

Vibration Control
Prof. Dr. S. P. Harsha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Module - 6
Principles of Passive Vibration Control
Lecture - 2
Design of Absorber

Hi this is Dr. S.P Harsha from Mechanical and Industrial Department IIT Roorkee, in the course of Vibration Control, we are discussing about the Principles of Passive Vibration Control. So, in the previous lecture, we discussed about the basics of absorber or we can say the isolator, and we know that when the vibrations are being generated from the source or it is being transmitted through the pipe.

And when it is you know like approaching to the receiver, we can certainly optimize that, which method or which way is you know like the fruitful thing either to reduce the vibration at the source itself. To deviate the path of the vibration waves or we the sound waves or the vibration through this, you know like the entire the molecular feature or else we can straightaway reduce the vibration at the receiver end.

And in this particular isolator, which we are saying the passive vibration control the isolator can be designed in such a way that, it can be act as not only you know like the spring feature, but also the damping feature. So, when we are talking about such features, means the elastomers, we can say in which you know like not only, we have the viscosity you know like, but we have the spring features together.

And they are you know like one of the best suited material now a days, because we can straightaway adopt such kind of materials in designing of various components, in the vehicles and even the industrial machines in which the rotations are there. So, in general when we are talking about the passive device; that means, you see here we need to put some kind of, we need to add some kind of you know like the additive features in terms of the spring, in terms of the viscosity or in terms of the mass.

Again, we need to check it out that what kind of the exciting frequencies are there from the system or when the exciting, you know like the forces are there. Then what exactly the amplitude of the vibration, and the exciting frequencies from the system, accordingly

we can choose the best suited part like we can simply adopt the mass or a spring. The damping is one of the important phenomena, when we know that the high amount of energy is being you know like created or the high excitations are there, at the time of resonance.

So, we know that the mass by adding the mass, we can suppress the vibration or by adding the spring, we can control the low frequency vibration, so accordingly we can adopt those things. And even as i told you that the elastomers, which are being there now a days, they can be even provided a integrated effect of the spring as well as the damping. So, we discussed you know like some of the cases there itself, that you know like how absorber can be applied straightaway, and what exactly the best suited methods for that.

And even you know like some of the numerical features in which you see we discussed about that how the transmissibility ratio or the transmission feature, can be you know like immediately just put under the suppression by adopting, you see a proper isolation at the natural frequency or before or after the resonant condition. So, in this case now, in which you know like we are going to discuss about the principles of this passive vibration controller, this chapter mainly deals with the design of absorber in which you see here.

(Refer Slide Time: 03:54)

INTRODUCTION

The term 'vibration absorber' is used for passive devices attached to the vibrating structure. Such devices are made up of masses, springs and dampers. From a control point of view, vibration absorbers can be considered as passive controllers. (The term 'passive' is used loosely here, meaning that the controller can be constructed using masses, springs and dampers).

For such controllers stability is not an issue, since the closed loop system is also a passive one. Vibration absorbers are devices attached to flexible structures in order to minimize the vibration amplitudes at a specified set of points.

When we are talking about you know like the absorber; that means, you see here again we need to go with the passive device, as we discussed. In which we need to attach or we need to add that kind of structure, which can immediately suppress the vibration by absorbing or by dissipating the energy. So, we know that when there is a dissipation the energy is being dissipated by the two ways one the kinetic energy and the strain energy.

So, when we are just designing the vibration absorber we need to take care that, when the dissipation is occur in these two forms of energy it is then being converted into the heat. So, our this vibration absorber should not be you know like affected by this heat generation, during the dissipation feature. So, such device which are being acted, as the vibration absorber and we are adding to the main source are made up of the mass spring and the dampers, and from the control point of view we need to see the vibration absorber can be considered as the passive controller.

Passive means we can say you know like it is a pre normal term, just that you see the controller can be constructed using all 3 features either the mass or the spring or the damper itself. So, such controller in these you know like cases the one of the important phenomena is the stability is not an issue there, because we know that when we are just you know applying the passive vibration control features.

The closed loop system is you know like one of the basic feature of the passive vibration, so vibration absorber are devices attached to the flexible structure in order to minimise, the vibration amplitude at an specific points or specific set of features. So, we can say that when, we are just you know like striking a localised reason of the machine or an object we can say, these passive structures which are being added there are an effective, you know like we can say the system through which we can at least suppress or absorb the energy of the vibration.

So, design of these vibration absorber has a long history actually, somewhere starting in the early 19th century by the Den Hartog even he introduced that the Frahm, which was you know like one of the scientist. He simply says that why cannot we adopt those things to suppress that, and in which you see here we simply assumed a second mass spring device, which simply attached to the main devices.

(Refer Slide Time: 06:01)

Cont.....

Design of vibration absorbers has a long history. First vibration absorber proposed by Frahm in 1909 [Den Hartog, 1956] consists of a second mass-spring device attached to the main device, also modeled as a mass-spring system, which prevents it from vibrating at the frequency of the sinusoidal forcing acting on the main device.

If the absorber is tuned so that its natural frequency coincides with the frequency of the external forcing, the steady state vibration amplitude of the main device becomes zero.

And in this you know like the model in which you see already, there was there was a system and mass and spring is to be attached there, it simply prevents it from the vibrating as the frequency of the sinusoidal forcing part from the main device. So, you know like this mass spring is simply you know like putting to illuminate, whatever the vibration features or the transmission is coming from the machine to the other parts of that.

So, if the absorber is tuned that the one of the specific property, if this absorber is exactly tuned, so that it is natural frequency coincide with the frequency of the external forcing. The steady state vibration amplitude of the main device becomes almost 0, because then entire energy is being gone to the absorber, because now it is tuned with the entire systems frequency.

So, when we are thinking from the control's perspective, the absorber acts like a controller that has an internal model of the disturbance, which therefore cancels the effect of the disturbance outside. So, this is one of the basic mechanism, which is working from the absorber to the device, because we know that whatever the internal disturbances are there. And through these internal disturbances, when the resonant conditions are being appeared, it is clearly absorbed the entire amount of the energy which is being there or which is being transferred from the main device.

(Refer Slide Time: 07:21)

Cont.....

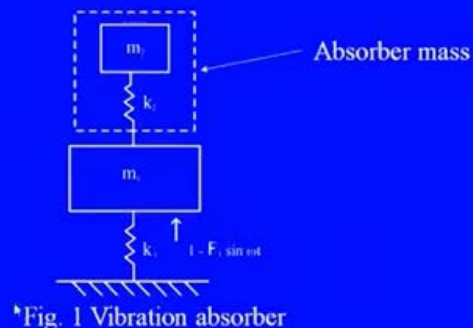
From a control perspective, the absorber acts like a controller that has an internal model of the disturbance, which therefore cancels the effect of the disturbance. The vibrating systems generally consist of a mass, linked to the structure by a spring and a damper. The vibration absorbers are set up so that the mass can vibrate at a specific frequency, close to that of the structure as shown in Fig. 1.

Thus, when the structure starts to move under an excitation, the mass of the vibration absorber move as well, this creates a new system with a different behavior.

So, the vibrating system generally consist of the mass linked to the structure by the spring and a damper, and the vibration absorber are just setup, in such a way that the mass can vibrate at an specific frequency. Just close to the main structure, which I am going to show you the next slide, so when the structure starts to move under the excitation the mass of vibration absorber, also move with the same structure and this creates a new system with the different behaviour.

