Vibration Control
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Module - 1
Review of Basics of Mechanical Vibrations
Lecture - 4
Multi Degree of Freedom Systems

This is Dr. S. P. Harsha from Mechanical and Industrial Department IIT, Roorkee, in previous lecture we discussed about the 2 degrees of freedom systems, that how you know like the damping or the masses or the stiffness are being varied, when we are taking the 2 degrees in that. We discussed about that how the Eigen frequencies can be calculated when you have the 2 degrees, and how you can make you see the coupled equations in that.

And we know that when the system is in the 2 degree of freedom systems, there are two natural frequencies, which are simply a reflection of our characteristic roots of the equations. And then we have the corresponding mode shapes for that, and these mode shapes are absolutely reflecting a individual natural frequencies for that.

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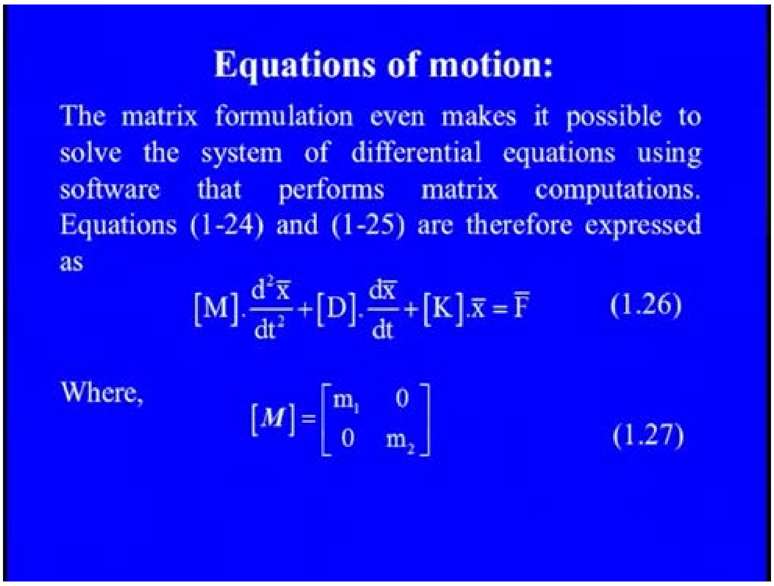

And then also we discussed about when you have a damping involved in that, then certainly we can simply like calculate, the undamped natural frequency and the damped natural frequencies for that. Lately we solved some numerical problems for along you know like with these kind of concepts, in the today's lecture again we will, like further carry out the similar kind of features that if we have the 2 degrees and more than 2 degrees of freedom system.

Generally we are referring more than 2 degrees of freedom system as the multi degree of freedom system, then how the things are being varied along with the masses or the stiffness or the damping for the high degrees. Like the natural frequency or the eigen vectors that means, the mode shapes, so in that same you see we are taking the same previous numerical problem in which, the two masses were simply under the oscillatory motion, which are being constrained by this spring motion.

So, we know that when you have more than 2 degrees of freedom system, there are you see here, like the matrix formulations for making the equations of motion. And these matrices are simply reflecting that, like what kind of arrangements are there, according to the system arrangements the elements are being coming into the matrix formulations. So, the matrix formulations even makes it possible to solve the system differential equations using any kind of you see the solving techniques or the software particular.

We need to first go to the basic equations like, the inertia force, the damping force or the restoring forces and when it is being excited by external forces, we can simply go to the Newton's law. So, you can see that the equation 1.26, which is simply the reflection of the same equation, but only in the state space form or we can say only it is in the formulation of the matrix. You can see that the mass matrix is there, which is nothing but you see the symmetric matrix along the diagonal and it has to be.

If there are you see some problems in the formulation, it seems that the whatever like the degrees of freedom or whatever like the formulations are not being captured correctly. So, you can see 1.27 is nothing but showing the mass matrices along and it is you see, like the symmetric along the diagonal M 1 and M 2. And even if we are going for a high degrees it should be in the same manner only, even if we just see the damping or the stiffness matrices, they are also in the similar manner only. But, again if you just see the diagram that what exactly the diagram was, so that we can straightaway understand what the feasibility of the system arrangement is...

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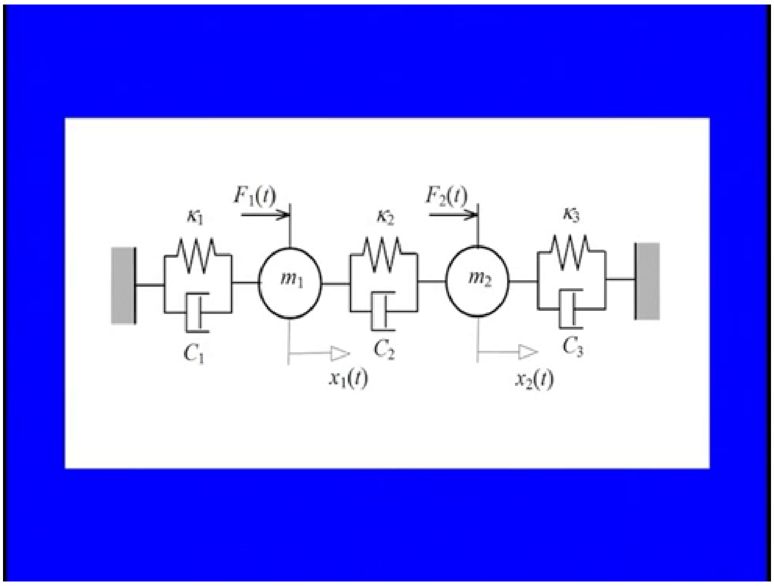

You can see that we have the two masses on which the two simple harmonic motion is being there, along with the displacement x 1 and x 2, and the forces are being acted on that F 1 (t) and F 2 (t). And with this you see, we have the stiffnesses k 1, k 2, k 3, we have the damping C 1, C 2, C 3, and then all these you see the motions are being occurred accordingly. So, now you see, if we want to just formulate this we know that the damping matrices is exactly in the same way just we derived stiffness matrices.

And they have to be positive along the diagonal only, if any negative factor is coming along the diagonal in the damping or the stiffness matrices that means, there is some problem in calculation. Or there are some problems in the formulation of these features, either in the free body diagrams or in capturing those forces with these elements.

So, you can see 1.28 equation is just reflecting that how the damping is being varied, like you see the first element it is you see here, as you can see in the diagram the M 1 is in between the C 1 and C 2, so certainly the damping is straightaway playing a key role in that. So, in the first it is C 1 plus C 2 and then on the second side it is minus C 2 and then you see here, our mass M 2 is also varied in between the C 2 and C 3, so certainly we have on one side it is minus C 2 and on another side it is C 2 plus C 3. And the same manner is coming in this stiffness matrices as well, because the arrangement of your our damper and the spring stiffness are in the similar manner in our previous diagram.

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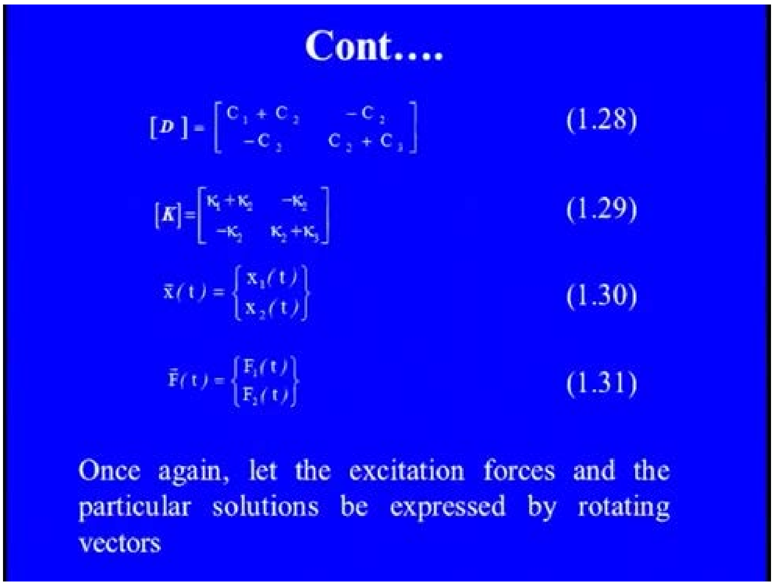

Then there are two displacements of the mass one and mass two, so certainly we have the mode shapes after being displaced by the F 1 and F 2 the forces. So, what we have we have x of t which is nothing but a reflection of our displacement outcome is x 1 (t) and x 2 (t) of two manner. And the same forces which we are applying the input forces are F 1 and F 2 (t) in the matrix form. So, now you see here, since we know the excitation forces are simply producing your homogeneous solution using the particular integral, or we can say this is something which like we want.

Because, we know that the complementary function is known homogeneous, simply giving you the, this transient feature our main part here is to calculate the steady state feature, which is simply bounded by this forcing factor or we can say a particular solution.

