Vibration Control

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Module - 1
Review of Basics of Mechanical Vibrations
Lecture - 2
Introduction to Damping in Free and Forced Vibrations

Hi, this is Dr. S. P. Harsha, again from the mechanical department of IIT Roorkee, I am going to deliver the lecture 2 under the national programs of technology enhanced learning on the Vibration Controls. As you see, yesterday we discussed about the basic features of vibrations that, you see like what are the basic causes of the vibrations then how it is coming under the solid mechanics. And you see here, like all the classical mechanics and also we discussed about that, when we are dealing with vibration then we can straightaway go to the single degree of freedom.

And in the single degree of freedom, what are the basic, we can say the elements are there, through which we can characterize the vibrations. Like we can say, the first is the displacement, then the velocity then the acceleration is there and then these all basic dynamic features or the elements. They are basically causing the different kinds of forces like we can say, the inertia forces, damping forces, restoring forces.

And then you see here, if the system is externally excited, we can termed as the forced vibration or if the system is due to it is inherent property, the system is excited then certainly we can say it is a free vibration. In the realistic system, there is no as such condition, which we can say that, like the free vibration, but when we disturb something and then we leave out, whatever the oscillation or the vibration is coming, it can termed as the free vibration.

And whatever the exciting frequencies are there through this free vibration feature is known as the natural frequency. So, we know that, we have a physical system, we can correspondingly get the mathematical description of that and then we can make the free body diagram. We can draw all that features, through which we can say that, at the equilibrium point, this is the equation of motion, which simply shows the realistic features.

So, yesterday we discussed mainly about the single degree of freedom system, in which we have the free vibration and in that, here there were three components. First it was the mass, second it was the damping, third it was the stiffness, mass and a spring is very common feature. Sometimes when the oscillation or the vibration is continuously occurring, we can say it is because of the mass and a spring combination. But, when you see they are dying that means, there is a dissipation of the energy, because of the damping is available. So, today in this our lecture, we are going to discuss about the damping, which is available in both free and the forced vibration.

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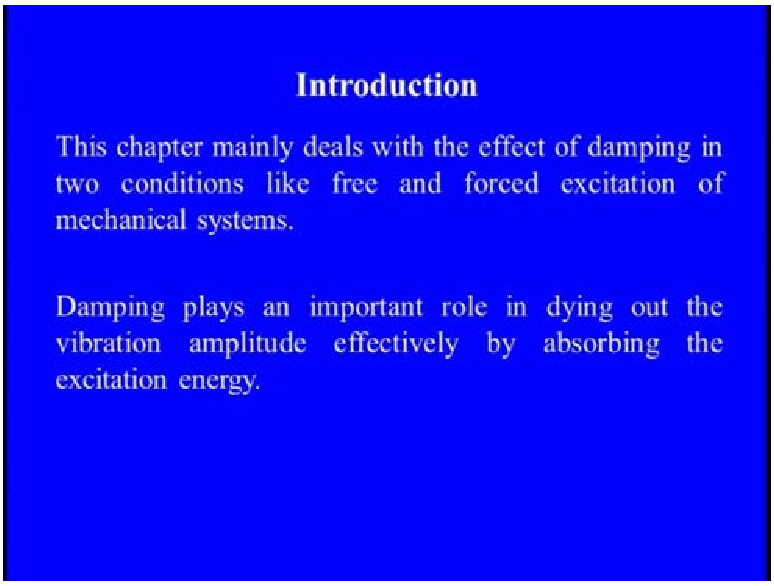

So, as I discussed, what exactly the effect of the damping is, sometimes when we are talking about the damping, the first thing is coming that, there is a liquid which is simply available to absorb the energy from the source of excitation. But, it is not all the time you see here, that we need to have a kind of lube oil or any kind of liquid, through which we can take the energy back out from the system. So, today we are going to discuss about all the modes, the basic modes of the damping.

The damping is there from the material also, the damping is there through the rubbing actions, the damping is there through the liquid formation, the fluid whatever the fluids are available. What are the exact, like the basic we can say, the mechanics is there or the mechanism is there, through which we can analyze the entire system. And again you see here, we are applying both the condition, the free vibration condition and the forced vibration condition.

So, damping we know that, the damping plays an important role in dying out the vibration oscillation, in terms of the displacement of the acceleration. So, how we can reduce that, what exactly the effective feature of the damping is, how much damping should be applied so that, we can immediately damp out the vibration at the critical feature. So, today you see here in this part, again prior to go to the effect of damping, let us simulate some of the numerical problem for the mass and spring system.

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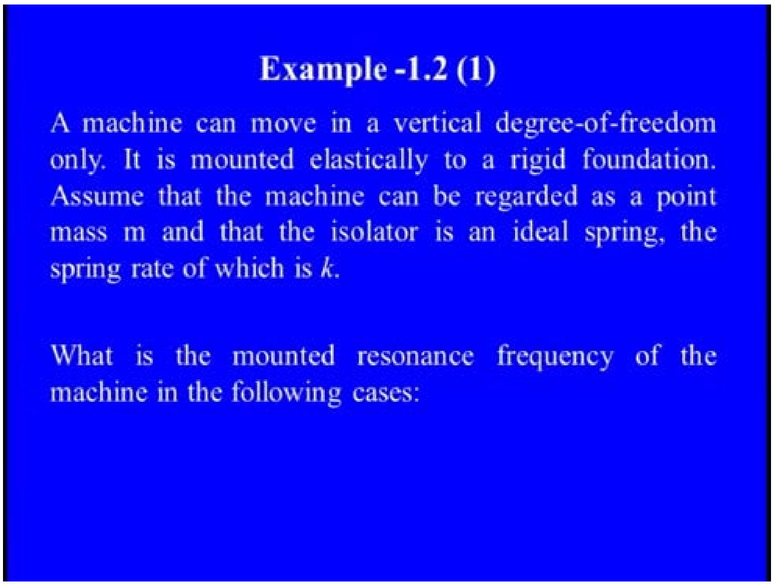

So, the first problem says that, we have a machine in which the vertical degree of freedom is there, since it is we are only considering right now, the single degree of freedom system, which is imply mounted in the elastic feature on a rigid foundation. Again we are assuming that, the machine can be regarded as the point mass, because you see there is a vertical mass, we can say through their center of mass a point, it is simply acted vertically downward.

And you see here, whatever the mass which is being acted towards the downward, is being balanced by the isolator or we can say an idle spring, which has the stiffness or the spring rate, we can say is k. So now, we just want to find out that, we have the mass, we have the spring, what exactly the exciting frequencies are.

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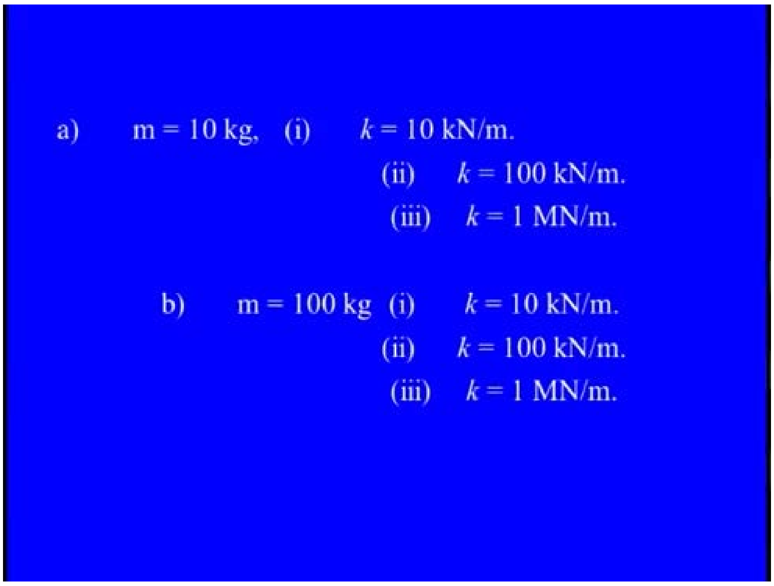

So, we are considering now the different numerical values here, first if our mass is of 10 kilogram. And now if you are taking the different grades of the spring based on their spring rate or the stiffness. So, our first case is the spring, when we have the spring 10 kilo Newton per meter, second when we have the spring of 100 kilo Newton per meter or third if we have the spring rate or the stiffness is 1 mega Newton per meter then what exactly the change in the exciting frequencies are.

In this numerical problem, we just want to simulate the effect of the stiffness, in the first part, like in the a part, the effect of the stiffness on the exciting frequency. In second, now if we just change the mass from 10 kilogram to 100 kilogram and then you see again we are taking the same stiffness values there, as we considered in a then what exactly the impact of this mass on the frequencies is. So, you see like in the solution we know that, first the natural frequency, the formula itself is square root of k by m, the stiffness over the mass distributed. And then you see here f 0, which is the exciting frequency in Hertz is nothing but equals to the omega 0, which is nothing but the radiant per second divided by 2 pi, whatever the circumferences.

