

**Vibration Control**  
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**Module - 3**  
**Vibration Isolation**  
**Lecture - 2**  
**Vibration Isolation – II**

Hi, this is Dr. S.P Harsha from Mechanical and Industrial Department IIT Roorkee, in the course of Vibration and Isolation, Vibration and Control we are basically discussing on the isolators. In the previous lecture, we discussed about the design concepts that what exactly the design stages should be adopted, for separation of vibrations. Irrespective of whether it is a high exciting frequencies or the low frequency excitations or even at the resonant conditions.

And then you see what could be the potential location for keeping the isolation, so that the effective isolation can be done, in terms of the source isolation or the shielding towards the receiver as well. So, in the previous lecture we have checked various examples in those we got to know that, what is the effective way of doing the isolation. And then whether we need to apply the spring or the damper or the masses to suppress those things.

And also you see along with that, we know that when we are talking about any isolation feature, the perfect isolator can be designed based on what is the insertion loss. So, even we have a discrete kind of systems or we have a continuous system, even we have steady state feature or the transient feature or even if we have, the impact loading which is one form of transient only, how we can design the system according to the isolators, so all these concepts which we discussed in our previous lecture. Now, we are going to apply these concepts into some of the real practices, and we are trying to solve some numerical problems.

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### Vibration Isolation of the Source [Part – I]

**Example 1:** Consider the machine arrangement illustrated in figure 6. An electric motor is elastically mounted, by way of 2 identical isolators, to a 2-mm thick steel plate. When the motor is driven, its rotating parts generate a vertically-oriented, sinusoidal exciting force between the machine and the joists. Calculate the ratio between the total force acting on the foundation with and without the vibration isolators.

So, in the first example we have a electric motor which is elastically mounted, by a way of 2 identical isolators, 2 millimeter thick steel plate is there. And when the motor is driven its rotating parts certainly generate the vertically oriented, the sinusoidal excitation forces in between the machine and the joists or which the or which the machine is being settled. We need to find out the ratio between the total force acting on the foundation when we have the isolators.

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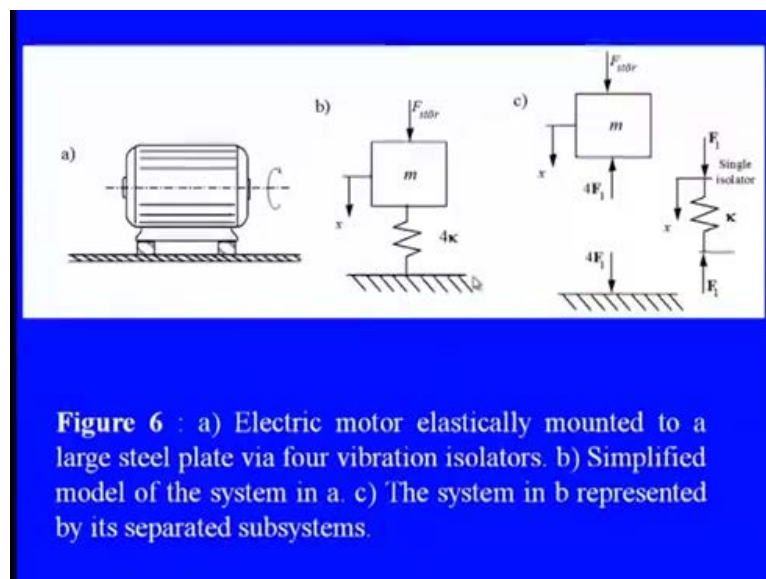
### Vibration Isolation of the Source [Part – I]

- Carry out the calculations at low frequencies under the assumption that the electric motor, when operating, generates a vertical harmonic exciting force with circular frequency  $\omega$  and amplitude .
- The mass of the motor is 100 kg, and each isolator's complex stiffness (see chapter 5, section 5.2.5) is  $(1.0 + 0.01i) \cdot 10^4$  N/m.

And in the other case when we do not have any other isolator in between along with this, we need to carry out. The calculations at low frequency under the assumption that the electric motor, when they are operating is just generating the steady state harmonic exciting forces, with the circular frequency of  $\omega$  and the amplitude is there. So, we are trying to limit right now the transient forces because if we adopt the transient feature then certainly the different amount of exciting forces are being coming out on the system.