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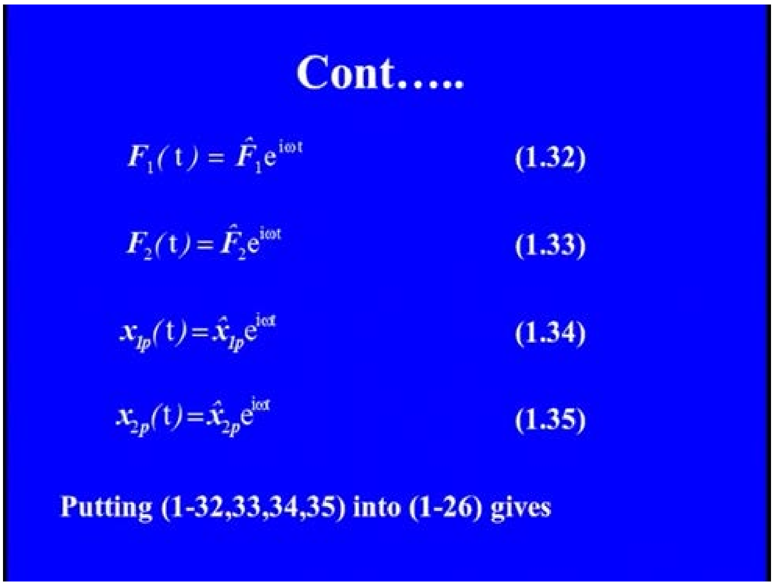

So, with that you see here, we can say that our force input F 1 and F 2 are simply showing the simple harmonic motion, so we have F 1 which is like, the time bounded the forces. So, it is f one e to the power i omega t, the force two which is acted on mass two is nothing but equals to F 2 into e to the power i omega t, so we have both the forces. And then we can expect the outcome in terms of the steady state formulation or we can say the particular integral, so x 1 p is nothing but equals to x 1 p, which is the amplitude into e to the power i omega t, which simply shows the simple harmonic, like motion in the outcome. Similarly, x 2 p which is a related to the displacement of mass m 2 is simply x 2 the maximum amplitude into e to the power i omega t, so we have both, we have you see the steady state solution with the x 1 p and x 2 p; and if we are combining all three, the input and output.

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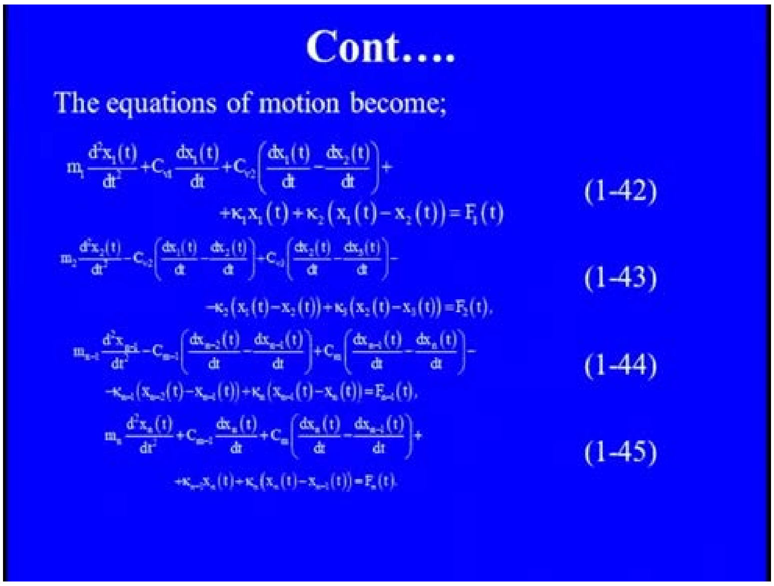

Then certainly we have simply end up with our main solution matrices minus omega square m plus x p, which is like which we want particularly like, so we have minus omega square m into this one, plus we have you see like i omega into damping matrices. And then you see we have this stiffness matrix and x part you see here and then it is equated to whatever the forces which is being excited, now our main theme here is to get the homogeneous solution.

So, far that we need to apply a force factor and we need to put a entire thing equals to 0 to get the eigen frequency, since you see in our equation we have the two main factor, one is the damping, one is the stiffness. And if you want the system performance in terms of the Eigen frequency, then first we need to keep the damping vector 0 to get the undamped natural frequency. And if the damping is there, certainly we have the damped natural frequencies, so first of all we need to keep this force equals to 0 to get, because you see the eigen frequencies are always coming for the free vibration conditions.

So, the force vector on the right hand side of the equation 1.36 makes an equals to 0 and then in order to calculate the undamped natural frequency, we need to keep the damping vector 0, so that we can calculate the undamped natural frequency. So, you see here, in general terms now, our equation 1.36 will be equal to minus omega square m x p plus k into x p equals to 0. And it will give you, this equation will give you the undamped natural frequency, the characteristic roots which is simply reflecting the spring and mass system.

Damping on other hand, simply brings the complex valued eigen frequencies that means, you see here if we have mass and spring system, whatever the roots which are coming which we are saying the characteristic roots or the eigen frequencies they are the real one. So, we can simply say that when the roots are real, they are simply showing the natural frequency and the roots are complexed values that means, they are showing the damped natural frequencies.

Again you see here both the frequencies being there, because our system has the spring and the damping together. So, with this particular feature now, if you are going towards first, the undamped natural frequency, certainly since the system is having 2 degrees, we have omega 1 and omega 2 as the two natural frequencies.

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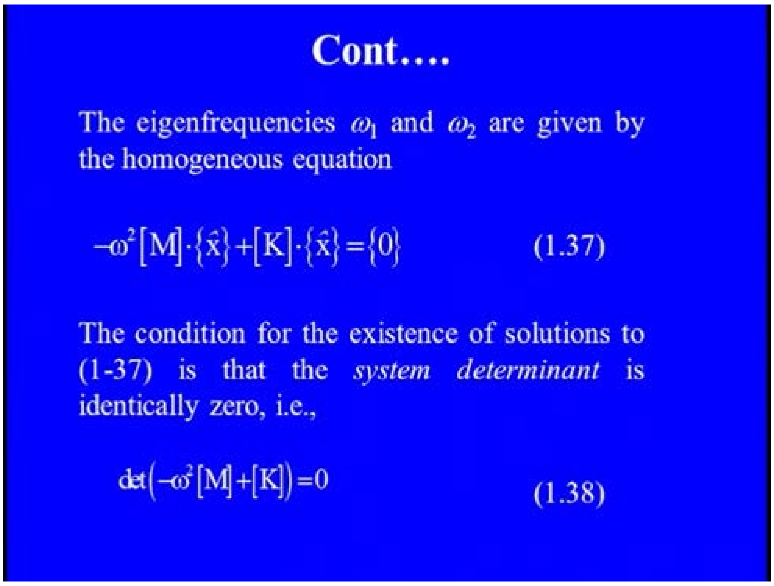

Then we can calculate you see here, minus omega square m x plus k x equals to 0 as we discussed, and to find out these things, as we already discussed that, we need to keep this matrix into the determined form and makes it equals to 0. Then we have determinant minus omega square m plus k equals to 0 and from that you see here, we can simply calculate the natural frequency. Or else you see here we can say the eigen frequency which are nothing but equals to the natural frequency square.

So, for a 2 degree of freedom system, certainly you see we have two omega 1, omega 2 as we discussed or even you see, if you are going for higher degrees. We can say the linear terminology for this, that if we have n degrees of freedom system certainly the n natural frequencies are there for that.

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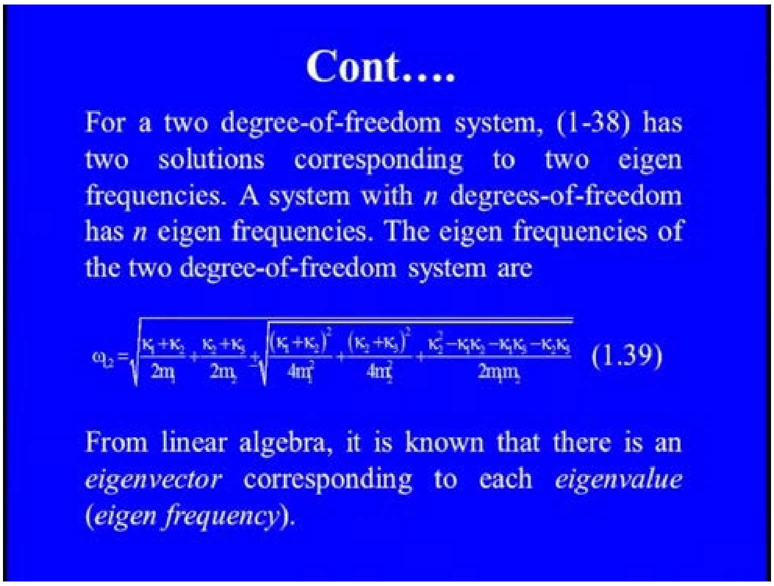

So, from this part you see here, the since it is a 2 degree of freedom system, so we have the omega 1, omega 2. And since it is are the motion of the mass m 1 and m 2, which we are saying x 1 and x 2, like is being restricted by the spring stiffness k 1, k 2, k 3 according to the arrangement in the previous diagram. We can see that omega 1, omega 2 is nothing but equals to the square root of k 1 plus k 2 by 2 m 1 k 2 plus k 3 by 2 m 2 and then you see here plus minus square root of, this is simply like an algebraic equation.

And from the algebraic equations we can get alpha 1, alpha 2 for that similar manner you see we can get this 1, so it is nothing but equals to this is square root of k 1 plus k 2 by 2 m 1 plus k 2 by k 3 our 2 m 2 plus minus square root of the whole square this one. So, it is k 2 square minus k 1 k 2 minus k 2 k 3 minus you see here, we have k 1 k 3 divided by 2 m 1 m 2. And again you see here, these Eigen frequencies are simply giving, like the real we can say the mode of this one, because this is undamped frequencies.

And the real roots are always you see like simply showing that, how the variation of the masses are in terms of like the relative positions means you see here, means if one mass or two masses are in phase or out of phased. And you see here, these things in which you see the in phased or out phased are there, we are saying these are the eigen vectors or the mode shapes of this one.

