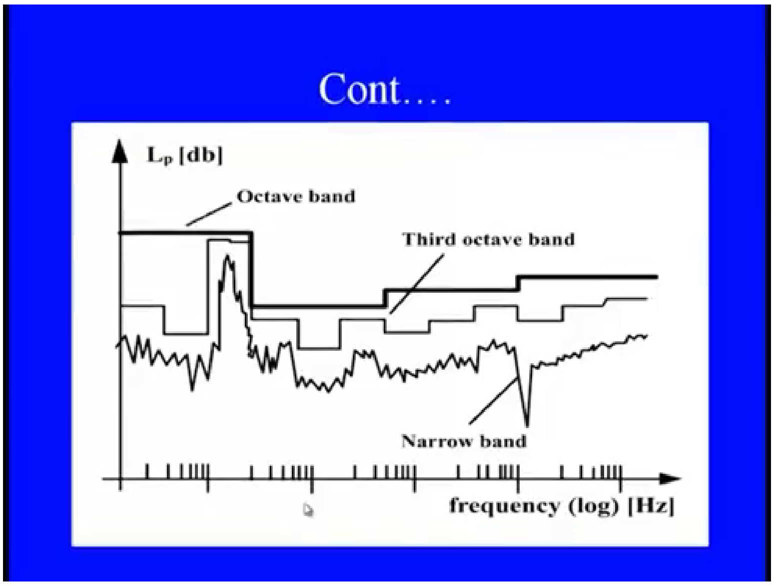

So, you see if we just want to show this part, then this is what it is you see here, this is the actual waveform which is being there, and if we are talking about the narrow bands this is what the narrow bands are there. And they are absolutely showing that at the lower amplitude, what is the this energy distributions are there, but the key feature if we are using the octave band. So, for the octave band that, this is being starting from this phase and then you see this is for this way, so when we have these say variation like that, the octave band starting from this to this.

And again when it is going dip and the variation is there, the new octave band in the second form is starting from here to here. So, this is the second form and then the next octave band because again there is a drastic change, so here we are again putting the octave band and then up to this before the narrow band this octave band is there. And then all the variations are there, so this is my octave band for the fourth octave band, and when we are putting the third octave band in individual the octave band.

So, we have these octave bands where the variation, then again an octave band and then octave band, so now this entire octave band is to be divided into three equal segments where the featured are being taken in such a way that, we can classify the bandwidth for you know like whatever the variations are there. So, for this octave band also, there are third octave band in that there are three components this and this and this one, so they are simply showing that how the variations are there.

So, as we discussed with the octave band and with the third octave band, and with the narrow band we can simply capture that what is the amount of , which is being there during excitation. And how much energy which we just want to pass, and we just want to see that the frequency band upper and lower frequency band, how they are being varied with these kinds of signals.

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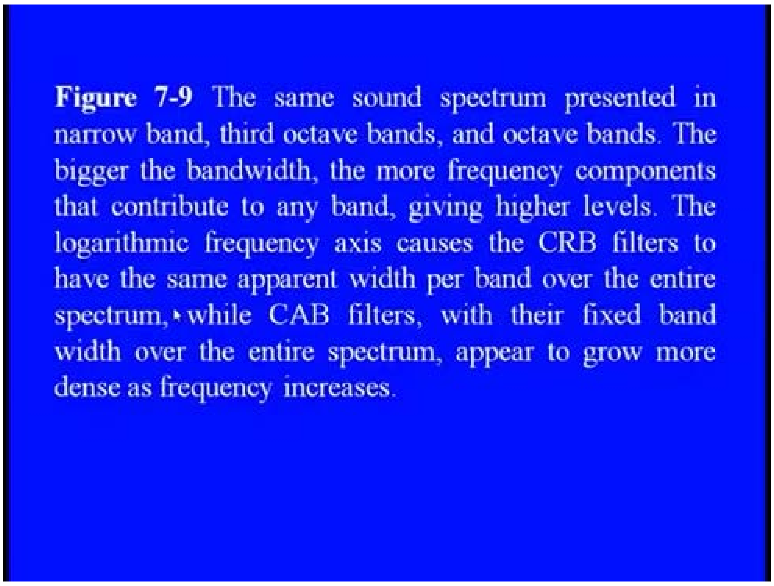

So, now in this particular the same sound spectrum which I was presented there with the narrow band, third octave band and octave band, and we know that the bigger bandwidth more frequency components can be contributed to any band, and they are always giving the higher levels. So, this logarithmic feature of the frequency axis can simply causes this constant relative bandwidth filters to have the same apparent width per band, whatever over the entire spectrum.

