Vibration Control
Prof. Dr. S. P. Harsha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

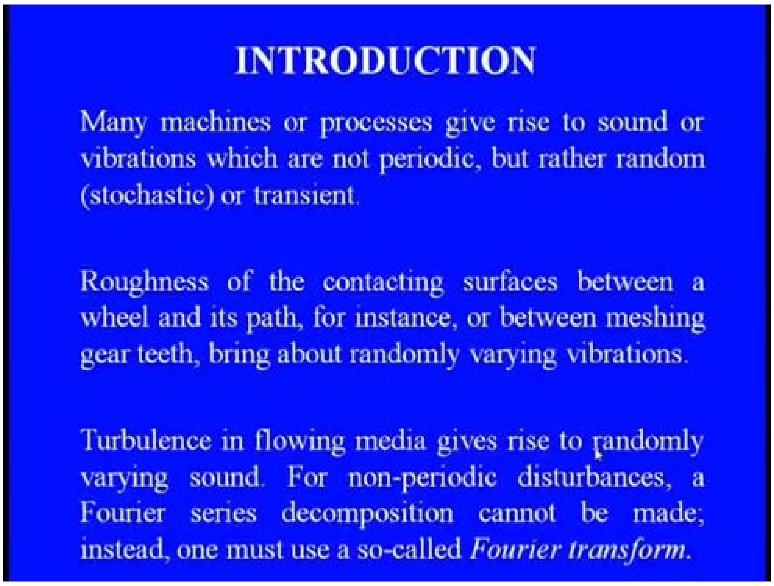
Module - 8
Vibration Measurement Techniques
Lecture - 3
Fourier Transformation

Hi, this is Dr. S P Harsha, from Mechanical and Industrial Department, IIT, Roorkee, in the course of Vibration Control, we are mainly discussing about the vibration measurement techniques. So, in last two lectures we discussed about, that what are the basic you know like the dynamic parameters which we can measure and we can characterize the vibration responses.

And in the last lecture we discussed about that you know, when we are capturing the data, what the exactly the data acquisition systems are and when we are capturing data in the time domain, which sometimes you see you are not giving the clear picture about the response features. We need to convert this into the frequency responses, then you see here we can use the Fourier transformation series in that, and whatever the information is there in the time domain with respect to the this displacement velocity or acceleration.

We can you know like convert this entire information, you know like into the Fourier transformation, where we can get the exciting frequency with the corresponding amplitude of the vibration masses. In this lecture now we are going to discuss about again the you know like the vibration measurement technique, in the Fourier transformation that what exactly you see here we need to you know like put those Fourier transforms, when the things are even sometimes you know like in the turbulence manner or sometimes, you know like when the abrupt changes are there in the signal, because of some external excitation then how do we measure those responses.

(Refer Slide Time: 01:54)

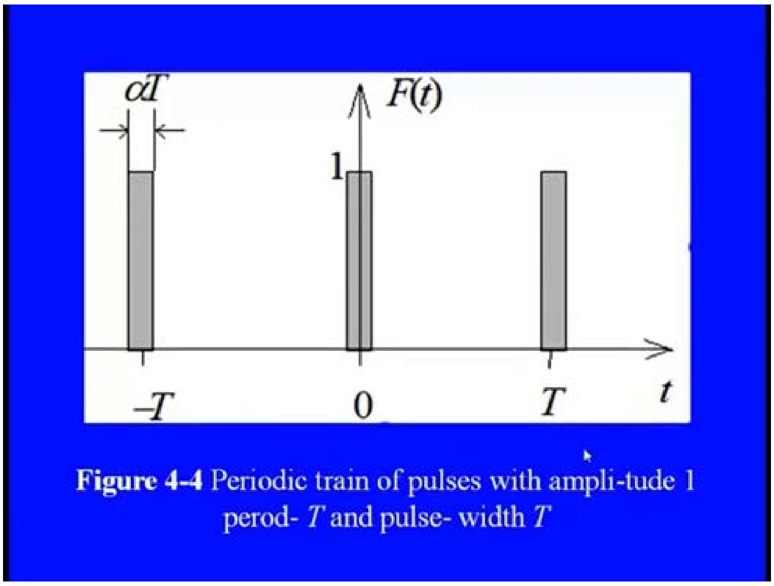


So, many machines or the processes give rise to sound or vibration which are not periodic, because you see sometimes even they are either the stochastic, or in some times we are saying it is a random or we can say the transient vibrations. So, when the responses are not you know like the steady state phase, or the periodic phase certainly you see here it is very tough to measure that. So, roughness of the contacting surface between the wheel or path for instance, we can say that you see here when the gear teethes are the meshing.

And some kind of you see disorientations are there or some you know like the different surface roughnesses are there, some irregularities are there on the surfaces of the gear. Then, certainly you see here we have a randomly varying vibration features are there, in terms of amplitude and the frequency excitations or even we can say when we have the turbulence in flowing media, through which you see here the randomly varying these vibrations are being coming out.

So, these random orientation the vibration which can be termed, as even the non periodic disturbances, in that the Fourier series decomposition which we discussed in the last lecture, that you see you know like how we can decomposes these information using the Fourier series. So, Fourier series decomposition cannot be made for that, so what is a solution for these kind of random in the signature, then we need to use the Fourier transform. Means you know like some segmental transforms are there, which can be immediately used for that particular time of period. And then we can simply you know like the classify or we can simply we can say analyze, what exactly the kind of you see the energy or we can say exciting frequency or the amplitudes are there in the signal itself.

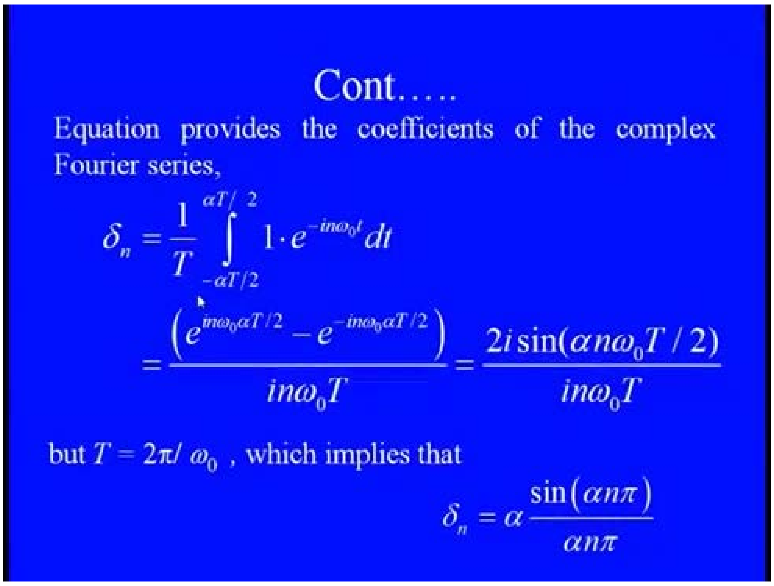
(Refer Slide Time: 03:46)



So, if you see that the periodic, you know like these pulses then we can simply find that you say you know like the at the 0 point, we have the amplitude 1 with the period t you see here. So, it is a well periodic feature, which can be immediately measure using these you know like the Fourier transformation series or the decomposition, and we can featured out. But, even we when we have the pulse width say you know like with the t, this pulse width and when we are simply multiplying with the alpha, which is something you see you know like the this constant feature. Then we have even at minus t, and t you see we have a clear variation, but with the width of this alpha t, we can say that you see straightaway we can find out the coefficient of these complex Fourier series.

So, if you are saying that you know like the complex coefficient is delta n for n-th type of you see you know like the signal, then it is nothing but equals to 1 by t, where the time period the total time period for this is capital t. So, 1 by t now here the yesterday we discussed about you see you know like the entire decomposition, that minus t by 2 to t by 2 here, now since we are using one of the coefficient here.

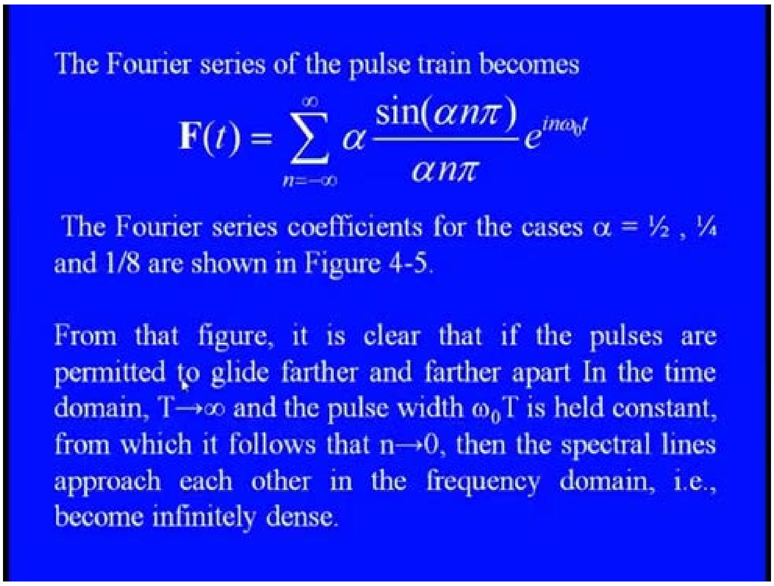
(Refer Slide Time: 04:27)



So, we have now the total feature, in which you see you know like the entire the signal is being considered it is minus alpha t by 2 to plus alpha t by 2, and in that we can say that since the main amplitude is 1, so one into e to the power minus n omega 0 t in to d t. So, then you see we can straightaway convert into you know like, these two special features. So, we have now e to the power i n omega 0 alpha t by 2 minus e to the power minus i omega i n omega 0 alpha t by 2 divided by i n omega 0 t, or else you see here when we are trying to convert these exponential series into you know like sinusoidal features.

Then we have the two iota because you see this cos figure will be canceled out, as we are simply putting this e to the power i n omega 0 this alpha t by 2 is nothing but equals to cos of you know like. This i n this cos of n omega 0 alpha t by 2 plus iota times of you see you know like sin of n omega 0 alpha t by 2. So, cos, cos will be cancelled out then we have you see the 2 iota sin of alpha n omega 0 t by 2 divided by i n omega 0 t, so this is what you see here the coefficient from the Fourier series which we can compute in terms of you see the sinusoidal feature between alpha t by 2 to minus alpha t by 2. If we are converting this you know like the total time period in terms of you see the frequency, then we have 2 pi by omega 0, when we are keeping, so we have now the coefficient delta n is equals to alpha times of sin alpha n pi divided by alpha n pi.