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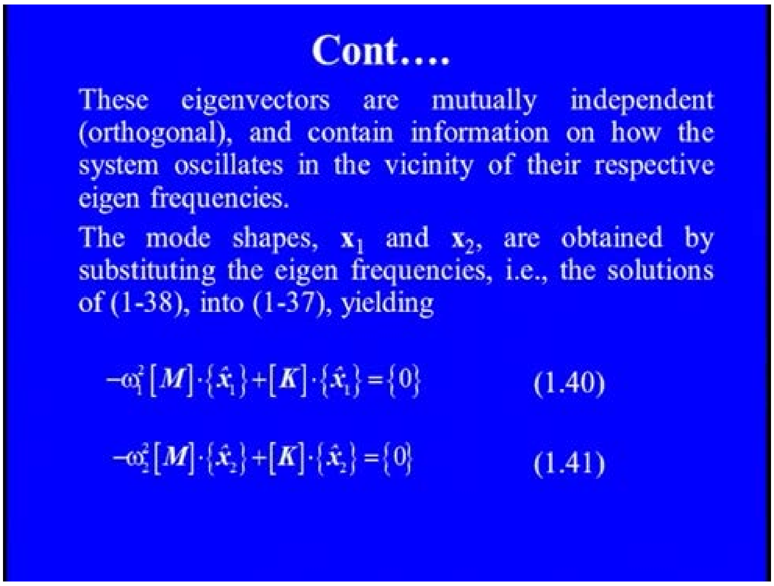

So, these Eigen vectors are mutually interdependent as I told you, the orthogonal feature is contained the information that how the system oscillates in the vicinity of their respective eigen frequencies. So, for that, either if we are talking about the omega 1 the first natural frequency or if we are talking about the natural frequency omega 2, we can simply find out the relative displacement of their masses. And the mode shapes x 1 and x 2 can be straightaway obtained by substituting these frequencies into the main equation, which simply corresponding say that you see in the equation 1.40 minus omega 1 square mass matrix plus k, this like the displacement matrix equals to 0.

So, these like the displacement for omega 1 we have x 1 and similarly, you see if we are going for omega 2, the next natural frequency or displacement vector is x 2, so minus omega square m x 2 plus k x 2 equals to 0. So, in these two equations you see, we know that there is a like the coupled equations, like we can simply get those values x 1 and x 2 in relation to these variation of the masses and natural frequencies. So, certainly we can calculate the mode shapes from these two, now we would we would like to apply this concept into the real application, so you see in the example this example which is there on your screen.

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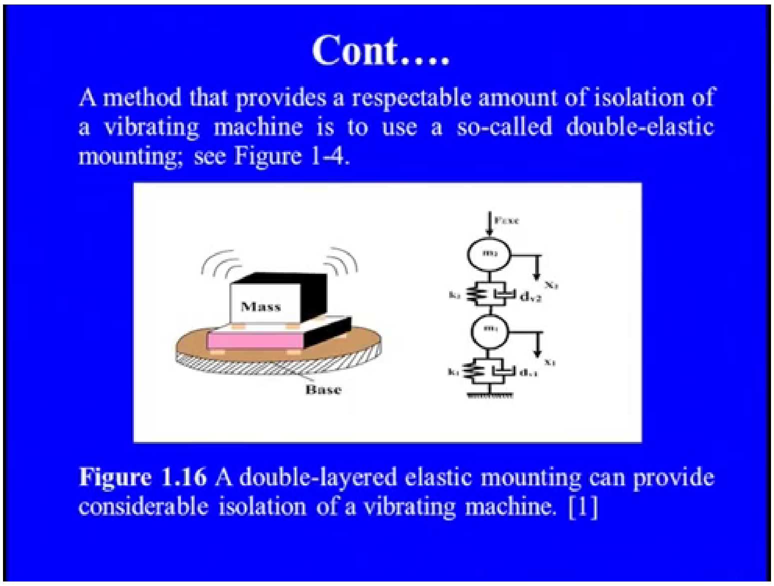

Simply a machine is mounted on the elastic foundation, this elastic foundation is not only providing the support, but the same time it is a kind of isolator to prevent the vibration transmission to the ground. So, you see here, if you look at that what we have, we have a mass which is being, like creating some kind of motion and through this motion you see here we have the oscillatory vibrations in that. And these vibrations are being, like just transmitted vertically downward, so that you see what we have, we have you see the base and with the base on n in between the mass.

We have some kind of you see, like the damper or the spring, so if you look at that the mass m is like we can say it is in the motion, so this mass is something we can say m 2 which is at the x 2. And similarly you see here the mass m 1 is our base feature you see look at that part this is the base, and this base is also, since it is coming under you know like this oscillatory mass, so it is also having some kind of displacement variation. So, x 1, so m 1 x 1 from the base and m 2 x 2 from the mass is having you see makes the equation or make the system in 2 degrees of freedom.

And on below of this mass or this base, we have like the damper and the spring, we can say this is simply a isolator just to prevent these things. So, with this formulation, we can say we have the double elastic mountings in between the mass and the base and then you see here these are simply providing both the stiffness and the damping together for that.

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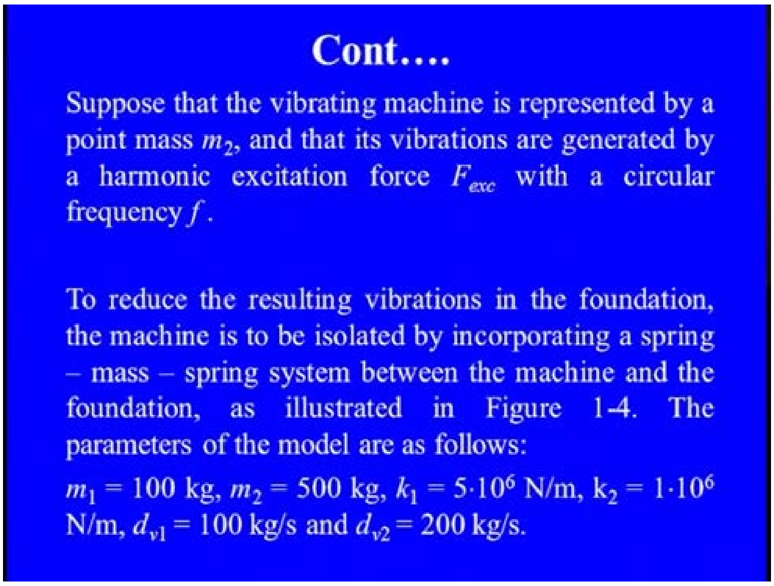

So, suppose that you see this machine, which we are saying that is representing by these two, so on top of that whatever the oscillatory mass is there it is m 2 and the base is m 1. And the force you see, whatever the force which is being excited or being applied this F excitation force is having the circular frequency of F. And our main theme is to reduce the amplitude vibration, which is being transmitted to the ground the this part, so we can incorporate the spring mass and the viscous arrangement in that.

So, now, certain parameters are there in this we have the mass m 1 and m 2, as 100 kilogram and 500 kilogram, the stiffness is also given to us 5 into 10 to the power 6 Newton per meter for first spring, and second is 1 into 10 to the power 6 Newton per meter. Similarly, we have the damping feature in this, like the d v 1 that is you see the damping in between you see, the base to the foundation that is you see here, 100 kilogram per second and d 2 that is you see in between the base and mass is 200 kilogram per second. So, we have a perfect isolator in between you see here mass to the ground, in the 2 degrees of freedom system.

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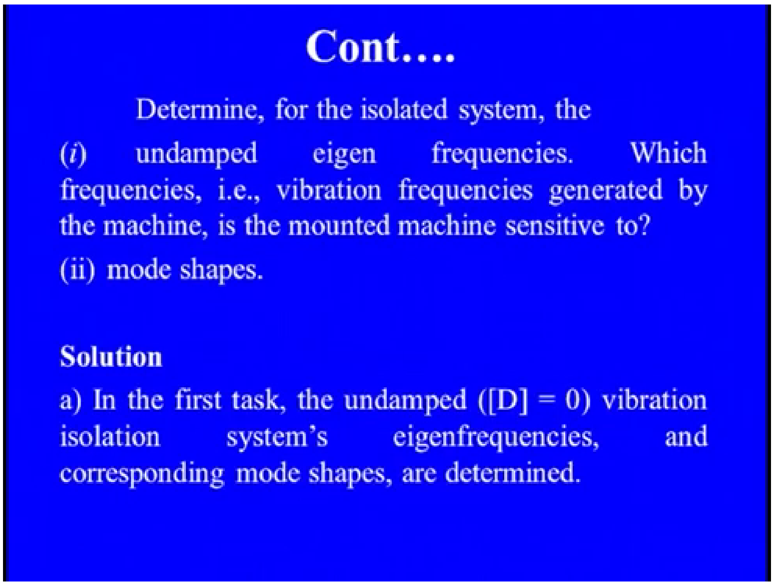

The main thing is to calculate the undamped, the natural frequency or the eigen frequency, in which you see here the vibration frequency that means, like just generated by the machine, which is being mounted on this system. And second what the mode shapes are, since we are interested in undamped one, so certainly would like to reduce the effect of these dampers. So, we need to keep mathematically the damping matrix equals to 0, so the system is undamped one and then you see here whatever the undamped systems are there only the effect of this is mass and the spring together.