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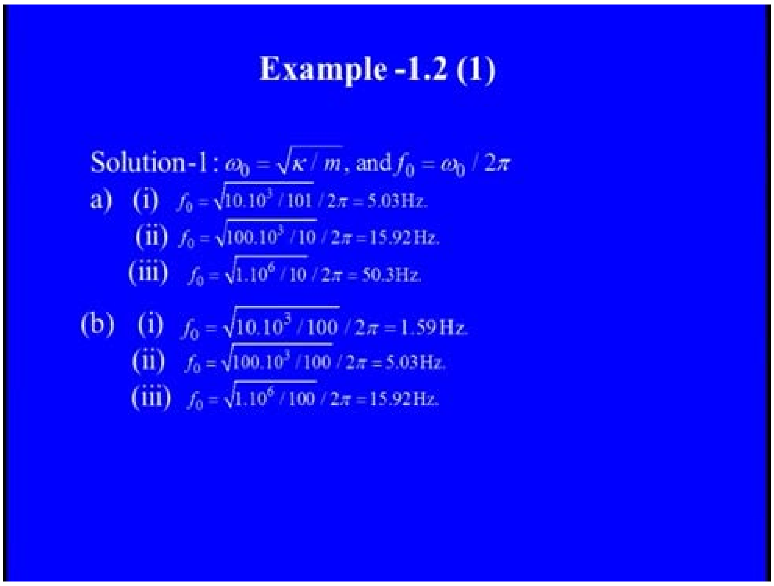

So, we can get first, when we have the mass which is of 10 kilogram and the stiffness is 10 kilo Newton per meter then through which we can simply calculate 10 into 10 to the power cube divided by 10 over 2, pi, through that we can know that, it is 5.03. And now, if you are increasing the stiffness, you can see that, by increasing the stiffness with the same mass, the exciting frequencies are been increasing. So, here through that, we can simply conclude that, if you are not changing the mass, but if you are changing the spring property, we are increasing the stiffness.

The exciting frequencies are becomes more, so more and more stiff feature means, if we have a rigid body with more stiff, the exciting frequencies are quite significant. Because, we know that, if there is a physics behind it, we know that, if we have more stiff feature, we are applying the same amount of load to a more stiff or less stiff body, certainly the kind of deformation is different. More stiff less deformation, less deformation certainly we have the force by deformation is there, the stiffness, so stiffness is quite more.

And if the stiffness is more, certainly we have more exciting frequencies, so like this from this numerical at least from first part, we could easily find out that, we have with this particular more stiffness, we have more exciting frequency, the b part. Now, we have simply added more mass, now we have 100 kilogram of mass and you can see that, there is a drastic reduction of the exciting frequencies are. The first f 0 is 1.59, second is 5.03 and third is 15.92, so there is one tenth of the previous exciting frequency.

So, we know that, whenever we are adding any mass just like the fly wheel in our usual cases, there is a clear separation of the frequencies are, but it is not applicable to all the kind of frequencies. So then you see here, we need to find out that, when the system is exciting at the low frequency, when the system is exciting at the higher modes of the frequencies or the system is exciting exactly at the resonant frequency then which kind of the devices either the spring damper or mass should be useful to control the exciting frequencies, so that you see, we are going to discuss in the coming lectures.

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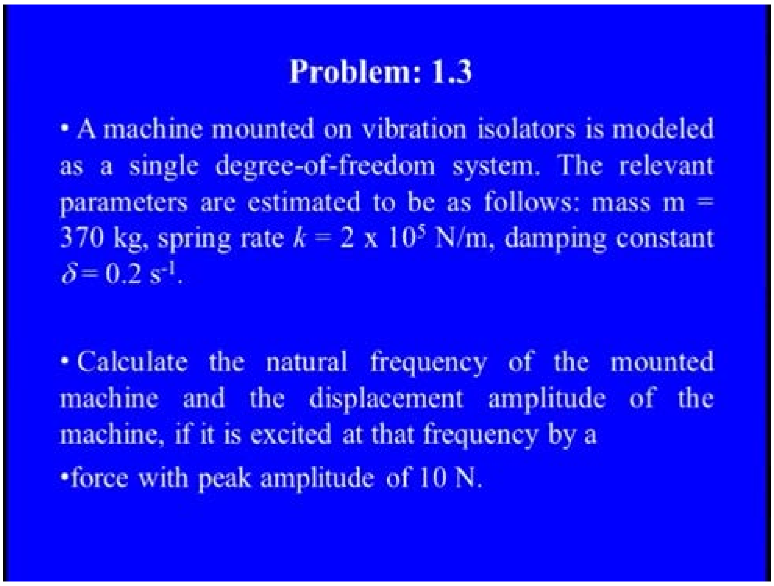

But now you see here, we are again taking one more example, in which we have a machine, which is mounted on the vibration isolator and is being modeled as the single degree of freedom system. And then you see the corresponding parameters, which relevant to the real feasible system is the mass is 370 kilogram, the spring rate which is nothing but the stiffness is 2 into 10 to the power 5 Newton per meter and the damping coefficient is there.

So now, we added the damping so that, we just want to see the dissipation of the energy from that. We just want to calculate, what exactly the natural frequency of this system is and if you see the displacement, which we just want to calculate for the machine. If we just want to find out that, we can straightaway calculate when the system is excited at the natural frequency. When the system is excited, we can say the external force is of 10 kilogram at the peak amplitude, what exactly the displacement is.

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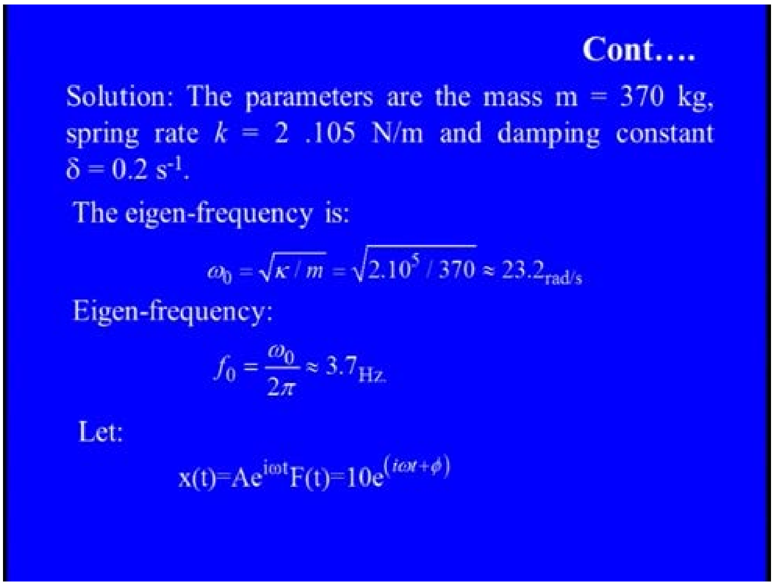

So, with this, now we can see the parameters, as we already discussed that, these are the given parameters. First of all we would like to calculate the natural frequency, which is nothing but the square root of k by m, so it is we can say, this 23.2 radiant per second or else 3.7 Hertz. Once we have these things, now our main, the second part is that, what exactly the maximum amplitude which we are saying that, though it is a resonant condition, the forcing exciting feature is exactly exciting at the natural frequency.

So, at the resonance, what could be the possible amplitude is, so for that, again we need to go to the basic equation of the periodic part like x of t is equals to A into exponential e to the power we can say omega t or else we can say, it is since the force is given to us, the maximum force that is, 10 Newton's of 10 into e to power i omega t plus phi.

Now, you see here, if you are applying these conditions we know that, the maximum amplitude is nothing but equals to f by m divided by square root of omega n square minus omega square whole square plus 2 delta omega square. The omega n and omega are two different thing, because we know that, there is a damping, the system is biperiodically excited. So, when the system is biperiodically excited, now we could easily figure out that, the main undamped natural frequency and the damped natural frequency.

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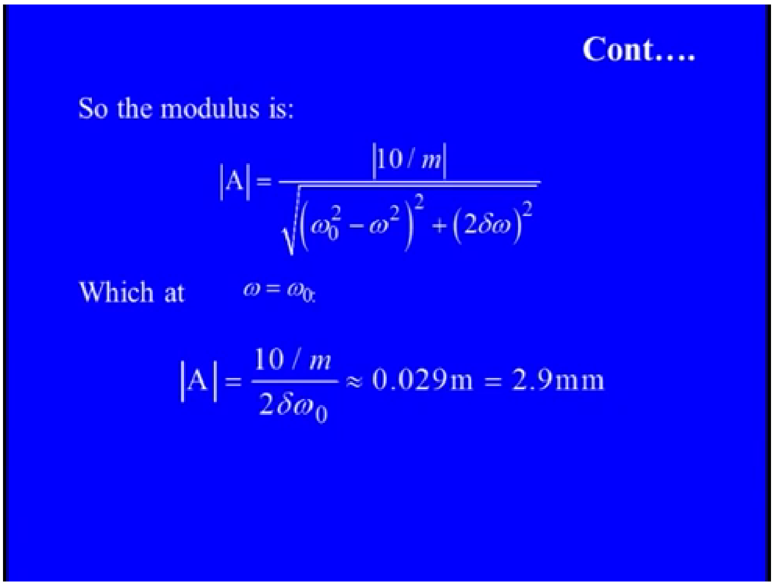

And when we are saying that, the system is exciting at the natural frequency, the omega equals to omega n and then you see since this term will be gone, so we have the maximum amplitude is nothing but equals to the peak force f by m divided by 2 delta omega n. And we can simply put that, we have 3.7 Hertz which is omega n, we have the force which is 10 Newton, we have the mass and the delta means, the damping coefficient is given to us. So, maximum amplitude in this case can be occurred at the resonant frequency is 2.9 millimeter.