So, just now you see we would like to start from the basic system, so in this the numerical data's are like that, the mass of motor of 100 kilogram. And each the isolator we have basically 4 isolators at the 4 junctions for complete mounting of the motor is having isolator the complex stiffness, which we discussed already in the previous lectures is  $1 + 0.01i$  into tens power of 4 Newton per meter.

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Such you can see on the screen that we have, the electric motor which is elastically mounted on the large steel plate via 4 vibration isolators. Now, this is our physical system, as we discussed in our first lecture that whenever the physical systems are there, we need to put the spring mass system as a discrete system like just to analogy of the actual systems are. So, here we have the mass and whatever the exciting forces are, and then you see here this mass which is being rested on this steel plate is having the 4 spring.

So, all the 4 spring since they are in absolutely at 4 junctions they have the stiffness of 4 k. And then you see here when we are trying to resolve with the free body diagram, according to the force is acting on the system, we have the f excitations which is being applied on top of that you can see this. And you see whatever the restoring forces, which are coming out due to the spring connection. So, we have this and equal and opposite amount of this spring forces are being acting on the mass as well as on the foundation.

So, even if we want to just look at the next diagram it just shows that how a clear representation of the spring, and the restoring forces which are being balanced at both the ends of the springs. So, this is what my system is and now you see here, we need to start to evaluate the losses in that and we see you know like what the other parameters are, so first of all we need to apply some of the assumptions, which can differ or which can make rather ease of the calculation from the realistic feature.

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**Solution:** Assumption(s)

- [i] the motor can be considered a rigid body;
- [ii] the foundation can be regarded as rigid; and,
- [iii] each isolator can be described as an ideal massless spring.

The Equation of Motion can be constructed considering with isolators for the mass  $m$ , as well as Hooke's law for spring. Thus,

$$m \frac{d^2 x}{dt^2} = F_{exc} - 4F_1 \quad \text{and} \quad F_1 = \kappa(x - 0)$$

where  $4F_1$  is the total force acting on the foundation, i.e., the force transmitted through all four isolators

So, first of all the motor the source of vibration is to be considered as the rigid body, second the same foundation whatever the you know like the steel flat plate is there, can also be considered as the rigid body. And each isolator the 4 isolators are there, which is being you know like acted as the spring is the mass less spring. So, when we are considering those things, then it is pretty simple that the inertia forces which are being generated due to the rotation or some kind of oscillation are being there from the 100 kilogram of the machine.

So, we have  $m \ddot{x}$  by  $\ddot{x}$  inertia force, which is well balanced by the differences of the force you know like the cumulative effect of 4 springs restoring forces and the exciting forces. So,  $F_{exc} - 4F_1$  is my force summation which should be balanced by the whatever the inertia forces are. So,  $4F_1$  we can say that these are nothing but the restoring forces coming out from the spring in terms of the foundation we can say, so 4 transmitted through all 4 isolators.

And since we are saying that, whatever the excitation features are being coming from the machine to this. Because, when we are operating the machine from the electric operations like this you know like the compressor or the pump, we know that whatever the exciting nature of the systems are is always according to the simple harmonic motion.

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Assuming a sinusoidal, complex-valued displacement and eliminating  $X$ , the exciting force become

$$\frac{4F_1}{F_{exc}} = \left( 1 - \frac{\omega^2}{4\kappa/m} \right)^{-1} = \left( 1 - \frac{\omega^2}{\omega_0^2} \right)^{-1}$$

Where,  $\omega_0^2$  is the machine's so-called mounting resonance.

Note: the first term in the equation, applies to machines with four mounting points. For machines mounted at  $n$  points, the term  $4\omega/m$  should be replaced by  $n\omega^2/m$ .

So, with this sinusoidal feature of this the complex value displacement can be straight way you see you know like we can say the eliminated, and the exciting forces becomes  $4F_1$  divided by the exciting forces  $F_{exc}$ . Excitation is nothing but equals to  $1 - \omega^2 / (4k/m)$  because this  $\omega^2$ , which is nothing but you see the circular frequency of the system excitation and  $4k/m$  is coming out from the machine excitations.

Because, you see here we know that the entire system is supported at the 4 supporters, so  $k/m$  is my natural frequency of the system. So, I can say that this is nothing but equals to  $1 - \omega^2 / \omega_0^2$  to the power minus 1, here  $\omega_0^2$  square

is nothing but it is mounting resonances. So, in this the first part is just saying that we need to apply the entire equations of these for all 4, we can say this rigid mounting parts. And for the machine if we have say the n points or which the machine is rigidly supported.