So, we have you see whatever the roots, which are being coming out in the equation from this mass and the spring system is our undamped natural frequency reflection. So, you see here and then whatever the corresponding eigen vectors are there, they are our the mode shapes, so in this figure you see here we know that since the damping is there, but we just want to ignore the effect of the damping to calculate the undamped natural frequency. So, for that we already discussed that for undamped natural frequency, we could easily calculate omega 1 and omega 2 in the previous example, we discussed that it is nothing but equals to the spring stiffness variation and divided by the mass.

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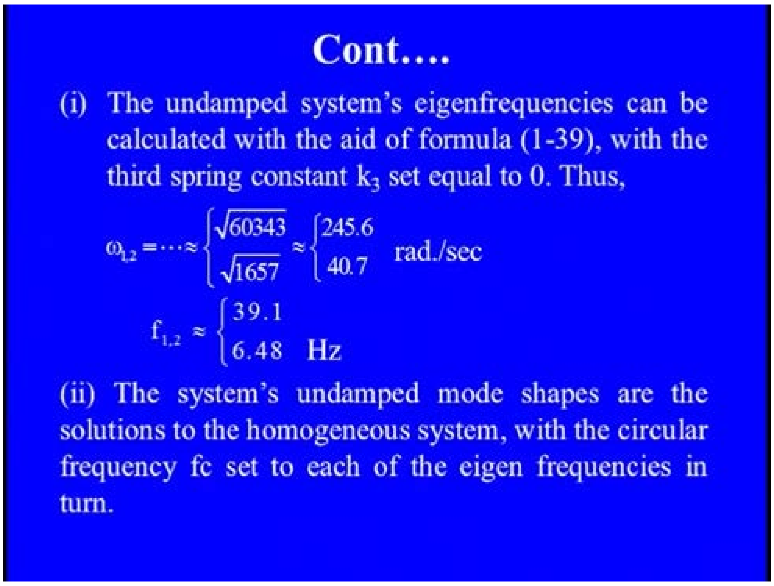

So, when we are doing these things you, we have omega 1 comma two is nothing but equals to square root of this 60343 and square root of 1657, or else you see it is almost nearly equals to 39.1 Hertz and 6.48 Hertz. That means, we have clear feature that the first natural frequency and the second natural frequencies are like that 6.48 first one and 39.1 is the second natural frequency. And the systems undamped mode shapes can also see like be easily calculate using the homogeneous system equations, with the circular frequency f c for each eigen frequencies.

That means, what do we have, we have you see the omega 1 and omega 2 and whatever the displacement which will be coming out from these say 6.48 Hertz and 39.1 Hertz are simply showing that how the mass displacements are there. And then you see we can easily figure out these things by putting this part into the main equation.

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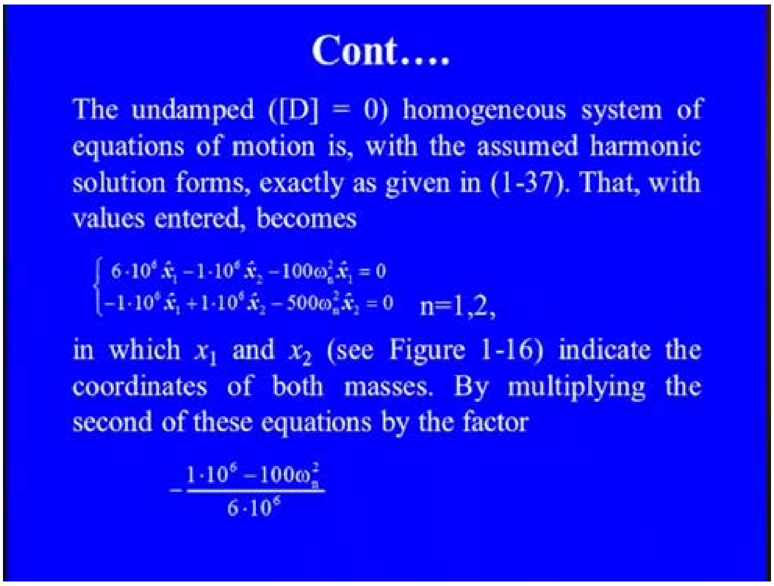

So, the first equation what we have, since the first natural frequency is 6.10, so we can keep that you say 6.10 x square, then you see whatever, like a this tends for 6 that is my stiffness you see here, into x 2 minus you see whatever the damping features are there; 100 omega square, like this x omega n square x 1 equals to 0. And similarly, we can keep this part as 1 into 10 is to power 6 into x 1 plus you see here whatever we are keeping the other terms, like we have this stiffness term at k 2, and this mass 500 and we are keeping and we are making equals to 0.

We can calculate the mode steps in terms of you see, the displacement x 1 and x 2 which simply indicates that, how the masses are being deviated. And by multiplying these things we can see that it is nothing but equals to minus 1 into 10 raise to power 6 minus 100 omega square omega n square divided by 6 into this. And when we are calculating this, we could easily figure out that how the masses are being displaced at first natural frequency 6.48 or at second natural frequency 39.1 Hertz.

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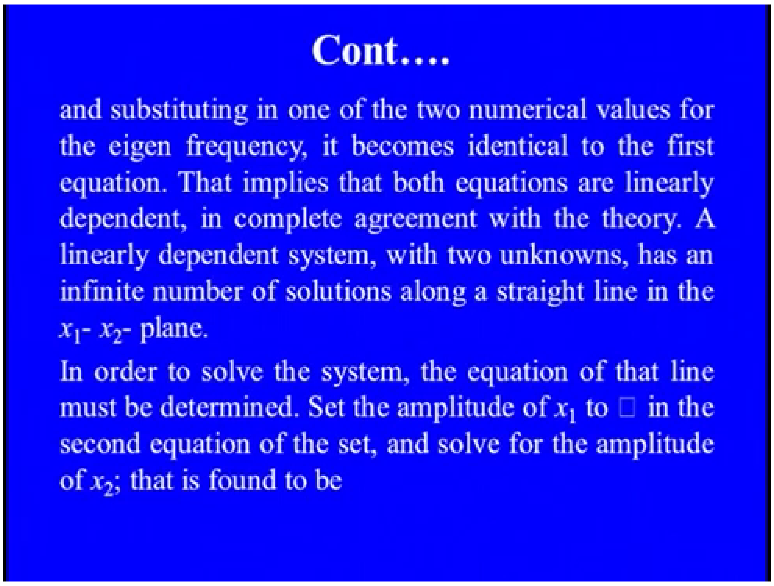

And substituting these numerical values in this we could easily figure out these eigen vectors, and it becomes identical to these things that, when we are evaluating those things it is a very symmetric process. And this means that you see both the equations are linearly depending, and in the complete arrangement they are simply giving the relative displacement in phase and out phase, in terms of the mode shapes. And the linear dependant system with two unknowns, has simply you see the infinite number of solutions along with that.

So, you see here in order to solve these equations, certainly we need to apply that what exactly you know like the numerical datas, which can be incorporated to get the final solution, in terms of x 1 and x 2. So, when we are trying to solve these things now, we can simply put that in terms for this x 1 amplitude we can keep alpha 1.

And in terms of x 2, we could simply find out that you see x 2 is nothing but equals to 1 into 10 raise to power 5 divided by you see here, 1 into 10 to raised to the power 6 minus 500, which was you see the mass m 2 omega n square alpha. And if this amplitude x 1 has the value of you see the omega n, then certainly we can say that the x 2 must be having the value, which simply 1 into 10 power 6 divided by this numerical digit. So, now we have you seen like the Eigen vector, which is simply corresponding the first natural frequency omega 1 in these terms, where the alpha is simply a constant.

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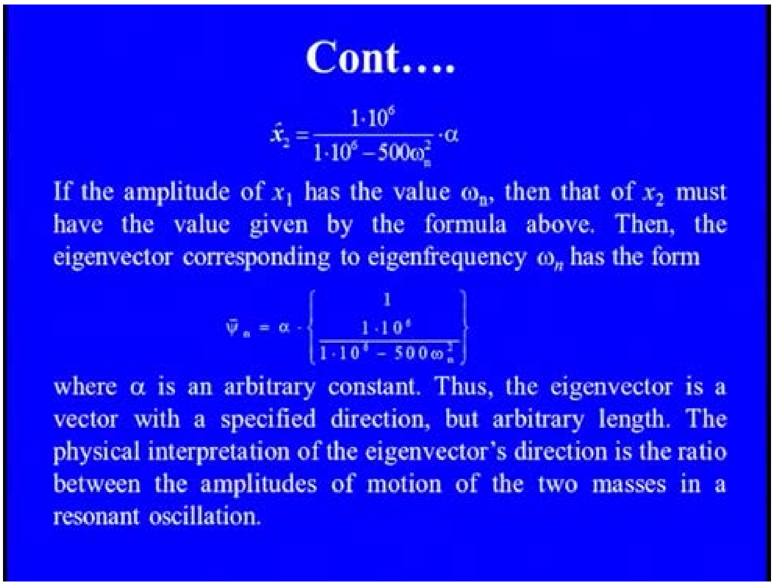

An Eigen vector is nothing but a vector which is specifically in a specified direction, like in phase or out phase. And we could easily figure out that what exactly the x 2 by x 1 is what exactly the ratio of that, which simply represented with the variation of this alpha 1.

