

Vibration Control
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Module - 6
Principles of Passive Vibration Control
Lecture - 4
Isolation with Stiffness and Damping

Hi, this is Dr. S. P. Harsha from mechanical and industrial department, IIT Roorkee. In the course of vibration control, we are discussing about the principles of passive vibration control. And in this, we are going to discuss about the isolators design along with the stiffness proper stiffness and the proper damping feature. Prior to go to this lecture, we know that in the last lecture, we mainly discussed about that you know like how we can design the shock absorber, what is the specific use of the shock absorber when we are coupled the spring with the damper. And then you see, if we are a lone spring is using then what exactly the feature means the features are being coming out from that.

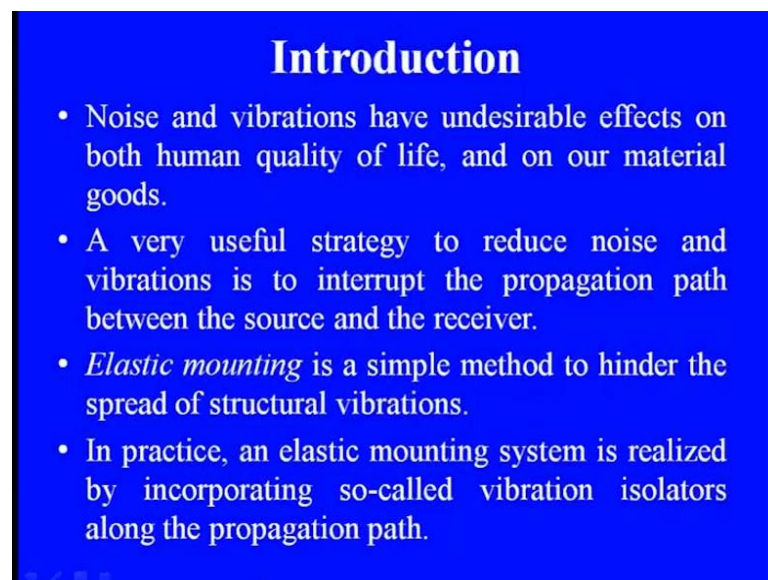
So, we discussed about the various types of the shock absorber, from the metallic spring to the pneumatic features, the hydraulic features and even you see here, we know that some self containing hydraulic features are there in this shock absorber. And even in the railway vehicles, we know that even prior to fail the things we are just using a specific kind of shock absorber, which is just you know like absorbing the high amount of impact energy under the plastic flow or any kind of we can say the elastic reason. We also discussed about that how we can design this shock absorber means, what kind of you see you know like rather rather we are using the hydraulic cylinder or in the pneumatic part or in the fluidic feature of the cylinder or even when we are using the coiled or leaf spring or the torsion bars.

Then how the integration can be coupled, the integrations of these two things are just you know like properly designed coupled in such a way that it can provide the smother part. Because we know that when the shock features, the impulsive forces are being coming due to the irregularity of the road or the bumps or any kind of thing then not only we need to absorb the amount of energy, but also it leads to dissipate. Or to you know like just we can say transfer or absorb the energy at the time of you see here the excitations.

So, all three features right from absorbing to dissipating to we can say you know like controlling the features by storing is being provided by these shock absorber.

So, you see here the previous two lectures were delicately given to the concept of shock absorber, because this is one of the most common device which is being there right from automobiles to the industrial machinery or even the air craft motion as well. So, in this lecture now, again the passive vibration control concept is there, but with the design of the isolators in which you see the stiffness and the damping features are being associated together.

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Introduction

- Noise and vibrations have undesirable effects on both human quality of life, and on our material goods.
- A very useful strategy to reduce noise and vibrations is to interrupt the propagation path between the source and the receiver.
- *Elastic mounting* is a simple method to hinder the spread of structural vibrations.
- In practice, an elastic mounting system is realized by incorporating so-called vibration isolators along the propagation path.

So, when we are talking about the noise or vibration we know that if they have the undesirable effects not only on the machine or on the material of the machine, but also on the human part. When the comfort, discomfort, the quality of life and everything is coming. So, we just want to adopt the strategy in which we can reduce the noise and vibration just by interrupting the progression path whatever the propagation things are there, and we need to just strike out this in between the source and receiver. And we know that this is one of the effective way the passive vibration control in which we are using elastic mounting just to hinder the spread of the structural vibrations.

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- In practice, an elastic mounting system is realized by incorporating so-called vibration isolators along the propagation path.
- Strongly vibrating machines in factories, dwellings and office buildings can be placed on elastic elements.
- The vibration isolation is yet another substructure incorporated between the two structures.
- The objective of vibration isolation is to reduce the vibrations in some specific portion of the receiver structure. It is apparent that vibration isolation can be realized in many different ways.

So, in practice an elastic mounting system can be realized by incorporating the vibration isolators along with the propagation path. So, when we are saying that the elastic mounting is being there, we need to check it out that what the strong vibrating features are there, in that an accordingly we can simply put the elasticity feature in the elastic mountings. So, vibration isolation is another we can say you know like the kind of feature in which the substructure is being incorporated between that vibrating and the ground itself or any two component we can say. So, the objective of this vibration isolation is to reduce the vibration at in a specific portion of the receiver structure. So, it is apparent that the vibration isolation can be realized in various ways, because through that we are striking not only on the source or the receiver, but also we are trying to deviate the path through the transmission feature.

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- It therefore falls upon the designer to arrive at an isolation system design well-suited to the specific situation.
- A vibration problem can also be nicely described by the same *source – path – receiver* model used to characterize the noise control problem.
- Source: a mechanical or fluid disturbance, generated internally by the machine, such as unbalance, torque pulsations, gear tooth meshing, fan blade passing, etc.