(Refer Slide Time: 08:38)

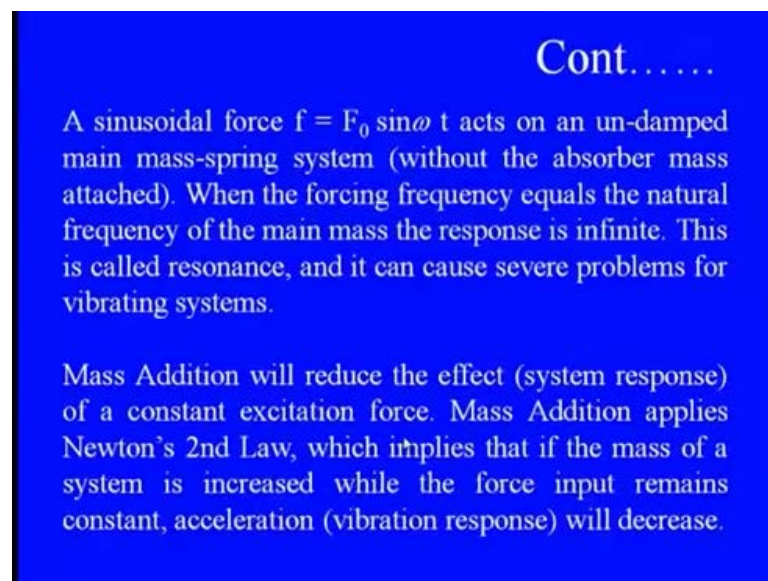
It is then impossible to reach the uncomfortable accelerations reached with the structure alone.



So, as you can see that, when you know like the entire absorber is being you see know like the covered the mass and spring, and then you see when it is attached to the basic system. Even whatever the forcing frequencies are there like $f \sin \omega t$ or $\cos \omega t$, whatever you see then it is straightaway going towards the absorber part, which consist the mass and spring together the undamped system.

So, whatever you see you know like the frequency which is being there from the exciting system, and it is attached you know like it is going towards the attached absorber mass. And when they are tuned to itself the entire energy is being absorbed by the absorber mass, so it is then impossible to reach an uncomfortable acceleration, within the structure alone. So, this is the basic concept of vibration absorber, which is then attached to the basic structure, so when we are saying that when there is in a main machine or the main part you see here. The entire component the vibrating mass, which is under sinusoidal force say $f_0 \sin \omega t$ acting on that you know like the undamped system, in which the mass and spring is there.

(Refer Slide Time: 09:39)



Cont.....

A sinusoidal force $f = F_0 \sin \omega t$ acts on an un-damped main mass-spring system (without the absorber mass attached). When the forcing frequency equals the natural frequency of the main mass the response is infinite. This is called resonance, and it can cause severe problems for vibrating systems.

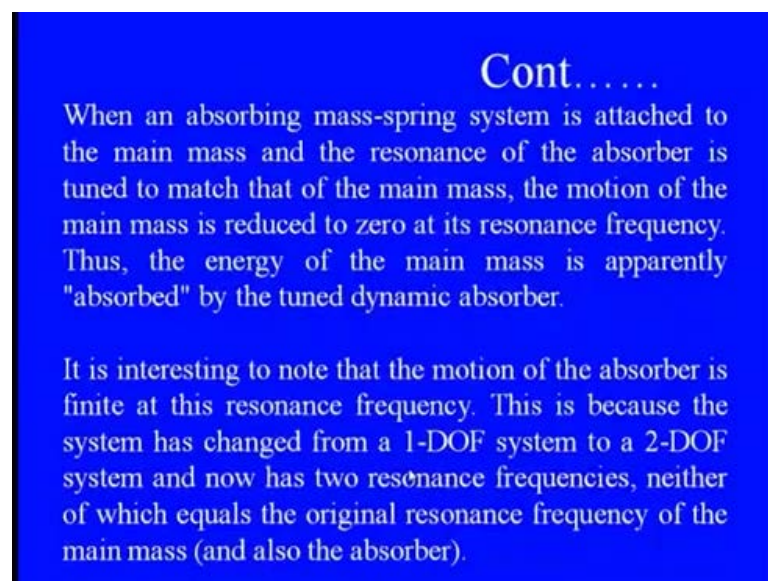
Mass Addition will reduce the effect (system response) of a constant excitation force. Mass Addition applies Newton's 2nd Law, which implies that if the mass of a system is increased while the force input remains constant, acceleration (vibration response) will decrease.

So, we can say that when the forcing frequency equals to the natural frequency of the main mass, where the force is being you know exerted the response is infinite, because of the resonant feature. And when you know like these situations are coming, we know that the severe vibrations are being occurred in that, so when we are adding the mass on the system just to reduce the effect of these vibration.

At, the same you know like we can say the excitation forces, then this particular mass, through this you know like whatever the attachment is there. Through the spring has clearly you know like absorbing the entire energy according to the Newton's second law, which says that, if a mass of the system is increased, while force input is same then certainly you see the acceleration will decrease according to the second law. Where you have seen like the rate of change of momentum, is giving the mass and mass is nothing but equals to the force is nothing but equals to the mass into acceleration.

So, we know that we are not changing the mass, in one point of time when the force is being acting and whatever the excitation is coming, but on the other hand with the same force when we simply adding the mass certainly the acceleration should be reduced. So, when such things are there, we can straightaway use these thing with the increase of mass feature.

(Refer Slide Time: 11:22)



Cont.....

When an absorbing mass-spring system is attached to the main mass and the resonance of the absorber is tuned to match that of the main mass, the motion of the main mass is reduced to zero at its resonance frequency. Thus, the energy of the main mass is apparently "absorbed" by the tuned dynamic absorber.

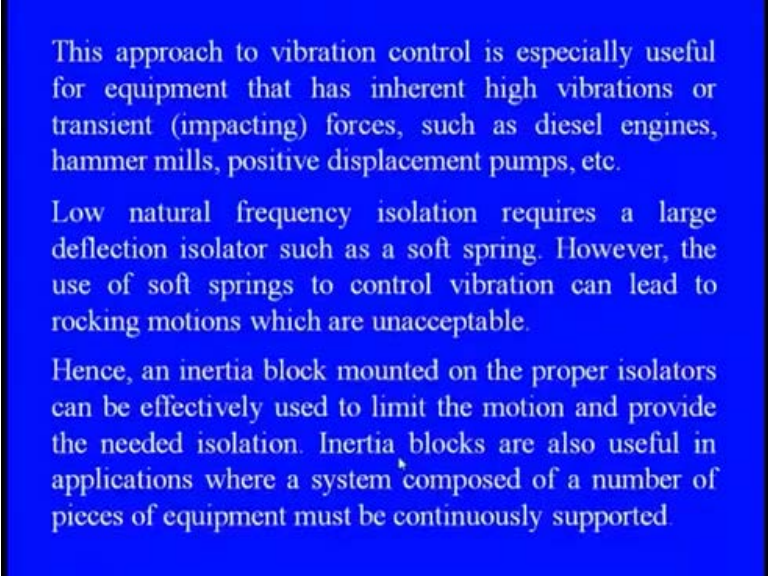
It is interesting to note that the motion of the absorber is finite at this resonance frequency. This is because the system has changed from a 1-DOF system to a 2-DOF system and now has two resonance frequencies, neither of which equals the original resonance frequency of the main mass (and also the absorber).

So, when absorbing mass spring system is attached to the main mass, then we can see that the resonance of absorber is absolutely tuned with the main mass, and the motion of main mass is almost reaching to the 0 at the resonant frequency. That means, whatever the energy or the excitation amplitude is being created or induced at the resonant condition, it is absolutely absorbed by the attached spring and mass. And the energy of the main mass is apparently, we can say that you know like this absorbed feature at the tuned dynamic absorber.

So, this is one of the interesting phenomena in the motion of the absorber, and we can say that at the resonant frequency, these absorber are just showing the finite behaviour under the steady state feature in absorbing the entire energy. And it is, so why because we know that when the system is changed from one degree of freedom, when the entire when the single mass is there to two degree of freedom, when the mass and spring is being attached.

So, now, you see here we can say that the 2 resonant frequencies are there, and both the frequencies which are being we can say that neither of which is equal to the original resonance there. So, certainly you see here when there is a shift in the frequency, the resonant feature of the entire system which is being converted from single degree to 2 degree is just gone down. So, this is the basic physics in the vibration says that, when something is being added as a mass and even that is tuned to the basic system, certainly you see here the system entire frequency is being shifted as per the Newton's second law.

