Vibration Control
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Module - 7
Principles of Active Vibration Control
Lecture - 10
Numerical Problems

Hi, this is Dr. S. P Harsha, from Mechanical and Industrial Department, IIT, Roorkee, in the course of Vibration Control, we are mainly discussing about the Principles of Active Vibration Control. In which, we discussed about the basics, what the basic principles are there also we discussed about various materials, which are being involved you know like in active vibration control.

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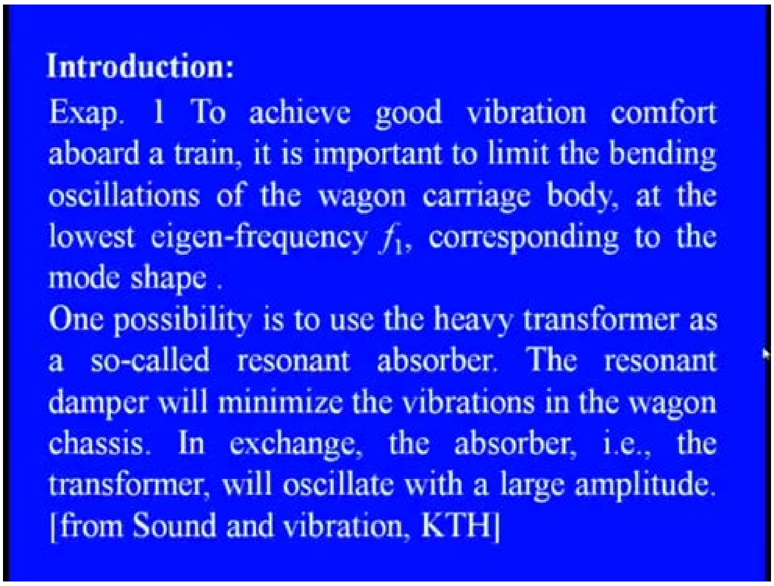

In which you see here with using of these smart materials or intelligent materials we can say that, we can achieve the you know, whatever the required purpose is there for the active vibration control. Means whatever the vibrating masses are there that, can be straightaway suppressed out with the using of these smart materials. So, it started from these piezoelectric materials, then we discussed about the ER electro-rheological fluid, magneto-rheological fluids, we also discussed about the electro and magnetostrictive materials.

And in that you see here, we discussed about the shape memory alloy, which has some you know anthropomorphic these features are there in their material. Even up to the plastic nature means plastic deformation, they can be recovered themselves into their original shape, and then you know in the last lecture we discussed about the special featured the electromagnetic dampers.

And we found that you see all these right from the materials to these, you know like these devices they can be straightaway act you know like as the damper. Means you see they can you know like added their viscosity by adding this magnetic, or electrical fields or even you see they can even sense those part, and then they can you know like go towards the actuation part.

So, the meaning is very simple from these materials or these you know like we can say are the devices or you know like the integration of the sensor actuator, and the control unit. We can suppress the vibration by introducing the force or the damper or anything like that, so we discussed you see you know like the two broader categories in entire vibration control mechanism.

At the passive vibration and active vibration, we solved some of the numerical problem, based on you see you know like the insertion losses and you see what the impedances are there. What the mobility is are there, and how we can design effectively the isolator, or the absorber against the continuous loading or the shock loading. So, today you see we are going to discuss about the second part of the numerical problems in the active vibration control.

So, here you see we have the first problem, which simply indicates that you know like absolutely based on the this wagon part, in which you see when the train is moving we know that even at the lowest frequency. We have you see the huge vibrations, which can be even transmitted to the platform and there is a huge noise together. So, you know like we will just try to see try to formulate the problem that, you see what exactly the things are being there, and how we can resolve the issues.

This problem is mainly taken from the standard book, which was adopted for the entire course development under the MOU between the IIT Roorkee, and the KTH Sweden, the fundamentals of sound and vibrations by the KTH book. So, if you will see that this problem, it is just saying that to achieve the good vibration comfort when we are aboarding a train, it is important to limit the banding oscillation of the wagon carriage body.

So, you see the entire carriage body, when you know like we see that we just want to you know like control the vibration, there is one mode in which you see you know like the banding operations are there. And at the lowest frequency if we are saying that the Eigen frequency, rather the natural frequency is F 1 which corresponds to this banding mode shape. So, now you see here when we are just trying to see that, what kind of you know like the banding these wagons are there.

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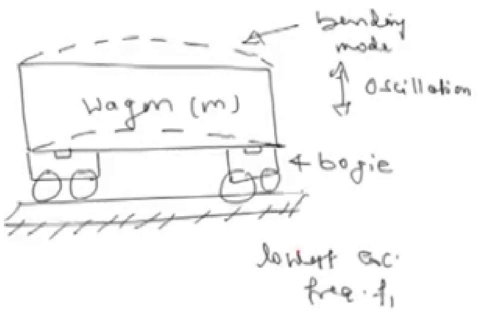

So, if you are saying that, this is what you see my wagon is you know like, and in this wagon, in this part you see here, if I am saying this is you know the wagon part, which has some mass say m. And when you see it is in the mode of say you know like the banding part, so we can say you see the this is what you see you know like the kind of mode shape, which can be there in this part.

So, this is I am saying that you see you know like when the oscillations are there, in that part this oscillatory feature is there we can say this is you see the banding mode. And in this banding mode you see here the entire wagon is absolutely, when it is oscillating you see the entire wagon is you know like in this particular phase, in which you see you know like it is being oscillated.

When, we are saying that this is being attached to say this is what you we have the bogies, so this is you see the bogie part is there, and this is being the mounted feature of the bogie. And below the bogie we have you say these our wheels are there, when I am saying that you know like this is are the rails, on which you see these you know like the wheels which are being moving.

So, say this is absolutely you know like grouted towards the, we can say the suspension system there in the soil or something you see, so this is you see the entire part, in which we can see that you know like these bogie part is absolutely being attached there. In which you see we have you see you know like both the kind of suspension, this primary suspension system and the secondary suspension system is there.

And this is you see you know like the this is what you see our problem, and we are saying that this banding mode is occurring at this lowest exciting frequency, we can say this F 1. So, this mode which corresponds to you know like, we can say the lowest frequency is absolutely with the banding mode, so we have you see this banding mode. Now, we are trying to resolve that this is you see the exact configuration of this problem, now we want to suppress this one, so when we want to suppress the vibration which is being there you see.

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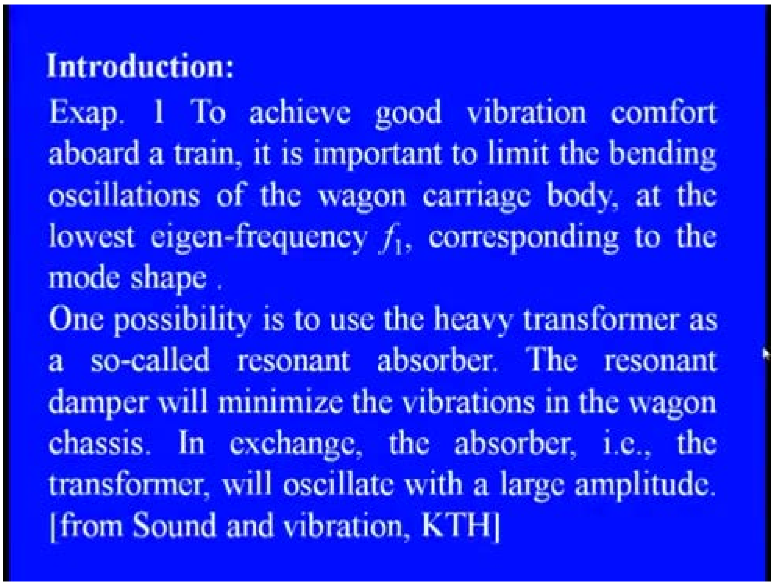

You know like creating by this movement under, you see the banding mode of oscillation, then we can say one of the possibility even in the question it is given clearly, one of the possibility is to use the heavy transformer, as, so called the resonant absorber. So, in one of our lecture we discussed that, if you are adding say attached mass or the seismic masses is, then we can reduce or we can suppress the vibration.

So, here in the question they are saying that one possibly, which we can use you see as a vibration absorber is the heavy transformer or we can say some added mass is there. And this resonant damper will minimize the vibration, in the this chassis, wagon chassis and in exchange the absorber that is you see the transformer will oscillate with the large amplitude.

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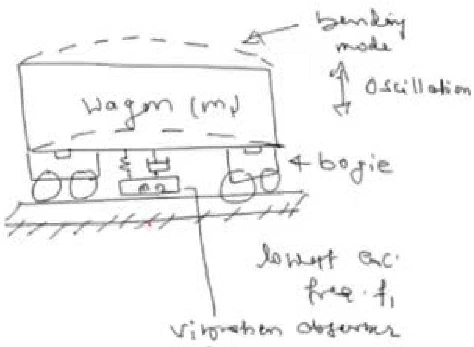

So, you see here you know like either of that you side effects, when we are talking about this, so now, if you are going back to our system, we know that now we need to add a kind of you see you know like the transformer. So, we can say this is what my you know like the transformer part in which you see the spring, and you see this damper is there and this is being attached to a mass.

So, if I am saying this is my mass m 1, I can say this is my mass m 2, and you see here these spring dampers are there, so this is what my we can say this vibration absorber, or we can say this is nothing but equals to the heavy transformer. So, this heavy transformer is being now attached, you know like with this we can say the banding mode of oscillation and this below part, there we can say that by adding these things can we suppress with the use, it can we suppress the vibration with the using of this spring and the damper.

So, now you see here this is you know like this our entire part, which basically we would like to discuss in our question, so this is what our problem is you know like again you see here we can repeat out this problem. What we have, we have you see you know like a kind of wagon this carriage body, which is under you see you know like the banding mode of oscillation.

And in that you see here when it is being you know like operating, under this situation where the banding mode of oscillation is there, we can say we have the exciting frequency F 1. And then this lowest exciting frequency can be you see you know like, absorbed with the using of this heavy transformer, which we are saying that the resonant you know like this absorber, and then you see you know like the entire things are there.

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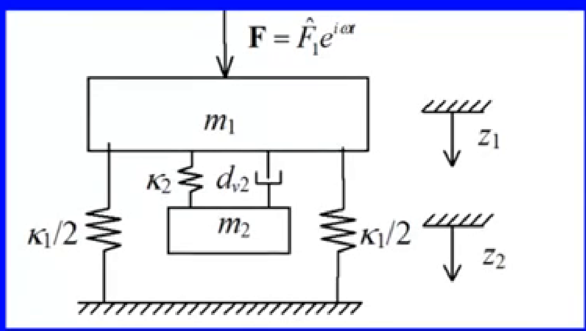

Fig.2 Model of the wagon with a absorber

So, now you see here now we would like to put, this into you know like the spring mass damper system, so if you look at this part, we have the same situation here the exciting frequencies are being there you see you know like the exciting forces are there. You see the f, now this m 1 is the mass, which we discussed about the wagon part, so this is my entire wagon.

This wagon is basically associated with the 2, we can say the bogies part, so this is you see the bogie, where the suspensions are coming, so you know like the suspension. We are saying that this is you know like the symmetrical feature absolutely you know like arranged in a in perfect way, so we can say the K 1 by 2 and K 1 by 2 the suspension is coming from the bogie part.