So, it is therefore falls upon the designer just to arrive an effective isolation system design which can be well-suited to the specific situations. But generalized, if you are talking about then we have three main things the source-path and the receiver. And through that we can effectively control the entire vibration features. So, when we are talking about the source, we know that the source may be of solid or the fluidic feature, may be unbalance or misalignment in the solid, may be looseness or maybe you see you know like the external force features are there. In the fluidic part the airborne structure where you see the flow induced vibrations are there. So, even the bearing, the defects in bearing, the gear meshing features, even there are various fan blade passing, various you see the solid medias are there through which the vibrations are being generated and the noise is being spread it out.

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- Path: the structural or airborne path by which the disturbance is transmitted to the receiver Receiver: the responding system, generally having many natural frequencies which can potentially be excited by vibration frequencies generated by the source.
- The best solution to a vibration problem is to avoid it in the first place. Intelligent design is far more cost effective than building a bad design and having to repair it later.

The path, this is something we can say is absolutely based on either the solid or liquid the structure or the airborne path by which the disturbance is simply being transmitted towards the receiver. And the receiver is nothing but the responding feature which generally you know like having various natural frequencies at which the entire system is excited and simply being going to the receiver end. So, the best solution to a vibration problem, just to avoid at the first place means we need to strike it out at the source only. And the intelligent design is far more cost effective then what the when we are building a bad design and having just repairing later on by deviating the path or something something like that, but when you see here it is unavoidable then the another thing is that how to minimize this.

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- Minimizing the vibration transmission generally involves using isolator springs and/or inertia blocks.
- The basic principle is to make the natural frequency of the machine on its foundation as far below the excitation frequency as possible.

So, minimizing the vibration transmission is generally involving the isolator springs or the inertia blocks, because ultimately we need to deviate the whatever the transmission features when the vibrations are being transmitted. So, the basic principle is to make the natural frequency of the machine on its foundation is far below the excitation frequency as possible, so that is why you see here we need to just strike it out that how we can make the natural frequency is just lower down. So that whatever the exciting frequencies are being coming you know like the resonance can be avoided, and this can be done either by using springs or by inertia blocks.

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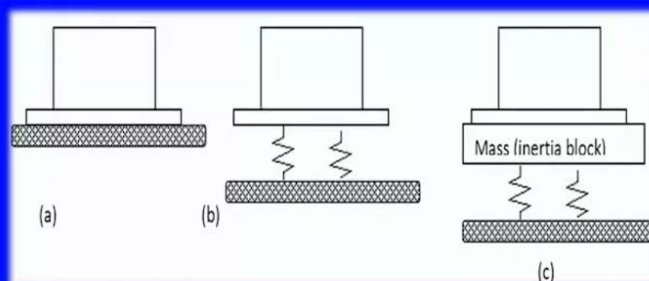


Figure 1. Vibration isolation systems: a) Machine bolted to a rigid foundation b) Supported on isolation springs, rigid foundation c) machine attached to an inertia block

How you can see that now we have three configurations even rather four the fourth is the next one, so three configurations, which is on your screen. First a simple machine, which is bolted to the rigid foundation. So, whatever the exciting part is coming through the vibrating of mass on top immediately transmit out, because we have a rigid connection through the bolt. Second, we are just providing a simple isolator using a spring in between the vibrating mass and the rigid foundation. And third, we are providing a machine, the vibrating we can say you know like the system which is being attached to the inertia block and then to the using a spring we are simply coupled with the rigid foundation. So, you see here, in between the rigid foundation and the vibrating mass, we have added mass which is we can say the inertia block and the spring itself.

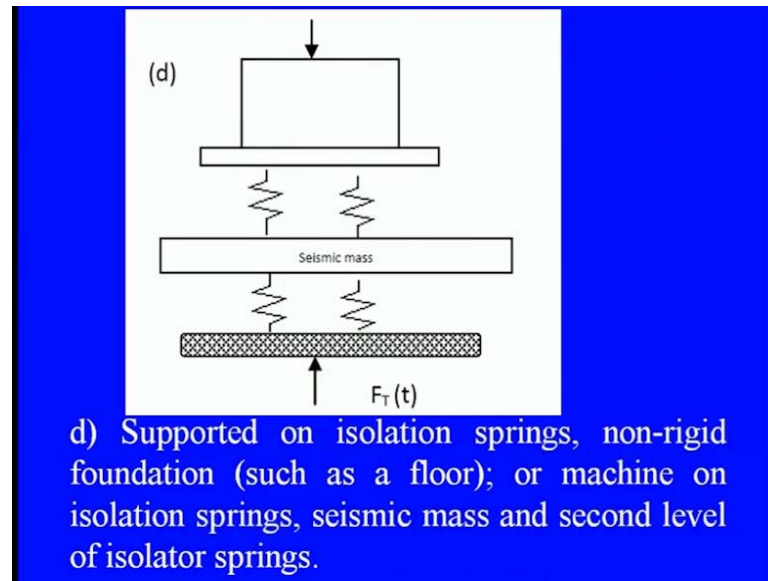
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- Consider a vibrating machine, bolted to a rigid floor (Figure 1a). The force transmitted to the floor is equal to the force generated in the machine.
- The transmitted force can be decreased by adding a suspension and damping elements (often called vibration isolators) Figure 1b , or by adding what is called an inertia block, a large mass (usually a block of cast concrete), directly attached to the machine (Figure 1c).

So, you see here when we are using you know like this bolted feature in the rigid floor we know that whatever the force which is being generated by the vibration mass immediately transmitted out to the foundation. The transmitted force can also be decreased by adding a suspension device of the spring in between the vibrating mass and the foundation as I shown in the – b. And also you see here some inertia block which has you know like the mass in terms of you know like the concrete or anything which can be directly attached to the machine, the vibrating machine and then you see the spring and the foundation can be provided.