So, in these two numerical we could easily figure out that, when we have this system, say the system is now exciting at the lower frequencies, the spring is a better control as you are increasing the stiffness, certainly you see here, we can have a effective manner of our exciting frequencies are. As the system is going towards the high frequencies, the mass is one of the good suppression, we can bring the entire exciting amplitude or the frequencies toward the lower down. And in the second numerical, if the system is exciting at the resonant condition, the damping is one of the best choice to control the amplitude. So, you see here we can say that, these are the effective tools through which we can at least control the amplitude of the vibration.

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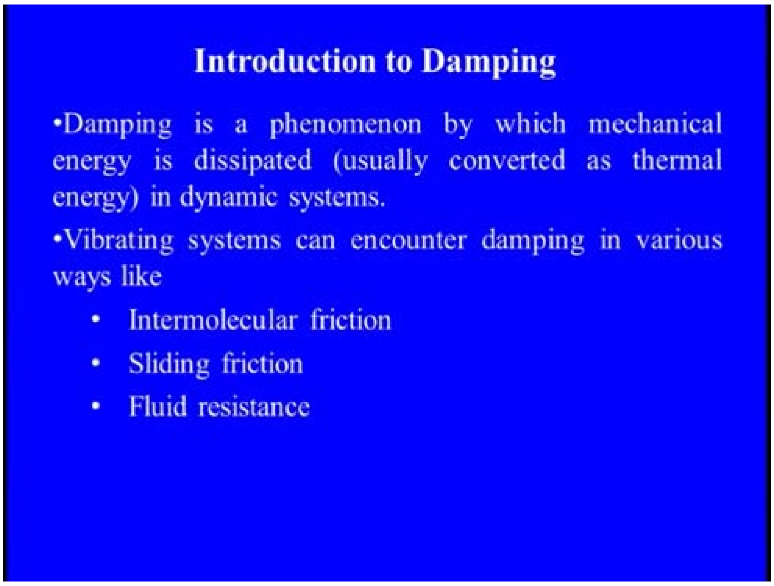

Now, coming into the main, we can say the feature of this chapter, that is the damping. As we have discussed, you see here, the damping is nothing but it is a phenomena by which the mechanical energy is dissipated or in other terms we can say that, the mechanical energy is converted into the thermal energy in the real dynamical system. So, how we can do that how we can dissipate the energy, how we can absorb the energy or extract the energy from the source itself.

There are mechanism for that, so vibrating system, which are being encountered these kind of damping, there are various ways through that. One is intermolecular friction, intermolecular friction is nothing but the molecular phenomena in which you see, when the molecules are being spaced out, they have some spacing to absorb the energy. It is basically a material property, what kind of material which we are choosing, say if we have a brittle material, we know that they are harder, but they are always providing good damping features as compared to the ductile material.

Second is sliding friction, in all the structural features when the vibrations are being transmitted through the structures, the joints or anything is the base and all, this structural friction due to the rubbing features of two surfaces, they are always applying some kind of damping at the, we are also saying it is a Coulomb damping. The third is the fluid resistance, which is a very common, we can say the representation of our damping.

When we have a fluid, it has one of the basic property through which we can control the vibration, is the viscosity. So, you see here, again it is property of the fluid, which kind of fluid which we are taking, because we know that, viscosity is nothing but a property by virtue of which, it offers the resistance against the flow. So, these are the three basic mechanism, which is always being there now in that and based on that, we can characterize this.

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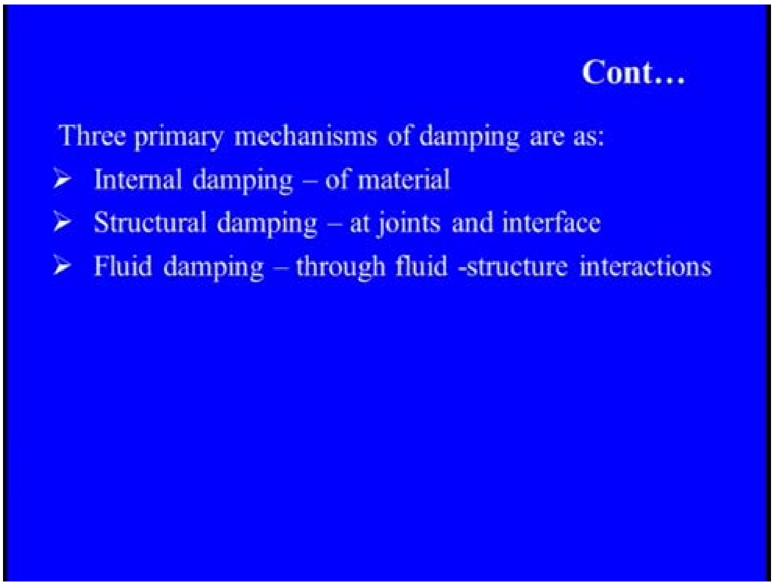

First is the internal damping which is nothing but a material damping, what exactly the material configuration is, whether it has a BCC structure, it has a HCC structure or whether it has HCP structures. Accordingly we can simply find out that, what exactly the configuration of their molecules are, how much energy can be absorbed by these molecules or you see how much damping is being provided by these materials.

Second, as we discussed is the structural damping, which is nothing but coming out at the joints or the interfaces of the two surfaces due to the rubbing action, we can termed as the coulomb damping. The third damping which we discussed as the fluid damping, that is nothing but through the fluid a structure interactions, the fluid molecules are there, they have a real good applications to absorb the energy, just like our viscous dampers are. We have very common device of our, say the door stopper, the door stopper in which we have a piston cylinder arrangement and there is a good kind of damping, this viscous damping are there according to our choice. So, these are the three basic mechanism of the damping, which can be applied to any of the things.

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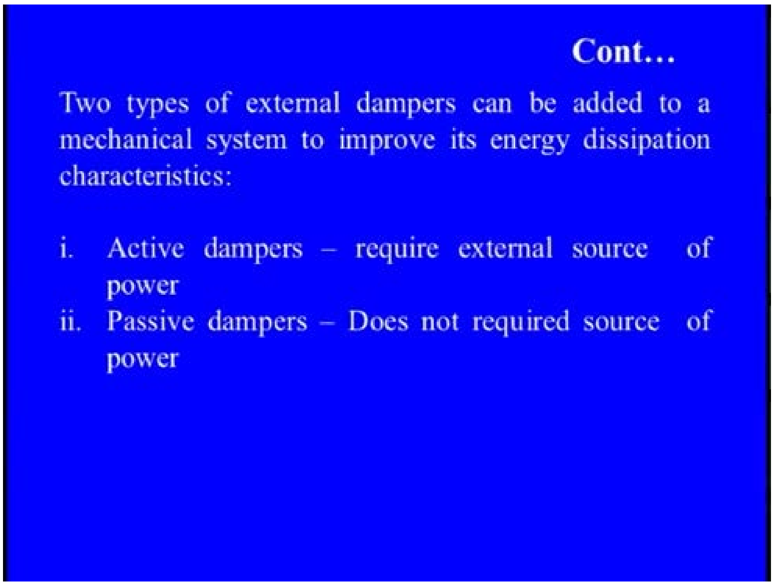

There are two external damping are also there, as we discussed in our first lecture also, which we can directly apply to any mechanical systems, just to absorb the energy just like the energy dissipation characteristics. So, in that, the two main parts are the active dampers, which always require an external source of power. Because at this point of time you see here, we are effectively applying our external power source to control the amplitude of vibration. Second is the passive dampers, as per it is name itself you see here, the passive means just we need an external material or external devices just to keep on that and they will simply absorb according to their material property, they will simply absorb the amplitude of vibrations, just like the insulation features. So, they do not require any kind of source of power.

Now, when we are just going little inside to the material damping, we know that the internal damping, which is also termed as the material damping, originate from the energy dissipation and they are associated with various things. First, they are associated with the microstructure of the material, since we know that, there were various microstructure defects are there.