And they are simply we can say the connector in between the machine and the foundation. And if we have these n points we can say that the first term should be 4 whatever you see the 4 omega by m is coming on the lower term, it can be replaced by n omega square by m. Means you see here whatever the number of points are there accordingly the things are being varied with the stiffness variation.

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In case of no isolators, the force on the foundation is equal to  $F_{exc}$ . The desired ratio between the force with and without isolators is therefore

$$\frac{F_u}{F_m} = 1 - \frac{\omega^2}{\omega_0^2}$$

If an insertion loss is defined on the basis of that ratio as;

$$D_{il} = 20 \cdot \log \left| 1 - \frac{\omega^2}{4\kappa/m} \right| = 20 \cdot \log \left| 1 - \frac{\omega^2}{\omega_0^2} \right| = 20 \cdot \log \left| \frac{Y_m + Y_f}{Y_m} \right|$$

In which  $Y_m$  and  $Y_f$  are the machine's and the isolator's respective mobilities. Note that that formula even applies to cases of *shielding isolation*.

So, in case of you see no oscillator now the first case, the force is directly transmitted through the foundation from the machine. And we are saying if this is equals to F excitation the desired ratio between the force, with and without can be just says that F u and F m is equals to 1 minus omega square by omega 0 square. So, this is what you see a direct relation in between, the forces which are being applied or transmitted you know like from machine to foundation, and there is you see n like in between we have the springs or not.

And we can even define the insertion losses on the basis of this particular formula that the D l is nothing but equals to 20 log as we discussed you see in the previous you know like the chapter. The insertion losses are nothing but equals to 20 log of 1 minus omega



square divided by whatever the natural frequencies are, so we can say  $20 \log 1 - \frac{\omega^2}{\omega_0^2}$  or even we can go with the mobility's. Because, we know that if the mobilities are high the isolator is perfect.

So,  $20 \log Y_m + Y_I$  divided by  $Y_m$ , where  $Y_m$  and  $Y_I$  are nothing but machines isolator with respect to their mobilities. And we know that this is absolutely applicable not only to the isolators, but also the shielding isolation towards even the receiver end. So, you see here in this particular mathematical description we could easily figure out that what exactly the ratios of the forces, when we are applying the isolation in between the machine and the foundation or not.

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The insertion loss, has several characteristics given below:

- (i) Firstly, no isolation is obtained for excitation frequencies far below the mounting resonance  $f_0$  corresponding to  $\omega_0$ .
- (ii) Secondly, the insertion loss takes on large, negative values for excitation frequencies near the mounting resonance in which the force on the foundation is amplified rather than reduced.
- (iii) Finally, large, positive insertion losses are obtained for excitation frequencies well above the mounting resonance increasing the insertion loss asymptotically approaches 40 dB/decade.

So, there are you see you know like various characteristics are there for the insertion losses just like the first, when no isolation is being obtained for any exciting frequency. Then the exciting frequencies are always far below the mounting resonances, just like in this case. So, we can say that this  $f_0$  which is being you know like we can say, the exciting frequencies are there, they are always far low than the mounting resonances.

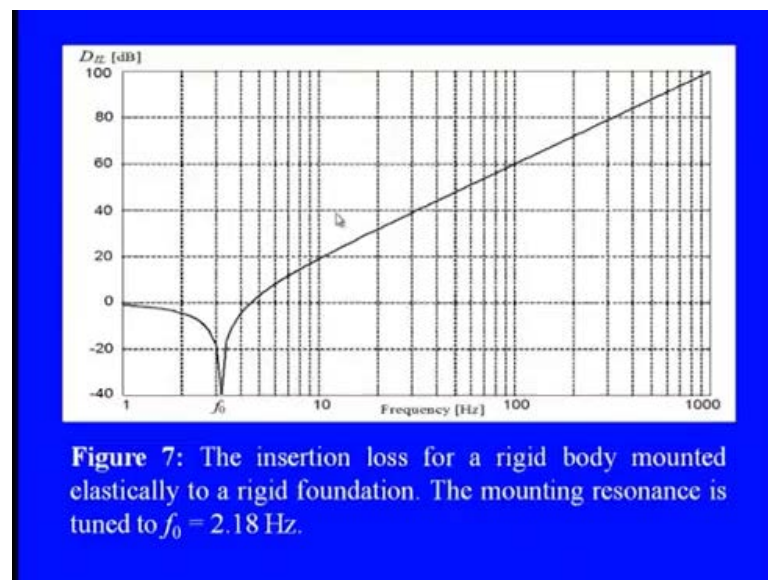
Secondly, when the insertion losses takes you know like we can say whatever just on the large or negative values for any excitation which is close to the mounting resonances, you see  $4k$  by  $m$ . When we can say that, the forces on the foundation which is being you amplified even in comparison to the reduction one, so more and more excitations are there. Because, it is pretty clear that when the forcing frequencies when they are excited