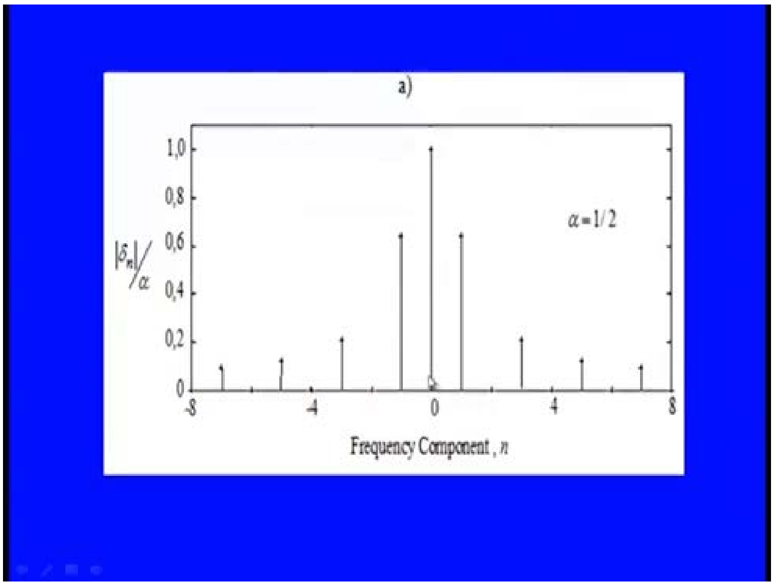
(Refer Slide Time: 06:34)



So, when we are keeping this into the Fourier series specially in terms of you see the entire wave formation is in the pulse, then we can say that the Fourier series part f of t is equals to summation for all. From minus infinite to infinite alpha into sin of alpha n pi divided by alpha n pi into, whatever the exponential features are means e to the power i n omega 0 t. So, these you see the Fourier series coefficients can be evaluated for various values of alpha, and we can find out that you know like when we are increasing the alpha or when we are decreasing the alpha, how they are straightaway affecting the entire signature.

So, you see here now we are taking three cases, in which the alpha is varying from say half one fourth, to and one eighth it is in the decreasing order of that. So, now, you see here I am going to show you first the alpha, and we would like to see that how the pulses are being you know like permitted straightaway. When they are simply showing from minus you know like infinite, this n equals to minus infinite to this total infinite part that you see how the things are how the frequency domains are being you know like capturing these frequencies.

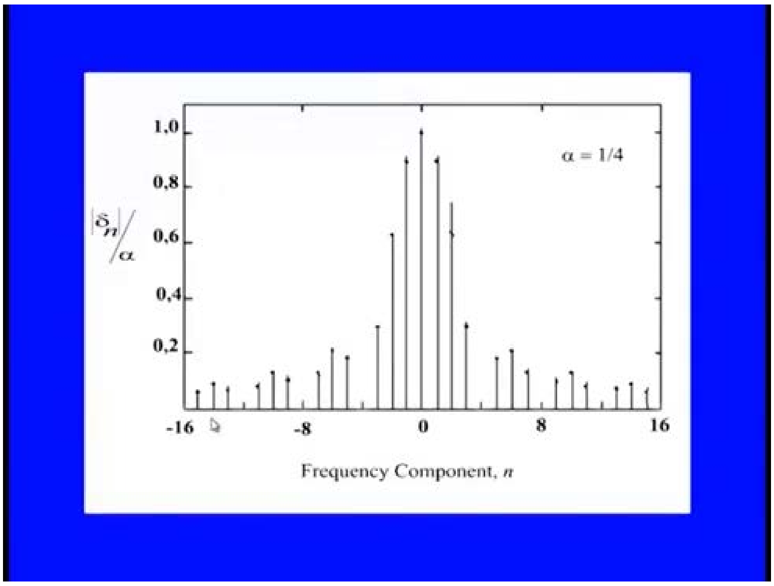
(Refer Slide Time: 07:50)



So, the first case which we are going to show that when the alpha is half, so when the alpha is half you see now this is a clear picture, that you know like this frequency component n in the x part. And you see here the delta n by alpha because you can straightaway take the delta n over alpha, which is nothing but equals to sin and pi alpha divided by n by alpha. So, when we are doing this now, we can straightaway vary with this delta n by alpha with the n and you can see that absolutely at z 0, we have the first amplitude that is the unique the period 1 amplitude. And then the corresponding features are being there with you know like, right from minus you know like alpha to plus alpha means up to the eighth part.

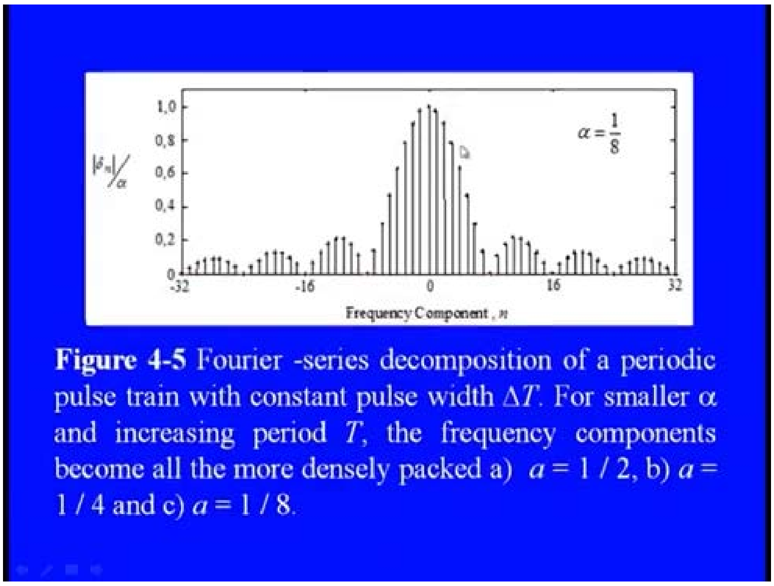
So, these are the clear excitations which are being there you know like with this constant amplitude and the frequency component, when we decrease the alpha, means one fourth now you can see the spectrum. The spectrum is showing the various frequency peaks which is very close to this part, because we know that you see when we just vary the alpha the discrete spectrum is clearly, you see the dense spectrum.

(Refer Slide Time: 08:40)



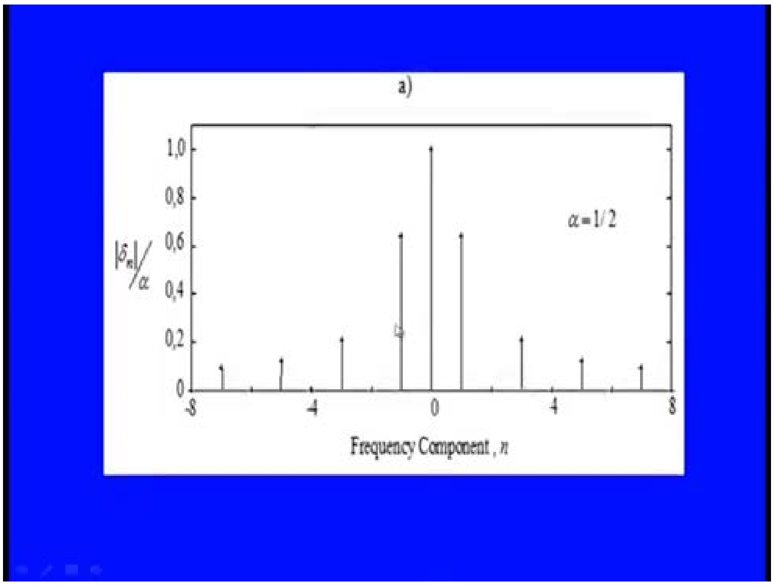
Rather I should say is clearly now just to you know like varying with the constant amplitude, so the first amplitude which is at 0, when you do not have any frequency component. You see we have just period 1 exciting frequency, but as we are increasing the frequency component towards positive or negative side. You see here this is a clear more number of frequency peaks are, and even we can now expect that the when we are decreasing now alpha from 1 by forth to even further half that is 1 by eighth.

(Refer Slide Time: 09:24)



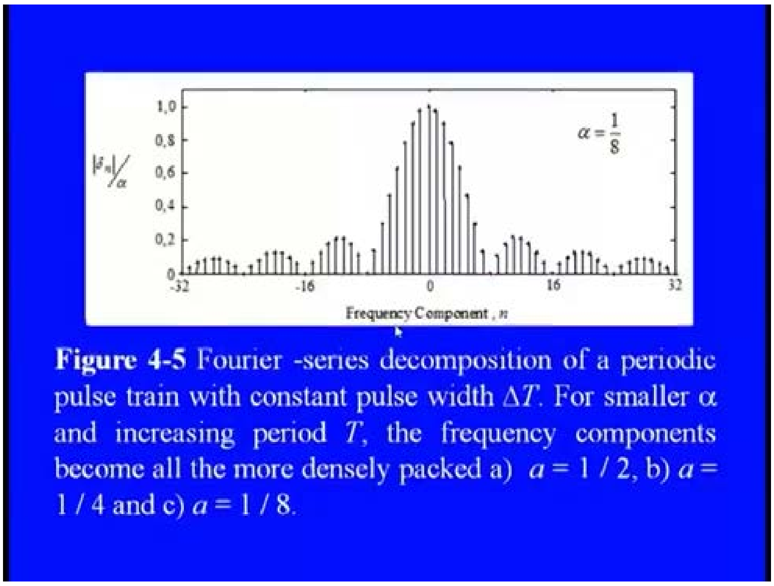
Then you can see here this is a clear form of you see you know like the Fourier series decomposition, when the periodic pulses are they are right from this is 0. And then these are you see you know like we look at that, they are clearly showing the pulse form in terms of you see delta n by alpha two the frequency component. So, now you see the frequency components are upto 32, the previous case it towards for 16, and the previous case it was for 8.

(Refer Slide Time: 09:48)



So, these are you see the clear variation that, you see how many frequency peaks are being allowed when you are decreasing the alpha.

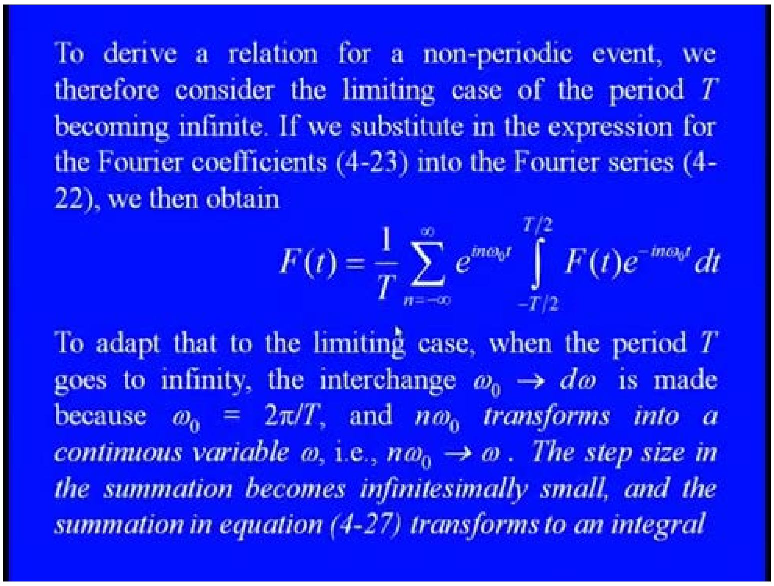
(Refer Slide Time: 09:54)



So, these diagrams which have clearly showing the few Fourier series decomposition for this periodic pulse train, with the constant pulse width that is delta t, so delta t is absolutely constant in that you see here. So, when we now you see from that we can conclude that for a smaller value of alpha say from you know like 1 by eighth, when we are just coming there is a clear increasing.