(Refer Slide Time: 13:13)



This approach to vibration control is especially useful for equipment that has inherent high vibrations or transient (impacting) forces, such as diesel engines, hammer mills, positive displacement pumps, etc.

Low natural frequency isolation requires a large deflection isolator such as a soft spring. However, the use of soft springs to control vibration can lead to rocking motions which are unacceptable.

Hence, an inertia block mounted on the proper isolators can be effectively used to limit the motion and provide the needed isolation. Inertia blocks are also useful in applications where a system composed of a number of pieces of equipment must be continuously supported.

So, this approach to vibration control is especially useful, when the when the equipment has inherent high vibration or we can say when it has the transient feature due to any impulsive or impactive forces. Like we can say that the diesel engine, the hammer mills the positive displacement pumps like you see reciprocating or centrifugal pumps. So, in

such kind of devices, in which you see they have the inherent nature, because of their rotation and other components.

We need to add these kind of the masses just to shift the frequency and to control the entire vibration by tuning the additional mass, but with the low natural frequency isolation. When we are just trying that you see the system is just you know like exciting at the low frequency, this isolation requires a large deflection isolator such as the soft spring.

And the soft the soft spring can be used to control the vibration, and which can then lead to some rocking motion, which are somewhat you see you know like, because ultimately that energy has to exchange. So, that is not sometimes acceptable, so we can say that an inertia block, which is mounting on a proper isolator can be effectively used to limit the motion, and then can be provided a simple solution to the vibration isolation.

So, inertia block are always being useful in the application, where system is composing a number of pieces of the equipment, and they are just you know like continuously supporting to the entire system. For vibration generation and even in the transmission they are continuously playing their key role.

(Refer Slide Time: 14:58)

An example of such equipment is a system employing calibrated optics. Thus, inertia blocks are important because they lower the center of gravity and thus offer an added degree of stability; they increase the mass and thus decrease vibration amplitudes and minimize rocking; they minimize alignment errors because of the inherent stiffness of the base; and they act as a noise barrier between the floor on which they are mounted and the equipment that is mounted on them.

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = f$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) = 0$$

So, just like we are saying that one example, in which we have various, you know like the these things are there and we just want these things means you know like these small,

small or we can say the masses are there. And it is in just in the you know like settled in the continuous way, so inertia block needs to be put, because they lower this centre of gravity and once they lower the centre of gravity.

We know that the axis of rotation, and the inertia the moment of inertia axis is just trying to settle down, so this you know like it is just offer towards some kind of additional feature of the stability. And they increase the mass thus decrease the vibration amplitude, and also you see here it is decreasing, the we can say whatever the rocking feature or the exciting feature in entire system. And also you see here, due to this there is a minimization of the alignment errors, because of this inherent stiffness of the base.

So, we can say that they are also acting as the noise barrier between the floor, on which they are mounted and the equipment, at which you see these things are being coupled to. So, you see when we are just talking about the system, in which you know like these isolator or the absorber are being there, and you know like as an inertia block, and they are just acted as the vibration controller in the same time the stability feature.

We can simply write the equation of motion with those as $m \ddot{x}_1 + k_1 x_1 = F_1 \cos \omega t$ is the inertia force due to the vibrating mass. And the $k_1 x_1$ and $k_2 x_2 - k_1 x_1$; that means, you see you know like this spring is being added, towards you see here you know like controlling the vibration with this spring. We can know that you see some kind of rocking features are there, and then this mass m_2 , which is being added with this with the other spring stiffness k_2 which is nothing but $x_2 - x_1$ is just for the balancing thing the inertia blocks.

So, when we are equating those you know like the equations in form of their force balance, we can say that these inertia blocks are just effective, when they are mounted on a perfect place as the isolator. And if there is no damping in the system, then we can say that the responses of these, two degree of freedom system is infinite at certain new frequencies, because now my system while adding these inertia block is now converting it to the two degrees of freedom system.

(Refer Slide Time: 17:05)

One must always keep in mind, however, that to be effective, inertia blocks must be mounted on isolators. If no damping is present in the system as shown, the response of the 2-DOF system is infinite at these new frequencies.

While this may not be a problem when the machine is running at its natural frequency, an infinite response can cause problems during startup and shutdown.

A finite amount of damping for both masses will prevent the motion of either mass from becoming infinite at either of the new resonance frequencies.

And also you see the system may not be a problem when the machine is running at its natural frequency, because an infinite response can also cause the problems during the start up and the shut down. So, certainly you see here we need to check it out, that how the frequencies can be shifted, so finite amount of the damping for both the masses for both. You see here either the added or the basic mass will prevent the motion of either mass, from becoming an infinite at either way we can say that the previous natural frequency or any new resonant frequencies.

(Refer Slide Time: 18:17)

However if damping is present in either mass-spring element, the response of the main mass will no longer be zero at the target frequency. Typically, the mass of the system is increased at the equipment foundation. Therefore, to successfully apply this method for vibration control, machines must be firmly connected to the foundation.

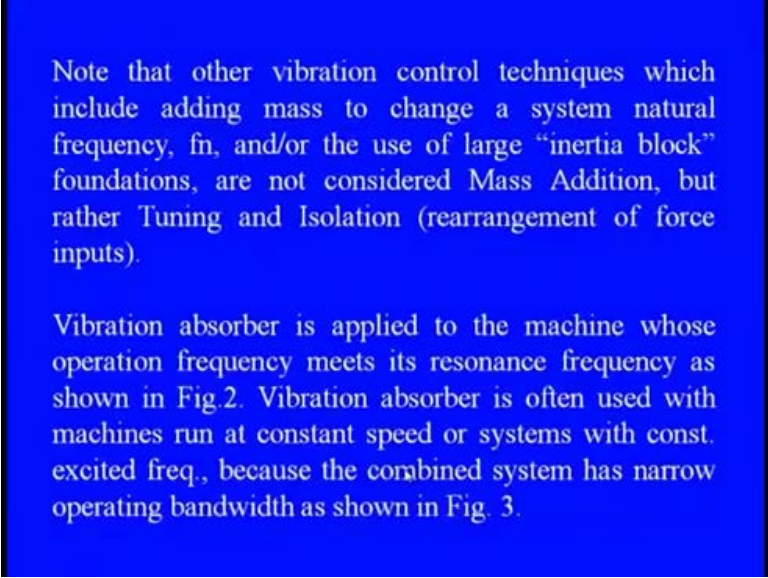
From a machine design perspective, foundations that include a well designed sole plate, epoxy grouted to a concrete base, will help to achieve vibration control and maintenance free equipment operation. One rule of thumb states that the weight of the foundation should be 5X the machine weight.

But, this is not you see the practical situation because we know that the damping is always present in either mass spring element, then we can say that the response of the main mass will no longer be will no longer be 0 at the targeted frequencies. So, the mass of the system is increased at an, we can say equipment foundations by simply adding that, so we can say that to just successfully apply this method for vibration control. The machine must be firmly connected to the foundation, and then it has to transmit the vibration in such a way that the mounting can be specially featured out.

So, from the machine design perspective the foundation that include a well designed solid plate, we can say that the epoxy grouted or we can say the concrete base that will be help out. Just control the excitation not only at the source and even in the propagation, and the proper maintenance can be done exactly at the looseness or the eccentricity or even the alignment feature.

So, the thumb rule says that the weight of the foundation should be almost 5 times of the machine weight, so that a proper damping feature can be added. And accordingly we can adopt the absorber in between the foundation, and the machine for suppression of the vibration.

(Refer Slide Time: 19:50)



Note that other vibration control techniques which include adding mass to change a system natural frequency, f_n , and/or the use of large “inertia block” foundations, are not considered Mass Addition, but rather Tuning and Isolation (rearrangement of force inputs).