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And the forth one we can rather add the seismic mass in between the two isolation, we can say the spring which we are saying that you see right from the foundation, and this foundation is rather not the rigid one, it has you see the flexible foundation at which that force is being transmitted. And then, you see, we can say that in between you see the foundation the flexible foundation and the vibrating mass, we have seismic mass in between these two springs. So, this is you see we can say a second level of isolator is being there along with the seismic mass in terms of the spring. So, these are the four features through which we can apply the isolator for an effective control on vibration by suppression it. So, this is what you see the suppression feature of vibration either by adding the spring or by adding the mass, so that the resonance feature of the exciting masses of the exciting object can be avoided.

Now, we are going for the mathematical treatment of this. So, we know that the equation of motion for such devices are all the force balance condition, the inertia force, the damping force and the restoring forces are being balanced by the external excitation force. And from that the response the final response can be simply taken as x of t is equals to F zero which is the input force by k divided by 1 minus r square sin ω t . So, this is what my output response of the system where the r is nothing but the frequency ratio ω by ω_n ; ω at which the system is exciting ω_n which is the natural undamped frequency of the system which can be calculated as square root of k by m .

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- The equation of motion for the above mass spring system is:

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

- The response of the system is:

$$x(t) = \frac{F_o / k}{1 - r^2} \sin \omega t \quad \text{where : } r = \frac{\omega}{\omega_n} \quad \omega_n = \sqrt{\frac{k}{m}}$$

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- The ratio of transmitted force to the input force is called *transmissibility, T*

$$T = \left| \frac{F_T}{F_o} \right| = \left| \frac{1}{r^2 - 1} \right| = \left| \frac{X}{Y} \right|$$

- This same equation can also be used to calculate the response of a machine *X* to displacement of the foundation, *Y*. The effectiveness of the isolator, expressed in dB is:

$$E = 10 \log_{10} \frac{1}{T}$$

So, when we are just going that what exactly the transmission is there through the excitation to the transmitted force then we can say that we can calculate the transmissibility which is nothing but equals to F_T transmission force divided by the input excitation force F_0 . And we can say that this is nothing but equals to one divided by $r^2 - 1$, where the r is the frequency ratio. Or else we can say that it is nothing but equals to that the responses of machine whatever you see the responses are there and the displacement of the foundation X by Y .

So, you see we can say that the transmission rather in terms of the machine response with the displacement of the foundation X by Y or with the force transmission F_T by F_0 , it can be easily evaluated based on $1 - r^2$. And the effectiveness of the isolator can also be expressed in terms of the sound propagation path and it is nothing but equals to $10 \log_{10} \frac{1}{T}$ means that the inverse part of the transmissibility. And you see this effectiveness is always being taken care that you see how much you see the sound propagations are there in terms of the sound level energy at the n in db.

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- The effectiveness of the isolator, expressed in percent is:

$$\% \text{ Isolation} = (1 - T) * 100$$
- The transmissibility as a function of frequency ratio.
- Vibration isolation (defined as $T < 1$) occurs when the excitation frequency is $> 1.4 f_n$.
- For minimum transmissibility (maximum isolation), the excitation frequency should be as high above the natural frequency as possible.

So, now you see we just want to see the effectiveness of the isolator we can simply expressed in terms of the percentage effect. So, percentage isolation is nothing but equals to one minus T into 100 you see. So, transmissibility is nothing but the function of r the frequency ratio because we know that the it is nothing but equals to one divided by r^2 minus one. So, the vibration isolation which we are defining as T is always occurs when the exciting frequency is greater than $1.5 f_n$ means when you know that the isolation is a purely effective tool. And it is acting as you know like as it is being designed only when transmissibility, or we can say that this entire exciting frequency is always greater than 1.4 times of f_n . And for minimum transmissibility, when the minimum transmissibility is there, when you have the maximum isolation effect the excitation frequency should be high as high as the natural frequency, so that the

resonance can be avoided and the absorption feature through this spring can be effectively done either by spring or mass whatever.

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- The transmissibility above resonance has a slope of – 20 dB/decade. The transmissibility including the effect of damping is:

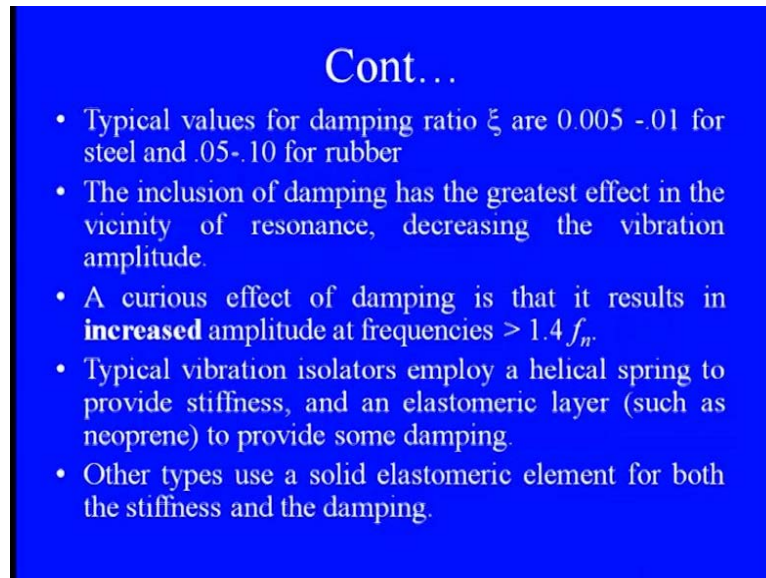
$$T = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(1 - r^2)^2 + (2\xi r)^2}}$$
$$\xi = \frac{c}{2m\omega_n} = \text{critical damping ratio}$$

So, transmissibility above resonance has a clear slope with the sound level of 20 db per decade and in that you see here when we just want to include the damping effect in the transmissibility then the formula for calculation of this is nothing but equals to square root of one square root of 1 plus two zeta r square. Where the zeta is the damping ratio, which is simply calculating based on the available damping divided by the critical damping. The r is the frequency ratio, the exciting frequency the forcing frequency we can say divided by the natural frequency divided by the square root of 1 minus r square plus 2 zeta r square.