And when we are considering the constant amplitude bandwidth with their fixed band along with the axis, and when the fixed bandwidth are there over the entire spectrum, they are appearing to grow more dense as the frequency being increases. So, that is why when we just want to see the specific or the discrete frequency features, certainly we are always going with the CRB, the constant relative bandwidth. And when we know that there are not many number of frequency peaks are there or the spectrums are there in the entire vibration signature, then certainly we are going with the this CAB type of filters.

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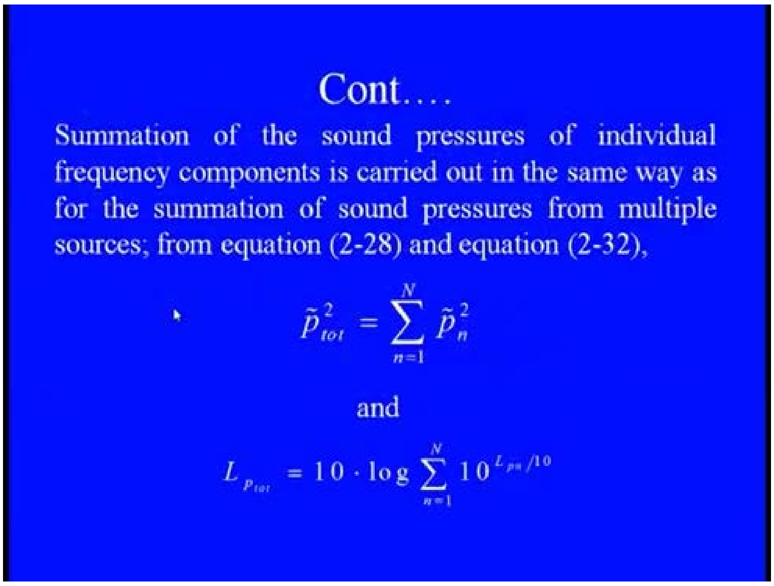

So, we can say that when we just want to calculate those things, we can simply use that what exactly the total pressure levels are there in this. So, we can say that total the sound pressure in that, is the summation of all these the p n square right from n equals to 1 to, this means we can say simply calculate with the individual frequency excitations we can sum up and we can calculate the total in our frequency band. So, we can say that as we already discussed the sound power is nothing but equals to 10 log like summation of all these power on the basis of 10, so it is 10 to the power L p n divided by 10.

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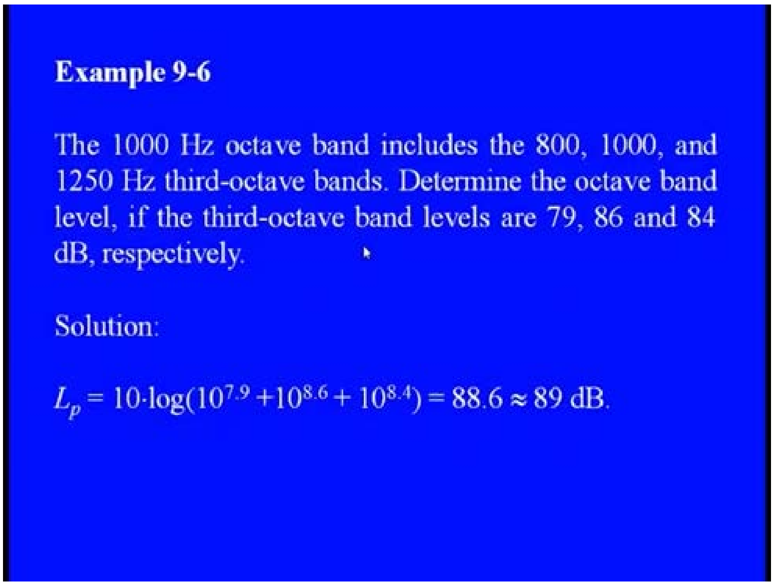

Because, now when we are talking about, the numerical problem this one say when we have this 1000 Hertz octave band, which includes you see 800, 1000 and 1250 third octave bands. So, within this octave band now 100 Hertz, now we have all three 100, 1000 and 1250 third octave band, we need to find out the octave band level, if the third octave band levels are 79, 86 and 84 d B respectively. So, now you see our sound level power which are simply we want to calculate this the total band level, so this is nothing but equals to L p equals to 10 log.