Vibration absorber is applied to the machine whose operation frequency meets its resonance frequency as shown in Fig.2. Vibration absorber is often used with machines run at constant speed or systems with const. excited freq., because the combined system has narrow operating bandwidth as shown in Fig. 3.

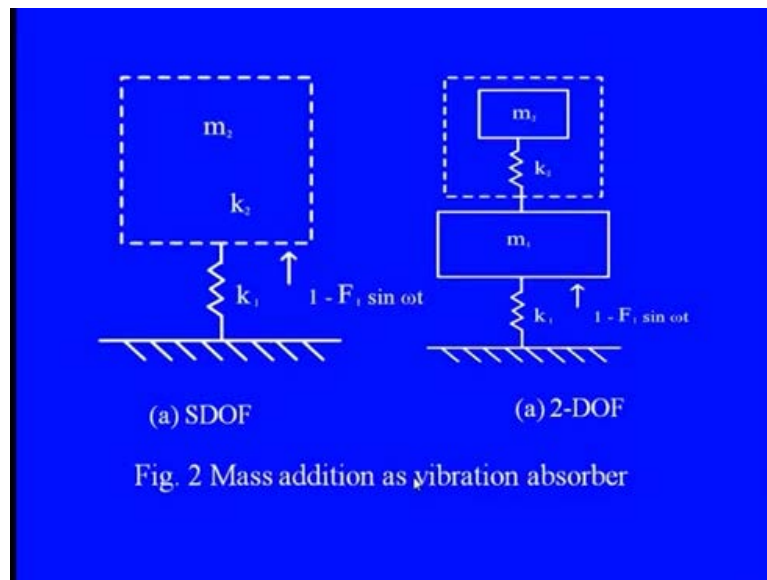
So, there are various other you see you know like the vibration control techniques are there, in which the adding of mass to change the systems natural frequency is being a pretty common part just by adding the inertia blocks to the foundation. So, this mass

addition is not you see all the time is a perfect solution, and can be act as a vibration absorber. So, vibration absorber is applied to the machine, whose operation frequency meets it its natural frequency, and then you see here you know like we can generally put the damper or just to shift the mass just to shift the frequency by adding the inertia block.

So, vibration absorber is often used with the machine, which runs at the constant speed or the system with the constant exciting frequency, because when you combine the system it just gives a narrow operating bandwidth at this part. So, this is what you see one of the basic feature, when we are just talking about the mass and spring system, we know that it is a single degree of freedom system and we have one undamped natural frequency.

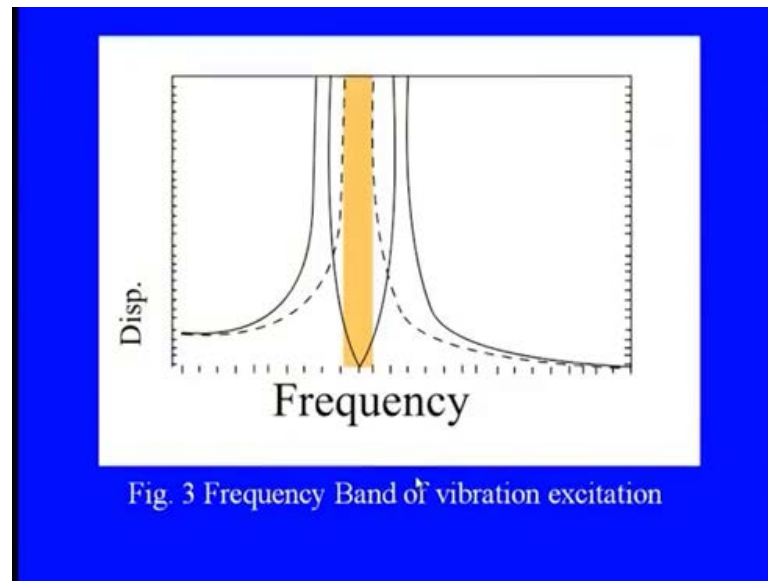
And when every time you see you know like this, this frequency generating in the machine, then we can straightaway adopt these vibration absorber in terms of the inertia block put it or mounting on the system. And when these absorber are just tuned, to the basic vibration system, because now we have the two degrees of system as you can see on your screen.

(Refer Slide Time: 21:20)



Then certainly you see here, these absorber are perfect in controlling the vibration by just shifting the frequency in the same time, the energy absorption can be you know like effectively done at the excitation source.

(Refer Slide Time: 21:38)



And in these cases you can see the displacement, the yellow region is clearly showing that when the system is exciting at the resonant condition, the huge amount of amplitude the energy you see that displacement feature is just infinite. But, if you want to bring it down to 0, then you see this closed curve this is what you see the closed curve, it just says that whatever the added mass the inertia block is being there.

It can simply tuned and you know see whatever the energy of excitation is there is completely taken away, and then you see here you know like all these frequency band in which you see the excitation of vibrations are there, it can be immediately absorbed during that time. So, if we just look at from the mathematical point of view, in which you see here we are converting a single degree of freedom system to the 2 degrees of freedom system. We can say that the inertia block which is to be added to the entire system, as the vibration absorber.

Then the equation is $m \ddot{x}_1 + k_1 x_1 = F_0 \sin \omega t$ and $m \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0$, and then you see here this stiffness matrix, which should be symmetric along the diagonal. Then it is k_1 plus k_2 minus k_2 minus k_2 , and then we can say that this is what my $x_1 \times x_2$ and whatever the force which is being exerted, on the main machine, so it is $F_0 \sin \omega t$.

(Refer Slide Time: 22:38)

Cont.....

When mass is added as vibration absorber, system acts as a two degree of freedom and the equation of motion in state space form is as;

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_1 \\ -k_1 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \sin \omega t \\ 0 \end{Bmatrix}$$

When the system is excited due to harmonic excitation, the input function is as; $x_1(t) = X_1 \sin \omega t$
 $x_2(t) = X_2 \sin \omega t$

$$\begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_1 \\ -k_1 & k_2-m_2\omega^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \sin \omega t = \begin{Bmatrix} F_0 \sin \omega t \\ 0 \end{Bmatrix}$$

So, when the system is excited due to the harmonic excitation as $F_0 \sin \omega t$, we know that the input function for the displacement is $x_1 \sin \omega t$, and the next which is we can say the displacement feature for the other mass is also $x_2 \sin \omega t$. So, when we are trying to solve it for the natural frequency in Eigen vectors, we know that we need to go and arrange the matrix, in such a way that the determinant of minus omega square m plus k becomes 0, and then you see here we can see simply get this equation.

(Refer Slide Time: 23:37)

Cont.....

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} X \begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_1 \\ -k_1 & k_2-m_2\omega^2 \end{bmatrix}^{-1}$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \frac{1}{\delta} \begin{bmatrix} k_1+k_2-m_1\omega^2 & -k_1 \\ -k_1 & k_2-m_2\omega^2 \end{bmatrix}^{-1}$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \frac{1}{\delta} \begin{bmatrix} (k_1+k_2-m_1\omega^2)F_0 \\ k_1F_0 \end{bmatrix}$$

So, when we are talking about the Eigen vectors x_1 x_2 , then it is nothing but equals to f_0 and 0 that is what the first matrix and the displacement into just minus $m \omega^2$ $k_1 + k_2$. So, it is $k_1 + k_2 - m \omega^2$ minus k_1 , k_1 and then $k_2 - m \omega^2$ inverse, or even we can say that it is equals to 1 by δ , $k_1 + k_2 - m \omega^2$ minus k_1 , and then it is $k_2 - m \omega^2$ inverse.

So, we are now trying to change, in such a way that you see the f_0 which is the input feature, and x which we just want to see here, to be just you know like that what is the outcome featured is there in that way. So, we can say finally, the x_1 x_2 is nothing but equals to 1 by δ $k_1 + k_2 - m \omega^2$ f_0 divided by k_1 f_0 , so when you are trying to calculate the delta the delta is nothing but equals to when we are just you know like opening the matrices.