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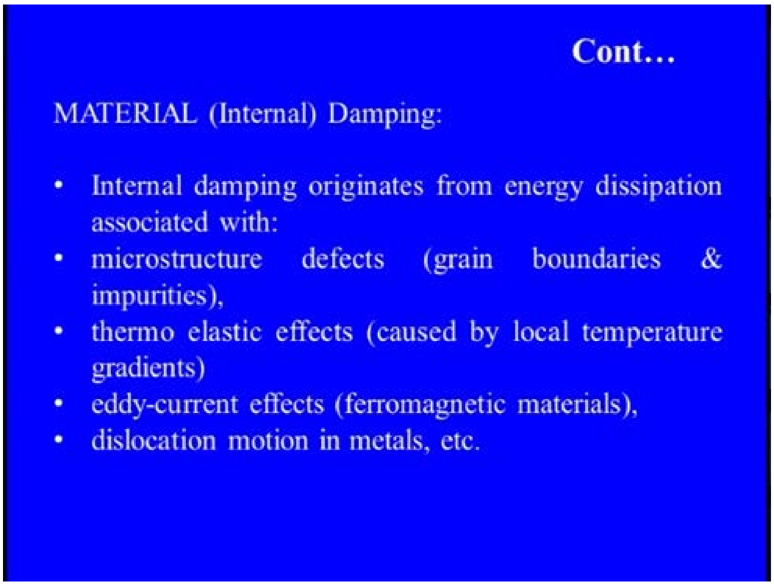

So, certainly it will affect like the material properties or the material damping properties like we have the grain boundaries, the grain structures, various kind of impurities between the grain sizes, so all these things, they are simply giving some kind of the dampers feature towards that. Second is thermo elastic effect, it is also affecting the material damping, because it is always causing from the temperature gradient at the different points. And we know that, when there is a different heat dissipation, certainly all the molecules of the material cannot exhibit the similar kind of material damping.

Third effect is eddy current effect, because we know that, there are various ferromagnetic kind of materials are there, where this eddy current effect is really significant. And we need to take care of this, because we just want to have an effective way, we just want to use effectively this material damping to damp out the vibrations. And then the last one is the dislocation motion, whatever during the application of load or the dynamic action, there are dislocation like the motion of the metals are or we cancel the layers of the fibers are and due to that, there is a clear change in the material dampings are.

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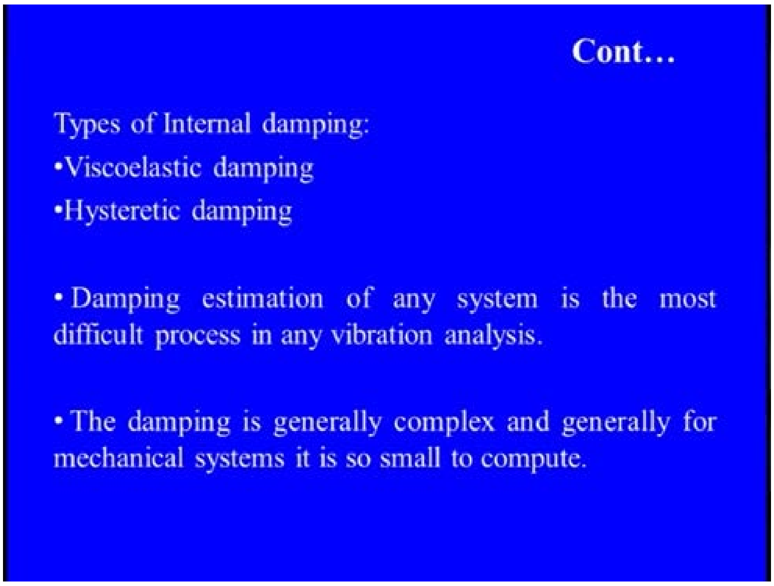

So, when we are talking about this, in that there are two types of this internal dampings are. One is the viscoelastic damping, in which the material structure is always providing according to the viscosity and their elastic behavior, that how much energy can be absorbed by these molecules of the material. Second is the hysteretic damping, because we know that, the eddy current effects are there, so hysteresis damping is also playing an key role in that.

But, again you see, when we are talking about all three types of damping, even through their mechanism we know that, the damping is not a simple phenomena, we cannot directly replace the damping by the dashpot or something, it is a very complicated phenomena. So, damping estimation for any system is not that simple, it is a very complex situation, but damping is generally we can say that, it is so small as compared to the other things, that we need to make an proper arrangement to justify the effect of damping.

So, with this particular consideration, now we are again moving towards our main mechanical system that, if we have a system consists of the mass, the spring and the damper then we can see on the screen that, we have equivalent mass, we have the equivalent spring and we have the equivalent damper for any kind of system. And correspondingly we can say that, if this is my physical system with these particular parts, we can simply find out the equation of motion based on the Newton’s law.

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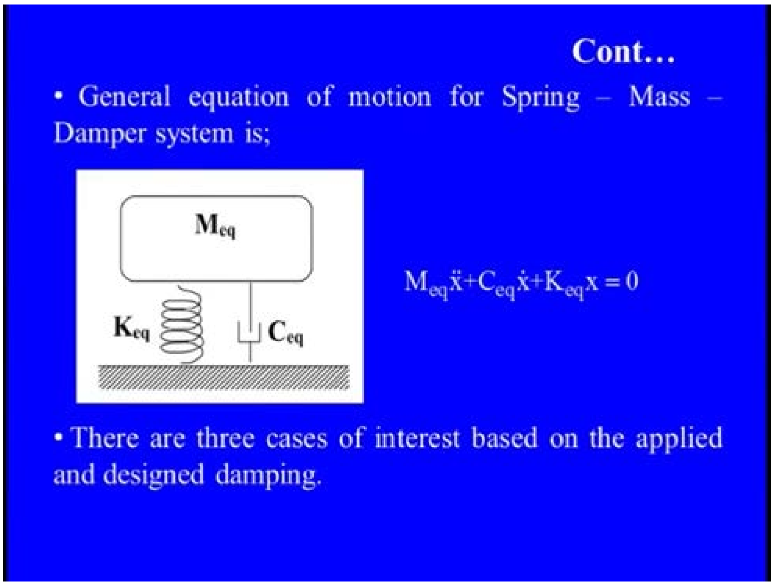

So, we have first, through the mass we have the inertia force, mass into acceleration, we have the damping force, this because we know that, the damping is basic characteristic of the velocity. So, we have this damping coefficient into velocity and we have this spring into deformation, so we have K into x, C into x dot and M into x double dot. So now, you see here, based on this, now we just want to characterize to see the system damping into three ways.

So, we know that, the damping ratio is one of the critical phenomena, so we can simply see that, the damping coefficient is nothing but equals to, that is the zeta is nothing but equals to the ratio of two damping. One is the C by C c or else we can say that, the C is nothing but equals to the applied damping, the damping which generally we are always offering to the system to dissipate the energy. Second the critical damping, the C c, as the name itself says that, this is the critical damping, critical damping is nothing but it is a design parameter.

It is always based on whatever the system, which we are taking is always coming here, so if we just want to calculate this, is nothing but equals to the C c is nothing but equals to 2 square root of K m. K is again the stiffness which is coming, according to the metal property, mass is the whatever, the mass which is being distributed amongst that. So, based on this one you see here, based on the this damping ratio, we could easily bifurcate the entire system into three categories.

If my zeta that means, the damping ratio is less than 1 that means, we are applying less damping, the C is less than C c. The applied damping is less than the critical damping, it is termed as the under damping phenomena means, we are not applying sufficient damping to damp out the entire vibration. The second category is, if I am applying more damping means, like the zeta is greater than 1, the over damping situation that means, the applied damping is greater than C, the critical damping then again this is something, which we require less and we are providing more.

The third case which is there is, when the zeta is absolutely the applied feature that means, the applied damping is equals to the critical damping. So now, you see now, we are going to bifurcate these things and we just want to see, what is the impact of these applied damping and the critical damping ratio on the system performance. How the dynamic parameters are there, how much time is really taking a system to die out with these kinds of things.

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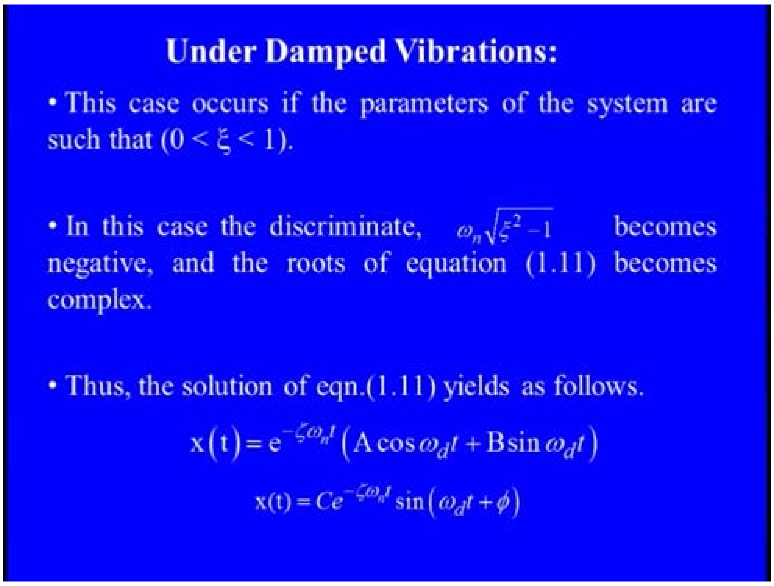

So now, if we have just go back to the slides then we will find that, you see that the first case as we discussed is the Zeta is less than 1. So, this case is simply coming as when the applied damping is less than the critical damping. So, we know that, since the damping is there, we have a damped frequency, so the discriminate feature that is, omega d or we can say this is nothing but equals to omega n square root of zeta square minus 1 becomes negative, because zeta is less than 1.