And you see you know like the increasing of period t as we are moving that, you see the frequency component become all more densely packed. And they are absolutely showing that, how these you know like the exciting frequencies are being varied in the band form, but when we have you see the higher value of alpha. Then there is a clear peak of excitation, and you can straightaway found that you see what the tuning part is there, when the frequencies you know the exact the frequency part with the frequency coefficients, or we can say the frequency component n is. So, you see here these you know like alpha one half, one fourth, or one eighth they are clearly showing about that how they are laying more number of frequency components, when we are just going with the decreasing value of alpha.

(Refer Slide Time: 11:01)



So, to derive the relation for non periodic event, when the abrupt changes are there or any kind of impulsive forces are being there in the shock part, we can simply consider a limiting case of period t, which becoming you know like the infinite. And if you substitute the expression for Fourier coefficient as we shown previously, now the Fourier series we can say f of t is nothing but equals to 1 by t summation of.

Again, you see and the Fourier this constant is n, which is minus infinite to infinite, e to the power i n omega 0 into t again you see here, now we need to integrate for minus t by 2 to plus t by 2 f of t into e to the power minus i n the iota basically, the iota n omega 0 t into d t. So, this is what you see you know like we need to keep, because you see we know that this non periodic event is absolutely for a very limiting case.

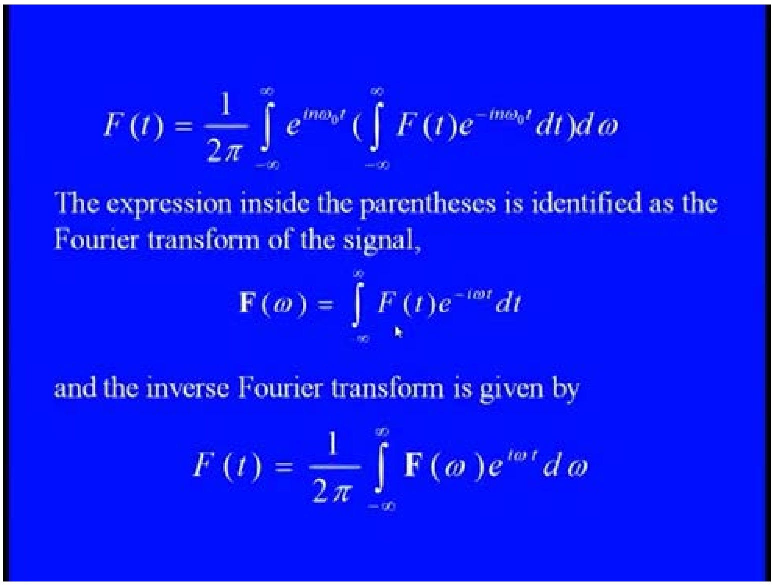
So, first of all we need to define that you see, where you know like what is the period in which you see this non periodic form of the responses appearing in that. And then we need to you know like induct those information in this particular Fourier series, where it is a clear variation right form you see you know like the coefficient infinite to minus infinite to infinite, and then you see in between minus t by 2 t by 2 what the information are there in terms of f of t.

So, to adopt this limiting case when the period t is just going to the infinite, we know that we need to interchange this omega 0 with what the delta omega a is for that particular this specific time period d t. Because, we know that when we have omega 0, which is 2 pi by t n omega 0 transforms into the continues variable omega, so we can say that n omega 0 is now referring to the entire omega.

So, step important thing here is the step size, the step size in summation becomes infinitesimally small, because ultimately we need to featured out that what exactly, because you see here. If you are taking you know like this our step size is bigger than we cannot featured out, exactly that what kind of variations are coming under non periodic form. So, we need to just capture you know like at the very small means infinite decimal is small, you know like the step size and then you see a we can simply sum up entire feature, in you know like in the transformer equation towards the integral form.

So, now you see here with this concept, now when we are going to the Fourier series f of t is nothing but equals to 1 by 2 pi, now it is being converting to 1 by 2 pi. As, we discussed already that you see this omega 0 is now being converted into pi by t, and then when we are just going with this we now the f of t is 1 by t. So, now, we have 1 by 2 pi into minus infinite to infinite e to the power iota n omega 0 t into, now you see whatever the information which we have saying that you see you know like the Fourier transform of the signal.

(Refer Slide Time: 13:34)

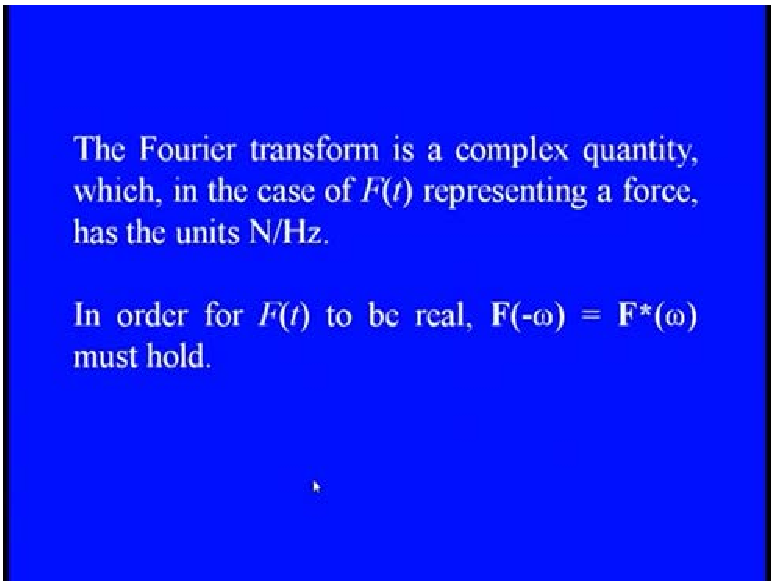


So, we need to go that what exactly the signal information, which is showing the non periodicity you know like, and what the informations are there towards that. So, we have into minus infinite to infinite f of t e to the power minus iota n omega 0 t into d t, and then it is d w, because ultimately we need to convert this entire omega 0 which is being here into d w as we discussed.

So, certainly you see here now an n omega 0 is referring to the omega part, so we can straightaway put here into d omega, and the expression inside this part means f of t into this e to the power minus iota n omega 0 t into d t. Can now, be identify as the Fourier transformation of the signal, and we can say f of now omega is nothing but equals to minus infinite to infinite f of t e to the power minus iota omega t into d t.

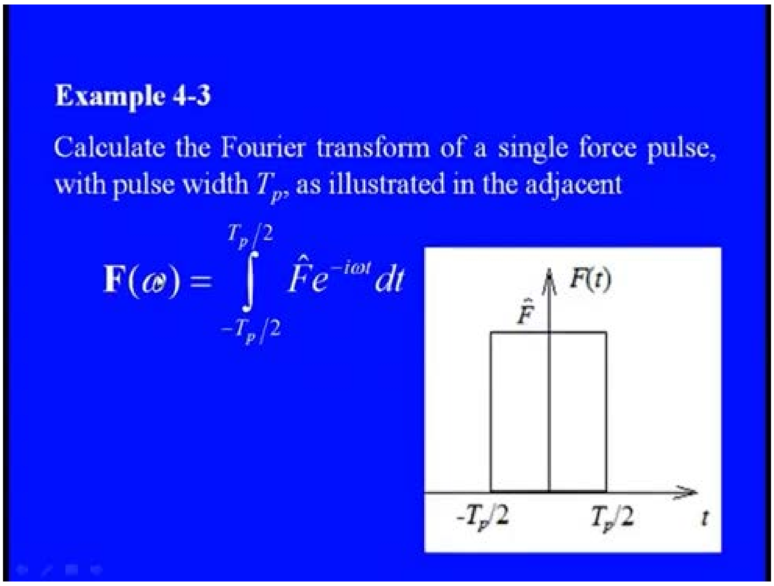
So, now when we are doing the inverse transformation, this Fourier transformation, then we have f of t, because of this you see the Fourier transform f of w is there. So, when we are doing this inverse transformation, we have f of t equals to 1 by 2 pi minus infinite to infinite f of omega e to the power iota omega t into d omega. So, this is what you see here the inverse Fourier transformation, which contains the you know like the non periodic form of the a signal information.

(Refer Slide Time: 15:24)



So, the Fourier transformation is certainly a complex quantity you know like, which in case f of t representing the force, and with this you see we have the Newton per unit hertz. And in order to you know like finding for f of t, which is to be very real we can say that f of minus omega is nothing but equals to f of this f of omega and you see here when the both are equal we can say the we can find out f of t for the real terms.

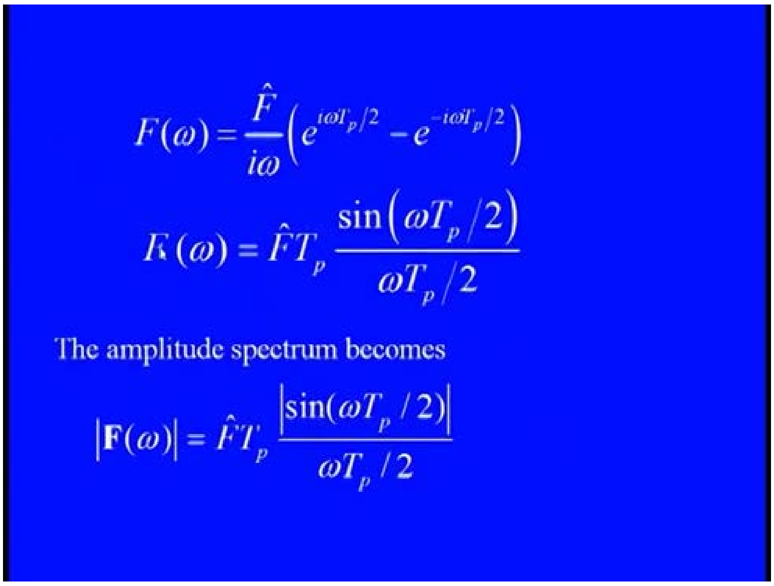
(Refer Slide Time: 15:51)



Now, we are taking the example in which you see here, we have the Fourier we need to calculate the Fourier transform, for a single force pulse, and the pulse width is given as t p, which you can see that its varying from minus t p by 2 to t p by 2. And this is what you see you know like the direction f of t with the t, and when we just want to find out you see we need to have the Fourier transform for this.