In between when we are saying that we have the heavy transformer, or we have you see you know like the resonant damper we can say this part which is used as the absorber have mass m 2, this has the mass m 2 and you see whatever the spring and damper features are there, we can say it is K 2 and d v 2.

Now, we know that when this is being excited, the entire things are being excited say it is a two degree of freedom system, because one mass which is the entire wagon mass is you know like moving with the z 1 oscillation. It is what my displacement feature, and m 2 mass is being you know like, we can say oscillating at z 2, so this is you see you know like we can say the modal of the wagon with the absorber, in the spring mass damper feature.

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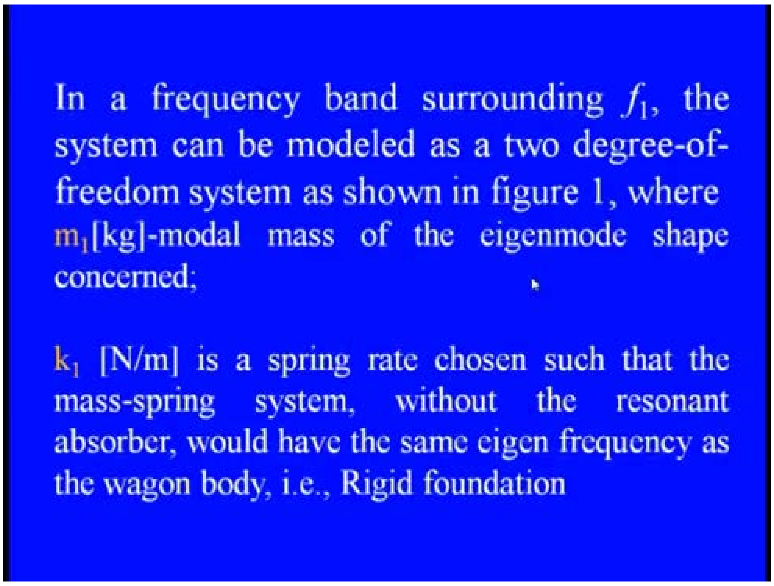

Now, we would like to you know like again go back to the question, they are saying that in the frequency band surrounding to lowest exciting frequency in the banding mode F 1 the system can be modeled as the 2 degree of freedom system. As, we discussed there where m 1 is the modal mass of Eigen mode shape connected with this part, and the K 1 is the spring rate chosen.

Such that the mass spring system without the resonant absorber, would have the same Eigen frequency as the this wagon body; that means, you see we are saying that whatever the foundation, which we have is a rigid foundation. And clear transformation the transmissibility is there from that, and that is why you see here we can simply put the analogue as K 1 by 2 and K 1 by 2 in both the side.

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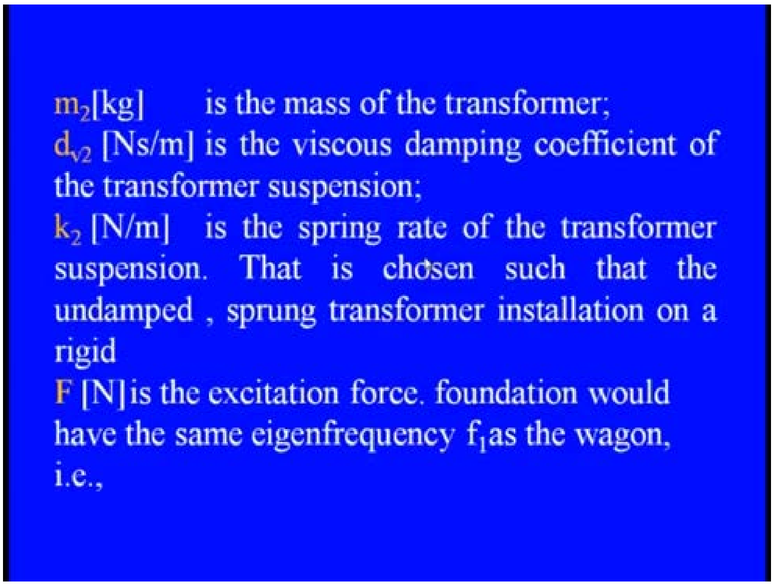

M 2 is the mass of transformer, which is simply being acted as the absorber d v 2 which we need to add towards you see the transformer feature. So, it is the viscous damping coefficient with the transformer suspension, and K 2 is the spring rate for the transformer suspension. So, we can say that you see the transformer, which is absolutely you know like if it is we are saying that the transformer is just suspended undamped part, so we have only the spring.

And if we are saying that now this is the entire suspension features are there against shock absorbing feature, then we need to add the damping and the spring together. The F is the excitation force and we know that when the train is moving it is absolutely we have the banding mode of oscillation, and the responsible figure for that is the capital F that is the excitation force. So, this is these all the parameters, which is being there you see in formulation of the problem, so when we are saying this is you know like the rigid foundation is there.

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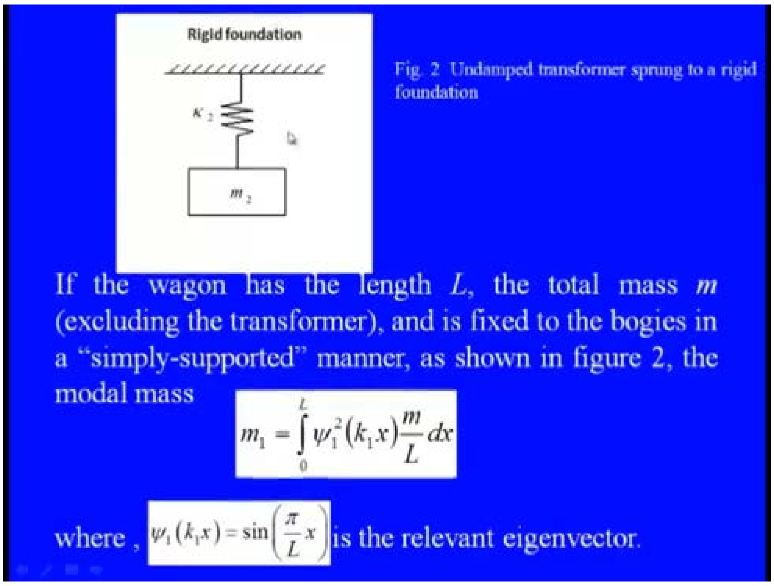

So, we can say that you see you know like for undammed feature, we have K 2 and m 2 only that we can straightaway say that if you know like, just it is being acting only at the undamped part, no shock absorbing is there. We can use this spring as you see one of the suspension feature with the added mass like for heavy transformer, now the question says that if the wagon has the length L, and the total mass m is there when we are just you know like excluding the transformer.

Then it is being fixed to the bogies in the simply supported manner; that means, you see here now we are saying that, if the added mass is not there. Then initially this entire bogie can be treated as the single mass m with the length L, and you see here since it is being supported at the bogie part, the entire wagon we can say that this can be treat as this. We can say simply supported beam, so this what you see the two supporters are there from the bogie, the entire mass is being you know like. Since, it is a rigid foundation, if they are simply you know like we can say exactly joined at this point, and this is the entire you know like means of the length.

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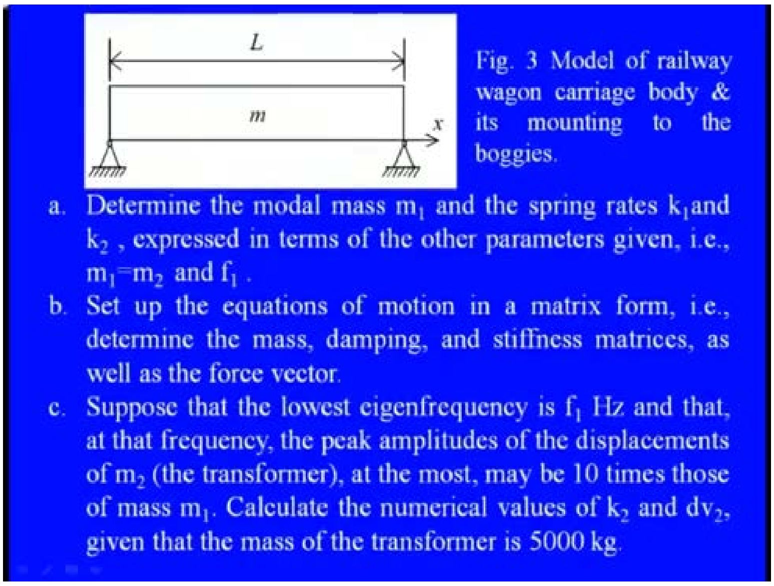

And they are just moving in the forward direction say the x is the degree of freedom, in which the movement is there. So, when we are considering this configuration without adding the mass, we can say you know like this total m 1 the modal mass is nothing but equals to the entire, you see whatever the distribution of the mass is there with the mode shape.

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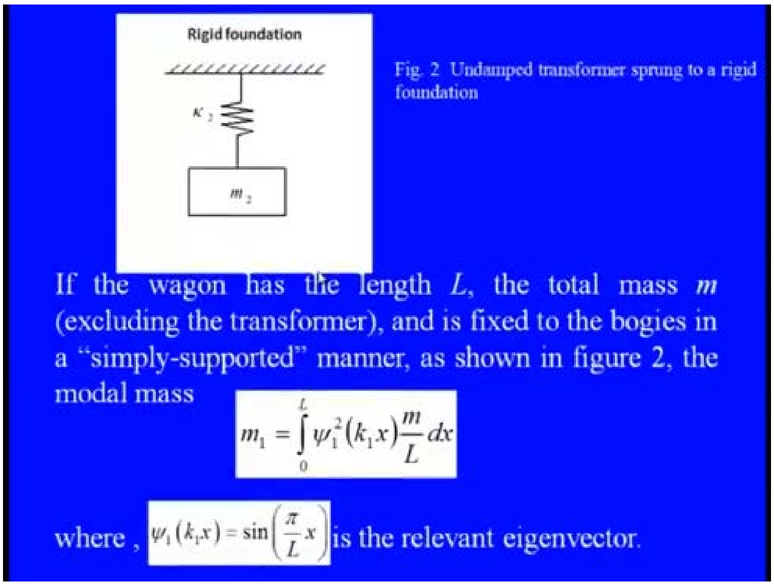

So, this 0 to L integration pi 1 square K 1 x into m by L into d x, so this is you see one of the simple configuration, when the distribution of mass is there along with the this mode shape. Where pi 1 K 1 comma x is nothing but equals to sin, because it has a sinusoidal feature, because the excitation feature is also the periodic nature, we can say pi 1 K 1 x is nothing but equals to sin of pi by L into x. So, it is a variation, so we can say this pi 1 K 1 x is nothing but equals to the relevant Eigen vector.