(Refer Slide Time: 24:45)

Cont.....

Where,

$$\delta = (k_1 + k_2 - m\omega^2)(k_2 - m_2\omega^2) - k_2^2$$

$$X_1 = \frac{(k_1 + k_2 - m\omega^2)F_0}{\delta}$$

$$X_2 = \frac{k_2 F_0}{\delta}$$

It is $k_1 + k_2 - m \omega^2$ and $k_2 - m_2 \omega^2$ minus k_2^2 square, so we have now both the modes x_1 and x_2 the x_1 is nothing but equals to $k_1 + k_2 - m \omega^2$ f_0 by δ and x_2 is $k_2 f_0$ δ . So, now you see we can say that the m_0 and k_2 , which are being the added mass and the spring and when they are just tuned in such a way that the overall displacement is 0.

(Refer Slide Time: 25:20)

Cont.....

Here, m_2 and k_2 can be chosen such that $X=0$, so

$$\omega^2 = \frac{k_2}{m_2}$$

Motion of absorber mass:

$$x_a(t) = -\frac{F_0}{k_a} \sin \omega t ; \quad X_a = -\frac{F_0}{k_a}$$

Force acting on the absorber mass:

$$k_a x_a = k_a \left(-F_0 / k_a \right) = -F_0$$

So, if i am saying that the x equals to 0 it is only when omega square equals to k 2 by m 2, so the motion of absorber mass is nothing but equals to say x of a into t is nothing but equals to minus f 0 by k a sin omega t where x i is nothing but equals to f 0 by k a. Since, you see you know like it is just absorbing part, so it is minus and when we are saying that the force which is being acting on now the absorbing mass through transmitted through the main masses. It is nothing but equals to k a x a equals to k a minus f 0 by k a, you see here which we are replacing or else we can say that it is nothing but equals to minus f 0.

(Refer Slide Time: 26:14)

Cont.....

Force provided by $m_2 = f_0$ Disturbance force; Hence, zero net force acting on the primary mass.

$$X = \frac{(k_a - m_a \omega^2) F_0}{(k + k_a - m \omega^2)(k_a - m_a \omega^2) - k_a^2}$$

$\omega_p = \sqrt{\frac{k}{m}}$ Original natural freq. of the primary System without the absorber

$\omega_a = \sqrt{\frac{k_a}{m_a}}$ Original natural freq. of the absorber before it is attached to the System

So; that means, you see here the total force is being transmitted to the absorbing mass and the force provided by this m_2 , if you are saying that it is 0, f_0 which is nothing but the disturbing forces you know like when it is just being there. So, the 0 net force acting on the primary mass is just giving in such a way that the entire force is being now transferred to the added mass. And there you see the displacement axis is nothing but equals to $k a$ minus $m a \omega^2 f_0$ divided by, now the total stiffness k plus $k a$ minus $m \omega^2$ into $k a$ minus $m a \omega^2$ minus $k a^2$.

So, this is what the arrangement of the interaction of the stiffness feature of the main mass, and the added mass, and then also you see here how the mass variations are there when we are just trying to compute the relative displacement. So, you see here we have the two main frequencies, one if I am saying the main frequency it is nothing but equals to the original we can say the natural frequency is square root of k by m .

And this primary system frequency is without the absorber, but when the absorbers are being added, then the frequency with the absorber is ω_a is square which is nothing but equal to square root of $k a$ by $m a$. So, you see here we what we have, we have the two frequencies ω_p and ω_a , and when we are just saying that, there is a ratio say mass ratio is there which we can show by μ $m a$ by m .

(Refer Slide Time: 27:45)

Cont.....

Normalize parameters $\mu = \frac{m_a}{m}$ $\beta = \frac{\omega_a}{\omega_p}$ $r = \frac{\omega}{\omega_a}$

↑
Normalize disp. of the primary mass

$$\left| \frac{Xk}{F_0} \right| = \left| \frac{1-r^2}{(1+\mu\beta^2-r^2)(1-r^2)-\mu\beta^2} \right|$$

- Damping can reduce the resonance amplitude of the system,
- Amplitude at operating point increase with increasing damping
- Select ω which will be tuned to zero amplitude
- Relation between k_2 and m_2 is obtained from $\omega^2 = ka/ma$
- Select ma and ka (consider restrictions: force, motion of absorber mass)

When we are saying that the frequency ratio is there in between you see the ω_a and ω_p means ω_a absorber, when the absorber and without absorber or also we can

say the ratio of frequency in between ω and ω_a . ω is the overall frequency of the system, and ω_a is the absorber feature absorber mass. Then we can say that the normalised displacement of the premier mass the primary which or you can see in the you know like the premier feature of that, in which first mass is there without the absorber is nothing but equals to x/k divided by f_0 .

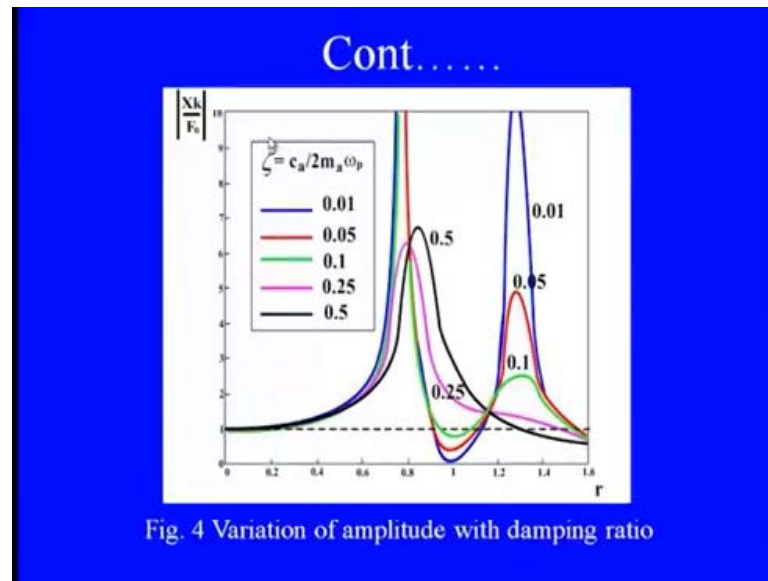
Sometimes, we are saying that this is what you see the transmission feature, when you have outcome the x and f_0 is the input is nothing but equals to the modulus the normalised feature of $1 - r^2$. R is nothing but equals to the frequency ratio ω by ω_a , $1 - r^2$ divided by $1 + \mu \times \beta^2 - r^2$, and $1 - r^2 - \mu \beta^2$. So, this is what you see here the normalised displacement of the primary mass, when the added mass is being there as a suppression feature of the passive vibration control.

So, we can conclude this, that the damping which can be reduce the resonance amplitude of the system is always applicable at the resonant feature. The amplitude at the operating point will certainly increase with the increasing, you know like at the damping feature, when these things are being added. So, select the natural frequency which will be tuned to 0 amplitude, so then you see here we need to just go, that how we can select the ω_a , so that the amplitude whatever the things are being coming it should be 0.

So, from this relation we can say that the relation between the k and $f_0^2 m$ can be straightaway obtained by the ω_a^2 equals to k/a and divided by m/a . So, we can select the added mass, and the you know like the spring property added k_a in such a way that that whatever, the resultant amplitude is coming at the main mass could be 0.

So, you see here you know like we can say that this is what, the amplification feature x of k divided by f_0 , and the r and you can see, that when you have this x of k y of 0 equals to 1. There is all the straight line in this part even when we are changing this part, but when we are changing the damping part, in such a way that you see the damping is here, which we are defining c_a divided by $2 m_a \omega_p$. That is nothing but equals to you see you know like it is clearly showing that, we have a clear feature of the damping the dashpot feature c_a divided by you see c $2 m_a \omega_p$.