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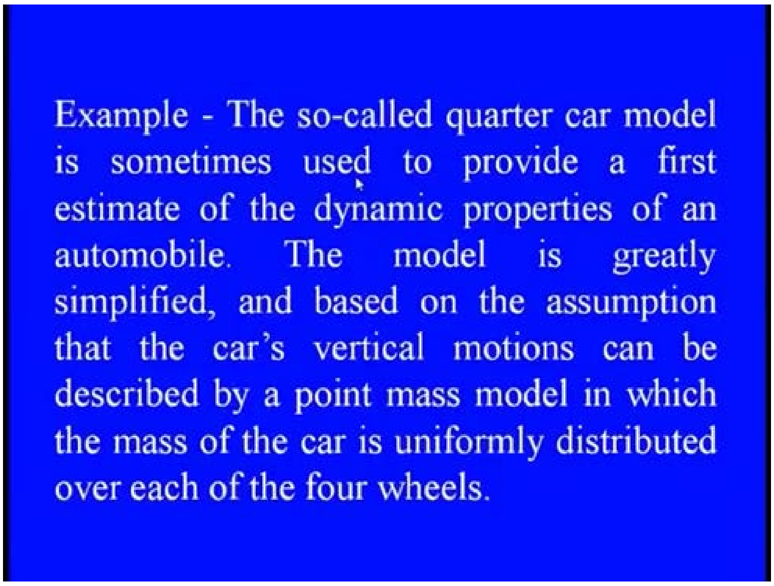

Now, 10 to the power now, we have this 79, 86 and 84 already third octave bands are there, so we can say 7.9 plus 10 to the power 8.6 plus 10 to the power 8.4, and when we are calculating this is almost nearly equals to 89 d B. Now, we are going to take the last example where we have a car model, say a quarter car model.

And we just want to find out the dynamic properties, using the vibration features and the model is greatly simplified based on the assumption that the car has the vertical motion, that can be described using the point mass model, in which the mass of the car is uniformly distributed on each of the 4 wheel. So, we are simply assuming that through this centre of mass, the car can be divided into four equal segment, and every segment is being supported with the tire part, because the 4 wheels are there, so the entire wheel is being supporting, so that is why we generally we are saying that this is the quarter car model for this.

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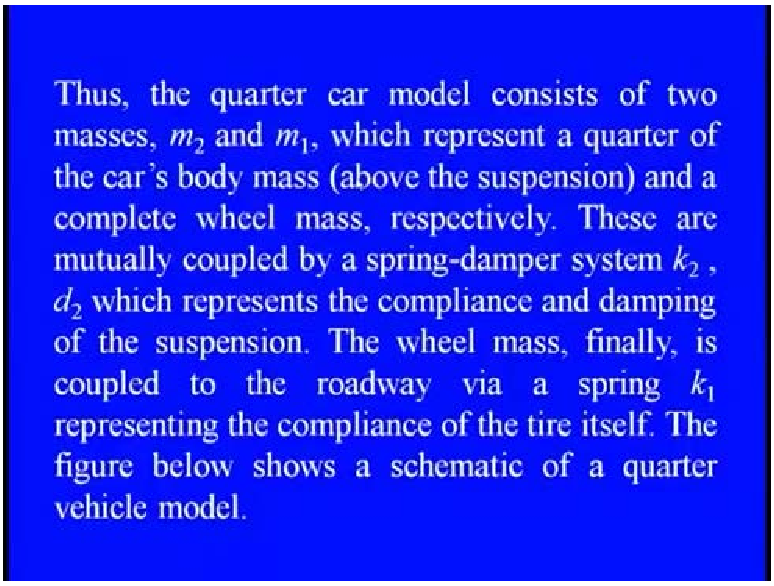

And the quarter car model consist of two masses m 1 and m 2, so we are saying that quarter of the car body mass above, which the suspension is there that is my m 2, and the complete wheel mass is my m 1. And correspondingly we can say that the mutually coupled spring damper system is there, so we can say k 2 and this d 2 they are simply showing the compliance, we can say the this spring and the damping features, while you can say when the wheel mass which is finally, coupled with the road way by the spring k 1 is being compliance with the tire itself.