And when you have the zeta is less than 1, so we know that, yesterday we discussed about the basic equation, our roots are now, the roots are nothing but equals to the Eigen values or the natural frequency in terms other way, they are the roots are now the complex one. And with these root now, if you want to calculate the generalized displacement, so x of t is nothing but equals to the e to the power minus zeta omega n t.

So, you see here, this exponential feature of decaying vibration, now we have two things, one e to the power minus zeta omega n t ,this is the decay of vibration amplitude x, but it is exponential decay. And the same time you see here, we have A cos omega d t plus B sin omega d t, so this sinusoidal feature of the decrement of the vibration amplitude is due to the damping frequency.

So now, this is very clear from this that, we have both the features in our displacement in the decrement of that, we have a oscillatory term and we have the periodic term in terms of the sinusoidal feature and there is an exponential decay is there. So, or in general, we can generally conclude that, x of t is nothing but equals to C which is nothing but the combined part of this, into e to the power minus zeta omega n t sin of omega d t plus phi, what exactly like the phase angles are.

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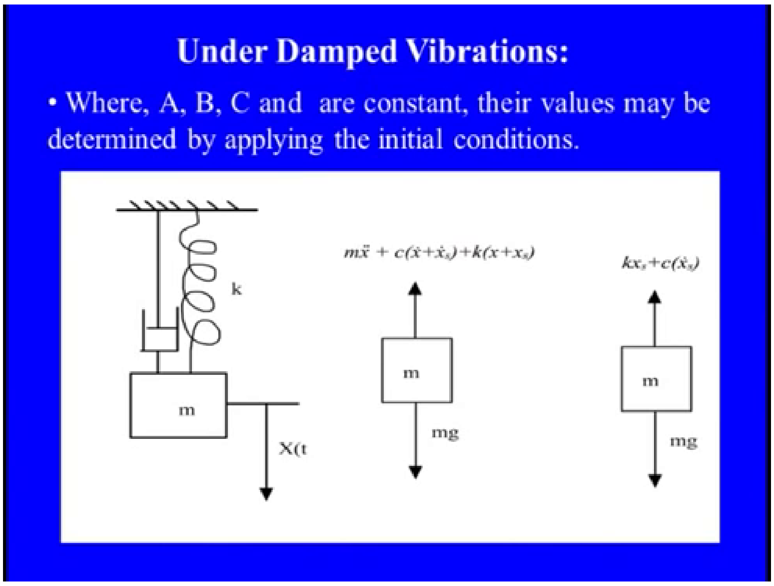

So, we can simply calculate the coefficient A, B or C, which are being closely related to the sin, cos or like the sinusoidal feature and these values can be easily get, when you know the initial condition. So, now you can look at that, what we have, we have clear a picture of mass, a spring and the damping configuration. And with that, we can simply find out that, when it is in the equilibrium position, there are the forces, the damping forces, the restoring forces and the inertia forces are well balanced.

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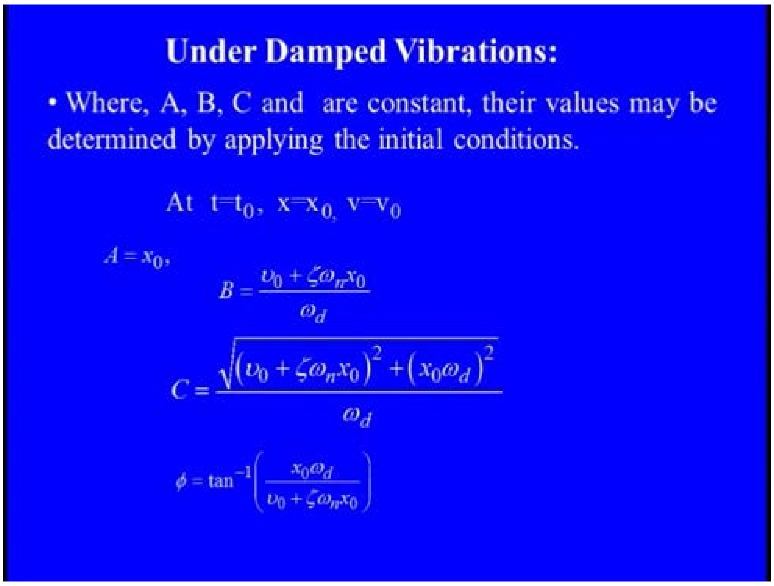

And with this, we can simply calculate the equation of motion and now if you are saying that, there are initial conditions like we have the initial time is t 0, the initial displacement is x 0 and we have the v equals to initial velocity. So, A can be easily find out, the coefficient A is nothing but equals to x 0, when we are keeping this into our main equation, we have the coefficient B is nothing but equals to v 0 plus zeta omega and this x 0 by omega d.

And C can also be calculated as you can see that, square root of v 0 plus omega plus zeta omega and omega n x 0 whole square plus omega, this omega d into x 0 square divided by omega d. So, what is it means that, we have a clear feature of both the thing means, the natural this exciting frequency and the damped natural exciting frequency. We can say natural frequencies are and both are playing a key role in calculation of either the B coefficient or in the C coefficient.

And then you see here, since there is in the final equation, which we discussed is nothing but equals to x of t equals to, we have the C into e to the power minus zeta omega n t. And then like the sinusoidal of this omega d t plus phi, the phi is the phase angle, which can be easily calculated using both the damping feature that is nothing but equals to tan inverse of x 0 omega d divided by x 0 plus zeta omega n x 0. So, both the factors are coming together in such a way that, when you know the initial condition in terms of displacement, in terms of time, in terms of velocity, they can be easily coming into the coefficients and we can find out.

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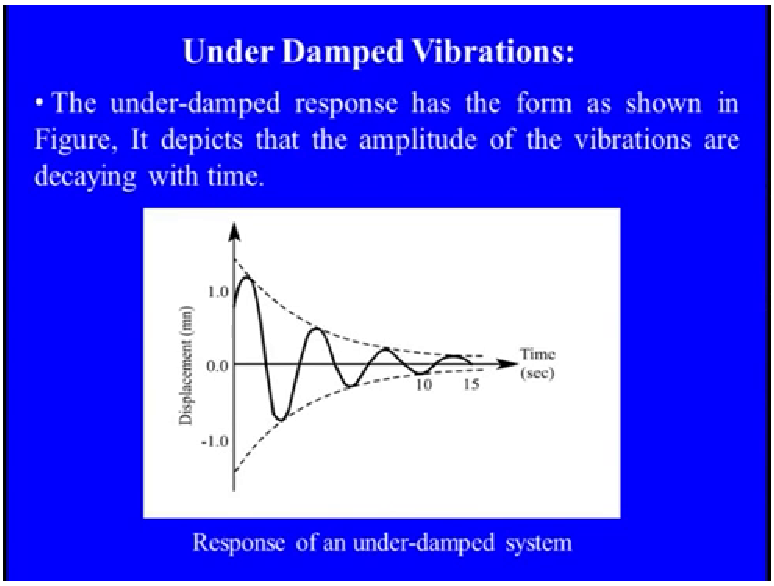

So, now we can see the effect of this under damping situation on the vibration oscillation, I told you that, it has a combination of two main features, one the exponential decay and one the sinusoidal feature. We can say that, the under damped responses can be easily shown here and there is a clear decay of the amplitude with respect to time, but they are taking the time in that manner. So, we have both the term, we have oscillatory term and we have periodic term and both are coming due to both the two different kind of exciting frequencies, the damped natural frequency, undamped natural frequency.

And you can see that, this is what my exponential decays are, we have and that this decay is having a clear feature, which is being preserving a sinusoidal peaks there itself. So, this is what, the effect of the under damping on the system performances, we can say we can die out the amplitude of vibration, but again it is taking too much time for it. Then you see here the over damping situation, the over damping means, when as we discussed, when we apply the damping more than the required one, the designed one.

So, in that case, we know that, our discriminate feature is positive, because zeta omega n square root of zeta square minus 1 is more than that, so it is positive. So, it gives our roots, the negative real numbers and when you know these things, certainly they will come straightaway into our equations in the exponential features. So, you can see that, my generalized solution of the equations with these over damping feature x of t equals to A into e to the power. So now, it is direct exponential features are there, minus zeta plus square root of zeta square minus 1 omega n square plus, you see here we can say that, omega n t plus again B into e to the power minus zeta plus square root of zeta square minus 1 omega n t. I can simply put there also the omega n square root of zeta square minus 1 is omega d.