(Refer Slide Time: 30:14)



So, with the variation of this zeta you can see there is a clear variation, when you have the omega by omega r is absolutely at 1, this is you see you know like the natural frequency there is no excitation. That means, the added mass is the perfect absorber at that point, and the entire amplitude is being washed out and you have x k divided by f 0 is almost 0 there, but when you have the frequency ratio omega by omega a. That is you see we are saying the r, is now you see you know like in just varying in such a way that you know like, when it is just going beyond 1.2, 1.4 or below that and with this damping.

(Refer Slide Time: 31:46)

Cont.....

Normalize parameters $\mu = \frac{m_a}{m}$ $\beta = \frac{\omega_a}{\omega_p}$ $r = \frac{\omega}{\omega_a}$

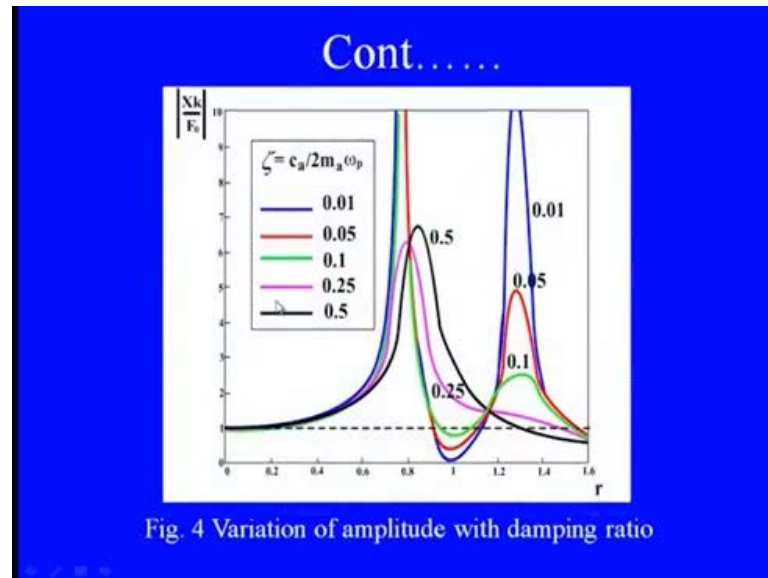
Normalize disp. of the primary mass

$$\left| \frac{Xk}{F_0} \right| = \left| \frac{1-r^2}{(1+\mu\beta^2-r^2)(1-r^2) - \mu\beta^2} \right|$$

- Damping can reduce the resonance amplitude of the system,
- Amplitude at operating point increase with increasing damping
- Select ω which will be tuned to zero amplitude
- Relation between k_2 and m_2 is obtained from $\omega^2 = ka/ma$
- Select ma and ka (consider restrictions: force, motion of absorber mass)

The damping ratio zeta, which is nothing but equals to c , which we have previously you know like I put here, that it is you know like just showing, the damping ratio at the available.

(Refer Slide Time: 31:51)



And you know like at the p part, we can say that with these features, there is a huge amount of we can say the these amplifications are there. And this is nothing but equals to you see just showing that whatever, the absorber mass are mass is there it is not able to suppress the vibration at those features. So, from this you see here we can say that when we are just putting the damping ratio along with the added masses part, then how we can control the vibration excitation, unlike in terms of see you know like the inertia blocks or in terms of the damping together.

So, the 2 degree of freedom system, which is simply showing here as a 2 natural frequencies just corresponding the 2 normal modes also at these frequencies, so we can say that at the lower frequency mode, both the masses are in phase as we discussed and you see at a higher frequency both the masses are absolutely orthogonal, or we can say out of phase. So, when such systems are there then it is somewhat drastic you see here to control the things, so we can say that the two degree of freedom system, they have the normalised forcing frequencies from this. When they are in phase is just 0.61, and when they are you know like out phase, where you see you know like the opposite modes are there it is just crossing 1.3 as you can see 0.67 is clearly showing here, and 1.67.

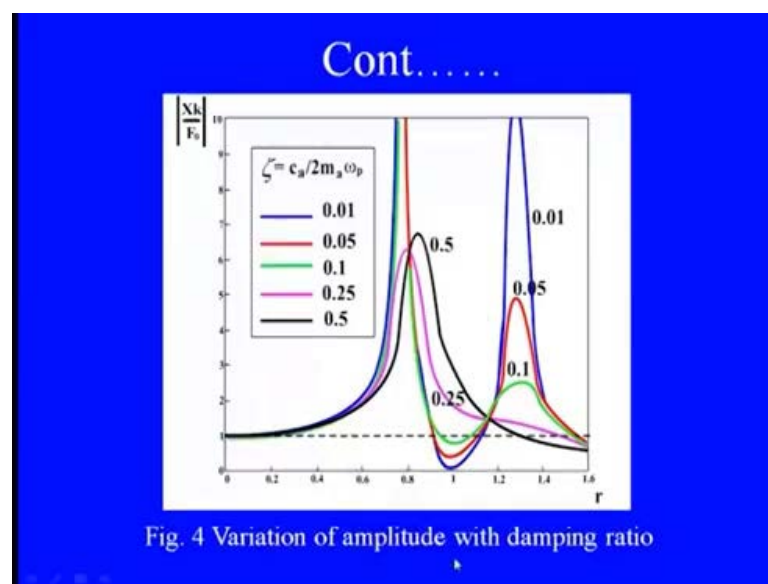
(Refer Slide Time: 32:31)

Cont.....

The 2-DOF system has two natural frequencies, corresponding to the two natural modes of vibration for the system. In the lower frequency mode both masses move in the same direction, in-phase with each other. In the higher frequency mode the two masses move in opposite direction, 180° out of phase with each other.

The animation below shows the motion of the 2-DOF system at normalized forcing frequencies of $f_{\text{left}}=0.67$ (in-phase mode), $f_{\text{middle}}=1$ (undamped classical tuned dynamic absorber), and $f_{\text{right}}=1.3$ (opposite-phase mode).

(Refer Slide Time: 33:18)



When you see the things are being varied in that way, and when you see here the undamped classical tuned you know like the things are there means, the damping is not available at that point. Then it is almost nearly equals to 1, and then you can see that when the damping ratio is unaffactive then the things are becomes almost 0 there. So, you see here in such kind of system, when we are simply adding the inertia blocks to suppress the vibration, and the damping is involved, there then we need to check it out. That whether the masses, because it is now the two degree of freedom system whether the masses are in phase or out phase or when the damping is not an effective part or it is

not present. Rather than you see here we can say that absolutely, the suppression of these vibration through this inertia block is absolutely occurred at this omega by omega equals to 1.

(Refer Slide Time: 34:17)

Cont.....

The arrows in the movie represent the magnitude and phase of the force applied to the main mass. Often in the design of systems, damping is introduced to achieve a reduced level of vibrations, or to perform vibration suppression. Consider a symmetric system of the form

$$M\ddot{x} + D\dot{x} + kx = 0$$

where M, D, and K are the usual symmetric, positive definite mass, damping, and stiffness matrices, to be adjusted so that the modal damping ratios, ζ_i , have desired values.

So, we can say that these you know like the magnitude and the phases, at which you see the force is being applied to the main masses are clearly shows in these part, that you see how the variations are there. And when the damping has been achieved at the same time, it is a clear reduction in the level of the vibration or we can say that we can perform the vibration suppression, at other than you see you know like omega by omega equals to 1. So, at the lower frequency at we can say 0.667 or the higher frequency 1.3, these you see the damping can be also act along with the added mass.

So, when we are considering the symmetric system, then we have $m \times \text{double dot plus d} \times \text{dot plus k x equals to 0}$, and when we are just adding those things, we know that the whatever the mass the damping or the stiffness matrices can be straightaway adjusted. So, that the model damping ratio can get the desired value, what the requirement is there to suppress the vibration.