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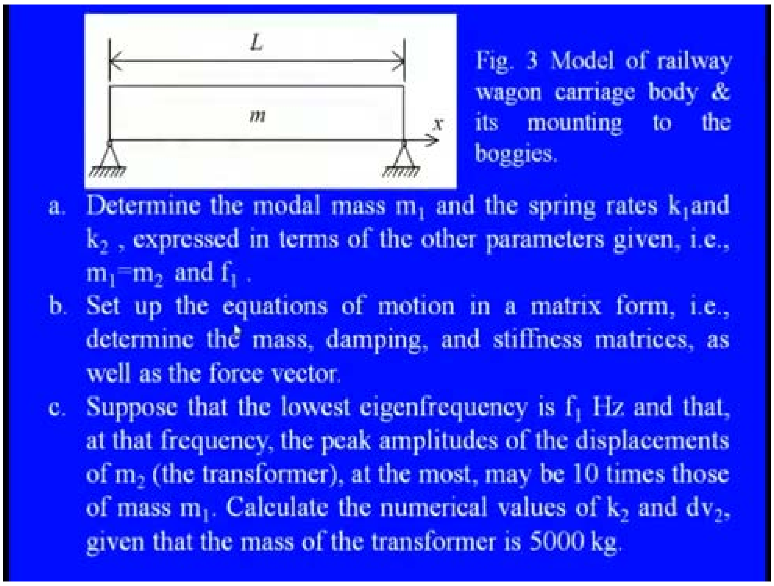

So, now you see the question say's that we need to find out the modal mass m 1, the spring rate K 1 and K 2, and we need to express these in terms of the parameters that is m 1 and F 1 is there. You see accordingly setup the equation of motion, in the matrix form; that means, you see here we need to find out the mass damping stiffness matrices with the force part, in the you know like we can say the matrix formation.

So, we can say we need to use the steady space variable, and we need to find out that, the third part saying that suppose the lowest frequency is F 1, which we discussed. And at that frequency, you see the peak amplitude of the displacement a of m 2, means the mass transformer at most we can say it is a ten times those to m 1. Then we need to find out you see you know like what the K 2, and d v 2 is there, when the transformer mass is 5000 kilo gram.

So, you see here, there is one condition where we can say that the peak amplitude of the displacement of the added mass or we can say the transformer is maximum is the 10 times of the even the mass m 1. Then what are the values of these K 2 and d v 2, the associated damper and you see this spring rate, when the total the mass of the transformer which is m 2 is 5000 kilo gram, so now, we are starting the problem.

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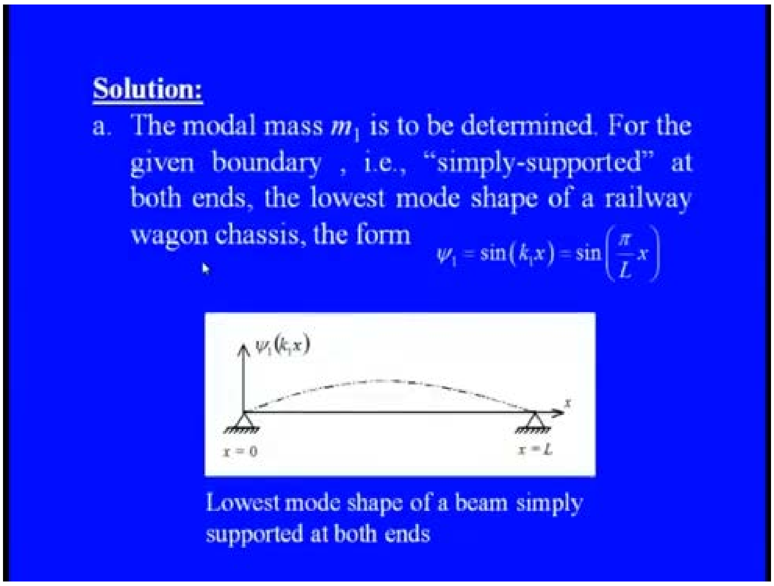

So, we need to first go that you see since the modal mass m 1 is supposed, to be determined, so for that we need to just go that what exactly the configuration is. And in the question it is clearly given that, the entire mass m 1 is distributed all across the 2 wagons or 2 bogies, so we have the two supported which can be acted as the simply supported beam. So, you see here the these simply supported beam starting from x equals to 0 to x equals to 1 x equals to L, so this is the total length is given to us, and we know that the exciting part is in the banding mode.

So, at the frequency F 1 we have you see the mode shape is psi 1, which is nothing but equals to the sin of K 1 x, so we have you see this is what you see the lowest mode of the entire banding, which is clearly showing that, this is the banding mode of this part. And then when we are showing these things, we can simply find out the lowest banding mode of these you know like railway wagon with the using of this psi 1 is nothing but equals to sin of pi by L into x. We can say this pi by L nothing but equals to whatever the deforming feature is there we can say it is nothing but equals to sin of K 1 into x.

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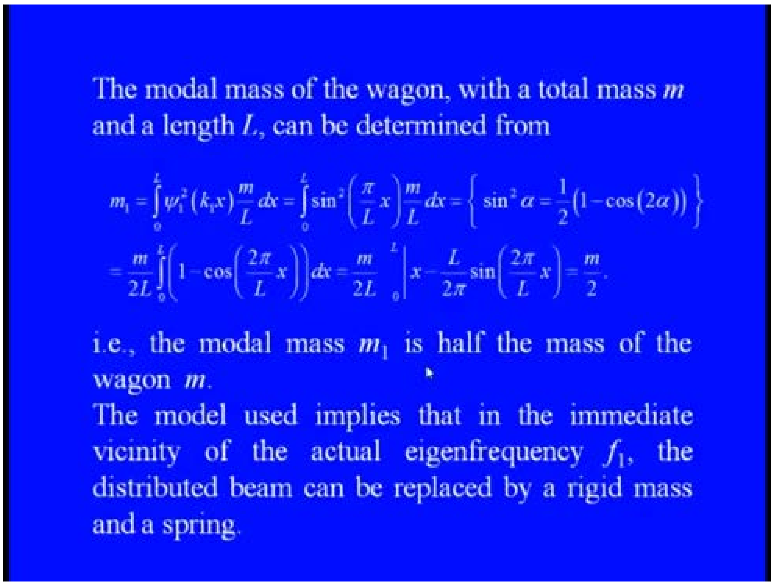

Now, we know that we can straightaway find out that, what is the modal mass is there of the entire wagon, so m 1 is nothing but equals to integration, which was given in the question also integration of 1 to L pi 1 square K 1 x into m by L d x. We can put this pi 1 as sin of pi by L into x, so we can say that even it is simply integration of 0 to L, means the integration of the total length, where you see the mass is being distributed in the wagon.

So, integration of 0 to L sin square of pi by L into x into this m by L into d x is given, now we can convert this sin square into cos of 2 x, which is nothing but equals to 1 minus cos of 2 you know like we can say x divided by half. So, when we are keeping this we know that it is nothing but equals to m by 2 L, which is the constant features m is constant two and L is constant. We can take out of them and now we can integrate the entire features, where you see the mode is being you know like the entire molecule, or entire fibers are under the this banding mode is there.

So, we can say it is integration of 0 to L one minus cos of 2 now two is there cos 2 L is there, so cos of 2 pi x by capital L into d x, and when we are integrating this. And when we are keeping when we are putting those limiting values 0 to L, we know that this m 1 is nothing but equals to m 1 is the modal mass of wagon. So, it is nothing but equals to m by 2, so we can say the modal mass of wagon m 1 is the half of the mass of the total wagon part.

So, when we are just trying find out that what exactly the modal mass, which is there under the banding vibration, we can simply make the configuration, when it is being uniformly distributed all across the length it is half of the total mass. So, modal used here implies that the immediate vicinity of the actual frequency F, which is the lowest frequency the distribution beam can be replaced by a rigid mass ,and a spring itself. So, we can when we are simply you know like putting those things, we can simply find out that you know like there is a clear analogue or clear representation of the entire beam and the spring features.

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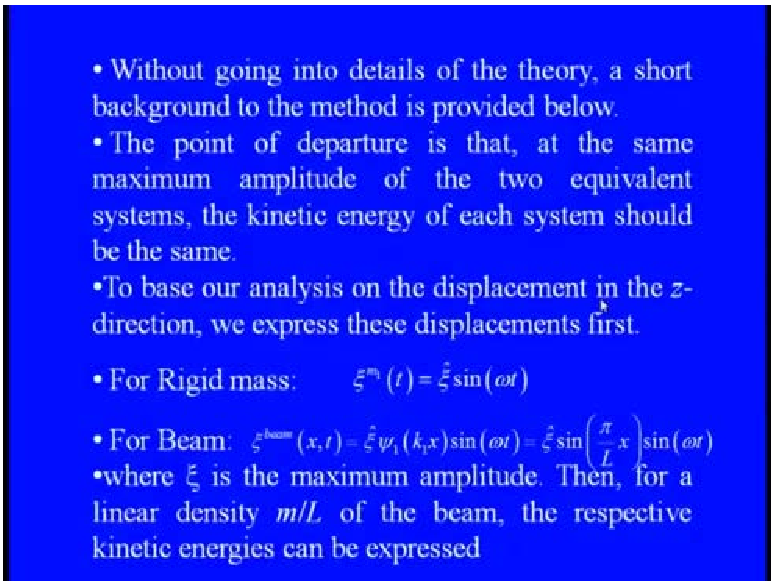

So, without going into detail of you know like these things, we can simply say that when the point of departure at same you see, means we can say at the same maximum amplitude of the two equivalent system. The kinetic energy of the each system must be same, because we are saying that you see whatever you know like the deflection features are there or with the maximum amplitude. When they are you see you know like just showing the two equivalent system, certainly you see here the kinetic energy should be same in that way.

So, to base our analysis we can that the displacement in this z direction, where we were showing you see you know like the rigid mass m 1 or you know like the beam part you see here, we can get the displacement feature first. So, the displacement for rigid mass, which is being there on that is nothing but equals to say if we are saying that the displacement. So, this epsilon this m 1, which is the dynamic parameters displacement is there, so it is a the epsilon m 1 t is nothing but equals to epsilon which is the maximum displacement into sin of omega t.

And when we are going for the beam, then certainly you see here this space coordinate, because you see you know like it has certain you know like the width is there. So, we can say that for the beam the displacement of the beam epsilon beam is x comma t, and we can say that you see here, the mode shape will be you know like coming together. So, we have epsilon which is the amplitude into psi one of this K 1 x and then sin of omega t, so we can say either this epsilon into sin pi x by L into we can say that sin omega t.

So, we know that the epsilon is nothing but equals to the maximum amplitude and for the linear density m by L, because we know that there is no change in the you know like the homogeneous, and the this isentropic properties are being considered. So, there is no change in the mass configuration in entire bogie, so we can say the linear density is being featured out for the beam. And when we are saying that these things then the kinetic energy can be immediately, this evaluated for the rigid mass and for the beam.