So, critical critical damping ratio is something, which simply shows the effect of damping on the exciting features. If the available damping is less than the critical damping, we know that this is the system where you see you know like the critical or we can say the higher or the lower damping features are there. So, this is you see the under critical damping features, so in that we can say the system is oscillating with the exponential, and sinusoidal feature. Second this is what the under damping second you see when the zeta is greater than 1, in which the damping is the available damping is more than the critical damping is the over damped phenomena. In which even we have the exponential decay, but you see here this feature is will take the infinite time to get it

the steady state response or it is just to dampen out. So, ultimately our system is to be designed on the critical damped feature in which the available damping and the critical damping should be equal to dampen out the entire vibration at the quicker quickest time.

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- Typical values for damping ratio ξ are 0.005 - 0.1 for steel and 0.05 - 0.10 for rubber
- The inclusion of damping has the greatest effect in the vicinity of resonance, decreasing the vibration amplitude.
- A curious effect of damping is that it results in **increased** amplitude at frequencies $> 1.4 f_n$.
- Typical vibration isolators employ a helical spring to provide stiffness, and an elastomeric layer (such as neoprene) to provide some damping.
- Other types use a solid elastomeric element for both the stiffness and the damping.

So, you know like we are saying that we are just keeping the damping ratio in between 0.005 to 0.1 for the steel and 0.05 to 0.1 for the rubber material. Because this inclusion of damping has the greatest effect in the vicinity of resonance, and they are absolutely decreasing the vibration amplitude by absorbing or dissipating the energy part. And the curious effect of the damping is that the result is simply increase the amplitude at the frequency which has greater than 1.4 times the natural frequency. So, when we are designing the vibration isolator and when we are straightaway adopting using the helical spring, there always been providing a good stiffness. But one you see here they are simply that they are not basically we can say transmitting feature, they are simply you know like we can say compressibility are there due to the spring part, they are just to saving the energy and releasing the energy accordingly.

But the elastomeric layer where you see the elastomeric properties are there which we can say the neoprene or any kind of you know like such things, they are simply providing some kind of damping along with the compressive or the we can say the elastic nature of this spring is. So, various other types of we can say the elastomeric elements are there

along with we can say in which we can say that they are simply provider of the stiffness and the damping together.

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MEASURES OF TRANSMISSION ISOLATION

- In order to be able to design in optimal vibration isolation, there is a need for, not only the determination of the vibration levels, but also for some measure of the vibration isolation obtained; that latter would permit comparison of alternative isolation strategies that may be applicable in a given situation.
- A number of different measures are in use for various specific applications. The most universally applied of these is the so-called *insertion loss* D_{IL} ; it is defined in either of the two following alternative ways:

So, measure of the transmission isolation. So, when we are talking about a isolation design, we need to see that what is the optimal vibration isolation should be there. Which is nothing but you see that how much vibration level can be suppressed out, how the energy is being dissipated and you see here you know like accordingly how the system is bringing towards the stable nature by doing that. So, some of the some of the measure is there for isolation, vibration isolation and that you see we can say that they are simply permitting that how the alternative strategies should be adopted, so that we can simply bring an effective solution of that. So, there are various measures according to the applications are being taken in that, the best in this is the insertion loss. The insertion loss is nothing but you see here, it is simply showing that how the sound level is being effectively damp out before and after being using.

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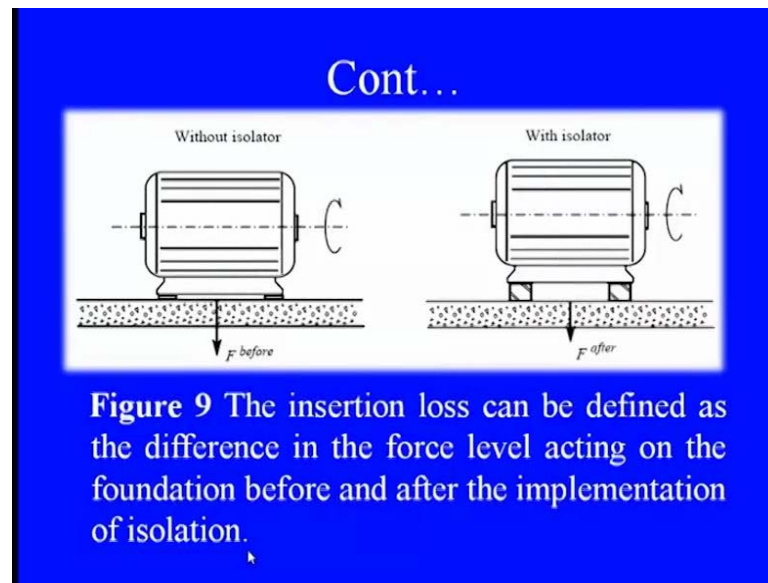
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$$D_{IL}^v = L_v^{before} - L_v^{after} \quad [dB]$$
$$D_{IL}^F = L_F^{before} - L_F^{after} \quad [dB]$$

- where the velocity and force levels L_v and L_F .
- The insertion loss is, therefore, defined as the difference in level at a given point before and after the vibration isolation is provided; see figure 9.