(Refer Slide Time: 35:25)

This in turn provides insight into how to adjust or design the individual elements m_i , C_i and k_i such that the desired damping ratios are achieved. Often in the design of a mechanical part, the damping in the structure is specified in terms of either a value for the loss factor or a percentage of critical damping, i.e., the damping ratio.

This is mainly true because these are easily understood concepts for a single degree-of-freedom model of a system. However, in many cases, of course, the behavior of a given structure may not be satisfactorily modeled by a single modal parameter.

So, we can say that it is simply providing the insight into how to adjust, how to design the individual element like the mass spring or damper, in such a way that we can achieve the desired damping ratio for the system. And when we are doing this especially towards the mechanical part then the damping in the structure is specified, in terms of either a value for the loss factor. Due to you see you know like the stationary or unstationary part or the percentage of critical damping, can also be just given with the damping ratio.

And we can say that when we are just trying for a single degree of freedom system or two degrees of freedom system, the things are quite feasible along with you see these parametric features mass. We can see the damping or the stiffness part; however, in many cases you see here we know that the behaviour of the given structure, is not be satisfactorily modelled when we are just going with the single degree of feature.

So, in such cases you see here, we need to go for higher degrees like n degree of freedom system, which is having the n -th damping ratio say ζ_n , and these damping ratios are just you know like for all the normal modes, which are being there. You see under any of the symmetricity of the matrices, that damping matrix mass inverse matrix, and the stiffness matrix which is symmetrical is just giving all these damping ratios, according to their the normal modes.

(Refer Slide Time: 36:40)

Cont.....

Hence, the question of how to interpret the damping ratio for a multiple-degree-of-freedom system such as the symmetric positive definite system of Equation arises. An n-degree-of-freedom system has n damping ratios, ζ_i . These damping ratios are, in fact, for the normal mode case (i.e., under the assumption that $DM^{-1}K$ is symmetric). Recall that, if the equations of motion decouple, then each mode has a damping ratio ζ_i defined by

$$\zeta_i = \frac{\lambda_i(D)}{2\omega_i}$$

So, if you are saying that, if you are just decoupling the motion then the damping ratio zeta can be defined as the lambda d divided by 2 omega i, so what we have here we have the two features in this. One the lambda that is the Eigen value along with you see here whatever the rotational frequencies are there, and these frequency ratios we can say that the zeta which is nothing but the damping ratio can be defined for the system. So, we know that the omega i which is in the denominator, it was nothing the undamped natural frequency of the system, at the lambda i it is also showing the Eigen value of the matrix along with the damping. And this ratio is clearly showing the effect of the damping, when we are just going in the damping ratio feature.

So, when we want to formulise this, and we can say that the definition which is just to you know like examine the normal modes just to be evaluate, whether the damping inverse mass matrix and the stiffness is symmetric or not. If it is symmetric then we can say that the damping inverse mass matrix and k is absolutely equals to the stiffness inverse mass, and damping part.

(Refer Slide Time: 37:55)

Cont.....

where ω_i is the i th undamped natural frequency of the system and $\lambda_i(D)$ denotes the i th eigenvalue of matrix D .

To formalize this definition and to examine the nonnormal mode case ($DM^{-1}K \neq KM^{-1}D$), the damping ratio matrix, denoted by Z , is defined in terms of the critical damping matrix D_{cr} of Section 3.6.

The damping ratio matrix is defined by

$$Z = D_{CR}^{-1/2} \tilde{D} D_{CR}^{-1/2}$$

But, if it is not for any normalised case then the damping ratio matrix can be defined as we have already discussed in that matter, it is just defined by the z . And in this way you see here we need to define the z , in terms of the critical damping matrices, as we discussed in the previous lectures d critical. So, in these cases here where the d critical is playing the key role, in defining the normal mode other than the symmetricity, we can simply define the damping ratio z is equals to the d critical half minus half to the power minus half d into d critical minus half.

So, this is what will see some kind of in the damping, it simply shows that with the critical part the critical damping, and the available damping how you see the symmetricities are there, and then accordingly we can calculate the damping ratio. So, you see here, when you have the normalised mode in the symmetric way with the damping inverse mass and stiffness, there is no problem we can straightaway go $zeta$ with the $\lambda_i d$ divided by ω_i .

But, if you see we want to examine the normal mode for a different cases in which the symmetricity of this matrix, in which you see the damping inverse mass. And the stiffness is not the symmetric way, then we need to go with the this you know like the damping ratio, along with the critical damping.

(Refer Slide Time: 39:53)

Cont.....

where \tilde{D} is the mass normalized damping matrix of the structure. Furthermore, define the matrix Z^1 to be the diagonal matrix of eigenvalues of matrix Z , i.e.,

$$Z^1 = \text{diag}[\lambda_i(Z)] = \text{diag}[\zeta_i^*]$$

Here, the ζ_i^* are damping ratios in that, if $0 < \zeta_i^* < 1$, the system is underdamped. Note, of course, that, if $DM^{-1}K = KM^{-1}D$, then $Z = Z^1$.

and when we are just trying to formulate this we know that this the d critical and the d , d bar which is nothing but the normalised damping matrix, with the mass for any you know like structure. We can simply calculate the you know like the matrix, for the damping ratio z , and in this you see here the Eigen values of this can be immediately find out along the diagonal of the entire z .

So, if you want to calculate the Eigen value, this is again the z i is nothing but equals to the diagonal, along the you know like the entire z matrices and it is nothing but λ_i into z or we can say it is sometimes the diagonal of the zeta star. The zeta star is nothing but the damping ratio, for such system in which the it is you know like we can say the ratio d i divided by ω_i is less than 1; that means, you see it is for the under damped system. And when you have the under damped system, and you want to calculate the you know like damping ratio, then we need to go with the diagonal matrix of the entire z matrix. But, when the system is the symmetric say d m inverse k is equals to k m inverse d , then certainly you see here we can simply calculate using the ω_i or the λ_i d straightaway.

(Refer Slide Time: 41:20)

Cont.....

On further consideration, it is also apparent that one can bring about a reflection by incorporating an element with a differing inertia from that of the medium. Since elements of that type are often idealized as rigid masses, they are referred to as *blocking masses*. Practical realizations of the concept of blocking masses are, for example, *seismic blocks* and *added masses* at compliant points; as shown in Fig. 5

So, we can say that from this consideration, it is simply you know like apparent that one can bring about the reflection by incorporating an element with the different inertia, from that same medium, where the excitations are there. And since, the element of that type are just often idealised with the rigid masses, we can say that they are simply the blocked masses. So, we can say with these you know like the practical application and the realisation of the entire system, the block masses concept can be immediately adopt according to the seismic blocks and the added masses. So, we can say that you know like, when we are trying to do these things either when we are you know like making, the added mass of the seismic masses.

The construction materials, because they are more stiff can be straightaway accommodated as steel or the concrete, and they can simply put you see here. You know like on this particular path just to provide the significant discontinuities, in the medium properties by adopting a compliant element approach. And for that reason it is much more common to use a complement than the stiff elements, and we can say that for this reason for the structural reason, they are saying that we need to just go with the more stiff material then the flexible one.

(Refer Slide Time: 42:06)

Cont.....

Considering, however, that the most common construction materials are relatively stiff, such as steel and concrete, it is often simpler to accomplish significant discontinuities in the medium properties by the compliant-element approach. For that reason, it is much more common to use compliant than stiff elements. Nevertheless, for structural reasons, there are some cases in which it is necessary to use stiff elements ...

(Refer Slide Time: 42:46)

Cont.....