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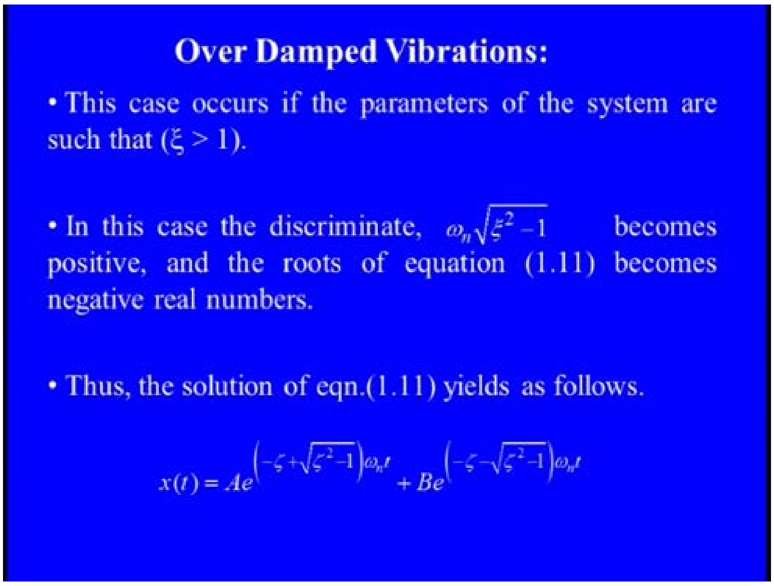

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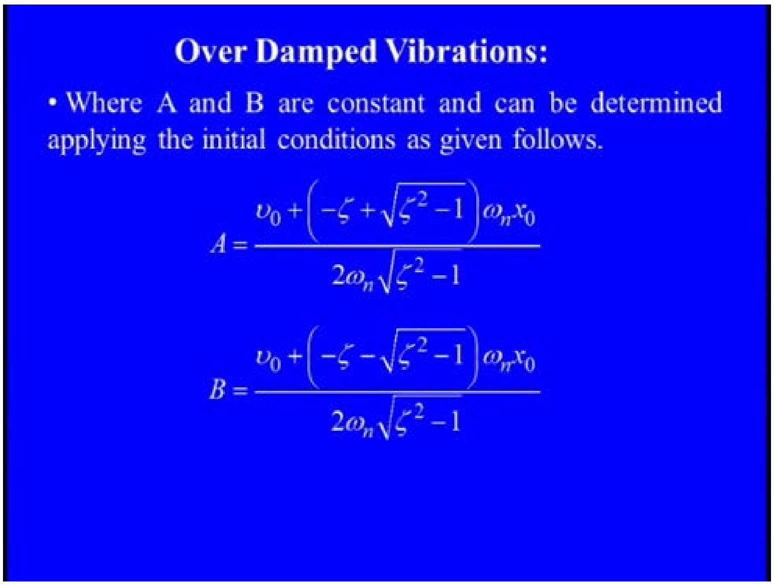

So, if I just apply these two to calculate A and B with the initial condition, I can simply find out A and B based on this value x 0, t 0 and v 0. So, A is nothing but equals to v 0 plus all the coefficient that is, minus zeta plus square root of zeta square minus 1 omega n t into x 0 divided by 2 omega n square root of zeta square minus 1. And similarly, we can get the B, only the difference comes you see, the minus in between zeta and square root of zeta square minus 1.

So, these two coefficients can be easily calculated and now, we can see the effect of this over damped phenomena on the vibration oscillation.

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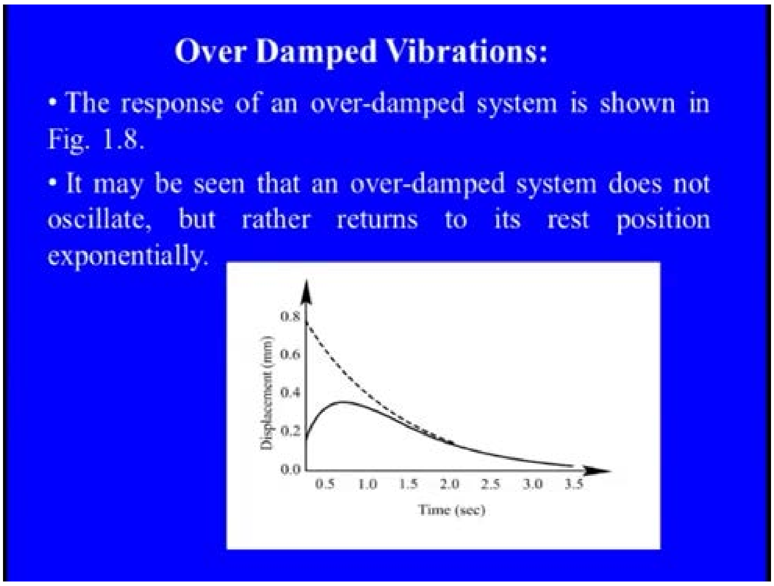

So, you can see that, the over damped system does not oscillate, we have a effective control on the oscillatory features, but rather you see here, it is coming towards the real position with the infinite time towards the exponential decay. So, there is a clear exponential decay you can see that, the dotted line of the form line, they are taking too much time to come to the steady state feature.

So, then again the things are coming that, what is the significance of over dampings are, though you see here, we are not able to get the final solution in a most conversing way through the over damping then why we are applying these things. So, you see again and you see if you are applying say more damping, more means say over damping is something say 10 Newton second per meter is over damping, if we have 12, if we have 15, say some numerical values then what is the effect.

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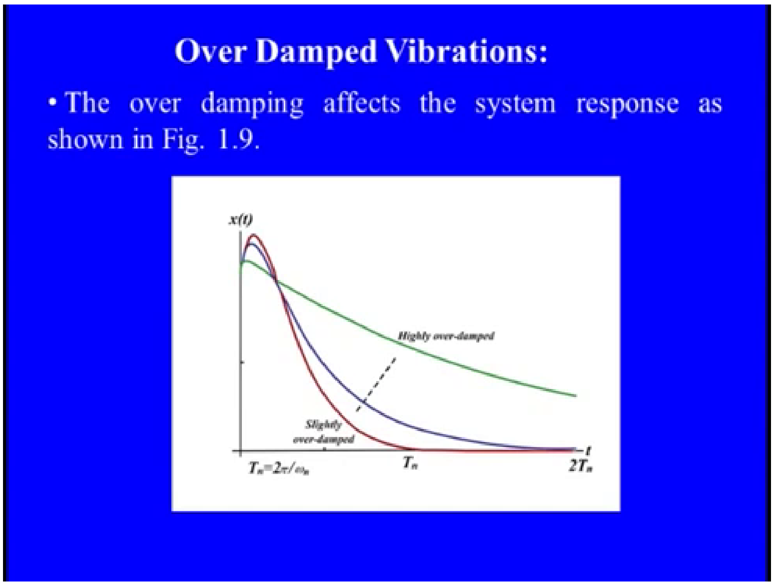

To see that, now you can see that, there are the different kind of over dampings are, these all three lines are simply showing the over damping phenomena. So, if it is a slightly over damp is there means, we know like we need say our critical damping is something around 10 Newton second per meter and we apply say, 12 or even the 15 time, certainly it is slightly one.

And if we apply 15 Newton second per meter of damping then you can see that, it is taking almost infinite time to come to the steady state or sometimes it would not come to the final solution, the converged solution in the numerical phenomena. So, even if you have the slightly over damped, you see though it is taking too much time as compared to this under this damping, whatever you see here. But, in the over damping phenomena, they are just if the red line, blue line or the green line, they are simply showing the real significance of the value of over damping on this situation is.

Now, the third part is over critical damping, the critical damping says that, now this is what we required and we provided accordingly. So, the C by C c is one, the damping coefficient you see, they are absolutely well balanced. So, we can say that, our discriminate that is nothing but equals to omega n square root of zeta square minus 1 is 0. And when it is 0, certainly we know that, all the real numbers are being there has the roots.

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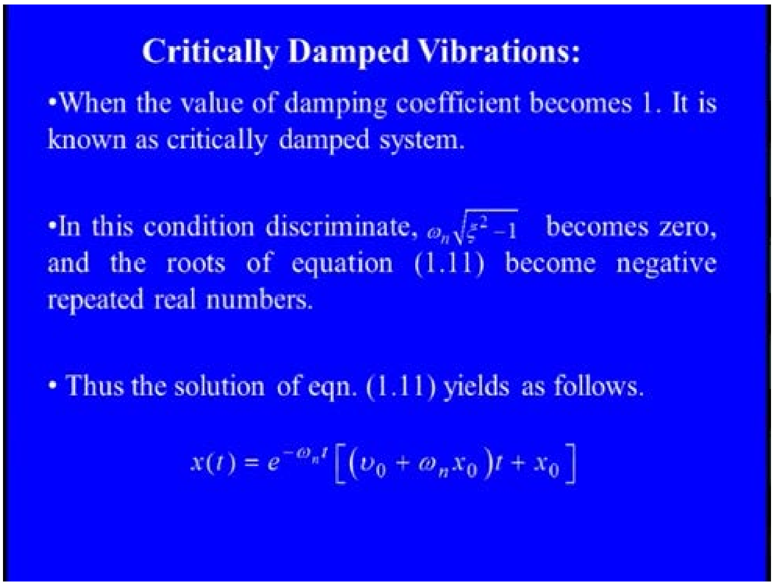

So, we can simply calculate that, the final solution of such kind of equations x of t is nothing but equals to e to the power minus omega n t, this is what my oscillatory term in the terms of the exponential feature plus v 0 plus omega n x 0 t and x 0. That means, you see here, the initial conditions of the displacement and the velocities are again playing a key role.