So, f of w is nothing but equals to minus t p by 2 to t p by 2 integration f, which is being applied here you can see this f bar, so bar into e to the power minus i omega t d t. So, you see here we can straightaway calculate, that what is the variation of this force with respect to you see here you know like the Fourier transformation, the we can say the pulse form of force.

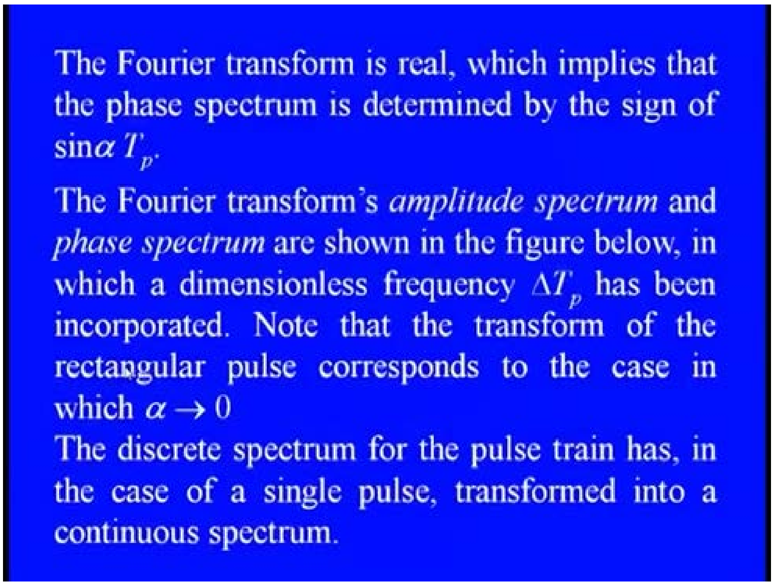
(Refer Slide Time: 16:42)



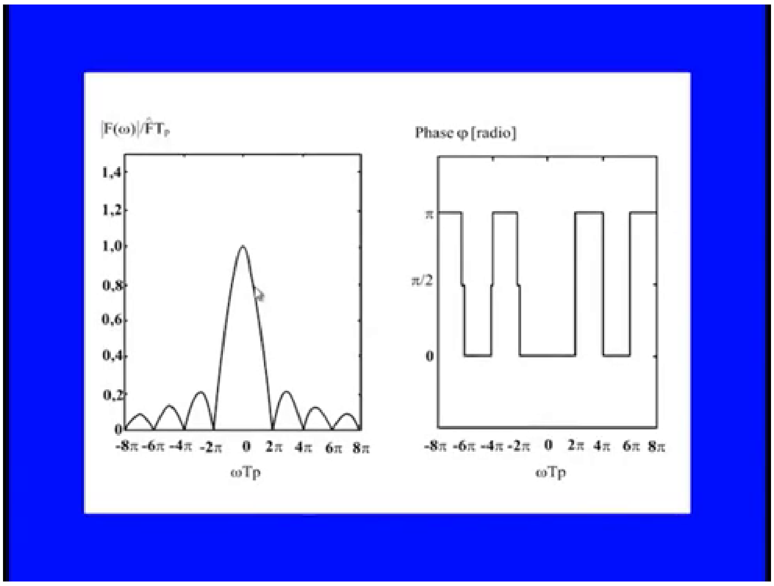
And now, when we are keeping this, so f of w is nothing but equals to f divided by i omega into e to the power i omega t p by 2 minus e to the power minus i omega t p by 2. So, when we are trying to calculate this f of w which is nothing but equals to f into t p now we know that, when we are just trying to formulate in terms of sin and cos omega you see here means omega t p by 2.

So, we have sin omega t p by 2 divided by omega t p by 2 this is clearly giving us not only the amplitude, but also you see here the phase. So, our interest is right now, to see the amplitude of the spectrum. So, amplitude of the spectrum becomes, now the modulus form of this f of w, so f of w modulus is nothing but equals to f that is you see you know like the whatever, the force in the single pulse magnitude f into t p modulus of sin omega t p by 2 divided by omega t p by 2. So, this is you see in the Fourier series my you know like the amplitude is there in the single pulse. So, Fourier transform is real which implies that the phase spectrum is determined by this sin of this sin alpha t p by 2.

(Refer Slide Time: 17:47)



(Refer Slide Time: 18:30)



So, Fourier transformations amplitude spectrum, and the phase spectrum these two are simply you know like we are going to discuss about that, which simply says that. When you have the dimensionless frequency with respect to you see you know like delta t p, which is being incorporated in that we need to transfer the rectangular form of pulse into you know like the real form with the delta the alpha times to 0. So, the discrete spectrum for this pulse train, now in this case particular single pulse is transformed into the continuous you know like this spectrum form.

So, when we are doing these things, now you can see that we have a clear this magnitude form f w by the this modulus of f w divided by f t p, and you see here at the 0, we have the maximum amplitude. And then you see here these you know like the wave transformations are there as we are, because we know that we can characterize with the using of the you know like this sin part especially with the sin of alpha into you know like t p.

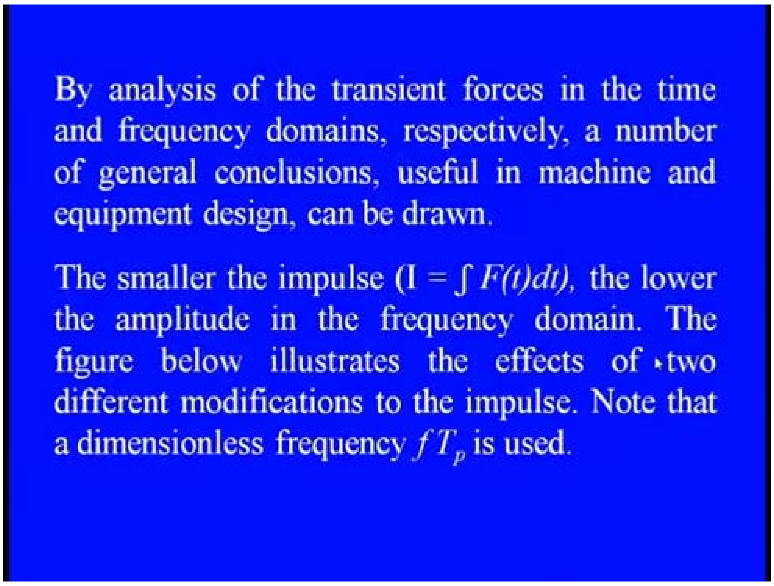
So, when we are just trying to move this omega t p, then we have you see here whatever the changes are there with the 2 pi, 4 pi, 6 pi and 8 pi, you can see there we have a clear this you know like the amplitude is 0. But, in between what are the variations are there dough you see the amplitude is clearly varying as we are moving towards even 0 to 8 pi, or 0 to minus 8 pi.

And these you see the amplitude is decreasing or when we are going with the you know this is what you see my amplitude, and when we are going to the phase part the phase is nothing but equals to you know like when we are just trying to capture. You know like for this Fourier transformation the phasor spectrum, so the phasor spectrum is clearly showing you see that, when we are just going from you see 0 to pi part 0 to pi, then you see this is you see at for every entire part this.

When we are just going from 0 to minus 8 pi or 0 to 8 pi the variation is very clear exactly, at this 2 pi you can see that the maximum phase differences are there, then they are being constant. Then again you see the phase difference from pi to 0, when we are just approaching to at the 4 pi, at the 6 pi, and at the 8 pi. So, similarly you see here at the minus 2 pi, minus 4 pi, minus 6 pi, and minus 8 pi, there is a clear phase shifting is there for these kind of Fourier transformation.

So, you know like when we are trying to simulate these two features, then we will find that the amplitude the you know like this particular part, the amplitude spectrum is clearly showing that. When we are trying to move from 0 to 2 pi means the this omega t p at the 0, at 2 pi at 4 pi and 6 pi and 8 pi, they are showing 0, but at that time there is a clear phase shifting is there in the phasor part.

(Refer Slide Time: 20:41)

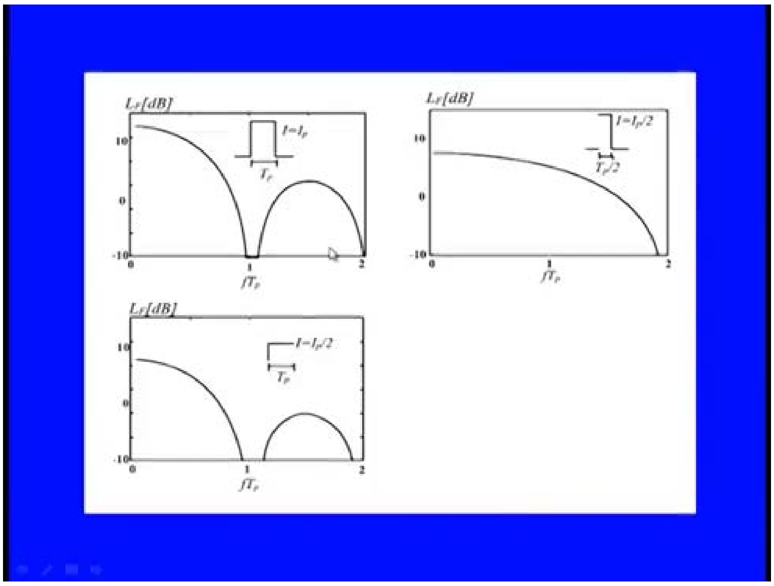


So, we can say that by the analysis of the transient forces in the time and the frequency domain, we can say the number of general conclusions can be made. And you see here through that we can clearly featured the response analysis that, you see here what exactly the response is saying in terms of the amplitude and in terms of the phasor, and that is very you know like we can say helpful information for designing part.

So, the smaller of the impulse means, when we are going for the small amplitude of the impulse part, and the lower amplitude can be clearly you know like featured out within the frequency domain. And you see you know like we just want to see the effect of the two different modification in the pulse, and what the modifications are there now we are going to use and here we are using you see the dimensionless frequency f into t p part there.

So, now, you see if you look at that, we are basically you know like focusing here that when we are trying to change, means when we are trying to modify the impulse then what exactly there. So, you look at that we have you see you know like, the sound part here, and you see this is what my frequency then a dimensionless frequency f into t p. So, as we are just moving from you see you know like with the this is what my t p feature, this t p and i equals to i p that is what my impulse feature, so when the impulse and you see the t these it has the clear relation is just like that. We can simply see that when f t p the non dimensional frequency is 1, now this amplitude whatever the you know like this amplitude is there it is just coming 0.