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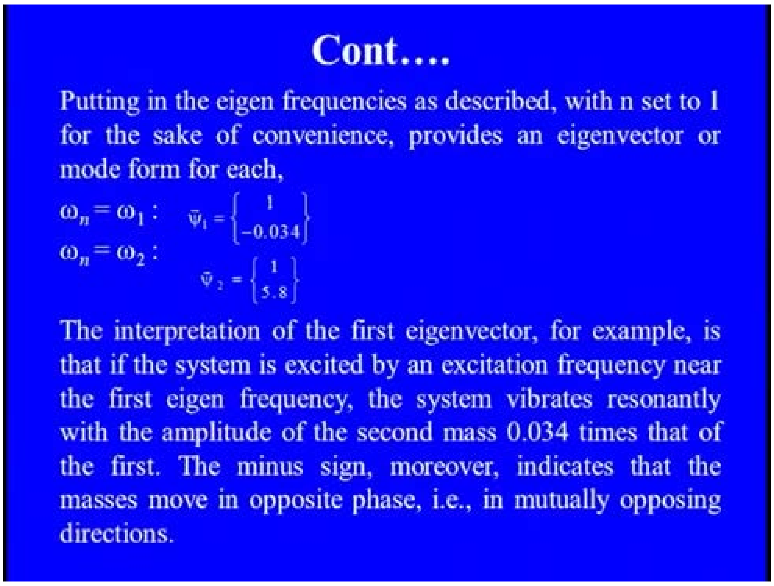

And then when we are keeping those values in this, we could easily say that, now we have omega n which is equals to omega 1 and you see whatever you see which is reflecting phi 1 over minus 0.034. And omega n which is simply for a second natural frequency is phi 2 is nothing but equals to 1 over 5.8, so if you look at these values, we know that in the first one omega 1, we have the mass the mass placing is in the opposite direction, so orthogonal feature, or out of the phase.

Second the both are the positive values 1 oblique 5.8, that means you see here we have in phased mass and they are absolutely moving together, so the interpretation of these the first eigen vector for this, is the system is excited by the eigen frequencies, just near the first we can say eigen frequency. And it simply we have the resonant conditions of the amplitude with you see the second mass is always being reflected with 0.034 times of the first one. So, the second mass is you see more and more oscillated at the resonant frequency, and the minus sign is simply indicates that both are in the opposite phase as we discussed.

So, you see here this diagram or this example simply showed that, we have a clear kind of a the arrangement and these arrangements are simply showing that, whatever the constraints are with the systems always the masses will be in phase or out phase with any degrees of freedom systems.

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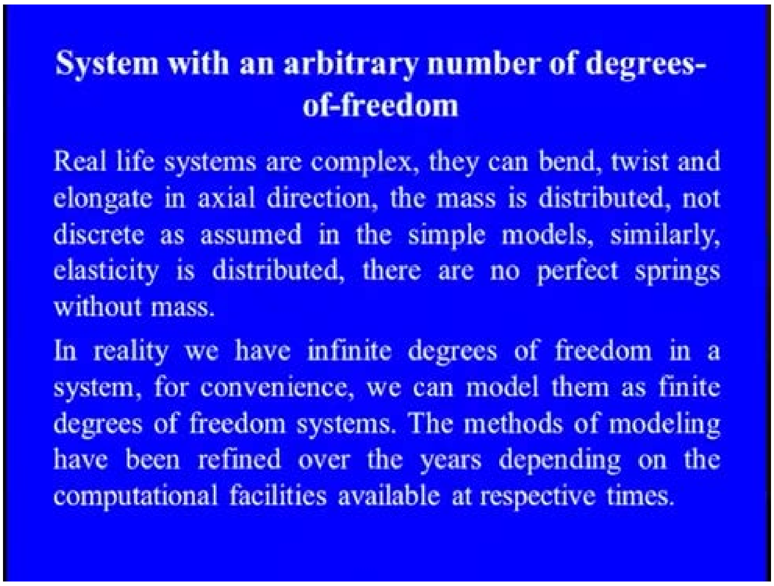

And now if you are going with any arbitrary number of degrees of freedom, means you see here if we are going to the any real time real life examples, we know that the system is very complex. May be you see here right now, we were simply formulating the equation of motion based on a linear propagation or linear motion that axial motion. But, if you see the band is there or if any twist is there or any elongation is there in two different directions, then certainly the mass which is being distributed is not the discreet one; and can be assumed to any of the different models.

And then the entire elasticity is straightaway disturbed by that, we cannot take you see the linear propagation of the spring as we discussed in the previous cases, because there is certain nonlinearity is there, due to the geometric feature. So, in the reality we have infinite degrees of freedom in the system, but for our convenience, because we just want to analyze the system performances under the dynamic action. So, certainly what we are trying to see, we are trying to put the finite degrees of freedom system in that.

And the methods of modelling, which is simply putting altogether is always depending on what exactly the computational facilities are, and accordingly we can go for the degrees of freedom systems. So, you see here right now, we discussed about the 2 degrees of freedom system, so even we can go with the these 2 degrees in the various configurations.

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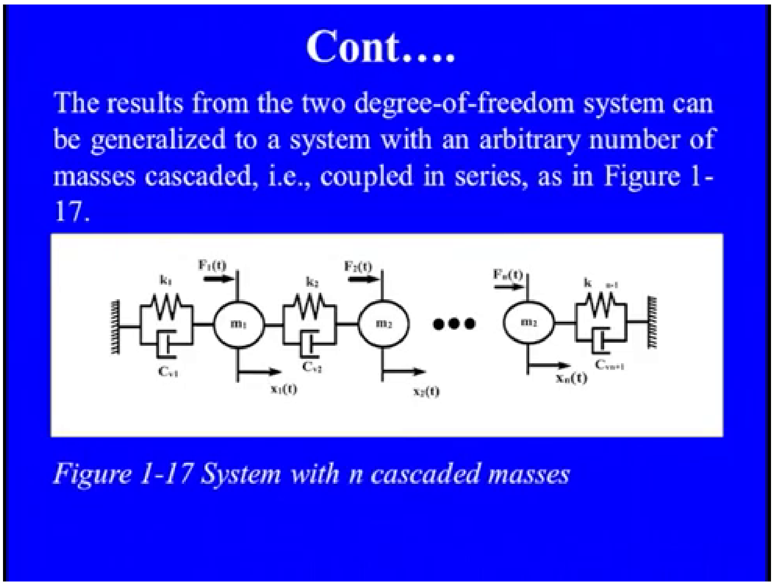

So, on your screen you can see that, we have one diagram which simply shows that, if we have the springs in the series formulation. That means, you see here we have say m 1, m 2, m 3, m 4 and all the n masses are being arranged in such a way that, all the spring and the dampers are coming in the series formulation. Like you see here for m 1, we have k 1 and C 1 on left hand side and on right hand side we have k 2 and C 2. So, for movement of m 1 is straightaway reflected or straightaway you see deviated from this path and similarly, you see for mass m 2 it is straightaway affected by these constraint C 2 m 2 on left hand side C 3 m 3 on other side.

And up to you see if we are going up to n'th mass, then certainly we know that on left hand side we have the spring stiffness k n, and this damping is C n, on right hand side we have k n plus 1 and C n plus 1 of the spring 1. And with these you see here cascaded masses, all these series solution or the series arrangement of the mass or this spring and damper is giving a perfect resolution of the forces, means the damping and the restoring forces along with the inertia forces of the mass. And say if you want to arrange these masses along with the masses or the damper and this stiffness is, along with their own forces.

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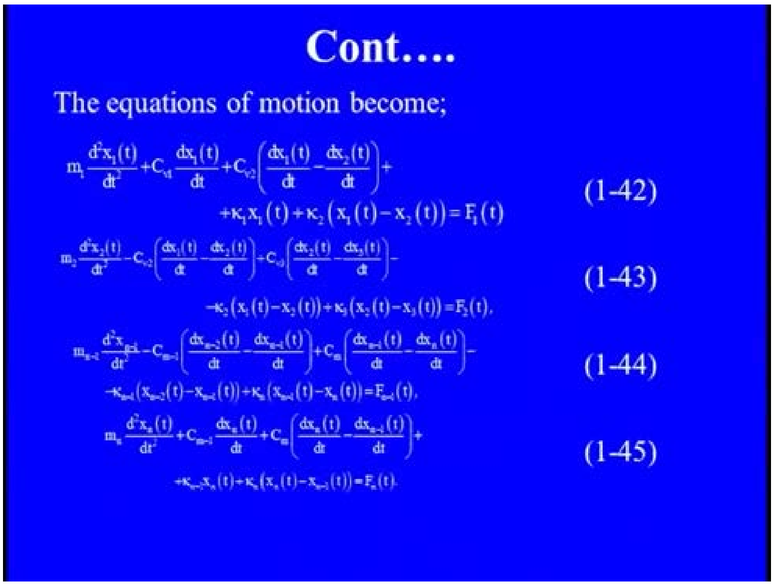

And we can say that first, the equation of motion becomes for the first type of arrangement, it is mass m 1 into d x 1 t over d t square d 2 x 1 t over d t square plus C 1 d x 1 t over d t plus you see here, whatever the damping which is in between. That means, you see here we have the C 2 and if this C 2 is coming in between d x 1 by d x 1 t by d t minus d x 2 t by d t. And similarly, you see here we have both k 1, which is absolutely related to x 1 t and k 2 which is related to the difference of these x 1 t minus x 2 t.

And then since the force is being applied on the mass m 1, so you see on right hand side we have the foreseen factor. And similarly you see here, we can easily formulate all these equations up to the n'th mass, because ultimately this is a discreet system in which the system motion is absolutely constrained by these springs, as well as the dampers part. So, when we are simply configuring all these forces, we can apply straightaway the Newton's law to individual masses and we can get you see all the force balance equations for that.