and when they are close to the systems frequencies there is a resonance huge amount of energies are there.

So, the spring cannot handle that amount of excitation level or the energy, so certainly you see here we need to check it out that, the insertion losses are now significant in this case. Finally, large and positive insertion losses can be obtained for those exciting frequency, which is well above than the mounting resonances the insertion loss is as asymptotically approaches up to even 40 d B per decade. So, we can see that if the exciting frequency is less than, whatever you see we can say the mounting frequency.

Then certainly you see, this is one of the effective way and insertion losses are almost negligible. Because, this is what the basic physics of the vibration is, when the system is exciting well below the natural frequency, the spring connectors are perfect one, they can straight away absorb the kind of energy. And they can even you see control the oscillation of the vibration whatever the amplitude is there. But, even when the system is exciting at the higher, higher frequencies or even when the system is exciting at the resonant condition, the insertion losses are quite significant with the spring controllers.

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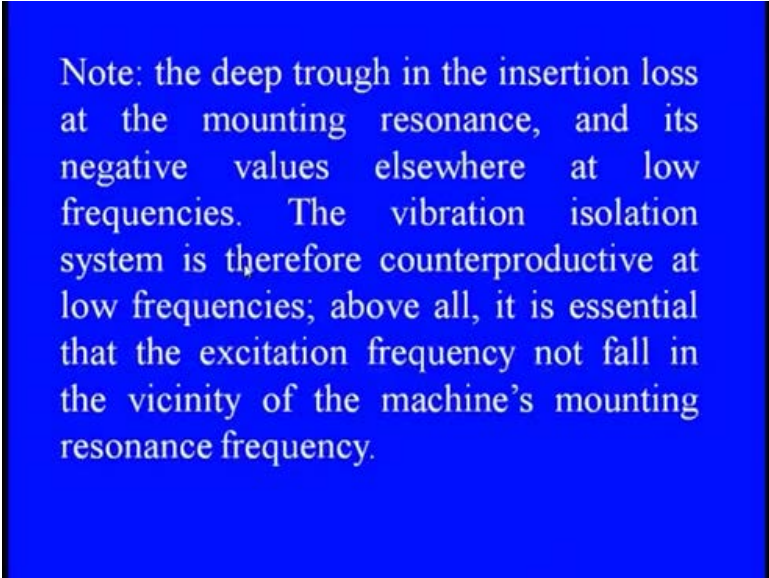


So, you can see that the insertion losses for rigid body, which is being mounted on the rigid foundation with these in between these things, the mounting resonances can be termed as 2.18 in this example. So, we can see that this is what the insertion losses they are being coming down to negative, and then you see here after that when the exciting



frequencies becomes more and more the insertion losses are almost approaching up to the 100 d B you see. So, this diagram clearly shows that at the lower frequency at the resonant frequency and at the higher frequencies, how the insertion losses are being varied with this, along with the mounting resonances.

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Note: the deep trough in the insertion loss at the mounting resonance, and its negative values elsewhere at low frequencies. The vibration isolation system is therefore counterproductive at low frequencies; above all, it is essential that the excitation frequency not fall in the vicinity of the machine's mounting resonance frequency.

The deep trough in the insertion losses at the mounting resonance, its negative value clearly shows in between the lower frequencies. And the vibration isolation system can now be counterproductive at the low frequency, and above all it is essential that the exciting frequency not fall in the vicinity of the machine's mounting resonance frequency, otherwise the insertion losses will be of high.

So, we can conclude from this part that the vibration isolators must be designed in order to prevent the coincidence of machine mounting frequencies with any other exciting frequencies of the part. Otherwise you see here, the things will be of the insertion losses will be of different nature, moreover it is clear that a positive effect is obtained from the isolator at the frequency which are above than the mounting frequencies, this is a great conclusion.