(Refer Slide Time: 21:30)

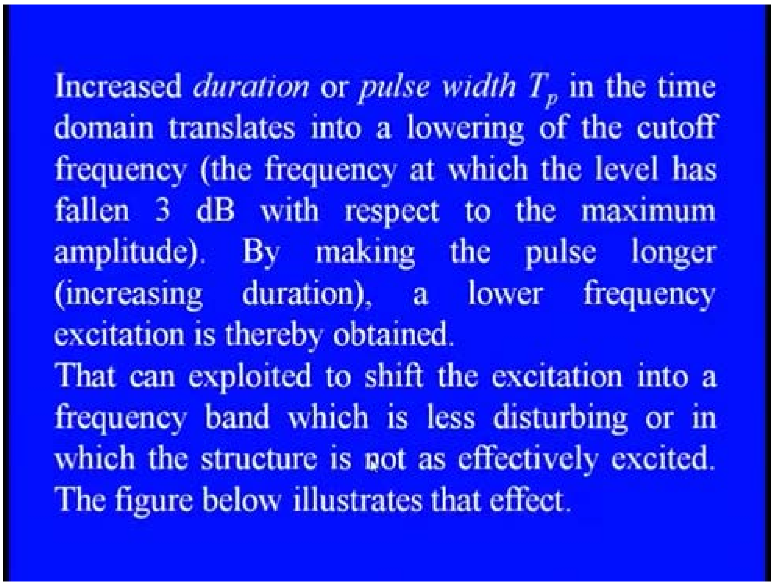


And then you see here it just remain you see for constant some time minus 10, I should say and then you see again the pulse is being there which can be again formed in this way, because of my this time period t p now you see here it becomes you know like going like that. In the second case when you can see that, when we are just going with the t p by 2 to i p by 2, so now, you see we simply bifurcate this entire signal into say the positive side where the half if t

by 2. And you see the i p by 2 is the intensity part is there of the impulse, then we can see that at f t p, you see this the you know like the there is a clear amplitude of this you see you know like the signal is. The signal is absolutely going upto the maximum the two, when the total completion is there, and when we are going with the you see you know like i equals to i p by 2 means we are simply you know like just lowering.

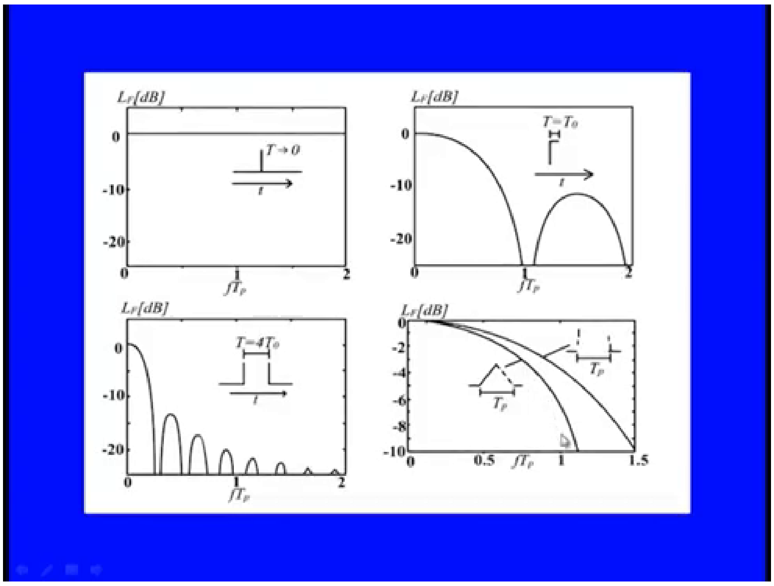
The amplitude of the impulse say I which is nothing but equals to integration of f t into d t, so this you see here when we are just trying to you know like go with this integration of f t into d t with the half i p by half. And but we are not changing the time period just like in the previous case we converted both into the half, but here we are keeping this one. We can see that you know like this amplitude is just coming down and you see here, it will be at the lower phase for maximum time, and then the another you see the peak will be formed in this way.

(Refer Slide Time: 23:45)



So, when we increase the duration or we can say rather the pulse width t p in the time domain, which simply the translates into the lowering of cutoff frequency. In generally we are just taking the frequency, at which you see the level has fallen 3 decibel you know like point with respect to the maximum amplitude, that is what we are saying that the cutoff frequency.

(Refer Slide Time: 24:33)



So, the 3 d b sound you know like the sound level, you know like this is one of the standard feature at which, you see whatever you see you know like the maximum amplitudes are being there, we can say that this is my cutoff frequency. So, by making the pulse with the longer you see you know like duration, means when we are increasing the duration the lower frequency excitation can be easy immediately obtained.

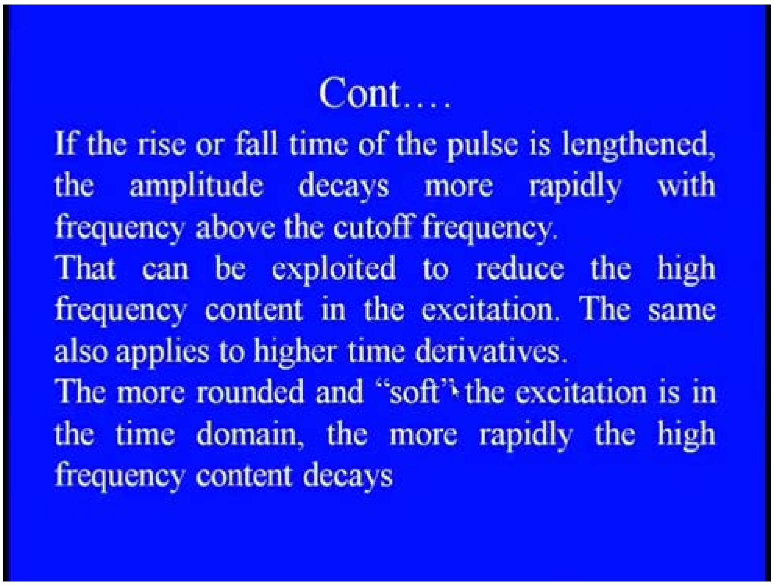
And this can be you know like and that is why you see here, when we are just going with the half of t p, you see that we cannot get that what exactly the lowered form of the exciting frequency. The total you see the blurred features are there and it is just bypassing the entire f t p part, so that can be exploited to shift the excitation in the frequency band, which is less disturbing or we can say in which the structure is not as effectively excited at the lower frequencies.

So, you see rather we can simply show that when the t is infinite, you certainly you see here you know like these Fourier series is there is nothing you see here it is absolutely the 0, but when we are now going with this t equals to t 0. That means, you see here this time strap is very short, now you can see this information is absolutely you know like coming down, and then this is clear this is clearly showing that at what frequency feature we have a excitations.

And then you see now we are increasing this time period, you see say in the feasible manner 4 times of t 0, you can see that all the number of frequency excitations are clearly exhibiting in the frequency spectrum with the variation of their amplitudes. And when you see here we are just taking that you see when we have the t p, and in this t p particularly you see when we are just trying to see that what exactly the variations are there.

Below, you see this is what you see you know like we can say 2, now we are changing only upto 1.5 you can see that it is clearly showing the variation with the time response t p. So, this is beauty of the time you know like amplitude, means when we are just going with the you know like more the increased form of the pulse width. Certainly, you see here we need to check it out that, how the information is to be transmitted into the lower form of this cutoff frequencies.

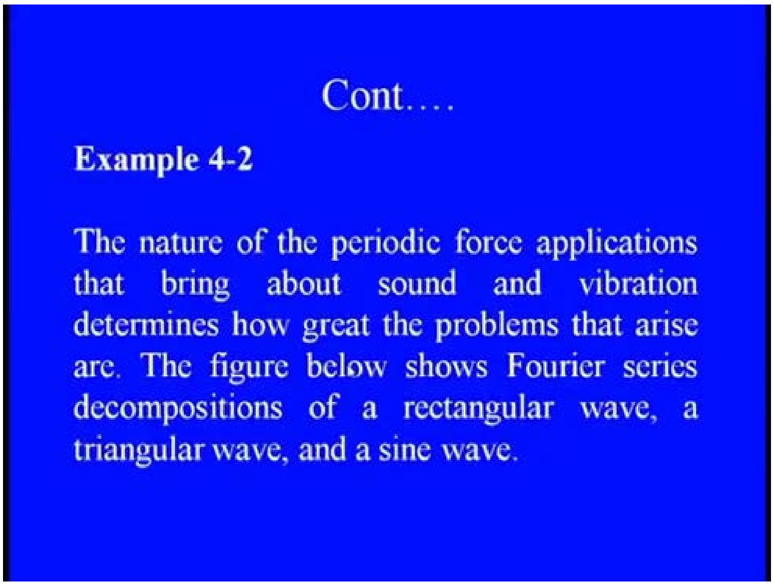
(Refer Slide Time: 26:22)



So, if the rise or fall time of the pulse is lengthened, means if we are just going towards more length of that the amplitude decays more rapidly with the frequency, and you see here we can say that at above cutoff frequency. There is a clear we can say decays are there of the cutoff frequencies, so this can be exploited to reduce the high frequency contained in the excitations, and also we can apply to higher time derivatives for that. So, we need to check it out that how you know like the rise or fall is there with the pulse, and we know that since the amplitude is clearly decaying at a faster rate. When we are just you know like talking above you see the cutoff frequencies, then we need to check it out that you know like the high time derivatives, how we can apply these things to that.

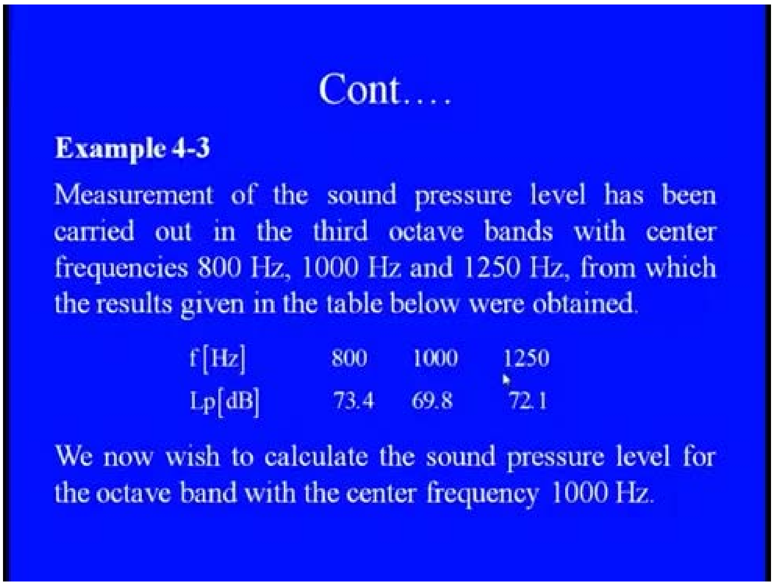
So, the more rounded or we can say the soft excitation in the time domain, more rapidly the high frequency content decays are there, so if you do not have you see you know like clear peak of excitations, we when we have the rounded form is there. Then certainly we can simply you know like justify that it is you know like, the fallen feature in these amplitude is more rapid you know like in terms of you see the high you know like frequency contents.