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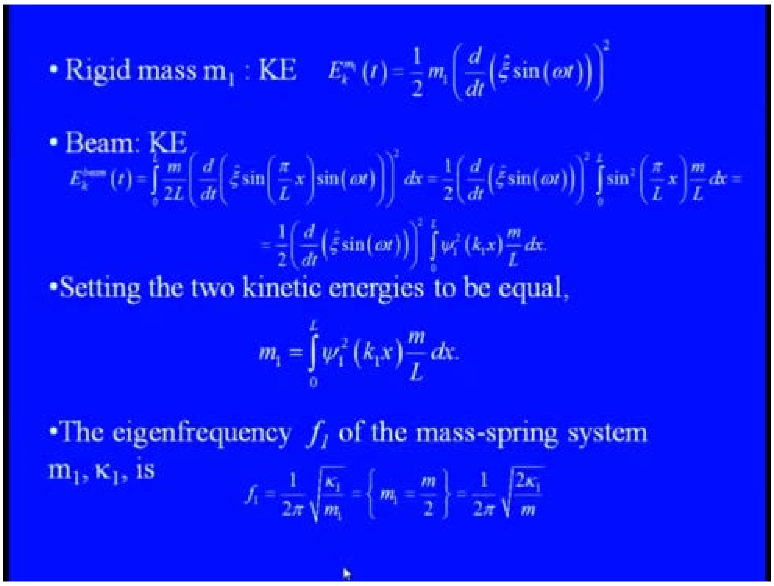

So, first of all for the kinetic energy for the rigid mass m 1, we can say that the kinetic energy k for m 1 is nothing but equals to half m 1, and then you see here this entire displacement is featured out. So, it is d by d t of epsilon sin omega t whole square, and for the beam also you see here, now we can straightaway put together the epsilon this K.

And for the beam entire is nothing but equals to integration of 0 to L, because there is a clear you see you know like the space is also there up to the x part, so 0 to L m by 2 L d by d t of epsilon sin pi x over L into sin omega t the whole square into d x.

So, when we are formulating this, now we know that it is nothing but equals to again when we are trying to formulate this, again we need to keep the two things together, one when we are saying that you see you know like the sin omega t. And second the sin of pi x by L, because ultimately we are multiplying this into d x. So, you see here when we are integrating and when when we are putting this, we have half of d by d t epsilon sin omega t whole square 0 to this whole square.

And then integration 0 to L pi 1 square of K 1 x into you see m by L into d x, so you see here you know like this we know that the mode shape is the function of d x. So, it is being there with the 0 to L integration, and you see since you know like we know that, whatever you see the sinusoidal features are there they are being epsilon sin omega t with the displacement, so d by d t is there this.

So, these two kinetic energies, when we are just keeping together, so we know that you know like, we can simply keep both the kinetic energy should be equal, so m 1 which is nothing but equals to 0 to L pi 1 square K 1 x m by L into d x. We can say that we can find out the Eigen frequency for the lowest, this is lowest Eigen frequency for the banding mode feature. Under this mass spring system, with the m 1 K 1 is F 1 is nothing but equals to 1 by 2 pi square root of K 1 by m 1, or we can say that m 1 which is nothing but equals to m by 2 we can say F 1 is nothing but equals to 1 by 2 pi 2 K 1 over m.

So, this is you see the Eigen frequency even when we are keeping this with the stiffness part, the stiffness is K 1 is nothing but equals to 2 pi F 1 square m by 2 or else we can say that you see here. The spring rate, which is nothing but you see you know like of whatever, the stiffness features are there of the resonant absorber can be get easily by F 1 equals to 1 by 2 pi K 2 by m 2. So, we can simply show the K 2 as 2 pi F 1 whole square into m 2, so this is you see you know like we can say these values, means these expressions 1.

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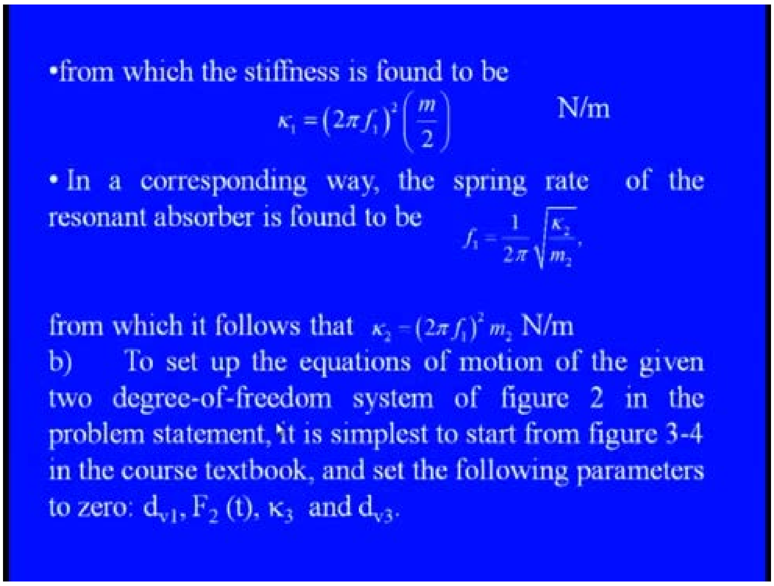

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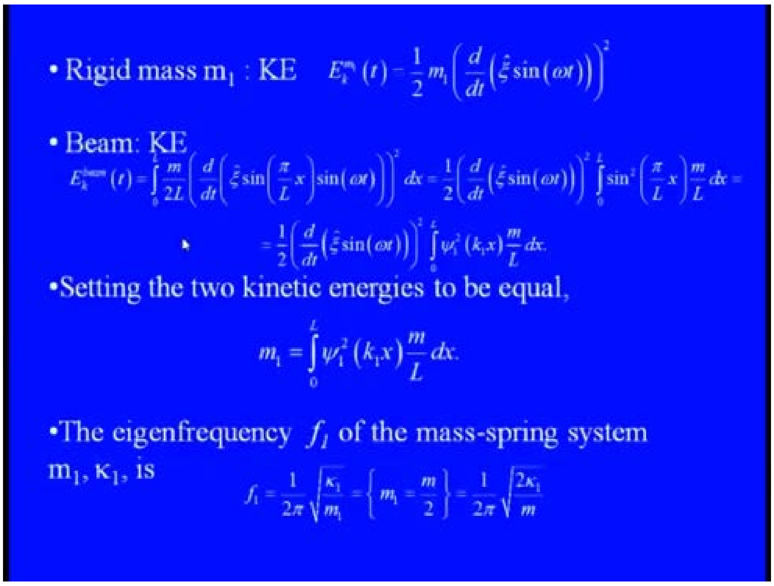

Where, we have you see clear what exactly the kinetic energies are there, so this is what you see the kinetic energy for mass m 1, the kinetic energy for mass m 2. What is the Eigen frequency, Eigen frequencies are like that, and we can get you see the spring rate or the stiffness. You know like for the absorber, and for you see the entire wagons wherever it is to be required, so K 1 and K 2 can be easily get. The b part was there, where we need to setup the equation of motion, for given 2 degrees of freedom system, as we discussed you see, where m 2 which was there as a you know like the transformer m 1 was you see the wagon part is there.

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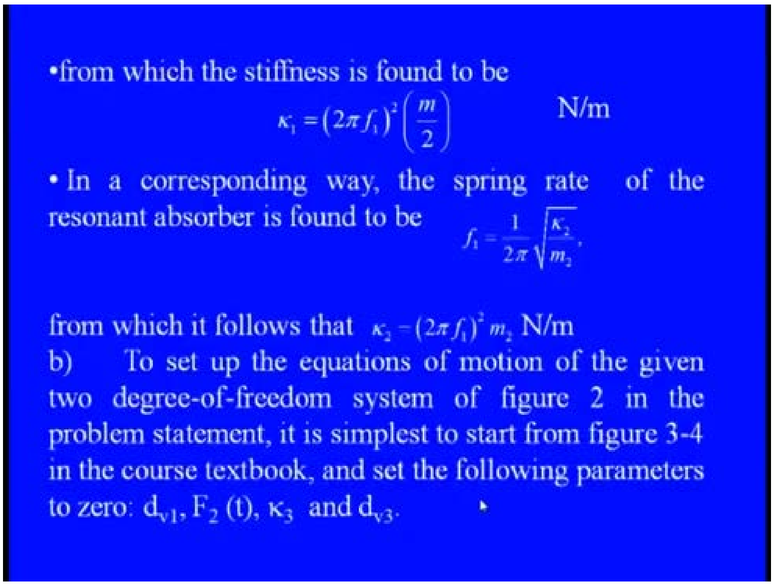

And then we are when we are starting from that you see when we have a rigid foundation, and you see the entire beam part is there. We can simply find out that you know like this in you know like that how we can get, those parameters means you know like the remaining parameters out of which. So, we know that it is a two degree of freedom system we can straightaway go to the equation of motion for that, in the matrix formation, so we know that the mass matrix must be symmetric all along you see the diagonal.

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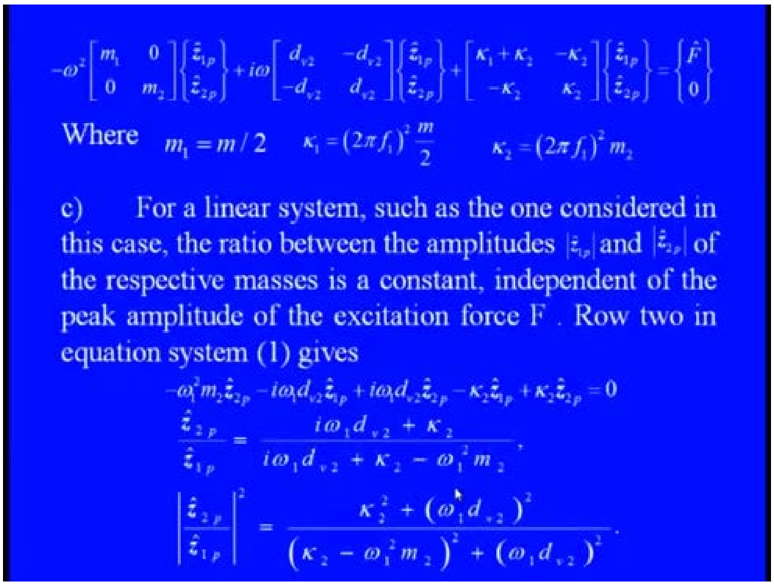

Because, you see we know that m 1 0 to 0 to m 1, so this is one thing is there even when we are talking about the stiffness matrix, it is K 1 plus K 2 then minus K 2 K 2 along this diagonal, and then K 2 is there. When we are talking about the damping which is only there with the mass m 2, so we can simply spread in that way, so when we are talking about this we have. Now, the equation of motion, which was required you see here as minus omega square matrix m, and matrix m is this m m 1 0 0 m 2 into you see the displacement.

So, displacements are given as you see the z 1 p and z 2 p, this is what you see here like the forced part is there, because we are applying the force, so we have you see you know like the displacement for the particular integral. Under this is z 1 p and z 2 p plus iota omega, this is what my natural frequency into d v 2 minus d v 2 minus d v 2 and d v 2, because there is no d v one as such you see here. So, you see epsilon 1 p this z 1 p and z 2 p is there, plus you see you know like the K 1 plus K 2 minus K 2 minus K 2 and the K 2 is there.

So, we can get you see all these mass damping, and a stiffness matrices with there you see you know like the mode shapes, and then this is what my forcing frequency F the force factor F and 0. Where, we can simply put m 1, which is you know like m by 2, as we know that K 1 as we discussed already 2 pi F 1 0 pi F whole square m by 2, and K 2 is 2 pi F 1 whole square m 2, so we can keep and we can get all the things.

The last part was there which is you know like for linear system, such as you see you know like, we are considering that whatever the ratio in between the amplitudes z 1 p, and z 2 p is absolutely at we can say you know like just moving. At the constant and independent of the peak amplitude, for the excitation force F, then we can get you see the ratios of that and we can get, that what exactly you know like the relative displacements are there of the masses of both m 1 and m 2 with these peak amplitude.