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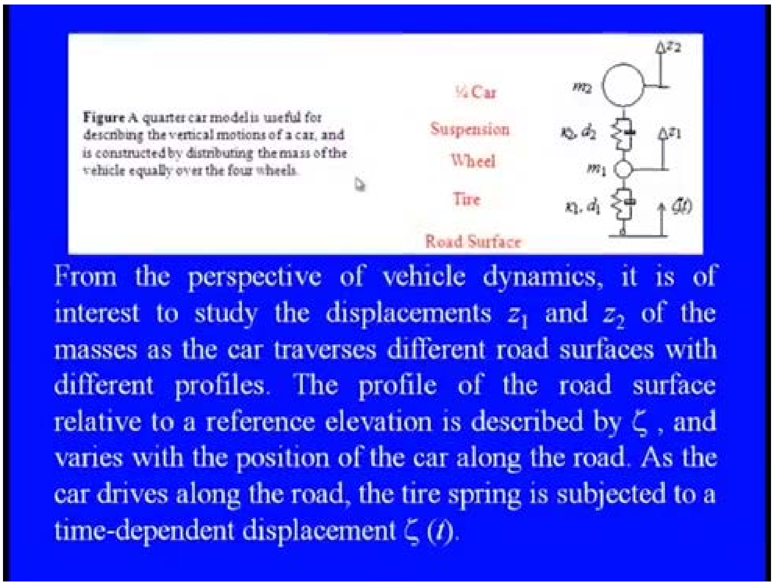

So, we can say the quarter car model which can be clearly shows here that we have you see the m 2, which is being here, so as we have already discussed that this the complete wheel mass is my m 1, and m 2 is the car body mass. So, this is what you see the car body mass is there, 1 4th of the car which is the car body mass, then it is being suspended and during this suspension we have the both k 2 and d B 2 is are there, which is being associated with the car model, the car body mass and with this wheel mass.

So, this is what my wheel mass m 1 is there and this is what wheel, and below the wheel we know that with the surface road surface contact, we have the damping and these this spring properties with the tire deformation. So, this tire is now providing the stiffness feature and the damping feature, just the k 1 and d v 1, so from the prospective vehicle dynamics it is of the great interest to study the displacement z 1 and z 2. You can see that we have a clear z 1 and z 2 when the car is travelling there is clear excitation of the wheel and the body suspension with this.

And the profile of the road surface relative to this elevation, now the profile, when we are just talking about the profile, we do not have a clear the smoother road surface. So, that means, we have a time dependent displacement from the ground and this is what the zeta of t.

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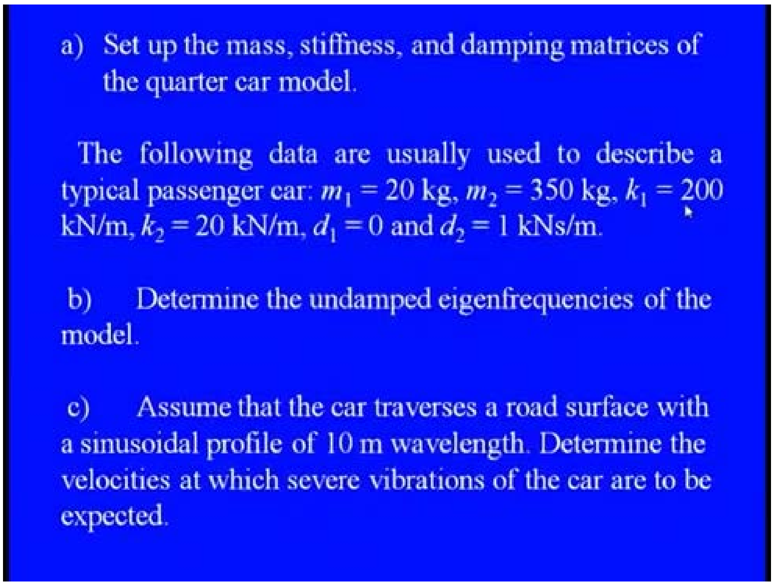

So, with this now our interest is to now find out the mass stiffness and the damping matrices of this car models, just to see that how formation of these matrices are. And now the data which we can straightaway use for the typical passenger car, say m 1 is our 20 kilo gram m 2, which is of the wheel is just 20 kilogram the entire body mass the vehicle means 1 4th of this is 350, not the entire means 1 4th of that 350 kilogram. The k 1 the stiffness associated with this the tire one is 200 kilo Newton per meter, and k 2 which is associated with this body to car suspension, they are of very less 20 kilo Newton per metre.