But, there are two path or there are two ways through which we can define the insertion loss one is based on the velocity component that what is the velocity level is they have. Because we know that velocity is also linearly depending on the frequency. So, we can straightway check that the insertion loss is nothing but equals to what is the before vibration of the entire structure, and after applying the isolation, what is the velocity part. And the difference is giving the insertion loss in terms of d b. And the same time you see the force which is the critical part in the excitation feature of the entire structure can also be taken for calculating the insertion loss, so the force before adopting the isolation and after putting the isolation we can measure and the difference is giving like that. So, we can say that insertion loss is nothing but the difference in the level at the given point before and after adopting the vibration isolation in terms of the velocity or the force and it is being measure in terms of d b.

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So, you can see the simple machine which is being mounted on the rigid foundation. Before you can see, this is the rigid connection in between the machine and the foundation, and vibration transmission is absolute criteria there. Whatever the vibration generation and transmission, they are perfect no loss of this force is there. So, either the force or the velocity, there is no loss. But when we are adopting, you see the vibration isolator we say in between the source as machine, and this particular path which is my foundation when we are adopting this, we can simply see the insertion loss in between you see here this force and this one. So, F_{after} whether the velocity and F_{after} whether the velocity or the force can be just taken care and see the effectiveness of this in terms of the insertion loss.

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- With these definitions as a model, it is of course possible to devise other such measures of the isolation effectiveness based on weighting different frequency components and bands, e.g., using A-weighting for instance.
- The choice of the relevant gauge of effectiveness, is ultimately determined by the specific application.

So, with these models, it is simply you know like we can say that what exactly the devise through which we can measure the isolation effectiveness just by weighting different frequency component. Or we can say the bands like a-weighting, b-weighting and you see it is all you see the weighting factors are there through which we can just see the effectiveness part. And the choice of relevant gauge of the effectiveness is just based on what are the specific application of the component, what are the service conditions towards that.

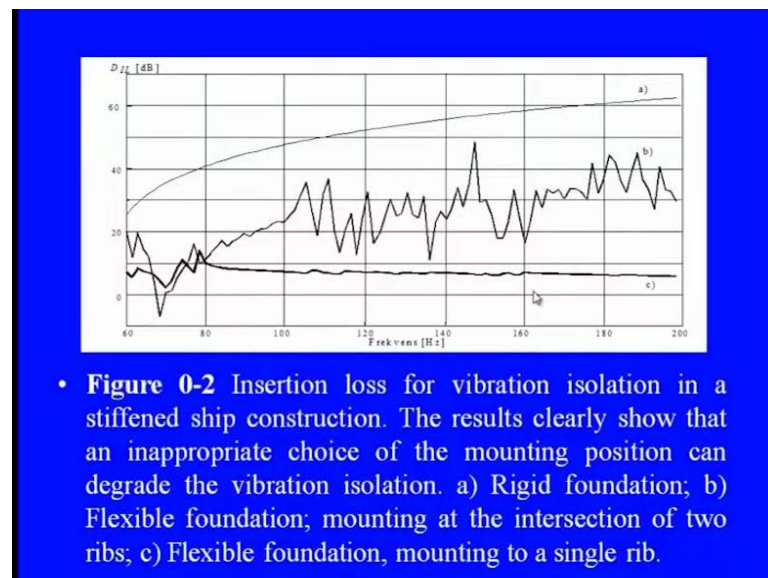
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- The machine is regarded as a rigid point mass, and the isolator as an ideal spring.
- Curve a) shows the insertion loss when the mounting positions are taken to be rigid.
- In both of the other curves, the measured stiffnesses of two alternate mounting positions, one stiff at the intersection of two ribs, and one softer on a single rib, are used.
- If the softer mounting position is chosen, the isolation obtained at higher frequencies is never more than 7 - 8 dB.

So, the machine is regarded as the rigid point mass, which I am going to show you see here and the isolator can be act as a ideal spring. So, this rigid mass, we can say rigid point mass and the ideal spring is one of the good we can say ideal perfect, we can say the compact feature in which we can simply adopt that what the insertion and the other losses are. So, now, I am going show, the figure in which the various you see you know like the types are there in which the insertion losses are there, the first is showing the insertion loss when the mounting positions are being taken to be a rigid. And you see here the other two curves, which are being you know like simply measured the stiffness of two alternative mounting positions; one as the stiff at the resonance of two ribs, and one is the softer on the same we can say single rib is being used. And if the softer mounting is just position accordingly the isolation obtained at the high frequency cannot be more than 7 to 8 db of the insertion loss.

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So, you can see that these a, b, c features are there, the a as we discussed about the rigid foundation b is the flexible foundation you know it is the mounting at the insertion losses with the two ribs. C is also you see here with you know like the flexible foundation, but the mounting using the single rib. So, you can see the fluctuations that how much effective controls are there in the vibration in the insertion losses along with you see the frequencies are there.

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- There are many different ways to design the foundation such that the mounting points have the desired properties, i.e., **low mobility**.
- Most of the methods used in practice are based on the use of added masses and stiffening beams applied in an appropriate way.
- In all such situations, it is important the plan such solutions right from design stage.
- Their incorporation at a later stage, to give the desired dynamic properties, is in most cases both expensive and time consuming.

So, there are various ways to design the foundation as we discussed about the rigid and flexible foundation, so that the mounting points have desired properties like the low mobility. And most of the methods in practice are just based on the use of the added masses or the stiffening beam applied to the system in an appropriate way. So, in first situations, we can say that it is important to plan the solution right from the designs stage. And there incorporation at then later stage, just to give you know like the desired dynamic property and we we know that this is not only the time consuming, but also you see effective expensive part in a effective less effective way.