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**Conclusion:**

- The vibration isolators must be designed to prevent the coincidence of the machine's mounting frequency with any important excitation frequency.
- Moreover, it is clear that a positive effect is obtained from the isolators at frequencies above the mounting frequency.
- The implication is that as low as possible a mounting resonance frequency must be sought.
- In practice, machine mounting is often designed so that the mounting resonance frequency falls in the 2-10 Hz band.

And the implications is that as low as possible the mounting resonant frequency should be short. So, generally you see here when we are designing these things the mounting resonant frequencies is always keeping in between 2 to 10 hertz band, so that we can avoid this condition in which you see here, the insertion losses are of this nature. So, this numerical was there for the rigid body and the rigid foundation, now if we are just taking the flexible foundation, instead of the rigid foundation.

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**Flexible foundation**

- As the excitation frequency increases, the deformation of the foundation due to the excitation force soon becomes too large to ignore.
- A model in which the *foundation* is *flexible* must then be used. A number of different models with differing characteristics are available for this situation.

So, when the exciting frequency increases, the deformation of the foundation due to exciting force becomes too large to ignore. Because you see here when the deformation of the foundation is greater certainly the exciting frequencies are of more importance, a model in which the foundation is flexible we need to use in that. So, the number of different modes with the different characteristic frequencies in such cases are available.

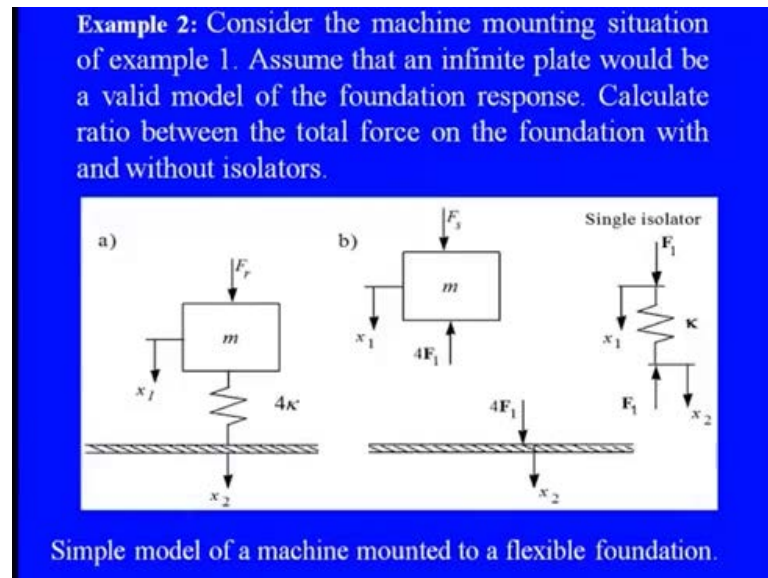
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#### **Flexible foundation**

- If, for example, the foundation is a system of joists with considerable dimensions, an infinite plate model might be used to describe the motions of the foundation.
- If the foundation exhibits a resonance, then a mass-damper system can be used as a first approximation to describe its behavior.

And if for example, the foundation of the system the entire system in which you see the joists and everything is there is to be considered in the dimensions, with an infinite plate model and straight away it can be used to describe the motion of the foundations with the flexible way. And if the foundation exhibits a resonance, then the mass system can be used as an first approximation which simply describe the real character of the systems. So, again it all depends on whether you see we are using with only to describe the motion of the I foundation or you see the mass dampers.

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So, in this case you see here now we are again considering the same previous example and now we are assuming that there is a infinite plate. The same steel plate of the joist you see here, and it is a valid model you see of the foundation response with the flexible feature. Now, we want to find out the total force on foundation with and without oscillator.

So, you can see that we have the same you see the motor you see here, which is joining at this part, the exciting frequencies are there. The same four rigid supporters are there in between the rigid body, and the flexible mountings or the supporting we can say that and you see the foundation is the flexible one, in which you see we have the displacement  $x_2$ . So, when we are applying these things on this now we need to put the 2 degrees there one at the masses, we have you see the force balance equation when the inertia forces are being suppressed by the difference of  $F$  excitation minus  $4 F_1$ .