So, again we need to go with the same equation, and we need to find out that what exactly the Eigen modes are. So, minus omega you see minus omega minus square m 2 z 2 p minus omega d v 2 z 1 p plus iota omega we can say d v 2 into z 2 p, and then we can say minus K 2 which was you see the stiffness of the mass m 2 associated with the z 1 p plus K 2, you see z 2 p is equals to 0, when we are just trying to keep these systems in that way.

So, we can say that you see the ratio between the you know like the mass amplitude z 2 p by z 1 p is nothing but equals to iota omega d v 2 plus K 2 and divided by iota omega 1 d v 2 plus K 2 minus omega 1 square m 2. So, this is all the you know like the manipulation from the above equation, we can get this z 2 p and z 1 p.

And when we are trying to do the square we know that it is nothing but equals to the square K 2 square plus omega 1 d v 2 whole square divided by the K 2 minus omega square m 2 whole square plus omega 1 d v 2 whole square. So, this is you see you know like the individual factor, when we are just trying to evaluate those things we can get this z 2 p by z 1 p whole square.

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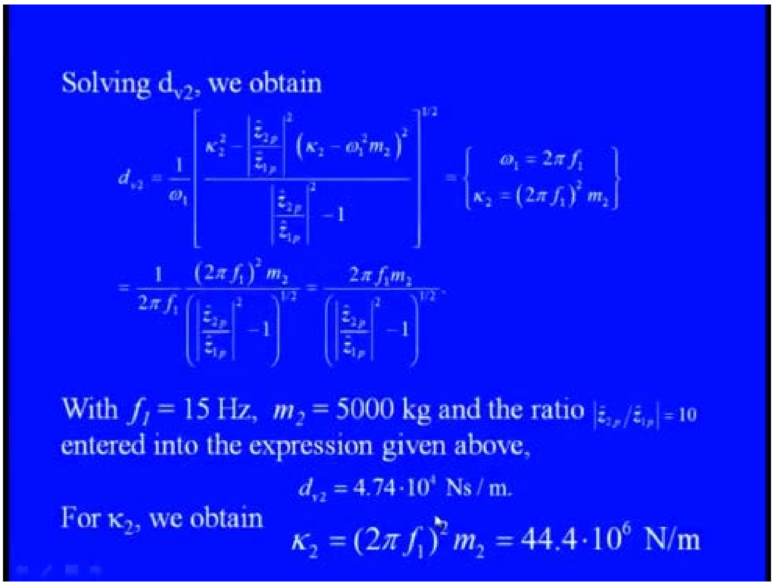

Our intention is to find out you see that what is the d v 2 here from that, so d v 2 can be straightaway calculated as one by omega 1 K 2 square minus z 2 p by z 1 p whole square. Whatever, the amplitudes ratios are there into K 2 minus this m 2 omega 1 square divided by z 2 p minus z 1 p square minus 1, when we are just trying to you know like modal this square root of that.

So, we can say that when we are trying to keep this omega 1, which is nothing but equals to the lowest natural frequency 2 pi F 1, and when we are keeping this K 2 as 2 pi F 1 whole square into m 1. Then we are concluding this d v 2 as nothing but equals to 2 pi F 1 m 2 divided by z 2 p minus z 2 p by z 1 p modulus of this square minus 1, and we know that you see here the first natural frequency is 15 Hertz. And m 2 is you know like given as the 5000 kilo gram, and we know that when the amplification or we you know that when these z 2 p is you know like exciting 10 times than z 1 p.

When we are keeping these things we know that you know like, these numerical part will simply give you that d v 2 is nothing but equals to 4.74 into 10 raise to the power 4, this Newton second per meter. That is the damping value, and when we are keeping into our K 2, which is nothing but equals to 2 pi F 1 whole square m 2, then we have you see the stiffness value that is 44.4 mega Newton per meter.

So, this is you see you know like we have all the values, means when we are keeping the mass value and all, this is what the relations are there with this relation, we can get all the value when we know the input values as you know like m 2 F 1 and z 2 by z 1. So, this is numerical this is something you know like the numerical problem, in which it is clearly showing that with the addition of mass as the absorber. How we can suppress the vibration or how you see the other things can be you know like evaluated with this, now we are going to the another problem of this, in this we have a machine, you can see on your screen.

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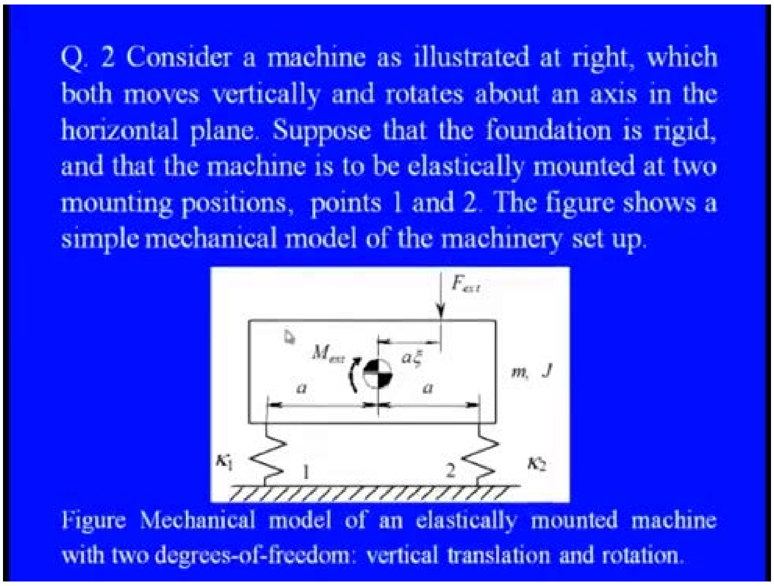

The machine is there, in which you see it is simply moving not only in the vertical direction, but also it is rotating, so now, we have it is you know like, because the 2 springs are there. So, it can oscillate to and fro part, but also you see here it can rotate as you see you know like you know like the junctions are there, at which you see it can rotate.

So, we have you see you know like the moment is being there, which is being excited, and the force is not you see you know like passing through the centre of the machine, this centered feature of the centre of mass, it is you know like the eccentric feature and this eccentricity can be measured as a into zeta. Now, suppose the foundation is rigid, so now, you see there is no deforming feature with the foundation part, and the machine is to be elastically mounted at two mounting position.

As, you can see K 1 and K 2 are there 1 and 2 the figure shows that, in this that there is a you know like the mechanical modal of an elastically mounted machine, with 2 degrees of freedom, where you see the vertical translation the translational feature, and the rotations are being considered. The key feature is that this rotation point and these two elastically mounted features are well balanced, because a and a distances are there from this point. As, you see you know like as it is moving, so we may have you see the kind of you know like, first mass m 1 and second is the movement of you know like inertia the j is there to gather this.

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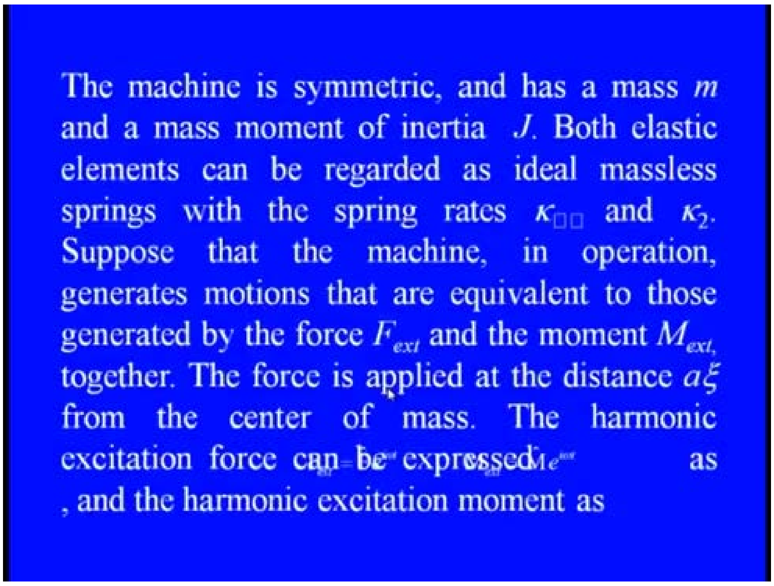

So, machine is symmetric we know that, because you see it is absolutely no because if we are not considering symmetric certainly, we would have the unbalance or some kind of misalignment can be there, because of this rotation or translation. We can say it has mass m 1, and the mass movement of inertia is j, both elastic element can be regarded as the ideal mass spring, so we are not considering any mass as generally we are considering you know like in other part with this stiffness.

So, we have K 1 and K 2 are the spring rates at this, and if you are saying that during the operation, the whatever the motions, which are being generated they are equivalent to those generated by this force excitation. So, we are now putting the analogue, that during operation this much mass and the moment can be generated, so this is we are saying F excitation and m excitation is the force the excited force and excited moment together.

The force is applied at the distance a zeta you see here from the centre of mass, and the harmonic force which is being you know like excited is F is nothing but equals to F e i omega t and m is nothing but equals to m e i omega t.

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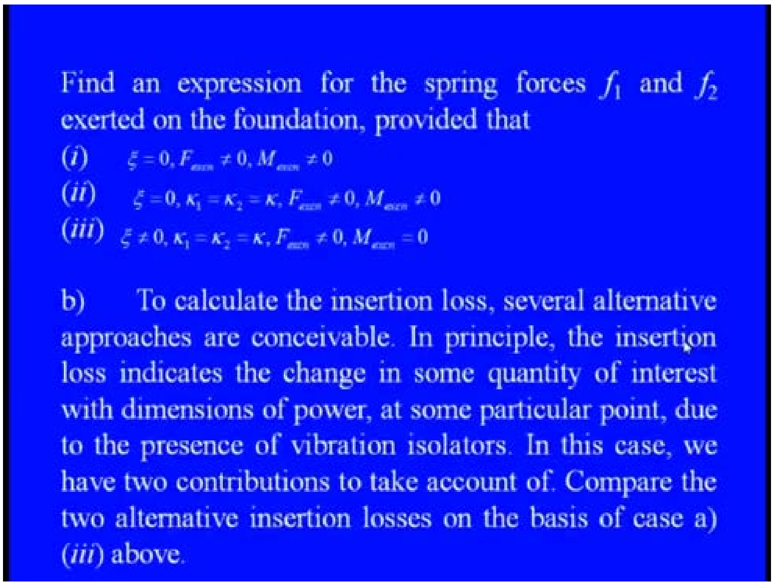

If these all harmonic excitations are there, so we can featured out this one, now the question is, now we need to find the expression for the spring forces F 1 and F 2, which are being there, at the these 2 location of the spring at these two.