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Important Considerations with Vibration Isolator Selection

- 1) Machine Location
 - As far away from sensitive areas as possible
 - And on as rigid a foundation as possible (on grade is best)
- 2) Proper sizing of isolator units
 - Correct stiffness (specified by the static deflection, more flexible is generally better)
 - Sufficient travel to prevent bottoming out during shock loads, or during system startup and shutdown

So, what are the considerations when we are just choosing the vibration isolator. First the machine location as we discussed. We need to see that we are simply putting the vibrating machines in such a way that there should be as far as possible from the other parts of the machine or the workers. Just like you see we know that when we are just going to power plant rather you see, it is gas based or the thermal power plant the turbo generator units in which the total generator feature are there, the heavy rotors are being used to transmit the entire power which is being generated say in that boiler or somewhere. It is creating a huge amount of inertia force and along with the huge amount of inertia forces, because of you know like the unbalanced feature or any feature you see when this huge mass is generating, the huge inertia we know that the high level of exciting vibrations and the sound levels are there. So that why we are keeping in such distance level that it should not affect the other machine's part first. Second, we can simply put the rigid foundation means we can say the high-grade foundation, so that we can provide the best isolation at the source itself prior to go for the transmission feature.

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- 3) Location of isolators – isolators should be equally loaded, and the machine should be level.
- 4) Stability – sideways motion should be restrained with snubbers. The diameter of the spring should also be greater than its compressed height. Isolator springs should occupy a wide footprint for stability.
- 5) Adjustment – springs should have free travel, should not be fully compressed, nor hitting a mechanical stop.
- 6) Eliminate vibration short circuits – any mechanical connection between machine and foundation which bypasses the isolators, such as pipes, conduits, binding springs, poorly adjusted snubbers or mechanical stops

Second is the proper sizing of the isolator units. The proper sizing means the stiffness, what are the stiffness of the spring which we are providing it should be correct, for the static deflection or even we can say for more flexible is always being better. And the sufficient travel to prevent the shock loads or you see vibration level should be you know like should be provide in such a way that it can absorb or it can deviate the path of this

transmission feature of the vibration. So, these you see the isolator units are being chosen according to the sizing of this feature.

The third is the location of isolator in this you see here, the isolator on the machine isolator means the source, isolator on the receiver or isolator on we can say the transmission part should be chosen according to the excitation level of the system. The fourth is very important there is stability. The sideway motion should be restrained with the snubbers; and these snubbers, they are always providing good stability to the entire systems and the diameter of the spring which we are you know like designing should also be greater than its compressed height. Otherwise you see you know like the buckling or any kind of act is there, isolator spring should also occupy a wide you know like footprint for the stability. It should not you see just going towards the banding or it should not going you see towards the torsion effect in that.

Fifth is the adjustment, the spring should you know like travel or should not you see know like fully compressed in such way that we cannot even go for any kind of adjustment and the hitting of the mechanical devices are being there due to the fluctuation loading when the spring is compressed or released. We need eliminate the short circuit of the vibration, so any mechanical connection between the machine and the foundation through which the you know like the isolator is being there.

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Design of absorber for continuous system

- A general beam is taken as the primary system with absorber attached to it and subjected to a harmonic force excitation.
- The point excitation is located at b ; and the absorber is placed at a :
- A uniform cross section is considered for the beam and Euler–Bernoulli assumptions are made. The beam parameters are all assumed to be constant and uniform.

Just like you see the pipes, conduits or we can say the spring or poorly adjusted snubbers or any mechanical stops, we need to check it out that the isolator should be you know like in such way that they should bypass. The thing in such a way that it is not transmitting entire vibration to these pipes or any conduits or any binding springs.

So, in this section, now we are designing the absorber for continuous system. So, we can say that the general beam can be taken as the primary system with the absorber attached to it and subjected to the harmonic force excitation. And the point of excitation is located in such way that that you see you know like the absorber can be placed absolutely along with this part. And uniform cross section is considered for the beam, and we know that the Euler-Bernoulli theorem can be straightway applied with all the assumptions. And if now you see, we are simply assuming that whatever the beam which is being taken as the constant and uniform cross section.

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- The elastic deformation from the un-deformed natural axis of the beam is denoted by (x, t) and, in the derivations that follow, the dot $(\dot{\cdot})$ and prime (\prime) symbols indicate a partial derivative with respect to the time variable, t ; and position variable x ; respectively.
- Under these assumptions, the kinetic energy of the system can be written as

$$T = \frac{1}{2} \rho \int_0^L \left(\frac{\partial y}{\partial t} \right)^2 dx + \frac{1}{2} m_a \dot{q}_a^2 + \frac{1}{2} m_e \dot{q}_e^2$$

And now you see we are considering the elastic deformation part along the neutral axis of the beam say in terms of the space and time x and t , and you see you know like these derivations are simply followed with the time and the space part. And they are simply say that the simply a partial derivative with respect to the time variable and the position variable. So, now, you see the kinetic energy can be computed using the material property of the stiffness is the beam is the density ρ , and you see here whatever the excitations are there it is being varied with the time, so dy by dt . So, what we have, we

have now the kinetic energy half rho for entire beam, whatever you see the deviations are there dy by dt square 0 to l integration into dx plus, the added mass, I am going to show you that that is m a q dot it is being you know like varied with the q dot. So, q dot square plus m e q e dot square half.

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- The potential energy of this system using linear strain is given by

$$U = \frac{1}{2} EI \int_0^L \left(\frac{\partial^2 y}{\partial x^2} \right)^2 dx + \frac{1}{2} k_a \{y(a, t) - q_a\}^2 + \frac{1}{2} k_e \{y(b, t) - q_e\}^2$$

So, you can see that we have you see this is what excitation, the beam is simply excited there. Now you see here since the beam has its own inherent property like the elastic module, the length, the area and the movement of inertia in that. And when we are saying that we are adopting the two different masses for absorbing feature along with the damper, the stiffness and also you see here we have special feature damping, so that the linear strain can be provided in this. So, we have at that location a and location b, and for this, we can say the potential energy is nothing but equals to half E I - the modulus rigidity into now this part in which you see you know like the entire you know like a deflection features are there.