So, you can see that even for m 2 or m n minus 1 or even for m n, we have all these you see the arrangement of the individual forces like from, this m n d 2 x n by d t square, so this is my inertia force for n’th mass, this is my damping force. As in a left hand side you see here, we have d x n over d t, on right hand side we have C m and this is all the differences of this d x n by d t and minus d x n minus 1 by d t. And similarly, you see the stiffness variation like for k n or k n whatever you see, the displacement differences are there for this and the displacement individual is coming together.

But, these are the individual equations which were simply reflecting that, there is a force balance equation for individual masses and all this setup, under this excitation is under the static equilibrium, even after the forcing F 1, F 2 or up to F n or x 1 or x n displacements are there. So, now you see here we can straightaway go, because it is a multiple degrees of freedom system, we can go to our the matrix systems. And we know that there is a straight formulation for that, like for mass the mass matrix should be identical along the diagonal feature, so you can see that on your screen.

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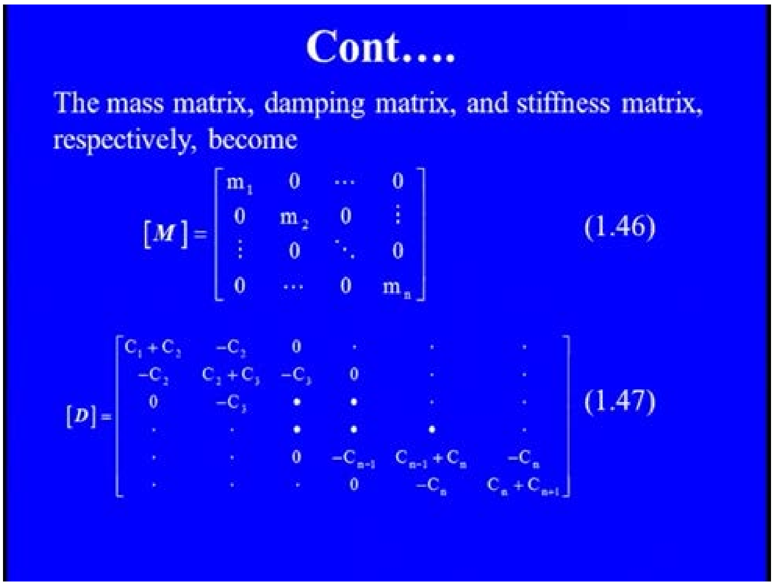

The equation 1.46 is nothing but the mass matrix in which all the elements along the diagonal is symmetric, right from m 1, m 2, m 3, m 4 and then m n along the diagonal and all other elements are simply the balanced equation. Since it is balanced equation, so it should be 0, similarly for damping we know that, when we are talking about the n'th degrees of freedom system, there is you see the diagonal feature must be positive.

So, you look at that we have C 1 plus C 2 starting from this and ending up to C n plus c m plus 1 and then you see the corresponding feature, if we are just on the first part, means the first mass which is the motion of the first mass is constrained by both c 1 and C 2. So, what we have c 1 plus C 2 on your screen you can see, equation 1.7, C 1 plus C 2 other side minus C 2, and if you are going towards the mass m 2, then you have minus C 2, C 2 plus C 3 n minus C 3. Similarly, if you are going towards further directions, we can simply find out that all the damping parts are coming exactly in the positive manner, when we are moving along the diagonal of that matrix.

And if we are going now towards the stiffness part, it has a similar feature, because the arrangement of the damping and the stiffness is, the deformation in the stiffness and the velocity in the this damper they are having the similar manner, no nonlinearity is there. So, along with that you see the elements, in the matrices are also the symmetric along the diagonal feature and they must be positive.

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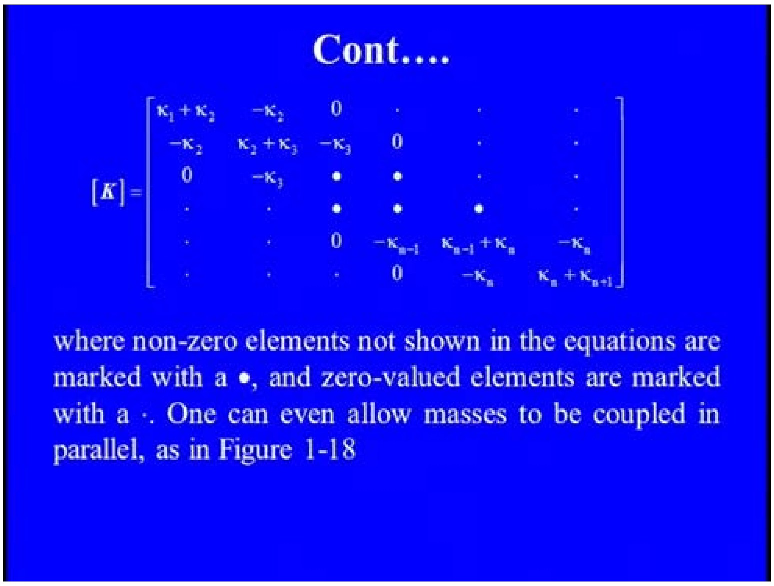

So, with that we can see the stiffness matrix k 1 plus k 2, similarly k 2 plus k 3 plus similarly you see, if we are moving towards you see the n’th part k n plus k n plus 1. So, this entire symmetricity is simply showing that this is a well balanced equation, and they are showing that the entire system is in equilibrium manner. Where the non zero element not shown in the equation are simply marked with the dot, and the zeroed value elements are marked with the a in these equations like that in these matrix elements. And one can even allow the masses to be coupled in parallel like you see now, if we just want to arrange our masses in the different configuration, like you can see that in this configuration.

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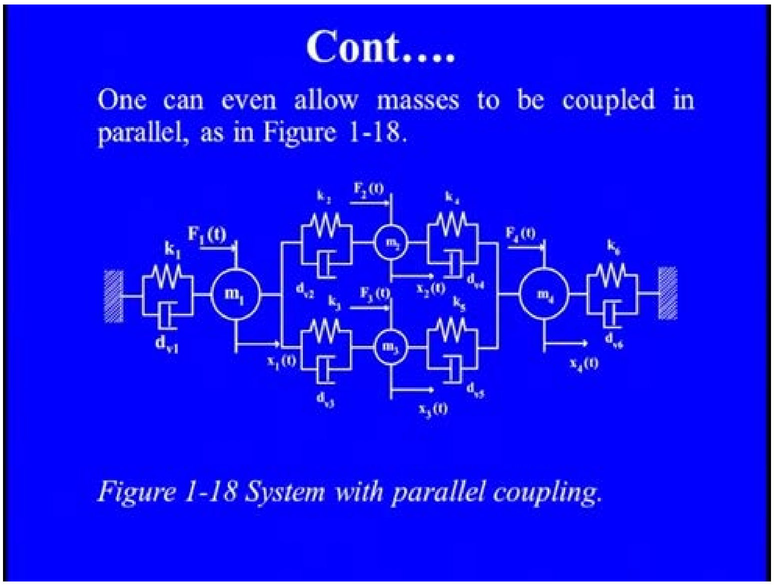

Figure 1-18 System with parallel coupling.

We have the masses, so the first mass m 1 is absolutely with our k 1 and c 1, but you see the masses m 2 and m 3, they are absolutely coupled with now over these k 2 and k 4, and if this is coupled with k 3 and this is my k 5. So, when these masses are now coupled parallelly, we can see that there is a clear impact of these coefficients in the matrix formulation or the equations of motion, because now we have a parallel coupling of the masses along with their stiffness and these damping coefficients.

And then later on you see if you are ending up with our you see m 4 mass along with our k 6 and damping coefficient d 6, so when now if we have this arrangement, we can again see that you see what exactly these forces, which are being balanced by this F 1 on m 1, F 2, F 3 and F 4 on these m 2, m 3 and m 4 masses and when it is, so then you see here the equations are like that.

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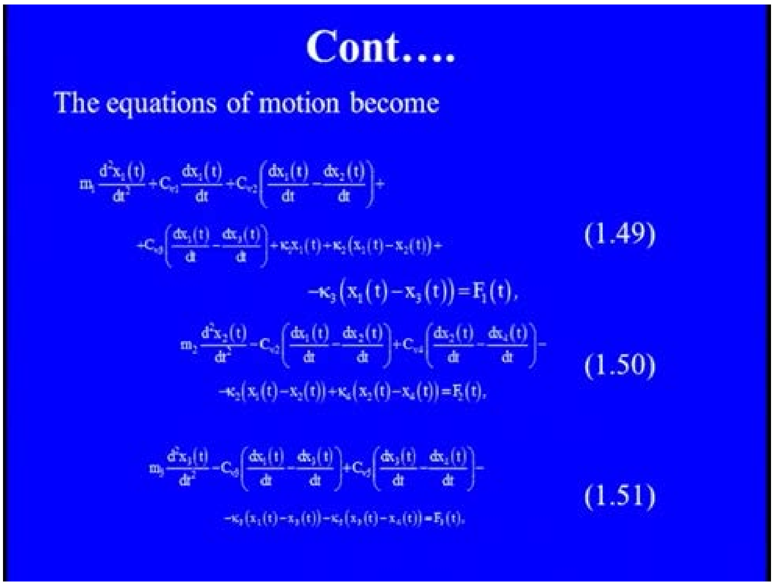

On the for the first masses there is no change as such you see here, because this is what it is you see the m 1 and this one, the damping here is somewhat change, because whatever the damping.