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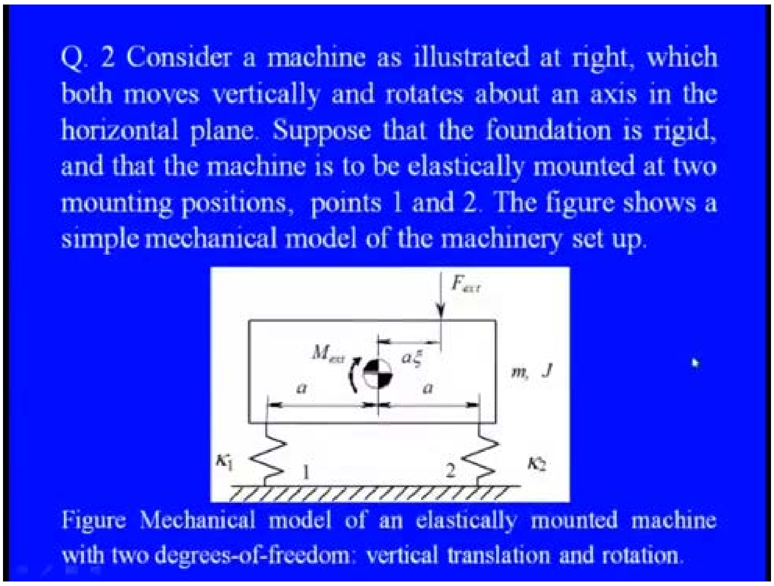

So, here F 1 is my restoring force F 2 is my restoring force at these two locations 1 and 2, so we need to find these F 1 and F 2 which are being exerted or from the foundation on the foundation.

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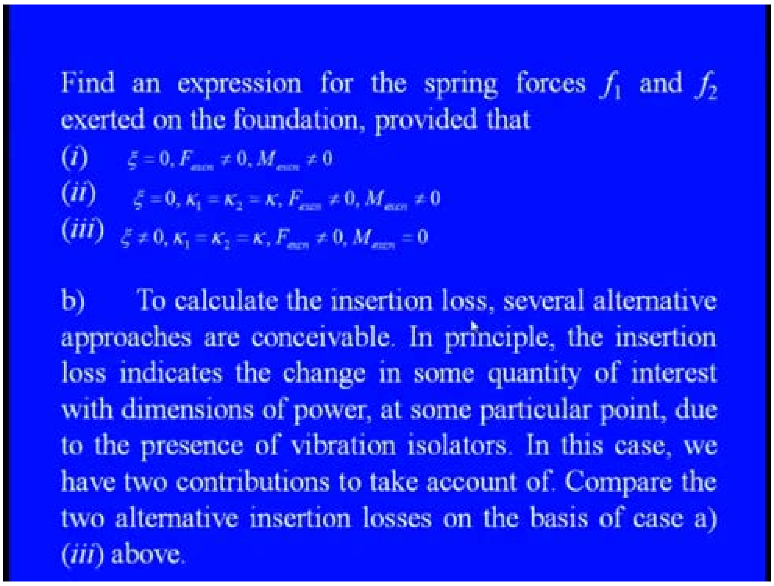

In if the first condition is the zeta is 0, and then you see here this F excitation and m excitation is not equals to 0; that means, you see here these excitation features are you know like being acted there the zeta is 0 there. Second the zeta is 0 and then K 1 K 2 and k the spring rate is constant from both the side, at one and two you know like with the rigidly mounted on the foundation.

And we do not have you see F excitation and m excitation is not equals to 0, and the third case is when the zeta is not equals to 0, again remember the zeta is nothing but a into zeta is the linear distance, where the F this F excitation is being acted. So, zeta is not equals to 0 now, but a spring rate K 1 K 2 and k is equals they are equal, F excitation is applied, but there is no moment, so m excitation is 0.

So, these are the three conditions, which we are going to apply to formulate F 1 and F 2 to spring rates, B part we need to calculate the insertion losses, where you see the several alternative approaches. You know like are conceivable, in principle if we are talking about the insertion losses, they are indicating the change in some of the quantity of the interest with the dimension of power.

And at some particular point due to the presence of vibration isolator, we know that there are you see you know like some kind of you see the insertion losses. So, in this case we can say that there are two contributions to take into account, and we need to compare both the features you know like in case of this a and we can say, in which you see. The third case where we have you know like the eccentric force is there, and the spring rates are same, but there is no moment is there, so in this case now we can say that the two main conditions are like this first.

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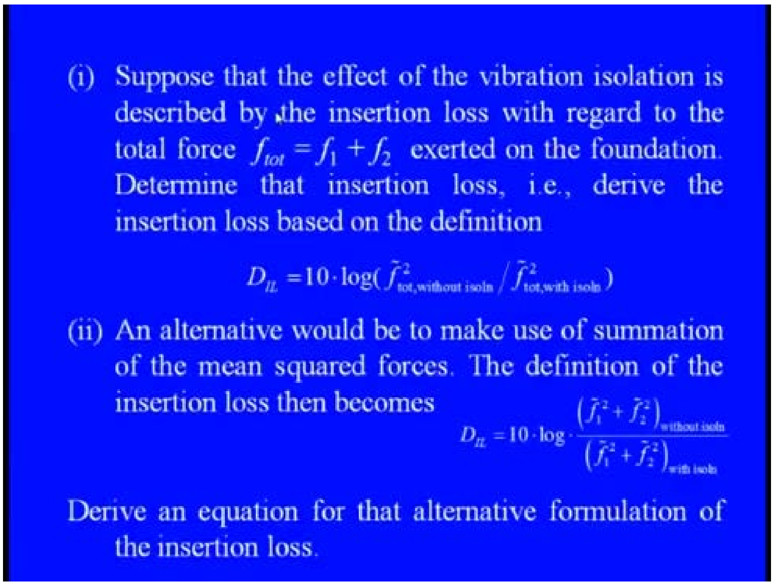

Suppose, the effect of vibration isolation is described by the insertion loss with regard to the total force, which is the linear summation of both the F 1 and F 2 exerted on the foundation. So, we can find out the insertion loss, which can be straightaway derived d i L is nothing but equals to ten log of this F square total without, you know like applying this isolation divided by F square total with isolation. Second solution is the alternative would be make the use of summation of the mean squared forces, whatever you see the forces which are being coming.

And the definition of the insertion losses can be of d i L 10 log, now the you know like. since the insertion loss is being coming out featured out from the square of this the mean square forces. So, it is F 1 square plus F 2 square without isolation divided by F 1 square plus F 2 square with isolation, and then you see here we need to find out that how we can get those things.

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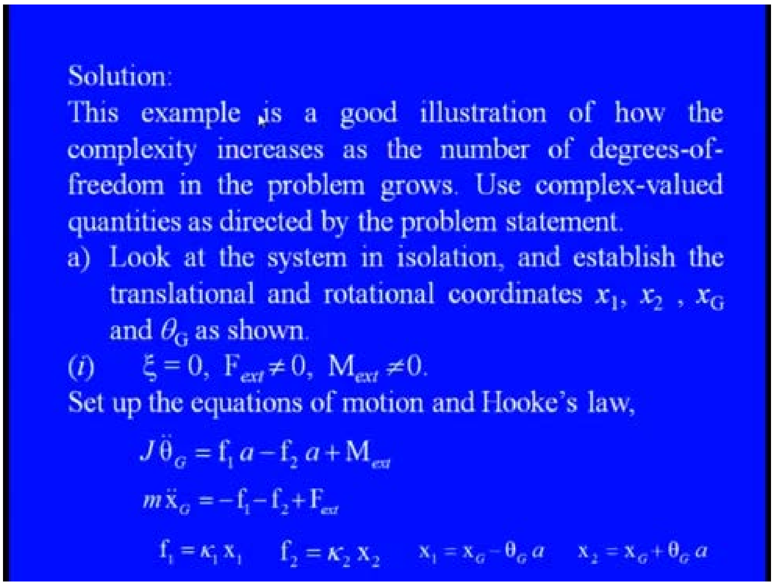

So, the first condition says that we need to find first F 1 and F 2 for these conditions, so in that we can say that when you know like the degrees of freedoms are being increased, we know that the complexity in the solving problem is more of the these increased. So, first we need say that we have the 2 main coordinate system, where the movements are there one is the translational, we can say you see x 1 and x 2 are there, because of m and j second is the rotation. So, x g and this theta g is there for the rotational features.

The first case was there where the zeta is 0, but F excitation and m excitation is present there, so now, you see here, when we are applying this and you see you know like when the oscillation and the translational features are being there together. The equation of motion first for rotation part is nothing but equals to j into theta j double dot, the inertia force during the rotation equals to f 1 a minus f 2 a the distance of a you know like from K 1 and K 2 are same a distance.

So, the moment from this is coming f 1 a in one direction and f 2 a from other direction, so f 1 a when the forces being acting from the distance they are absolute in the same direction where the m excitation are there. So, we can say the total moment on this side is f 1 a plus m, and then the you know like the reverse direction is minus f 2 a, so the equation of motion is j theta g double dot is equals to f 1 a minus f 2 a plus m excitation.

Now, if we are going towards the other dimension that is x, so we have m into x g double dot that is you see the inertia force, due to you know like the linear motion equals to now you see in the restoring forces, which are being you know like the opposing part. So, it is minus f 1 minus f 2, and you see the other part you know like the spring force which are being acted on top ward the forces, which are being there f excitation on downward, so plus f excitations.

So, now, we can say that this f 1 and f 2 is nothing but equals to K 1 x 1 and K 2 x 2, where x 1 x 2 are linear we can say displacement in K 1 K 2 are the spring rates or we can say nothing but equals to our stiffness features of this spring corresponding springs. And also we can say that the x 1 is nothing but equals to when we are just making the displacement feature linearly, so x 1 is nothing but equals to x g.

So, first x g whatever the displacement of the g factor at the g point the gravitational point, and then you see here minus theta g into a, because the distance is a here. So, theta g whatever the rotational into a. Similarly, x 2 is also same, but only the direction is difference here, so we have this x g plus theta g into a. So, these you see the linear summation in between you know like x 1 and x g, x 2 and x g from the this translational and the rotational coordinate.

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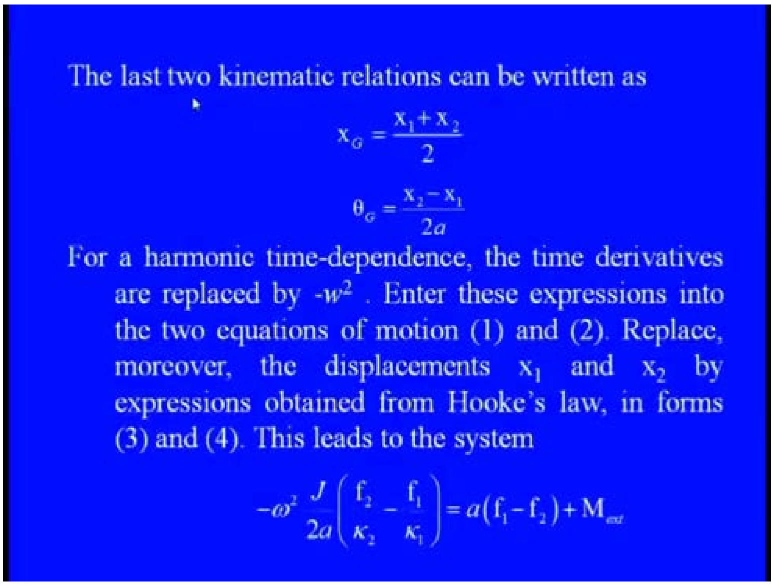

We can keep you see you know like these kinematic relations together in that way the x g is nothing but equals to x 1 plus x 2 by 2, and theta g is nothing but equals to x 2 minus x 1 what are the difference divided by 2 a. And we know that this is the harmonically time dependence problem, where the time derivatives are being replaced by our frequency minus omega square.