So, del 2 y by del x square, square into dx plus the mass which is being added. So, half k a, the y which is based on you know like a coma t minus q which is you see you know like the the whatever the displacement is there of the mass a the q a square simply when we are talking about mass e half k e y the b t, because this is the time and the space feature b coma t minus q e which is you see the mass e displacement square. So, I have both the kinetic energy and the potential energy together.

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The equations of motion may now be derived by applying Hamilton's Principle. However, to facilitate the stability analysis, we resort to an assumed-mode expansion and Lagrange's equations.

Specifically, is written as a finite sum "Galerkin approximation"

$$y(x, t) = \sum_{i=1}^n \Phi_i(x) q_{bi}(t)$$

And based on you see the Hamiltonian principle, we can simply write the equation of motion which is simply you know like providing or facilitating the stability analysis and we can simply get you see you know like the entire equation based on the Lagrangian dynamics. And for that first assumption which we are assuming that the Galerkin approximation for this that is nothing but equals to the y whatever you see the deflection which is based on the space and the time x coma t is nothing but equals to summation of i to 1 phi i q of b i times t.

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The orthogonality conditions between these mode shapes can also be derived as (Meirovitch, 1986)

$$\int_0^L \rho \Phi_i(x) \Phi_j(x) dx = N_i \delta_{ij}, \quad \int_0^L EI \Phi_i''(x) \Phi_j''(x) dx = S_i \delta_{ij}$$

where $i, j = 1, 2, \dots, n$, δ_{ij} is the Kronecker delta, and N_i and S_i are defined by setting $i = j$ in above equation.

So, it simply shows that when we are simply describing these things in which you see here, we have you know like the variations along with the angels we can simply you know like chosen using the Meirovitch part the orthogonality conditions in that for the mode shape. So, we have the integration of 0 to 1 rho phi i and rho phi j both based on the x part is nothing but equals to the two constant N i, delta i where the delta is the Kronecker delta, the delta part and this is simply showing that how variations are there in i and j feature. So, we can say that this is the integration of E I phi j x phi j into x dx is nothing but equals to the S delta i j. So, here we know that the Kronecker delta which simply shows you know like the unit vector deviation and N i and S i is simply you know like we are getting with the setting of i coma j.

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- The feedback of the absorber, the actuator excitation force, and the damping dissipating forces in both the absorber and the exciter are considered as non-conservative forces in Lagrange's formulation. Consequently, the equations of motion are derived.
- Absorber dynamics is governed by

$$m_a \ddot{q}_a(t) + c_a \left\{ \dot{q}_a(t) - \sum_{i=1}^n \Phi_i(a) \dot{q}_{b_i}(t) \right\} + k_a q_a(t) - \sum_{i=1}^n g_i \ddot{q}_a(t-\tau) = 0$$

So, we can say that now we can even put the excitation feature and when we are doing this the feedback of the absorber. When we are saying that you know like the when we are trying to actuate the exciting force and the damping dissipation forces both can be put you see here with the absorber and the exciter and then we can say what are the non conservative forces which are being there can be immediately formulated based on the langrage's dynamics. And accordingly we can say that the absorber dynamics which is being there you see on that particular entire beam is nothing but equals to the equation m a q a double dot the exciter part, you see here with inertia forces as the mass is being you know like oscillating with q a and c a that is the damper with the exciter into q dot a t q

dot a with the t minus all these summation which we are simply assumed previously that is summation of phi i into a q b i into t.

So, this you see the variation is clearly showing that the difference between whatever the damper which is the damping effect which is being provided there, and second you see here how the orthogonal conditions are being there using the Galerkin's approximation there itself. And then you see here the restoring forces k a q a dot the q a into t. So, this is you see whatever the you know like the restoring forces being coming out where this added mass is there minus summation of i to 1 n g into q a q double dot a t minus tau at the time you see you know like the time constant what exactly the differences are there, and how you see the absorption features are there where under the dynamic actions.

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- Finally, the beam is represented by

$$N_i \ddot{q}_{bi}(t) + S_i q_{bi}(t) + c_a \left\{ \sum_{i=1}^n \Phi_i(a) \dot{q}_{bi}(t) - \dot{q}_a(t) \right\}$$

$$\Phi_i(a) + c_e \left\{ - \sum_{i=1}^n \Phi_i \dot{q}_{bi}(t) - \dot{q}_e(t) \right\} \Phi_i(b)$$

$$+ k_a \left\{ \sum_{i=1}^n \Phi_i(a) \dot{q}_{bi}(t) - \dot{q}_a(t) \right\} \Phi_i(a) + k_e \left\{ \sum_{i=1}^n \Phi_i(b) q_{bi}(t) - q_e(t) \right\}$$

$$\Phi_i(b) + g \Phi_i(a) \ddot{q}_a(t - \tau)$$

$$= f(t) \Phi_i(b), \quad i=1, 2, \dots, n$$

And even you see using this Meirovitch conditions which we can also apply the same conditions there of this here. So, based on those when we are simply going with the Kronecker delta the coefficient at i equals to j you see N i and S i. And you know like we can simply represent the beam vibrations using this is the continuous vibration using you see you know like the partial derivative of and i q i double dot b i of t because you see at the b exciters are there plus s s plus s i q b i t. So, N i and S i are simply using as the constant plus c a into this Galerkin's displacement approximation feature minus whatever the exciting features are there in the damper using the velocity part. So, we have phi i a q this q of b i minus q of a q dot a which is the velocity of our added mass.