(Refer Slide Time: 27:36)



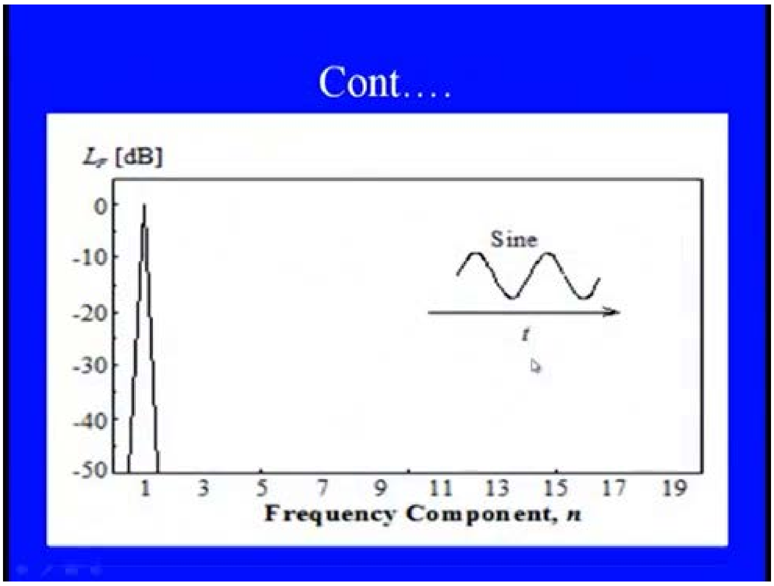
So, now, we are going to the another example, in this you see here the nature of periodic force applications you know like, just we can we can simply apply this force application that bring about the sound and vibration determination, how great the problems that arises are. So, now, you see we are simply showing the Fourier series decomposition of the rectangular wave, a triangular wave and the sine wave.

(Refer Slide Time: 27:57)

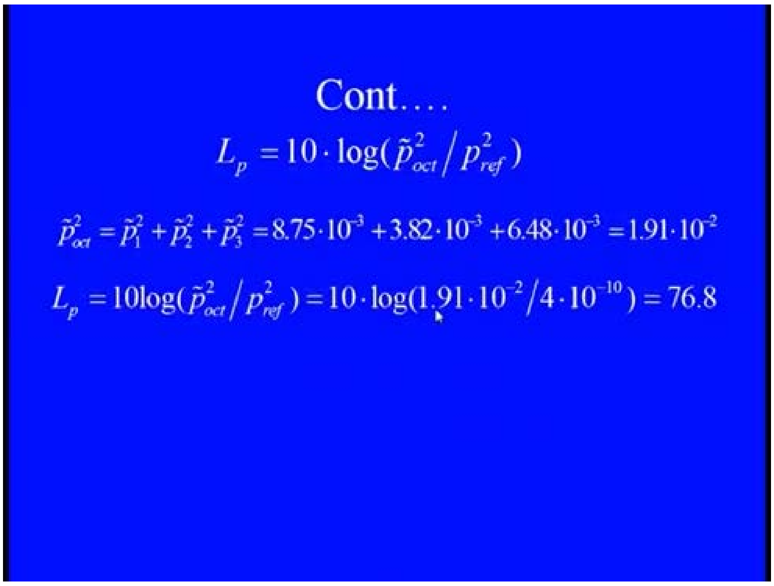


So, you can see that this is you see the first is like the this square wave is there, and this is what the feature of this square wave, and now we can see that what the variation of the amplitude is there with respect to the frequency component n. So, this variation is clearly you see you know like at all these component, upto say you know like the nineteen component these are the variation in the amplitude, when the frequency this waveform of this you know like propagation is the square wave, and when we have the triangular wave. The triangular wave feature is like that you can see that even for all the 19 exponent what the variations are there, when the decay is there of the amplitude, this is also decay, this is also decay.

(Refer Slide Time: 28:43)



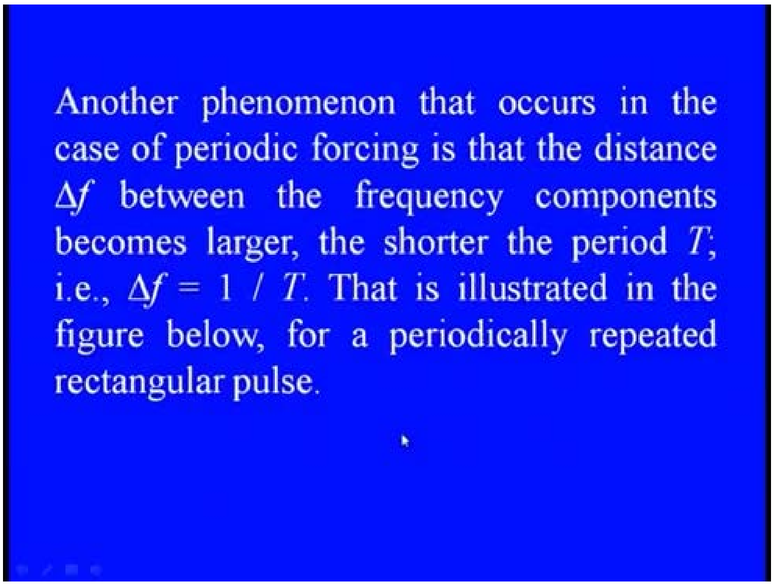
(Refer Slide Time: 28:53)



And the third form, when we are just taking you see here the sine wave, then this is you see here the frequency component 90 and you see this variation is there. So, the amplitude of these over tuned decay more slowly for rectangular wave, as we can see that you know like 1 by n is there for the decaying part, where the n is nothing but the n-th frequency component. Then for the triangular wave where the decay is 1 by n square, so this is the pretty clear picture is there, that when we are just looking to the rectangular or you see the square wave.

And when we are talking about the triangular wave, the decay is you see this is what this is what the drastic decays are there, because the dependent feature is 1 by n square, where you see the decay is very slow, because the dependent feature here. In this square or triangular wave is 1 by n, and because the over tuned you know like often fall in more disturbing this frequency band we can say it is a good design principle, which always make force you know like application.

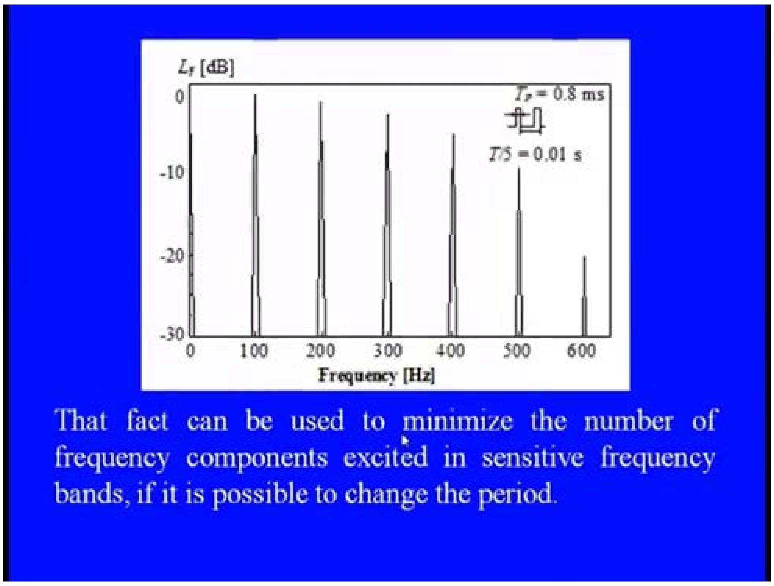
(Refer Slide Time: 30:15)



As, a soft or even more towards the sinusoidal feature of the decaying part in the you know like, all these variations with the time domain. So, when we are going towards that you can see that when the sine wave curves are there, they are the soften part, and they are simply showing a stream lined motion along the path. So, we can say that you see when we have the sinusoidal feature this is a clear peak of excitation clear tone is there, exactly at these particular you know like the dedicated frequency component, and the variations are alike that.

So, another phenomena that occurs in the case of periodic forcing is the distance is delta between the frequency component, and when it becomes larger the shorter period t can be clearly showing that, you see what exactly the exciting peaks are. So, we can say that for a periodically you know like the repeated rectangular pulse, we can simply featured out that you know like, when we have this square. When we have the triangular, and when we have this sinusoidal wave, what kind of you see the excitation peaks are being there.

(Refer Slide Time: 30:49)

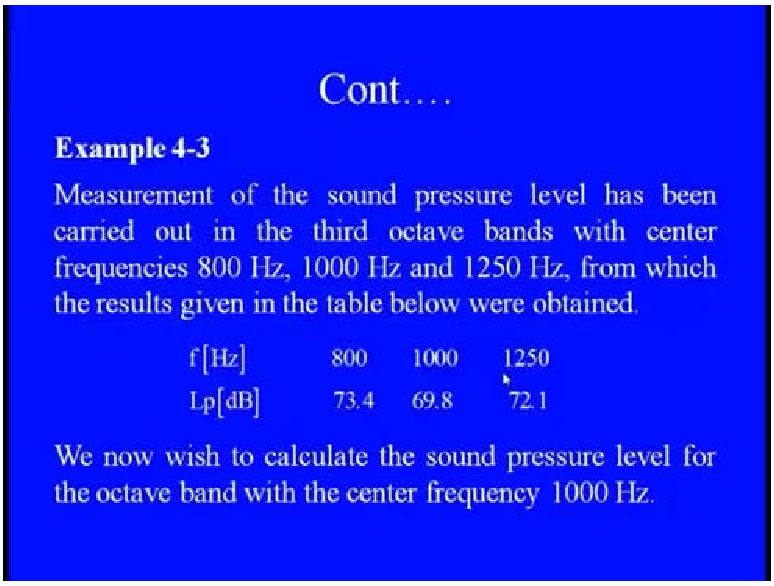


So, the fact that you see we can use to minimize the number of frequency component excited in the sensitive frequency band, only we need to check it out you see what exactly the peak forms are there. So, now, you see right now we have this t p is nothing but equals to 0.8, you know like the millisecond, and you see here we just want to find out you see for this t by 5 is nothing but equals to 0.01 s.