The d 1 is not there, because damping is not being acted as along with the tire, so we are assuming that there is only the restoring forces are being there, when they are being excited from ground to tire itself. And then d 2 is given as 1 kilo Newton second per meter, so this is one we need to now determine the undamped eigen frequency of the model. And also if you are assuming that the car travel on the road surface with the sinusoidal profile now, and that sinusoidal profile having the wavelength of 10 meter, then we need to find out the velocities at which the severe vibration of the car are to be expected. So, this is something a typical vibration problem and we need to formulate using this part, so we can say that, how we can put those things.

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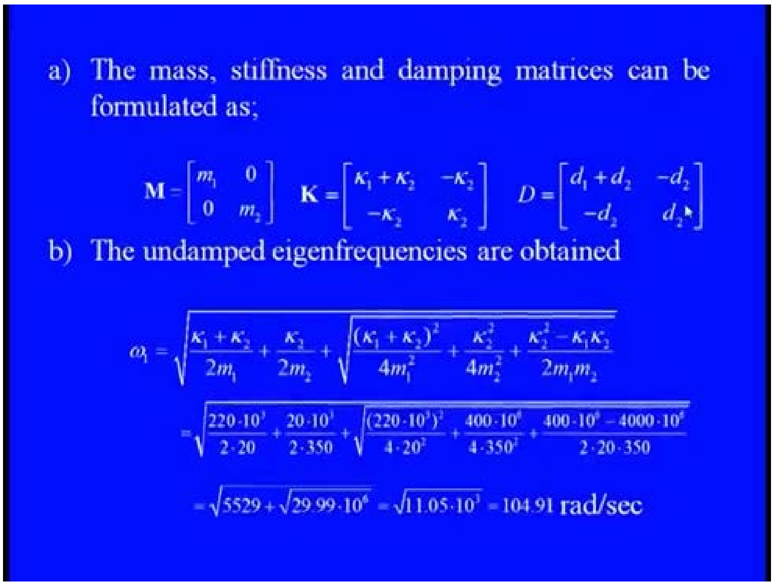

So, first thing the mass stiffness damping matrices that can be straightaway get with the usual term, so m this is along the diagonal they are being symmetric, so m 100 m 2, k is k 1 plus k 2 minus k 2 k 2 and this one, and this is what it is. Now, if you want to calculate undamped frequency, eigen frequencies we can calculate you see the k 1 plus k 2 by 2 m 1 plus k 2 by 2 m 2. And then the square root of both the square plus you see here like k 2 square and minus k 1 k 2 over 2 m 1 m 2. So, when we are keeping those values, we can get straightaway the first natural frequency of this is 104.91 radian per second.

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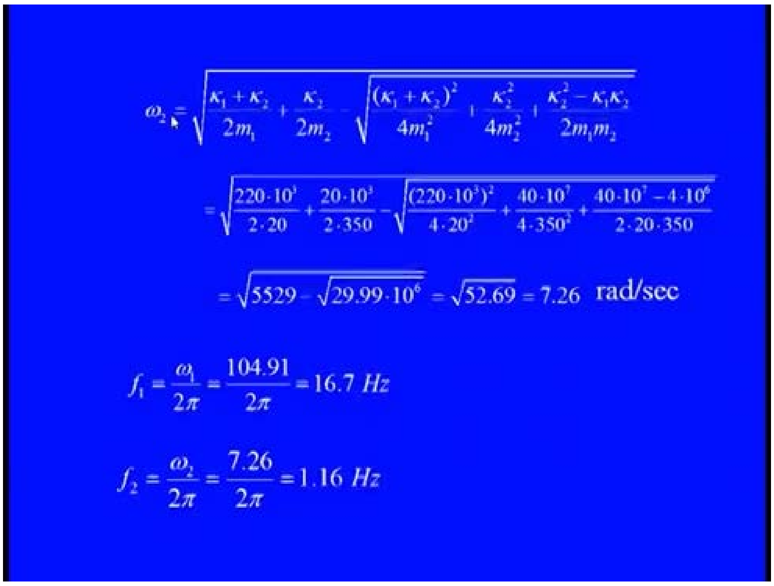