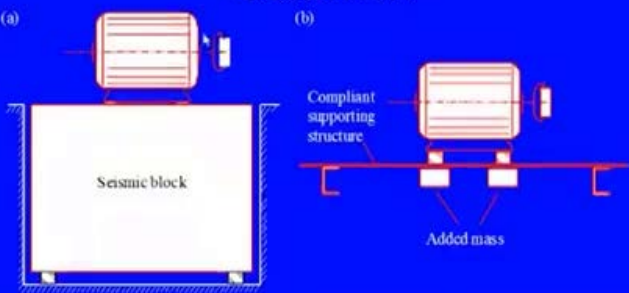


Figure 5 Blocking masses, preferably in combination with elastic elements, give very good vibration isolation and are frequently used in practice. a) Seismic block. b) Added mass at a compliant point. [1]

So, you see here these things are there we have a seismic block here, though you see this is a added mass on this particular entire system, which can be suppress the entire vibrations, but we need to check it out the compliant, structure support. Just like you see these added masses, and they are perfect and they can provide rather a better solution according to this space, and you see here just to absorb the vibration feature or the vibration excitation feature here.

(Refer Slide Time: 43:19)

Design of vibration isolators

There are a number of rules of thumb that should be followed when designing vibration isolators. If these are adhered to, the results should be acceptable.

- (i) The isolator's (static) stiffness must be chosen so low that the highest *mounting resonance* falls far below the lowest interesting excitation frequency.
- (ii) The mounting positions on the foundation should be as stiff as possible.
- (iii) The points at which the machine is coupled to the isolators should also be as stiff as possible.

So, the lastly you see when we are trying to design the vibration isolators, there are number of thumb rules in that and these are first the isolator, which is of the static nature we need to check it out the stiffness of that isolator must be chosen. So, low that the highest mounting resonance can fall far below the lowest intersecting, you see here whatever you know like this this interacting feature. You know like should be lower down means whatever the we can say the interesting exciting frequency, which are being coming on the system due to the interaction of the feature.

It should be as low, as the we can say you know like according to this stiffness feature, and the mounting positions second the location part, the mounting position on the foundation should be as stiff as possible. So, that the robustness between the added feature can come, the point at which the machine is coupled to the isolator, should also be as stiff as possible.

Then we can say that the coupling can be you know like can be done perfectly and whatever the energy, which is being dissipated during that point, the coupled feature can be immediately act as you see the absorber or the stiffness feature. So, what i mean to say that these are critical, you know like we can say are the rules for designing the vibration isolators, according to the whether we are saying that you know like the mounting resonances are there. The excitation features are there from the machine or even the location point of view, few more when we are talking about you see you know

like these exciting features, like you know like in the second case, when we are talking about the mounting position or in third case when we are just talking about the coupled feature.

(Refer Slide Time: 45:12)



Cont.....

Rules (ii) and (iii) are normally not difficult to fulfill at low frequencies; at high frequencies, however, internal resonances make them problematic.

(iv) The isolator should, if possible, be designed so that its first internal anti-resonance falls well above the highest excitation frequency of interest.

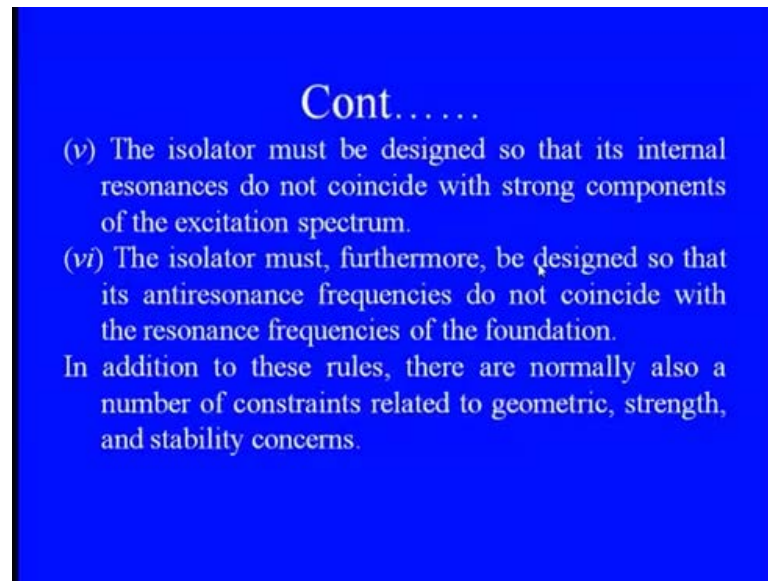
That rule is, in practice, very difficult to follow. If it cannot be followed, then one should ascertain by measurements or computations that at least the following alternative rules are fulfilled:

They are not difficult to fulfil at the lower frequency, but you see at a higher frequency, the internal resonances are always creating, so many problems there itself within the material itself. So, the isolator should be designed in such a way that, the first internal anti resonance falls well above the highest exciting energy of the entire machine. So, you know like, when we are designing the isolator we need to take care that, what could be the possible way to just put the anti resonance frequencies.

And you see here you know like, when we are just coming the entire exciting frequencies should be well far means well you know like higher than, whatever you see the anti resonances are there. So, this rule in practice is very difficult to follow, but if you are not trying to follow these things, then certainly you see here we can simply you know like calculate or measure.

So, that the added mass or anything you see the isolator can be put in such a way, that the anti resonant part should come immediately, and you see whatever the higher highest excitation frequencies are there, they are just to be go ahead in that. So, you see here you know like practically or in through computations, we need to check it out that what is the feasible way out to design, the best possible way of our vibration isolators.

(Refer Slide Time: 46:48)



Cont.....

- (v) The isolator must be designed so that its internal resonances do not coincide with strong components of the excitation spectrum.
- (vi) The isolator must, furthermore, be designed so that its antiresonance frequencies do not coincide with the resonance frequencies of the foundation.

In addition to these rules, there are normally also a number of constraints related to geometric, strength, and stability concerns.

And then you see here when we are talking about the isolator it must be designed, in such a way that internal resonances should not coincide, with the strong component of the excitation. Otherwise, you see here in the spectrum the entire feature will come out, and you see here it may even supply the energy to make the system or to lead the system towards unstable part.

So, the isolator must be designed, so that its anti resonance frequencies do not coincide with the resonant frequency of the foundation, and in addition to these rules there are normally you know like number of constraints, which can be adopted, according, to the geometry or the stability, or the strain point of view of the structure, because you see here the local feature and the operating conditions.

And whatever you see you know like we can say the environmental features are also playing a critical role in designing of the isolators. So, if you are talking about the entire chapter, then we could easily figure out that when we are designing the absorber, there are various ways through that we can straightaway adopt a best suited method. Like simply you know like adopting a spring then what kind of springs are there by adopting the damper or it is an integrated effect of the damping, and the stiffness or even when we are just trying to bifurcate.

These degrees of freedom from 1 degree to 2 degree, and then you see here the corresponding shift is there in the frequency 1 natural frequency to 2 natural frequency.

And when we are tuned over you know like the added mass or even sometimes even we say that the seismic mass, to the exciting frequency of the main part, then we know that we can rather lead the entire vibrating system the main mass towards the 0 amplitude.

So, these are the best possible ways, through which we can design our absorber according to the condition, and according to you know like whatever the exciting features are there from the machines. So, this is all about this chapter, and in the next chapter now we would like to see that how these concepts are being applied, for the principles of passive vibration control.

That what kind of you see the spring alone is suitable or a spring with the damper is suitable, just like you see in the strut of our motor bike, which you can see just you know like just connected the entire suspension feature from. You see the you know like the entire frame to the wheel, and there it is you see here we have a combination of the damper and the spring, and even you see here when we are just putting you see you know like your, when we are choosing the vehicle wheel for you know like the two wheeler or four wheeler or bicycle or anything.

The material of the wheel itself is playing a critical role, not only from its reliability point of view when it is running or the life you see here, but also from the damping features. So, or even the you know like the restoring features, when it is being deformed under the air compressors, so what i mean to say that now in the next lecture we are going to discuss about the practical applications of the absorber. Whatever, we discussed in basic or in the design absorber, we are trying to apply and we will see that how they are being used in that practical applications.

Thank you.