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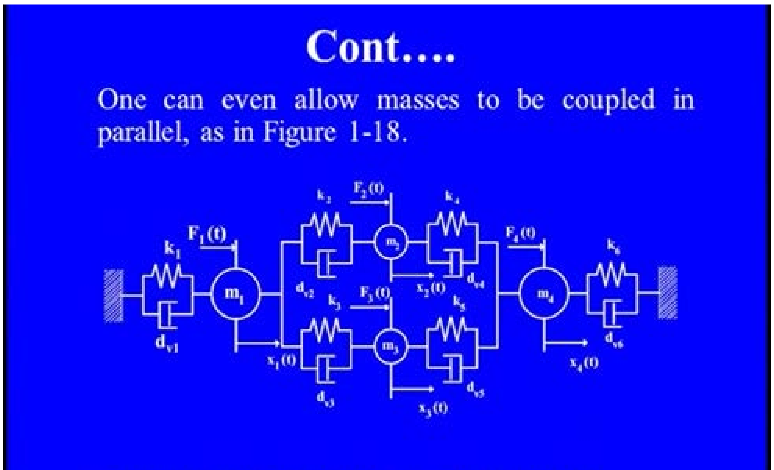

Figure 1-18 System with parallel coupling.

If you look at the previous diagram, whatever the damping is coming for this first mass is an integral feature of this first k 2, and you see the k 3 the stiffness and the damping is coming from this d 2 and d 3 here.

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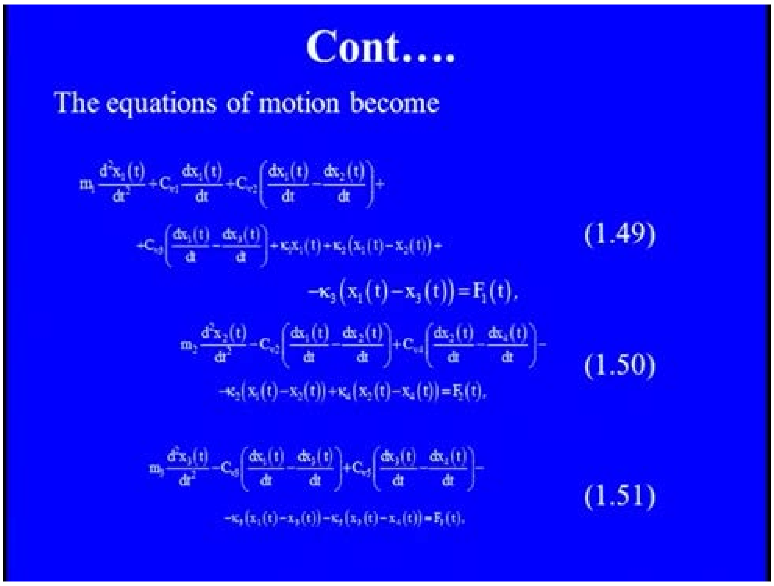

So, we need to calculate accordingly, so if you are going towards that you know that, the first C v 1 is coming with the d x 1 by d t there is no problem in that, it is a straight damping force. But, as far as you see the C v 2 is concerned we have d x 1 and d x 2 the differences and similarly for here, it is d x 1 and d x 3, because you see here on right hand side there is a clear interaction of x 2 and x 3 velocity vector. And similarly, if you are going for our stiffness matrices, now the stiffness we can say the restoring forces.

Then again the restoring forces on the left hand side, we have a straight k 1 x 1, but on the right hand side we have the k 2 and k 3, they are in the integral feature of x 1 x 2 and x 1 x 3, and then you see we are making equal. So, only what we need to see, we need to make in a free body diagram in such a way that, that how the forces the restoring forces or the damping forces are coming on mass m 1 from left hand side, or from right hand side. And once we are making balance of these things, then we could easily figure out that under these force balance conditions.

There is you see, whatever the mass, whatever usually, the oscillation is there in the masses with the m 1, m 2, m 3, m 4 that could be easily figured out. So, now, if you are going towards the other masses m 2 and m 3, so far m 2 we know that since the m 2 is moving with the x 2 only. So, the inertia force has no problem, but the damping here, the damping force is something you see integrated with x 1 and x 2 part here, and here it is x 2 and x 4 part.

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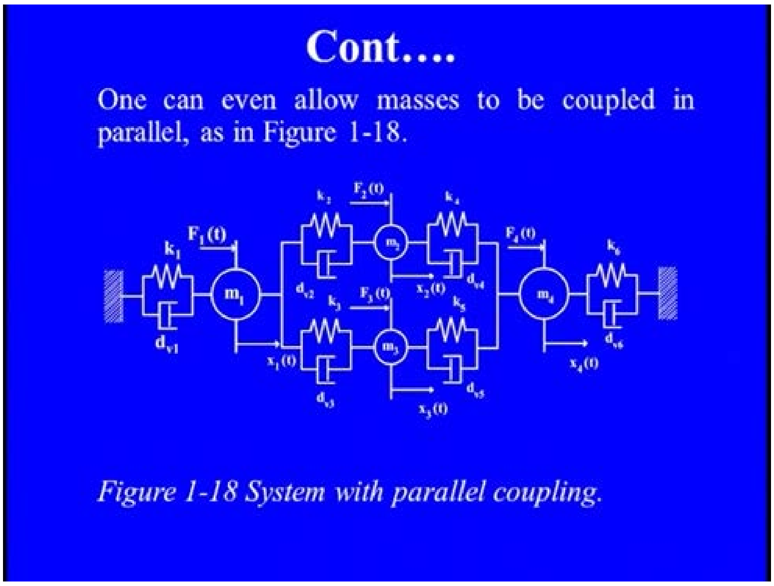

Figure 1-18 System with parallel coupling.

So, you look at that you see here, this is something here we have k 2 and k 4 see this is d v 2 and d v 4 and these both the part you see here is straightaway affecting the motion of this one. Similarly, if we are to m 3, we have you see k 3 and here we have the k 5, here we have d v 3 and d v 5, which are simply making the entire balance of this m 3.

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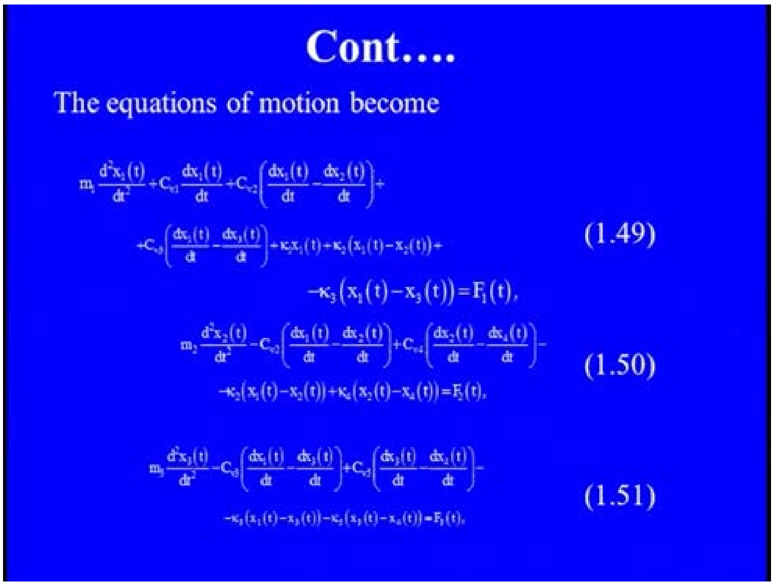

$$m_1\frac{d^2x_1(t)}{dt^2}+C_{v1}\frac{dx_1(t)}{dt}+C_{v2}\left(\frac{dx_1(t)}{dt}-\frac{dx_2(t)}{dt}\right)+ +C_{v3}\left(\frac{dx_1(t)}{dt}-\frac{dx_3(t)}{dt}\right)+\kappa_1x_1(t)+\kappa_2\left(x_1(t)-x_2(t)\right)+ +\kappa_3\left(x_1(t)-x_3(t)\right)=F_1(t), \quad (1.49)$$

$$m_2\frac{d^2x_2(t)}{dt^2}-C_{v2}\left(\frac{dx_1(t)}{dt}-\frac{dx_2(t)}{dt}\right)+C_{v4}\left(\frac{dx_2(t)}{dt}-\frac{dx_4(t)}{dt}\right)- -\kappa_2\left(x_1(t)-x_2(t)\right)+\kappa_4\left(x_2(t)-x_4(t)\right)=F_2(t), \quad (1.50)$$

$$m_3\frac{d^2x_3(t)}{dt^2}-C_{v3}\left(\frac{dx_1(t)}{dt}-\frac{dx_3(t)}{dt}\right)+C_{v5}\left(\frac{dx_3(t)}{dt}-\frac{dx_4(t)}{dt}\right)- -\kappa_3\left(x_1(t)-x_3(t)\right)-\kappa_5\left(x_3(t)-x_4(t)\right)=F_3(t). \quad (1.51)$$

So, with this you see here, if you are trying to see the relative motion or the relative effect of this part you see here, we can say the damping forces or the restoring forces all the forces are being resembled here in a proper way. Similarly, for mass m you see 3, which is again the independent masses here, we could easily figure out this part.