And similarly, you see here we can get the omega 2, because it is a 2 degree of freedom system, so again it is being there along with the k 2 and k 1. So, we can simply put those values, because k 1, k 2 are given to us and m 1, m 2 is there with us, so we can calculate the omega 2 as 7.26 radian per second. So, this is what the excitation frequencies are there at the for the wheel and for the body mass. So, correspondingly we can say that f 1 will be excited at 16.7 Hertz and f 2, means the body the car body mass at the 1 4th will be excited at 1.16 Hertz.

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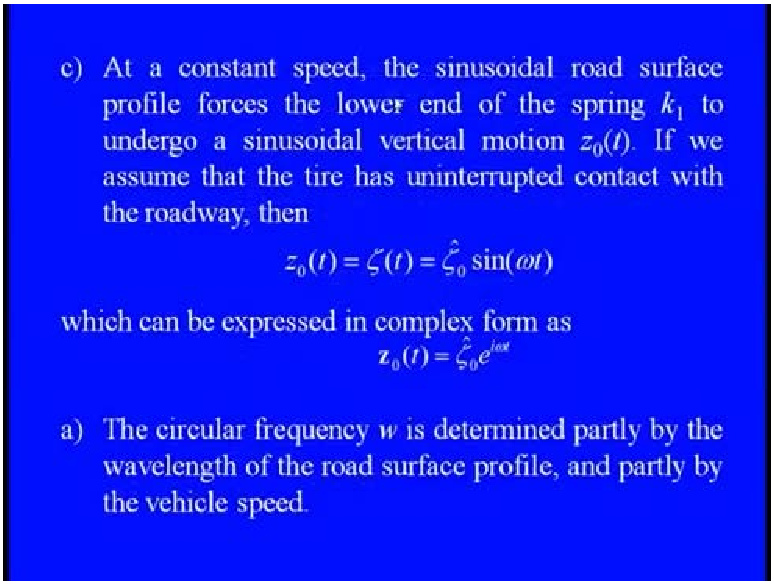

So, that is what one formation how do we calculate this, but the main feature is coming when the car is moving at the constant speed, and we have a sinusoidal road surface profile at the lower end. And they are being coming to the our tire part for which the spring k 1 is going under the sinusoidal vertical motion say z 0, so now we need to assume that z 0 which is nothing but the epsilon t which is given the question is nothing but the epsilon 0 sin omega t. Because, all these features are being there, all big feature of the road is being coming to the vehicle, and even we can explain this in the complex form generally we are using the z 0, is the epsilon 0 e to the power i omega t. So, we can calculate the circular frequency based on what the wavelength is giving of the road surface, through which this sinusoidal feature of excitation is coming on the vehicle.

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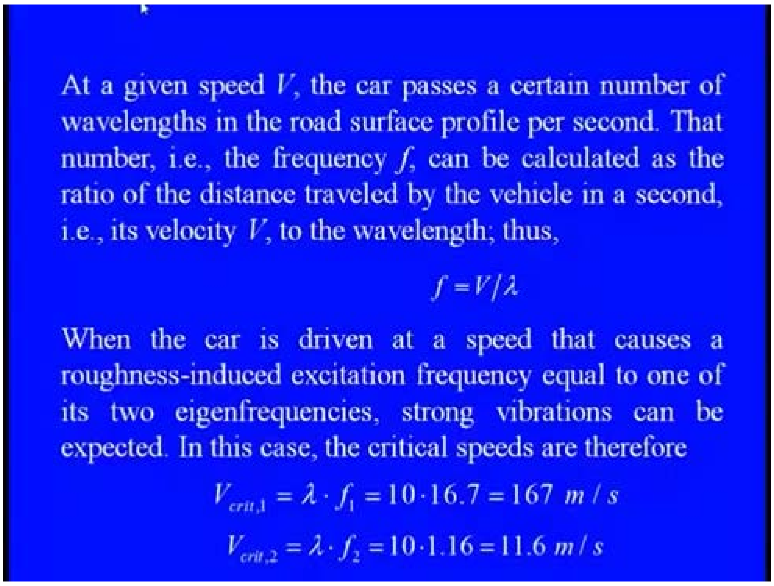