And then you see we can adopt you see ϕ of a plus c this is what of the exciter feature into summation of all these. In between you see the differences are the ϕ into q b i t minus q dot e plus you see the restoring force which can be incorporated using k a and k e is the absorber and the exciter, and then you see here we can say that this is equals to f of t ϕ i b . Where we know that you see the entire feature are being just taken care with the exciting force into this ϕ i b .

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- By proper selection of the feedback gain, the absorber can be tuned to the desired resonant frequency, f_c . This condition, in turn, forces the beam to be motionless at a ; when the beam is excited by atonal force at frequency f_c . This conclusion is reached by taking the Laplace transform of above equation and using feedback control law for the absorber. In short,

$$Y(a, s) = \sum_{i=1}^n \Phi_i(a) Q_{bi}(s) = 0$$

So, with the proper selection of the gain, feedback gain which was shown there you see here the absorber can be tuned to the desired resonant frequency and when it is being tuned we know that the entire amount of excitation which is being there is being absorbed by this absorber mass. So, this condition, in turn, forces the beam to motionless when within the tune feature is there and when then beam is excited at say you know like a the atonal forces at the frequency f_c we can say that you know like the added mass is simply reached with the using of you know like the Laplace transformation. We can say y of a com s when we are just using the you know like the feedback control law for the absorber the y of a com s whatever the displacement in the absorber part is nothing but equals to using the Galerkin's part summation of i equals to one to n ϕ i a into q b i s equals to zero. So, the entire amount of energy is being absorbed by this attached mass, when this attach mass is tuned to the entire beam.

And we can say that this y of s is nothing but equals to the function of a $\cos \omega t$ $\cos \omega t$ Q a s of t into then you see whatever the displacement is there q a $\cos \omega t$ and we can say that the q a which is the generalized feature is nothing but equals to which is we can say the the generalized coordinate for that is nothing but equals to the function of Q of b i into t . This is you see what is the exciter displacement are there of this. So, ultimately, you see, when we are talking about the displacement of our absorber which is depending on a $\cos \omega t$ the a is its own you see the coordinate the absorber and s is the generalized coordinate. We can say that it is clearly showing the dependency on y and q a of this particular you know like the entire absorber. And we can say the y of a $\cos \omega t$ is nothing but equals to in this case of the entire we can say at the point a is nothing but equals to summation of i $\cos \omega t$ i equals to one to n ϕ_i a q b i t equals to zero.

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- Which indicates that the steady-state vibration of the point of attachment of the absorber is eliminated.
- Hence, the absorber mimics a resonator at the frequency of excitation and absorbs all the vibratory energy at the point of attachment.

So, this is what the condition for special case in which we are simply mounting the exciter and the absorber and we can tune the absorber along with the beam vibration to get the beam excitation zero. And we can say that it is simply indicates that we have steady state vibration at the point of attachment of the absorber and it is can be straightway eliminated. So, absorber mimics a resonator at the frequency of excitation and absorber you see you know like absorb all the energy, vibration energy at the point of attachment only. So, this is one of the specific mathematical and we can say you know like the physical treatment of such problems in which we can simply defined that what exactly the way of the absorber is to be design. So, that it can be tuned with the even

continuous system because we have say continuous plate or any kind of you see the structure is there. We can simply attach the mass in such a way that it should be tuned with the excitation frequency of the entire continuous structure and it can be immediately dampened out even up to the zero amplitude of the vibration at the source.

Or even at you see here when we know that it is being transmitted even at the localized reason, we can add these you know like the isolator to dampened out the vibration. So, you know like in this, we discussed about the isolator which is supposed to be design along with the spring and the damper and which can be act as you see you know like the vibration isolation features. So, either at the source or at the transmission path or at the receiver, the many things can be designed and we discussed you see in various way especially under the category of passive vibration control.

So, this is all about our passive vibration control which we started from the basics then we design the shock absorber which has a greater application, because we know that whatever the forces which are being coming to the system or from the system to the surrounding, they are always being either the non-linear random or even the impact feature. So, all the time, we cannot said that the vibration which are being generated during the rotation of the component or the machine excitation is always having the simple harmonic motion feature or the steady state feature. They may have and that is even there, you see because of the basics of dynamics says that you may not have all time steady state response you may have the transient feature.

So, these transient feature of the vibration can be effectively controlled using the passive devices. So, straightway, we design right from the basic to shock absorber to the isolator and then you see here either we are using you see the mass or the spring, you know like they can be effectively used according to the exciting energy or excitation frequencies which should not be you know like close to the resonance. And when the things are being there at the resonance that the damper is one of the effective way, and in the damper you see here. We even discuss whether it is you know like the dashpot based on the pneumatic part or based on the oil feature or even you see the elastomeric features are there in that.

In which the viscous elastic features can be added towards that or even you see here, we can go with the very specific part in this that you see whether we are using silicon based

or any other ways, you see in which the temperature sensitivities are there. So, in the entire passive vibration control devices, we discussed about that how effectively we can use these isolators or the absorber in such a way that it can you know like suppress the vibration at the source even at it can suppressed the vibration at the receiver end. And the same time you see here it can you know like control the vibration at the transmission path by adopting mass, the seismic mass or we can say some added mass or even by adopting the springs, may be you see the coiled spring, leaf spring torsion bars or even by adopting the various types of dampers there.

So, this is you see you know like this chapter was mainly focused on the passive vibration control. Now the next lecture will be based on the active vibration control in which these devices are not directly applied, they are now we need to sense and we need actuate based on some electronic forces. So, this is something we can say a smart kind of controlling, where you see we are creating the anti resonant forces using some electronic features, so that is why you see here the sensor and actuator they have a direct linked with this particular active vibration control. And you see there are many places where the active vibration control either by using some smart materials or by using these you know like the sensing feature or actuation feature that active vibration control can be effectively implemented at the vibration excitations.

Thank you very much.