So, we can say you know like when we are keeping those equations together we know that with the using of the Hooke's law, our equations are now become minus omega square j by 2 a f 2 by k minus f 1 by k. That is what my displacement features and when we are multiplying with this, equals to a into whatever the distance a into f 2 minus f 1, that is what you see the total difference of the moment is f 2 minus f 1 the resultant force into a that is a moment plus m excitations.

So, you see here when we are making balance of these things, we can say that under the Hooke's law this moment balance equation is clearly giving. That what exactly the rotation is there when the moment is being applied, and when the force balance conditions are like that.

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$$-\omega^2 \frac{m}{2}\left(\frac{f_1}{\kappa_1}+\frac{f_2}{\kappa_2}\right) = -f_1 - f_2 + F_{ext}$$

Solving for f_1 and f_2 from (9) and (10),

$$f_1 = \frac{\dfrac{a-\omega^2 \dfrac{J}{2a\kappa_2}}{a-\omega^2 \dfrac{J}{2a\kappa_1}}}{\left(1-\omega^2 \dfrac{m}{2\kappa_2}\right)\left(1+\dfrac{1-\omega^2 \dfrac{m}{2\kappa_1}}{1-\omega^2 \dfrac{m}{2\kappa_2}} \cdot \dfrac{a-\omega^2 \dfrac{J}{2a\kappa_2}}{a-\omega^2 \dfrac{J}{2a\kappa_1}}\right)} F_{ext} - \frac{1}{\left(a-\omega^2 \dfrac{J}{2a\kappa_1}\right)\left(1+\dfrac{1-\omega^2 \dfrac{m}{2\kappa_1}}{1-\omega^2 \dfrac{m}{2\kappa_2}} \cdot \dfrac{a-\omega^2 \dfrac{J}{2a\kappa_2}}{a-\omega^2 \dfrac{J}{2a\kappa_1}}\right)} M_{ext}.$$

So, we can say with these particular equations, we can get minus m minus m by 2 omega square into f 1 k by f plus f 1 by K 1 plus f 2 by K 2 is equals to minus f minus f 1 minus f 2 plus f excitation. This is you see here the moment and the force balance conditions, when we are equating both the equations. We can have you see here a minus this f 1 is nothing but equals to the force f 1, which we need to calculate is nothing but equals to you see this a minus omega square j by 2 k j j by 2 a K 2 divided by a minus omega square j over 2 a K 1 divided by 1 minus omega square m by 2 K 1.

And this you see whole is coming from you know like as the just denominator, 1 plus 1 minus omega square m by 2 K 1 divided by 1 minus omega square m by 2 K 2 into a minus omega square. Same you see on the top of that you see the same here, a minus omega square j over 2 a K 2 divided by a minus omega square j over two a K 1 f excitation.

So, this is you see this part is coming from the f excitation, and similarly you see this m excitation features are also being there, so this is this part, we can say 1 plus this is you see 1 minus. So, it is a minus omega square j over this 2 a K 1 into 1 plus 1 minus omega square we can say m by 2 K 1, and 1 minus omega square m over 2 K 2 into, now a minus omega square j over 2 K 2 a K 2 divided by a minus omega square j over 2 a K 1. So, this is you see you know like with the coefficients in with respect to omega square, m and k we can simply calculate using f excitation and m excitation features.

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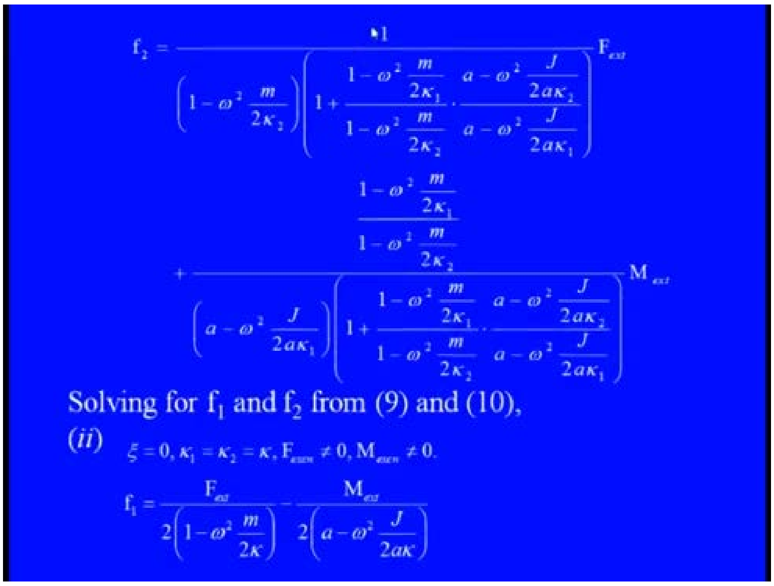

And similarly, f 2 can be also evaluated using these features one by this, into f excitation plus this 1 minus omega square m by 2 K 1, and one minus omega square m by 2 K 2 divided by this into m excitations. So, this is what you see the first condition, where both you see the f this f excitations are being there, both are being you know like present their case. Now, when we are saying that the zeta equals to 0, and K 1 K 2 are equal means stiffness are being equal now, so we can keep this stiffness is there equal and you see both this m excitation and f excitations are being present. So, we can simply get you see you know like in this f 1, and here f 2 when we are keeping this is spring stiffness constant same, or spring rate same. We can have f 1 is nothing but equals to f excitation divided by 2 times of 1 minus omega square m by 2 k minus m excitation divided by this two into a minus omega square j over 2 a k.

So, now, you see the k is same here and similarly we can calculate the f 2 as well the f excitation divided by 2 into 1 minus omega square m by 2 k plus m excitation 2 into a minus omega square j over 2 a k. The third case in which you see here we do not have you know like the zeta equals to 0, it is the zeta is present, but the moment is 0 here, f excitation is there spring rate is same, but there is no moment.

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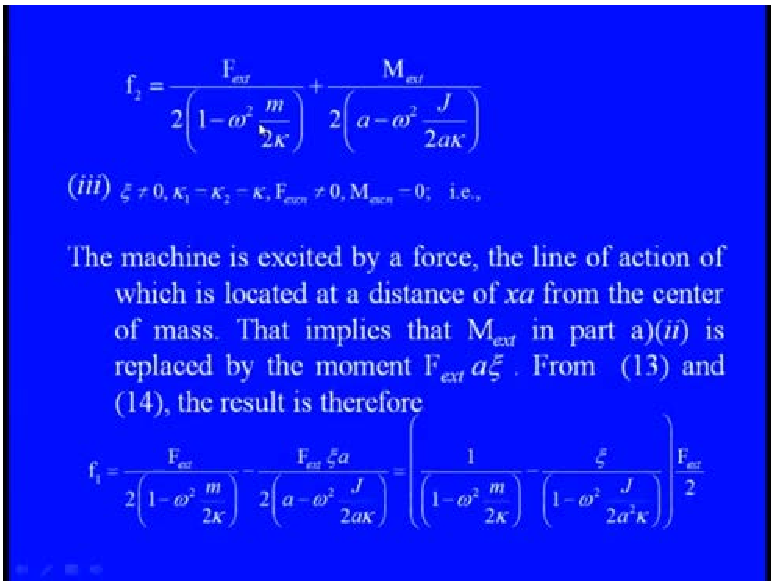

So, the machine is now excited by the force, only the line of action at which you see it is being located, which is absolutely at the distance of this you know like x into a from the centre of mass. And we can say now you see here this you know like m excitation, which was being calculated in the previous part is now being replaced by the f excitation into the distance and this distance is a into zeta, because zeta is now present here.

So, this is the total moment which is being acted by the applied force, so when we are now calculating this part we have f 1 is nothing but equals to f excitation divided by 2 into 1 minus omega square two m by 2 k minus. Now, instead of m excitation, now we have this f excitation zeta into a divided by 2 times of a minus omega square j over 2 a k, or else you see when we are putting the manipulation we know that f excitation is common in both the case. So, f 1 is nothing but equals to f excitation divided by 2 into 1 over 1 minus omega square m by 2 K minus zeta over 1 minus omega square j over 2 a k.

So, this is what you see the features are and in the second case also we can simply calculate you see when the first part is coming that, both the forces f 1 and f 2 is they are being there without isolator. So, we can simply calculate you see here in the equilibrium position because we need to derived equations, where you know like the f 1 f 2 are there you know without isolator.

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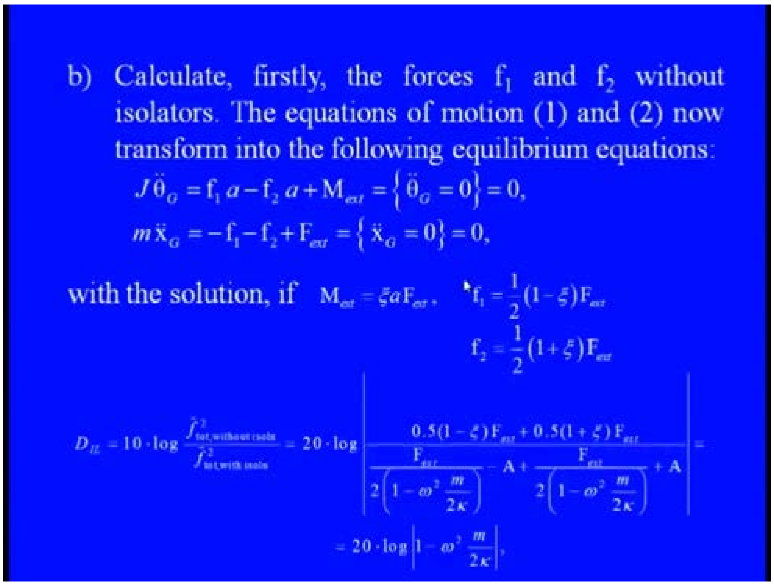

So, we can say the equations are the j into theta g double dot that is the inertial forces during the rotation part is equals to f 1 a minus f 2 a plus m excitation, where now we are saying that this is absolutely equals to theta g equals to 0. There and f is this x double dot means m into x g double dot equals to minus f 1 minus f 2, the reactive spring forces plus f excitation where you see here x g double dot is 0.

So, when you see we are saying that there is no isolation features these are being there, when we have you know like when we just want to solve these things, if m excitation is zeta a f excitation. We can say f 1 is nothing but equals to 1 minus zeta f excitation divided by 2, f 2 is nothing but equals to 1 plus zeta f excitation divided by 2. And then we can calculate the insertion losses, by keeping 10 log you know like this f square, this total without isolation, and f square total with isolation we can get these things this is nothing but equals to 20 log 1 minus omega square m by k.

So, we can calculate you see here the insertion losses in those things, when we are keeping those values there. Ultimately, we are ending with d i L is nothing but equals to 10 L, 1 minus zeta square f excitation square by 4 plus 1 plus zeta whole square f excitation by 4 and divided by this whole is there. Else, we can say that this is nothing but equals to 10 log 1 plus zeta whole square, divided by 1 minus omega over omega square into whole square, plus 1 minus omega square by omega r square.