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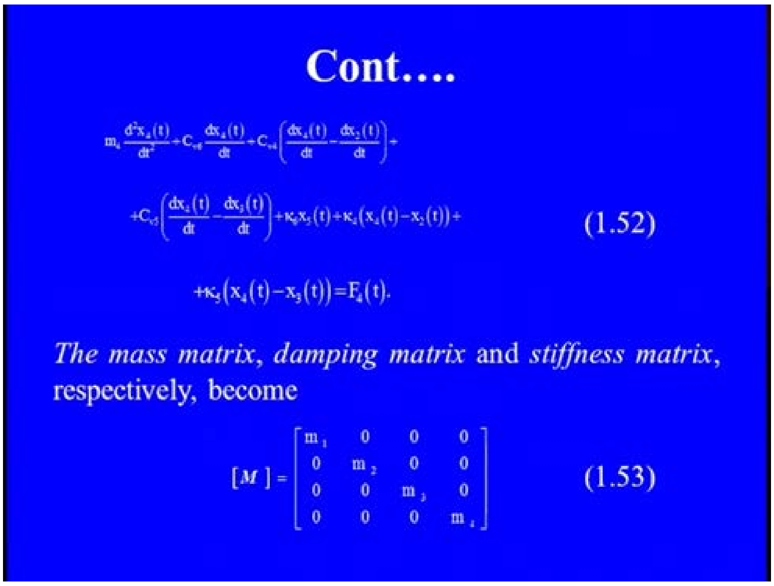

And m 4 which is the separate part the last part you see here, here on the left hand side of this m 4, we have a clear interaction of the two main a spring, this is from the fourth and the fifth one. And also you see here, we have a clear interaction on the right hand side we have the this whatever the springs are there like you see here, whatever the spring forces are there from the 6 and d v 6 from the damping forces. So, you see here all these features are being straightaway incorporated here, now if you are just going towards the masses, that how the mass matrixes are being arranged along with the masses which are being placed in the equations.

So, you can see that the mass matrix is again the same m 2 to m 4 is the four masses are being there, so we can see that since it is a 4 degree of freedom system, and we have you know like the mass matrices. It is again it is symmetric along the diagonal one, and now if you are going towards the other features, like the damping matrices.

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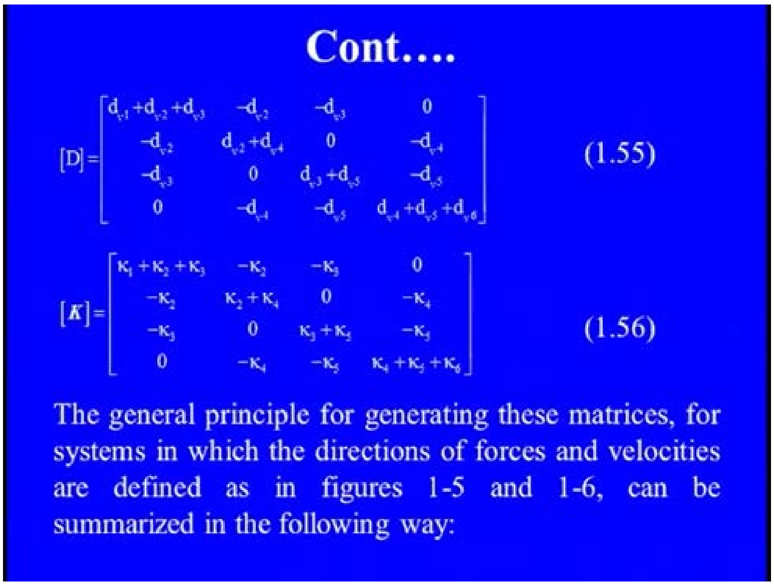

We can see that even in the damping matrix right from the first one, because you see the first feature of our the m 1 which is straightaway affecting by three damping forces, one d v 1 the damping coefficient on left hand side, on right hand side there was a straight effect of d v 2 and d v 3. So, this part is straightaway you see coming here, we have d v 1, d v 2 and d v 3 altogether here, and on the other side we have since, there was you see a clear dissipation on the right hand side, it is minus d v 2 and minus d v 3 on the other features.

Similarly, if I am going towards the other part it is minus d v 2 which is being there, and then you see in the middle of one, since it is being straightaway we have the two main part on m 2, one on these d v 2 and d v 4. So, certainly you see the featured, all the featured will come into the main part d v 2 and d v 4 here and similarly, you see all the diagonal feature is you see it is positive and they are being arranged in this way. And since you see as per the damping feature, the stiffness the spring is also arranged in same way.

So, we have the spring, we can say coefficients are k 1 plus k 2 plus k 3 as far as you see the left and right hand side of m 1 is concerned. And similarly if you are going up to the n’th part, up to the fourth mass we have both k 4, k 5 and k 6 for this one, so in this also we could easily figure out that even you see here, whatever the arrangements are. The general principle for generating these matrices for the system in which you see the direction of forces and the velocities are defined in this one, can also be summarized in this way.

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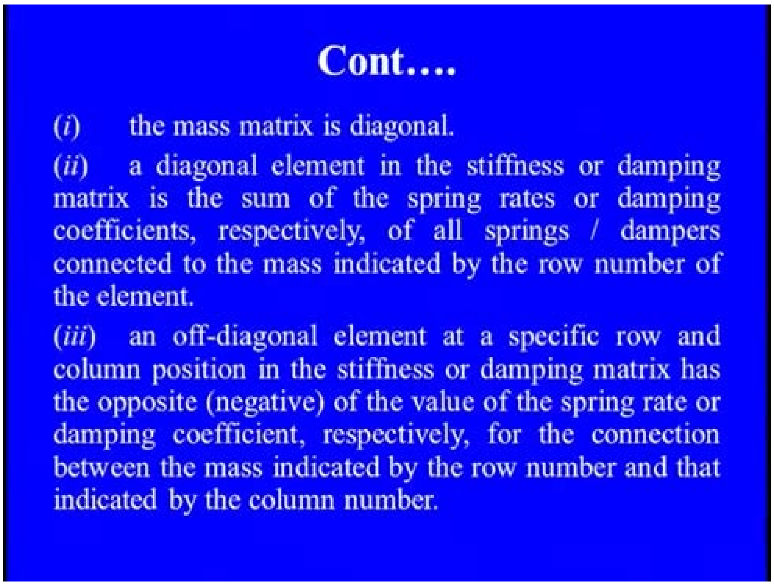

First the mass matrix must be diagonal, that means you see here you like whatever you see even the masses are arranged in the parallel way, or the masses are arranged in the we can say the coupled one, the series one just you see in the first case which we discussed, the mass matrix must be diagonal. So, this is one of the thumb rule for that, or else we can say it is a checking point for this, second the diagonal element in the stiffness matrix or the damping.

Because, you see here both the spring and damper is arranged is arranged in such a way that, they are absolutely occurring in the similar manner as far as the restoring force, or damping forces are concerned. So, the stiffness or damping matrix is sum of the spring rates or the damping coefficient respectively of all the springs, and the dampers connected to the mass indicated by the row number of elements.

That means, you see here even the first mass as just in the previous case, which we discussed the first mass either in the series feature, if it is being constrained by k 1, d v 1 or k 2 d v 2. Or else in the second case where the coupled one is there the parallel one, if this m 1 was like on left hand side it was k 1 and d v 1 or on another side it was k 2 d v 2 and k 3 d v 3. But, their arrangement whatever coefficients are coming, they are coming exactly along the diagonal feature, that what exactly the contributing these forces or the elements are.

And an off diagonal element at a specific row or the column position, in the stiffness or any damping matrix has an opposite or negative value of the spring rate or damping coefficient respectively. For that, the connection between the masses simply indicate by row number and that indicated by the column number, so what does it means that, it means that whatever you see the off diagonal elements are there. At any row you see here, they are simply giving the position of any spring or the damper features, by the negative way, minus d v 2, minus d v 3 in the first part.

Or even the second row when it is starting from the second element in the first column, minus d v 2 and then you see here the other things are coming d v 2 plus d v 3. So, what I mean to say that in these or either we are in the inter 2 degree of freedom system or multi degree of freedom system. The arrangement of the elements in these matrices are pretty standard one and this is what you see making a thumb rule for all these arrangements of the elements in a perfect positions in the matrices.

And it is even you see a check point for this, that if the things are not coming in these ways that means, either the forces are not properly configured or there is some problem in calculation features. Or in degree when we are just doing the free body diagram, we are not able to characterize properly, so you see in this today’s lecture, we discussed mainly about that you see, if we are talking about the 2 degree of freedom system or multi degree of freedom system, does not matter.

Only we need to see that how these forces, because this is a discreet system, so how the forces like the inertia forces are being balanced by the stiffness force, this restoring forces through the stiffness of the spring or the damping forces through this damping coefficients. And when all these being under balanced condition, even the force excitation is there then what exactly the elements are being arranged in the matrices. Because, we are now playing with more degrees of freedom system, the multi degree of freedom system, so there was a straight rule which we discussed already that in the elements. Either we are talking about the mass matrix, either we are talking about the stiffness matrix or the damping matrix, how the elements are being arranged in a proper way. So, you see here in these 4 lectures till date, we were just discussing, it was a it all these lectures were the review feature, because you see here ultimately our main theme of this course is to control the vibration the vibration control.

So, until and unless if you are not able to characterize the vibration properly with the single degree, 2 degree or multi degree of freedom system, then we cannot say that, we can effectively control or we can even effectively adopt any control strategy for this systems. So, in the next lecture now, our another feature will start the second module that simply shows that, how we can control the vibration. So, first of all we would like to discuss in the next lecture, that you see what exactly the formation of vibrations in the different systems for a mechanical machinery.

How we can strike on the root cause of vibration, and once you find out that, this is my cause of vibration, then how what kind of control strategies are there, because ultimately in the vibration the three main features are there. First the source second the receiver and the sink, and you see the transmission means what is the path of that, the transmission feature. So, you see here where do we need to put the isolators, if the passive vibration control, in active vibration control, the first thing is there strike immediately on the root cause of vibration. So, in the next lecture we would like to discuss the aspects of control of vibration, along with the transmission the root or the, means the source or the sink of that.

Thank you.