So, when we are trying to do we know that say the given speed V is there for car, which is passing through the certain number of wavelength on the road surface profile per second. So, that number that means, the frequency f can be calculated from the ratio of the distance travelled by the vehicle in per second that means, we can say that the frequency is nothing but equals to the velocity V divided by the wavelength lambda. So, when the car is driven to the speed that, causes the roughness induced the exciting frequency must be equal to one of the two eigen frequencies.

So, we can say that the strong vibrations can be expected at these two exciting frequencies are there, so we can calculate the critical speed V critical is nothing but equals to lambda into f 1 plus and lambda into f 2. The lambda is given the wavelength is given as 10 and the exciting frequency first was coming at 16.7, so the first is 167 meter per second and second the critical where the V critical at two part. So, lambda into this f 2, lambda is 10 and f 2 is 1.16 exciting frequency, so we have 11.6 meter per second, so this is what the speed, the critical speeds are there where we can expect the huge we can say vibrations at that.

So, this is the last numerical of this course vibration control, in which we discussed almost you see all types of signatures signature analysis, using the filters, data acquisition systems, fast Fourier transformation, time domain frequency analysis.

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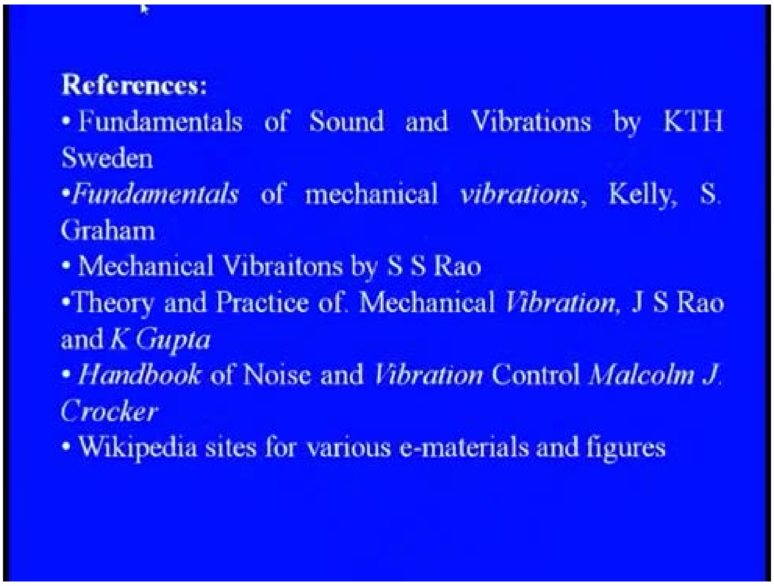

And for developing the entire course, we have used various references and I have already quoted in many of you see the citations wherever it was there, but also especially we are so thankful to from the book which we have adopted, in our one of the MOU with the KTH Sweden, and IIT Roorkee for developing the courses. And just flourishing this awareness about the sound and vibration to this Indian conditions, so that MOU was under the Euro, Asia project. So, that book was provided by them by professor Metsberg and team, so we are so thankful and we are simply citing these fundamentals of sound and vibration by KTH Sweden book, that is one of the basic book.

We also used the main core books of the basic vibrations like, the fundamentals of mechanical vibration by Kelly, fundamentals of we can say the vibrations by meirovitch, and then the mechanical vibrations by S. S. Rao theory and practices of mechanical vibrations by J. S. Rao and professor Gupta. Handbook of noise and vibration control especially by Malcolm Crocker, and also the active vibration control by we can say, by this professor Nader there in the Clemson university.

And also professor Inman, Denial Inman the vibration control we have also refereed those various there, and we have cited there, and also lastly, but not least the Wikipedia sites also providing some kind of the basic informative features, and we have taken the e-materials from that itself. So, thank you very much from team, through which we generated those courses, and either from the web or the video part.Thank you very much.