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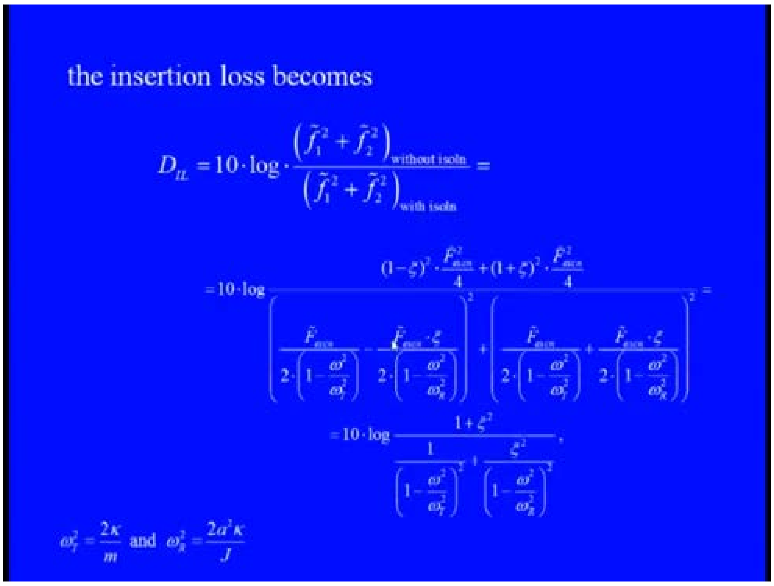

This equals to you know like we can say the entire whole square, where omega 1 square is nothing but equals to we can say two k by m and omega r is nothing but equals to 2 a square k by j. So, these are you see the rotational frequency and the translational frequencies are there, which can be straightaway put it together just to represent that how you see these analogues are there.

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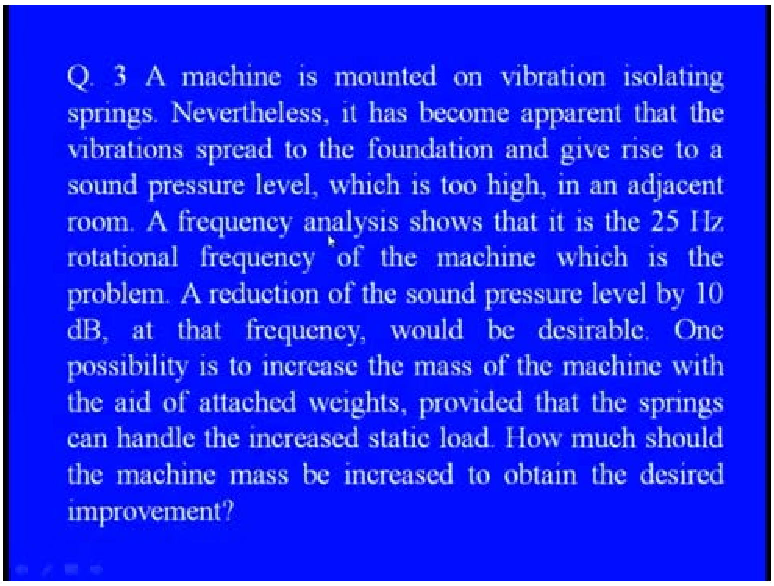

So, this is what the expressions are and we can get that, so in the last numerical now, we are taking one more example in which the machine is now mounted on you know like the vibration isolation is spring. And we are saying that you see you know like the vibration is simply being spread it in out, to the foundation and give rise to the sound pressure level, because you see here this is just the you know like the absorbing and releasing features are there.

So, we can stored the energy, and when the expansion is there it is being realized no absorption, no dissipation features are there, and frequency analysis which we can say you know like we just want to describe these things. It is at 25 Hertz rotational frequency of the machine, which is you know like the main problem that, you see at when the machine is approaching up to this the huge amount of you see the sound or we can say the vibration levels are being there.

A reduction of sound pressure level by ten d b at this frequency would be the required one, one of the possibility now here which we can say that to increase the mass of machine, with the aid or some additional weight. Provided that this spring can be handle to increase the static load, and how much should be the mass to be increase to obtain the desired improvement, so that the ten d b sound can be reduce their itself.

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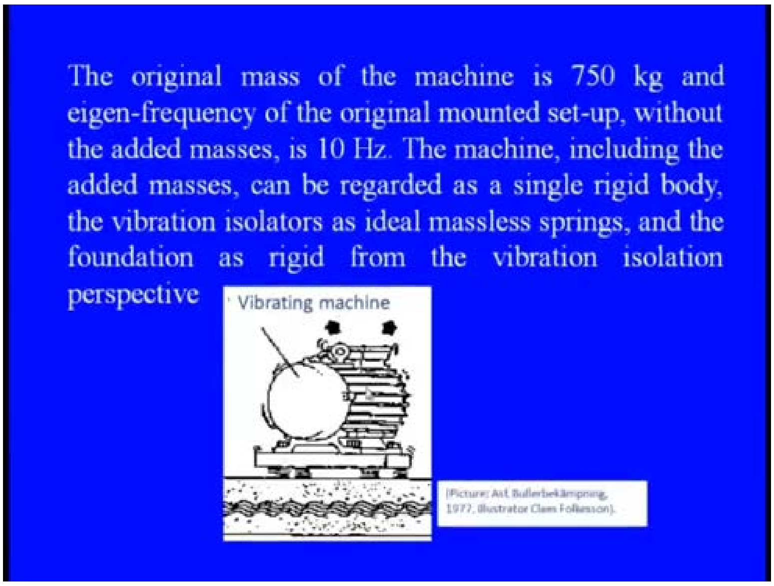

You can see that this is what you see the vibrating machine, which is a very common machine, and it is you know like absolutely at 25 hertz the huge amount of you see you know like the sound is being there. And it is being transmitted straightaway to the ground and the surrounding, the original mass is 75 kilogram and the Eigen frequency of the original mounted set up without any additional mass, which is supposed to be there is 10 hertz.

So, you see we know that the first Eigen frequencies are there, but when they are approaching up to 25 Hertz, then the things are more disaster. The machine including the added mass can be regarded as the single rigid body, because the added mass is you know like supposed to be act in the integral part of the machine. And the vibration isolator as the mass less spring is considered and the foundation is rigid, because we know that you see you know like the things are being transmitting in a speeder manner, so certainly it is there the foundation is rigid.

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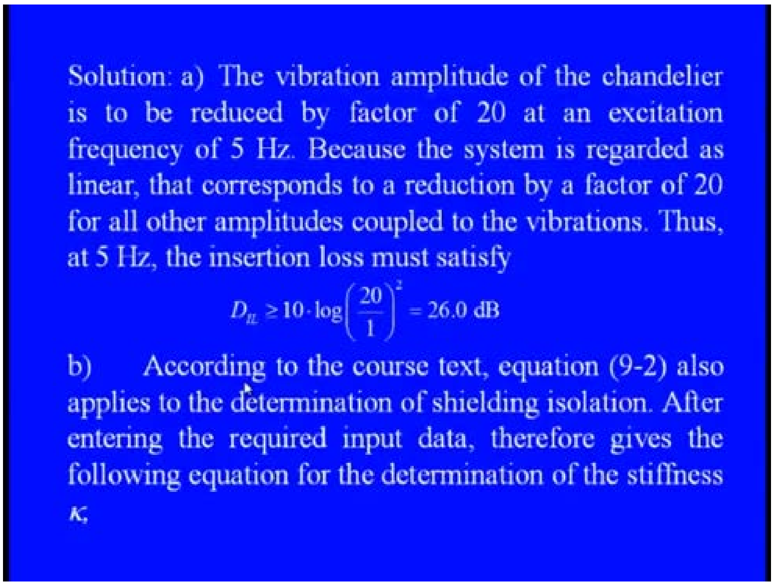

Now, we are going you see we just wanted to find out the vibration amplitude of the entire, you see chandelier which is to be reduced to the factor by 20 db at an exciting frequency of these Hertz. And you see here because the system is regarded as the linear and corresponding reduction by the factor of 20, when these amplitudes are there we can say the d i L the insertion loss is greater than equals to 10 log 20 by 1 square or we can say 26 d b.

So, this is what you see the insertion loss, when they are exciting at their own frequency, according to this now we can say that we can straightaway put the shielding isolations there. And after the you know like entering the required input data, we can simply find out you know like the. what exactly the stiffness parameters are there through, which we can calculate this.

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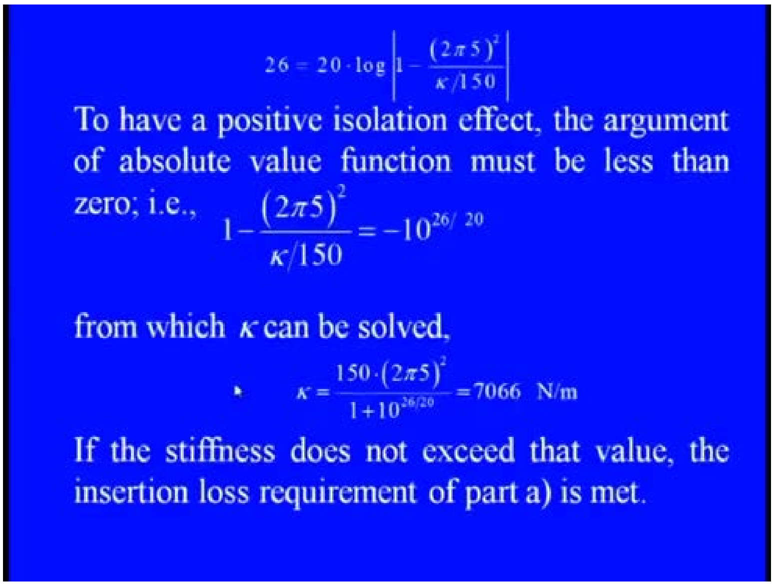

So, we can say that 26 d b, which is you know like generating at that insertion losses is equals to 20 log 1 minus 2 pi the exciting frequency 5 hertz divided by k over 150. So, we can say that you see to have the positive isolation effect, we know that the absolute value of these function must be equals to 0 or we can say 1 minus 2 pi f whole square.

You know like divided by k by 150 must be equals to 10 to minus 10 to the power 26 by this 20, and when we are trying to solve for k, the k must be equals to because we have right now 150 into 2 pi f whole square divided by 1 plus 10 to the power 20 by you see we can say the 26 by 20, we can say it is 7066 Newton per meter. So, when we want to see that the value of this should not be increased means the value of the stiffness should not be increase from this part, we can say the insertion losses must these 26 d b supposed to be met at this point.

So, you see here we can straightaway relate, that you know like when we are keeping the isolator, then what exactly you see you know like the values of the stiffness or we can say the damper or any other things are there, so that we can you know like met the insertion losses up to a certain sound level. So, in this chapter we have mainly discussed about the numerical problems, there were three numerical problems related to the system parameters.

And when we are talking about you see in that vibration isolation either by added mass or either by you know like the spring part or either by you see you know like the damping part. What exactly the key features through, which we can do this. So, this is the last part of our this module where you see the active and passive vibration controls are there, how we can achieve those you know like by adding the material or by adding the spring mass damper. Even by putting you see the sensor actuators, and the control unit you know like that we discussed.

Now, in the next lecture, we are going to discuss about the last module which is you see the measurement that, how the measurement devices are there, what are the basics what are the sensing. And the you know like we can say the data acquisition systems are there, and when you see we are getting those signals then how do we process the signals. So, that we can say that this is what the vibration signatures or the vibration signals are there, and we want to control these vibrations. So, measurement is also one of important part in the vibration features, whether the concept or the controlled part.

Thank you.