So, now, you can see that at the frequency, whatever the you know like the exciting frequency we have a clear tone, and you see here the sensitivity of these frequency. You know like this sound with the frequency components can be clearly showing, the kind of exciting frequency component n. So, you see here that can be straightaway show that what the exciting frequencies are there, and how we can get that part, so this was you see the example in which you see it was clearly showing that when you have a different frequency component.

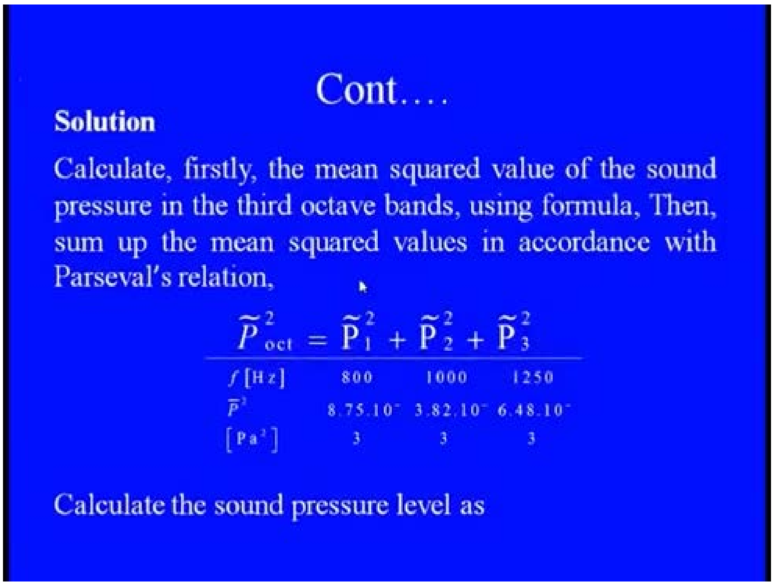
Like you see when you have a rectangular or the square wave or the triangular wave, and the sine wave then how the features are being there, in terms of you see you know like the amplitude and the corresponding exciting frequency. So, this is you see all the diagram they are clearly showing, now in the next example we are taking the sound pressure level.

(Refer Slide Time: 31:59)



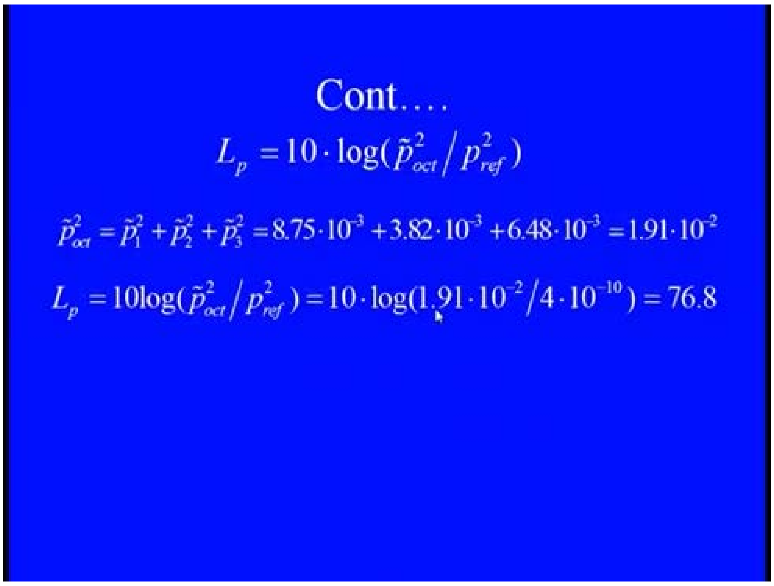
Which, is being carried out you see you know like always at the third octave band with center frequency 800 Hertz, 1000 Hertz, and 1250 Hertz. So, whenever we are trying to see that you see, when the octave bands are there what is my central frequency, and then what us corresponding you see the sound power is means in terms of d b. So, when we are saying that when we have the central frequency say 800, so the sound power in this, the sound pressure level is you see the 73.4 decibel, when we have the this central frequency 1000 Hertz, and if we have a 69.8 d b, 1250 hertz my central frequency, correspondingly 72.1 is my sound pressure level d b. Now, we just want to calculate the sound pressure level, for octave band with the central frequencies if we have the 1000 Hertz.

(Refer Slide Time: 32:51)



So, certainly here we need to just go first that what, mean squared value of my sound pressure in the third octave band, and then you see here we can sum up the mean squared values in accordance with the you know like, those Parseval relations. So, we know that the Parseval relation says that, this p square you know like these octave band is nothing but equals to p 1 square. If we have you see the three values so; that means, the 3 sources are there, so p 1 square plus p 2 square plus p 3 square.

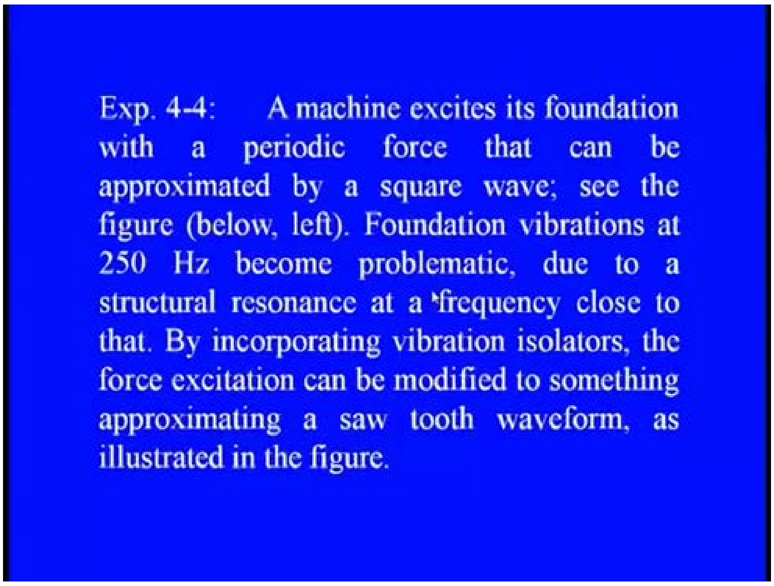
(Refer Slide Time: 33:40)



So, first of all we need to find out you see, what the square terms are there with this, so we have you see the frequency at 8000 and 1250, the p square is clearly showing that it is you know like the 8.75 into 10 to the power 3 3.82 into 10 to the power minus 3 and 6.48 into 10 to the power minus 3.

So, you see now, we can simply calculate the sound pressure level l p, which is nothing but equals to ten into log of p octave square divided by p reference square. And when we are you know like calculating the p octave that is nothing but equals to p 1 square plus p 2 square plus p 3 square. So, this we can simply find out the 1.91 into 10 raise to power minus 2, and when we are keeping this now with the p reference that, was you see 4 into 10 is power minus 10. So, now, we have the 10 log 1.91 into 10 is to power minus 2, which was being calculated as p octave square, and p reference square is 4 into 10 is power minus 10. So, we can get you see the some pressure level 76.8 d b, so we can calculate the you know like this part specially, when you have you see all the frequency levels together there itself.

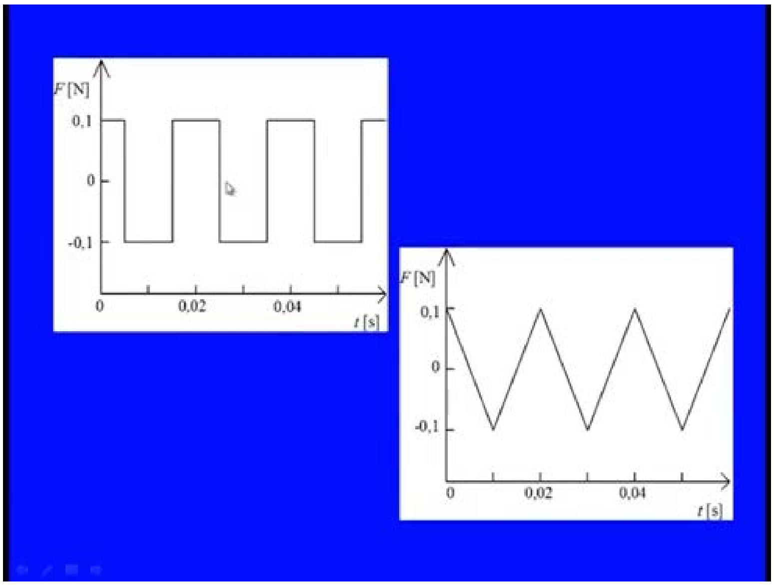
(Refer Slide Time: 34:37)



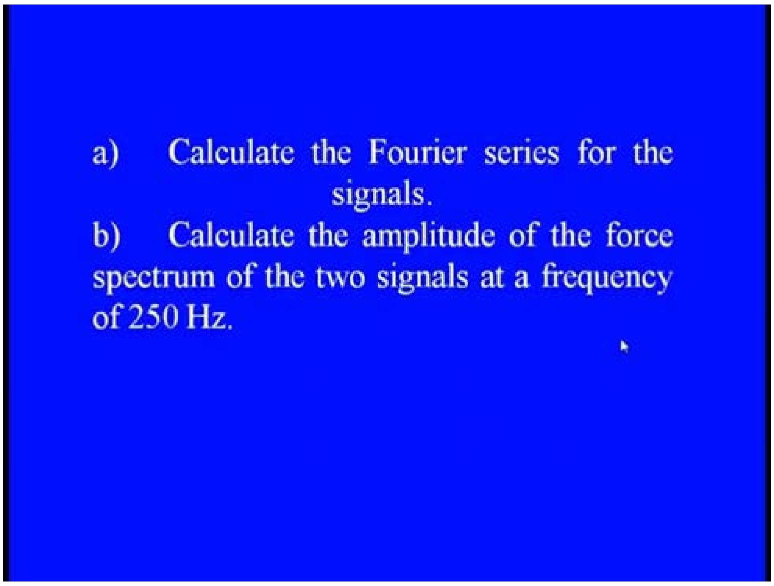
Now, in the last example we would like to see that when we are just you know like exciting the machine, and how we can design the foundation specially with terms of you see you know like the isolator. So, machine with its you know like foundation is you know like just exciting the periodic force, and we can approximated this by the sine by the square wave.

As, we are going to show you that and foundation vibration at 250 Hertz becomes the problematic, so now, this is my threshold limit that I need to go upto 250 Hertz like that when I am going to 250 Hertz. Then the problematic features are being there, due to structural resonance at the frequency close to that 250 Hertz by incorporating the vibration isolator. The force excitation can be modified to some of the approximation a saw tooth waveform, so now we are changing the waveform from the square to the saw tooth waveform.