And you can see that, there is no importance of here omega d, because that component is gone in this case. We have a clear natural frequency, undamped natural frequencies are there and they are simply giving, the vibrations are simply die out according to our choice. So, you can see that, this is what our if we change the initial conditions say, if we have initial velocity is 0, x dot 0 is 0, we can simply see that, immediately it is coming with the very low profile and it is gone, it is just 0. If you have seen initial velocity is very low less than 0, certainly you see here, it is taking some time to come to the steady state. And if my initial velocity is more than 0, certainly you see here, it will oscillate certain thing and then it is coming to the exponential decay.

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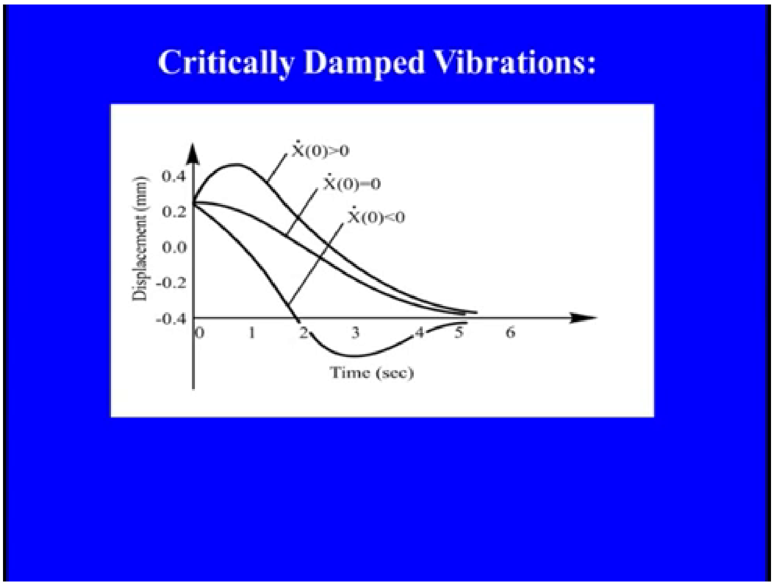

So, this is what the initial condition feature, but in all, now if you want to see the real features of all three part, the over damping, under damping and critical damping, you can see that, here what we have this is like the form line is simply showing that, this is my undamped situation. That means, there is no damping is available, simply spring mass system is and certainly when the spring mass system is there, it is imply oscillating at the simple harmonic motion and you have all the periodic features of that.

Now, if you are applying say under damped situation, so when it is under damped situation, this is my dotted lines are there, the first dotted line. Though you see here, with the periods, there is a clear decay in the amplitude of vibration and this decay is my exponential decay. And when it is over damped situation, you can see that, this is my over damped situation, it is taking the time and then here it is going upto the infinite feature. And when it is a critically damped, it will oscillate and then immediately die out at the quickest one, so this is what the real significance of our damping features on that.

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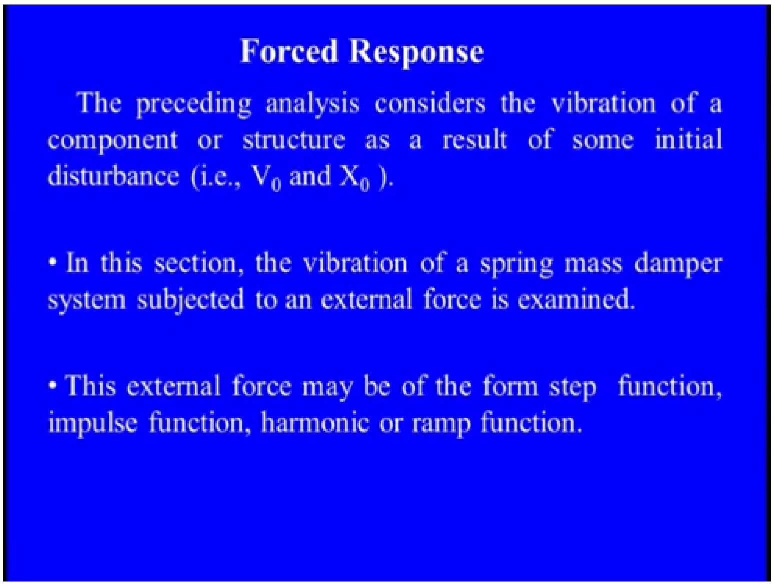

So now, we just discussed about the real damping part on the free vibration concept, now if you are going towards the forced vibration that means, there is a external force, which is exciting force, you see here apply to the system and it has its own frequency. So, we can say, even this system is also biperiodically excited, the system excitation, because of the inherent nature that is, the natural frequency and there is an external excitation frequency through the force it is coming into the system.

So, the previous equations was simply based on the initial disturbances v 0, x 0, now in this system, we are basically going towards the external force like excitation. We are applying the force and we just want to see, the effect of this force on the dynamic characteristic of the system. So, this external force may be from the step function, may be impulse function, may be harmonic, may be ramp. Again you see here, there are various types of, say when we are moving on the track or our vehicle is moving on the track, whatever the excitation is coming, that is somewhat the periodic excitation.

Because, the road is good, our tyres are good, there is no different kind of excitation in coming into the system. But, say if we have some disorientation or if we have discontinuities on our road, certainly a different kind of or if we have the speed breaker, the additional excitation is coming, we can simply consider into ramp or into some kind of impulsive or the step inputs are there, as in terms of the force part.

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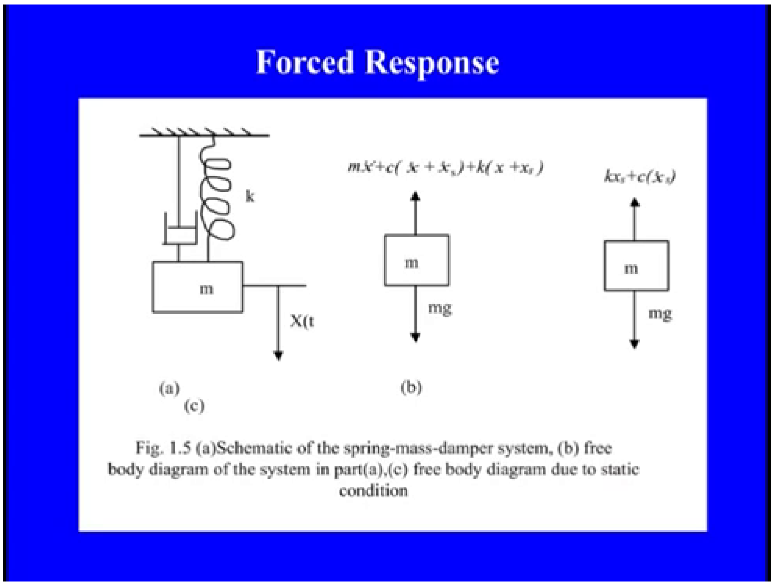

You can look at this the system, what we have, we have the same configuration, the mass, the spring and the damping and the force is there, which is being externally applied to the mass itself, just to disturb that. And when there is a disturbance, there is a displacement is there of the x of t. Now, what we have, we have all the same features like inertia force, like damping force, like the restoring force, along with that we have an external force is there.

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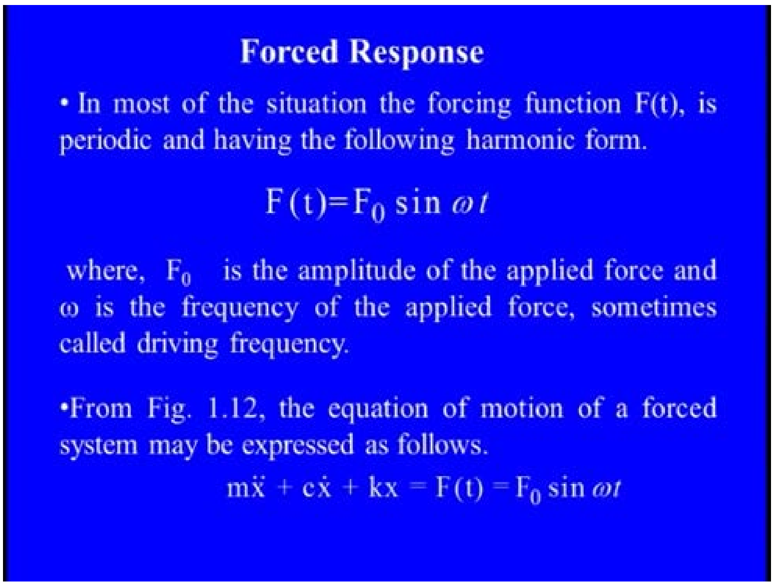

So, when we apply to the same system here, we know that, the external applied force is of, since it is a dynamic force, it has F 0 sin omega t. F 0 is the initial maximum amplitude of the force and you see here, since it is coming as in the harmonic form, it is sin omega t is there. So, we can simply calculate now, since it is a omega t, so this omega is the frequency of applied force, sometimes we are saying that, this is the driving frequency of the system.

So now, you see here, when we are making equivalency of this system, we know that, all the forces are well balanced. So, we can say that, the inertia force plus damping force plus restoring force equals to F 0 sin omega t or F of t, so all the forces are well balanced according to the Newton's law. When we are going towards the solution of this, we know that the solution, since you see like as I yesterday told you, that there are two components in any of the ordinary differential equation solution, we have complementary function and we have the particular integral. The complementary function is nothing but is always giving you the free vibration condition, the particular integral is always supplied by the forces. So, whenever we are talking about the free vibration we know that, we have a transient condition.

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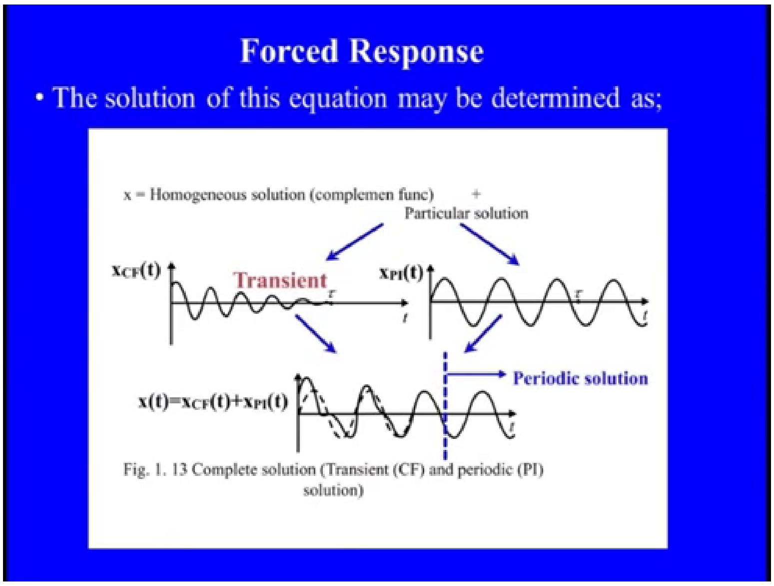

And in these transient conditions we know that, you can look at on the screen, the transient conditions are something, which is being dying out through their homogeneous solution, we can say the complementary function. So, in that, you look at that, the vibrations are being the clear vibration excitations are there and it is being dying out like that. Secondary you see, we have a particular integral, it is a well bounded solution, because we know the known forces are there and the known force solution is always a bounded solution.

So, we can get a straightaway, the kind of the real solution, which can simply gives you a clear peak of excitation, so it is like that, the periodic solution is there. When we combine both the thing means, when we are combining free and forced vibration, when we are combining complimentary function and particular integral, when we are combining homogeneous solution and non homogeneous solution, our last feature is, this is what it is x of t is x complimentary function plus x particular integral and this is what the clear depiction of that.

So, you see here, like this is what the basic feature of our dynamic systems are, that initially when they are starting, they are always showing the transient nature in it. So, certainly we can say that, this nature is always being occurred in the forces. When we apply the force, it always gives the maximum energy to the system in the beginning, but later on you see, after sometime the steady state features are coming. So, the same thing is there, since we have, why it is coming so because initially we have both the thing, we have the free vibration and the forced vibration.

And this free vibration, which has a oscillatory term is always giving you some kind of transient phenomena in the complimentary part and then you see here, because of my particular integral, which is again bounded by the external force is giving you the steady state part. So, if you are just trying to map the mathematical feature into the real vibration part, we can get that, there is a clear free and forced vibration feature.

There is a clear appearance of the transient nature and the periodic nature and this transient nature is coming into this system, because of this complimentary function or we can say, because of the free vibration condition. At that time you see here, we are not having effective control on the system dynamics, so it has the maximum energy at that point of time. So, we can get effectively the things, which are being more dominant feature.

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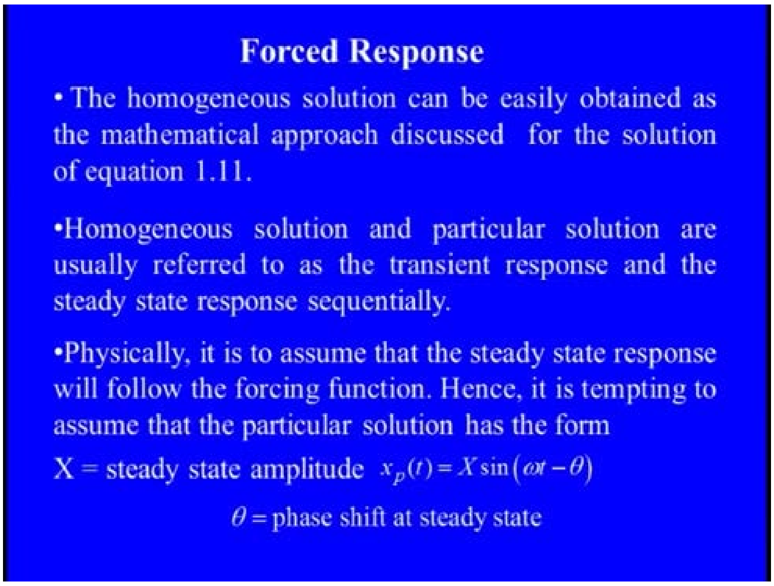

Then, you see here, if we applied these things then we know that the things are periodic and aperiodic phenomena. So, homogeneous solution can be easily obtained, as we discussed when we are applying the free vibration condition and when the free vibration condition is there, we have a natural frequency which is nothing but equals to square root of k by l. So, it is a system's inherent natural excitation, which does not have any control by any external source, is always giving you though homogeneous solution, but it has a transient nature.

Second you see here, the homogeneous solution and particular solutions are usually referred as a transient response and the steady state response, as I told you. So, steady state responses are well behaved responses, because we know the effectiveness of our force excitation and the exciting frequencies together. So, when you are taking about the particular integral we know that, though the system is exciting at omega n, but the same time, the system has the effect of the driving frequency omega f 0 sin omega t.

So, in particular integral, the system is biperiodically excited, physically we can say that, it is to assume the steady state response will always follow a forcing function in effective way. Hence, you see we can say that, it is tempting to assume the particular solution in the form of x of p, which is the particular all the steady state solutions is nothing but equals to X sin omega t plus phi, where the X is your sinusoidal feature, in which X is your amplitude, which is always bound by the sinusoidal feature and it is the steady state amplitude and phi is the phase shift or the phase features are there due to the steady state.

So, in this lecture, we discussed mainly about the effect of damping, the effect of stiffness and you see the masses also in our numerical problem, that if the system is exciting at some lower frequency, a spring is a very good we can say phenomena, through which we can control. If the system is exciting at higher frequencies, certainly the mass is a required parameter to suppress that.

But, when in the second numerical, we discussed about, when the system is exciting at the critical feature that means, at the resonant feature is there, the damping is the only way to absorb the huge amount of energy to control the amplitude of vibration. And then we discussed about the various phases of the dampings are, the damping can be coming out from the material due to the internal properties of the molecular structures. Second, the damping can be there at, we can say the interfacing of two surfaces.

The damping can be coming out due to fluid, which we are applying to the system and based on that, we can simply configure three parts under damping, over damping and critical damping. And accordingly, we can simply die out the vibration oscillation and then we discussed about, when the system is forced vibration excited, means you see the system is biperiodically excited one due to the inherent nature that is, natural frequency. One, due to the forcing frequency as we are applying the force from the outside then how the system is behaving, this is clear you see, the two nature of the solutions are.

The periodic nature, because of the particular integral or the forcing factor and we have that is called the steady state feature and we have the transient nature that is, the complimentary fraction. So, this is a clear mapping of our solution of the ordinary differential equation into the two main categories and how we can interpret the things in our vibration phenomena. So, in our next lecture now, we are going to solve some of the numerical problems of the forcing features.

When the force is being applied then how we can analyze the systems, dynamic characteristics then also now, we are going to discuss about the two degrees of freedom. Till now, there was only you see the single degree of freedom system, in which we are only considering the one independent parameters are there, in which entire dynamics is there in terms of the damping or in terms of, we can say the displacement, velocity and acceleration.

Now, if we have the two different parameters, through which we are just trying to describe the dynamics then how the forces, either the inertia force, damping force or restoring force is, are being configured out. And then even you see here, once we configured out that then how these equations, which are say the two equations, because if have two degree of freedom system, certainly we have the two equations of motion then how we can resolve the issues with that, if we have the two orientations.

And then once we calculate that then how many number of frequencies are there, till now you see there was only one exciting frequency, which is called the undammed natural frequency. If we have such kind of system with the two degrees then how many number of frequencies are there. And when we have these then what is the relative position of these, say whatever the masses or something, which we are generally saying as the Eigen vectors or Moore shapes. So, all these features, so in the next lecture, we are going to discuss.

Thank you.