(Refer Slide Time: 35:31)

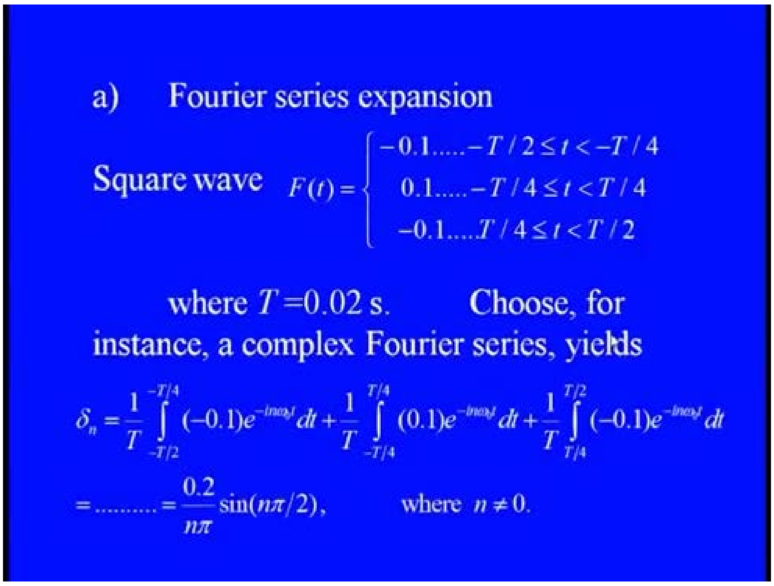


(Refer Slide Time: 35:53)



So, you can see that this is what you see here, the two main forms are there the waveform is this you see here, where we have the squared feature, and this is what my you know like the time, and this is the amplitude variation of the forces are. And when it is being saw tooth feature, then again you see here this is my another featured of when we are simply adopting the vibration isolator. Now, we need to calculate the Fourier series for these signals, and also we need to calculate the amplitude of the force spectrum of the two signals at 250 Hertz.

(Refer Slide Time: 36:07)



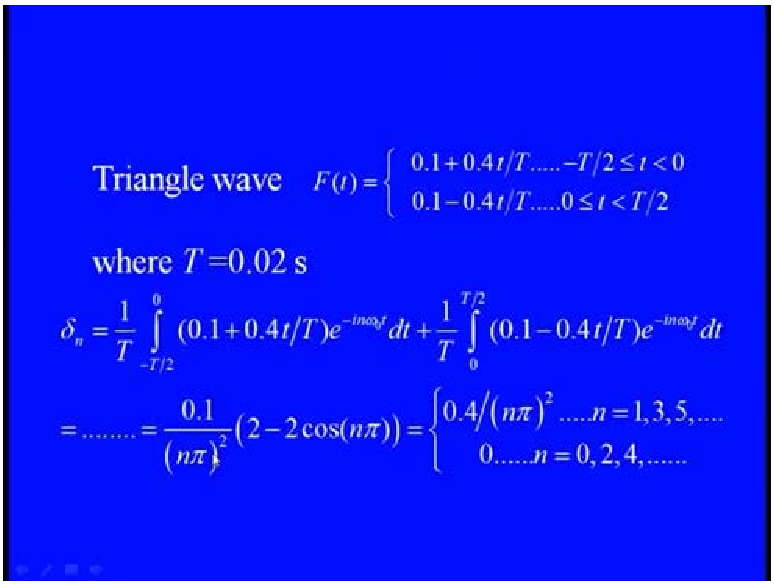
So, first the Fourier series expansion for the square wave, so when we have the square wave we know that the f of t is equals to you know like 0.1 t in between minus t by 4 to plus t by 4. As, you can see that this is what it is when I am saying that my t is this, so minus t by 4 to plus t by 4, you see here in this entire you know like we can say the spectrum, we can get you see the pure form of you see that the total information of this Fourier series component.

So, if I am just talking about minus t by 4 to you know like t by 4, I have f of t 0.1, and when I am going below means minus t by 2 to minus t by 4 then it is minus 0.1. And when I am just crossing this means t by 4 to t by 2, then you see here, I have minus 0.1, so this is what the variation is there in the square waveform, when i am just you know like crossing the right from 0 to minus t by 4 and plus t by 4. It is just 0.1, and then before that below and after we have minus 0.1, as f of t because you see we have the t as 0.02 second.

Now, we need to take the any instance and we need to calculate the complex Fourier series, so we are saying that we need to just go with the coefficient delta n. As, discussed already it is nothing but equals to 1 by t integration of minus t by 2 to say minus t by 4. The first one I know that my f of t is minus 0.1, so I can keep you see here minus 0.1 e to power minus iota, and omega 0 t into d t, the second 1 by t now it is from minus t by 4 to plus t by 4 and you see here it is the positive f of t.

So, we have 0.1 e to the power minus i omega 0, and t into d t plus now you see here, now we are going towards further part right from the t by 4 to t by 2. Then it is you see again f of t is minus 1 into the power minus iota n omega 0 t into d t, so when we are summing of all these things, now we have delta n which is coefficient is 0.2 divided by n pi sin of n pi by 2.

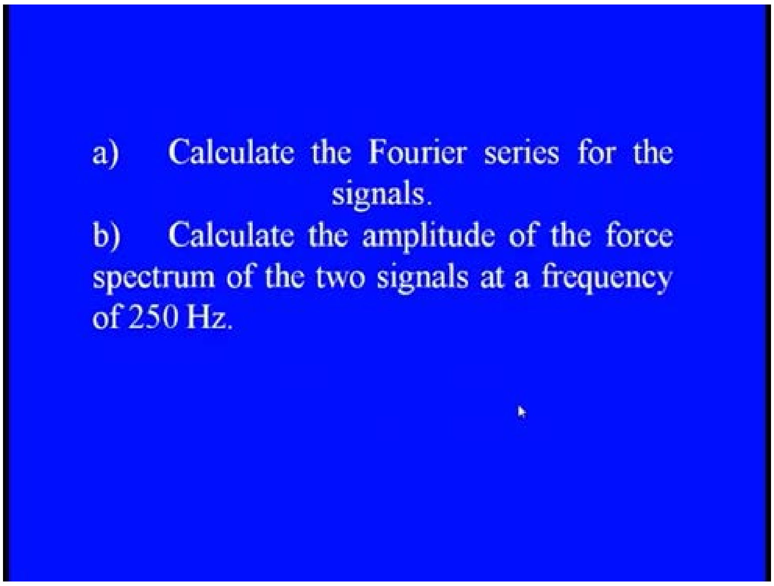
(Refer Slide Time: 38:33)



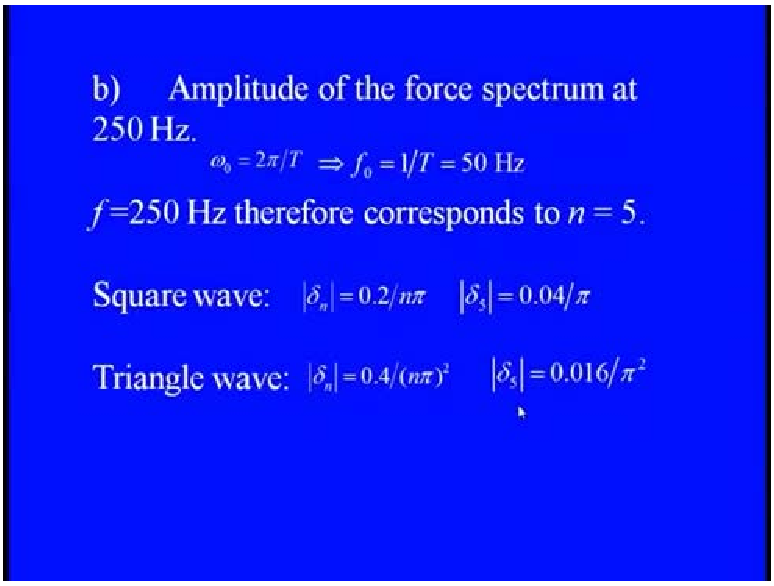
So, these you see the variation of these particular we can say the complex form when we are considering the square waveform, now if you are going with the triangular waveform then certainly I have the f of t the Fourier transformation is 0.1 plus 0.4 t by t. When, we are just going with minus t by 2 to 0, and when they are going from 0 to t by 2, now this is what my width for the triangular waves are, then it is 0.1 minus 0.4 into t by 2, so with this t 0.02 second.

Now, we can calculate you see both the things in my coefficient, the complex coefficient, so dealt n is nothing but equals to 1 by t integration of minus t by 2 to 0.1 plus 0.4 t by t e to the power minus iota omega n omega 0, and t d t plus 1 by t. Now, 0 to t by 2, so now, it is 0.1 minus 0.4 t by t, into you see we can say that e to the power minus iota n omega 0 t into d t, so when we are trying to do these things now, we have you see the two main features. When, the n is you know like the odd number, then we have 0.4 divided by n pi square, and when the n is of even number 2 4 6, then the delta n is equals to 0.

(Refer Slide Time: 39:50)



(Refer Slide Time: 40:05)



So, this is what you see the first part in which we can simply calculate the Fourier coefficients, the Fourier series particular for both the signals, when we have this square part and when we have the triangular part. Now, the second is we need to calculate the amplitude of the force, you know like the force spectrum for both the signals.

So, now, we are going to this the amplitude of the force spectrum at 250 Hertz, so first of all we need to go that what is the exciting frequency for this. So, omega 0, which is being there you see you know like 2 pi by t or the t is given as 0.02 second, we have the exciting frequency 50 Hertz, so we know that, when you have the natural frequency 50 Hertz. So, certainly at 250 Hertz the coefficient this Fourier coefficient is n equals to 5, and when it is there you see now we can keep and we can get the for square wave, the delta n is nothing but equals to 0.2 divided by n pi.

And delta 5 is nothing but equals to when we are keeping you see n, then it is 0.04 by pi and for triangular wave which is nothing but delta n modulus is nothing but equals to 0.4 divided by n pi square, we can say delta 5 is 0.016 by pi square. So, this is all about you see you know like this particular chapter in which you see we simply analyzed the information, whenever we are capturing, through our sensors in terms on the time domain.

And you see the information is not periodic one, which is one of the specific you see you know like, we can say application for that, when the machine is running all the time we cannot get you see you know like the periodic form. The turbulence or the non periodicity is there, then we need to take the Fourier transform we just pick that part we need to featured out, with the using of you know like this coefficient alpha, when we need to go with the lower value of alpha.

So, that we can you know like in incorporate the broader band of the frequency, and then we can just take the this infinite decimal step at that point, wherever you see this abrupt changes are coming. And we can simply get the Fourier transforms, for that particular value in terms of you see whether our sine wave, our square wave, triangular waves are there of the wave feature. Now, in my last lecture I am going to discuss about the filters, that how we can select the filters, what exactly the basic you know like these featuring parts are there in the filters. And then you see some of the numerical problems, you know like again for you know like the vibration measurement, and the control part.

Thank you.