And at the same time this foundation, which was ignored with the previous case is simply  $4 F_1$ , and then you see we have the excitation  $x_2$  is there. So, you see here in the single isolator now we can see that we have on top of one end, we have one displacement on bottom of that we have  $x_2$  and  $F_1$  and this you know like the top force  $F_1$  to  $F_1$  is to be balanced with the stiffness  $k$ . Now, we can apply straight way the Newton's law for this, and we know that you see since this elastic deformation is being you see you know like varied with the kind of you see the flexible mounting.

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The equation of motion, Hooke's law, and the mobility of a plate yield the following system of equations:

$$m \frac{d^2 x_1}{dt^2} = F_{exc} - 4 F_1$$

$$F_1 = \kappa (x_1 - x_2)$$

$$x_2 = (i\omega)^{-1} Y_{plate} 4 F_1$$

Eliminate  $x_1$  and  $x_2$ ,

$$\frac{4F_1^{with}}{F_{exc}} = \frac{-4\kappa / m\omega^2}{1 - 4\kappa / m\omega^2 + 4\kappa / (i\omega) Y_{plate}}$$

$$= \left\{ \begin{array}{l} 1 / i\omega m = Y_m \\ i\omega / 4\kappa = Y_1 \end{array} \right\} = \frac{Y_m}{Y_m + Y_1 + Y_{plate}}$$

So, Hooke's law can be applied straight way and the mobility of the plate which is at the yield point can be straight way formed in the equations. So, we have  $m \frac{d^2 x_1}{dt^2}$  is nothing but equals to this inertia force is being balanced by  $x F_{exc}$  minus  $4 F_1$ . And where the  $F_1$  is nothing but equals to the restoring force  $k$  into  $x_1$  minus  $x_2$  and this  $x_2$  which is being coming out due to the flexible mountings can be put as  $i \omega$  inverse  $Y_{plate}$  into  $4 F_1$ .

Now, we are trying to eliminate the  $x_1$  and  $x_2$  in such a way that we can get straight way the force relations. So, if am saying that when the isolator is there. So,  $4 F_1$  and when isolator is not there then force excitation, this is a very simple thing you see here when you do not have any isolator in between the machine and the foundation straight way the exciting forces are being transmitted.

But, if we have the springs then  $4 F_1$  the forces with the isolators of  $4 F_1$  by  $4 F_{exc}$  is nothing but equals to minus  $4 k$  over  $m \omega^2$  divided by  $1 - 4 k / m \omega^2 + 4 k / i \omega Y_{plate}$ . And we already discussed that this is the yield feature of the plate is in that because we need to apply the Hooke's law at that point. So, we can say that it is nothing but equals to  $1 / i \omega m$  which should be equals to  $Y_m$  or else  $1 / i \omega$  by  $4 k$  is equals to  $Y_1$ .

So, these are you see nothing but the two you know like as we already discussed with the previous diagram that you see here, we have  $Y_1$  and  $Y_m$  and they are straight way



showing here a clear insertion of these you see that how the machine and the isolator mobilities are being varied.

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In case of no isolators, the force on the foundation is equal to  $F_{exc}$ . The desired ratio between the force with and without isolators is therefore

$$\frac{F_u}{F_m} = 1 - \frac{\omega^2}{\omega_0^2}$$

If an insertion loss is defined on the basis of that ratio as;

$$D_{il} = 20 \cdot \log \left| 1 - \frac{\omega^2}{4\kappa/m} \right| = 20 \cdot \log \left| 1 - \frac{\omega^2}{\omega_0^2} \right| = 20 \cdot \log \left| \frac{Y_m + Y_I}{Y_m} \right|$$

In which  $Y_m$  and  $Y_I$  are the machine's and the isolator's respective mobilities. Note that that formula even applies to cases of *shielding isolation*.

So, this machine and you know like  $Y_m$  is mobility of machine and  $Y_I$  is the mobility of the isolator, when they are being varied here.

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The equation of motion, Hooke's law, and the mobility of a plate yield the following system of equations:

$$m \frac{d^2 x_1}{dt^2} = F_{exc} - 4F_1$$

$$F_1 = \kappa (x_1 - x_2)$$

$$x_2 = (i\omega)^{-1} Y_{plate} 4F_1$$

Eliminate  $x_1$  and  $x_2$ ,

$$\frac{4F_1^{with}}{F_{exc}} = \frac{-4\kappa / m\omega^2}{1 - 4\kappa / m\omega^2 + 4\kappa / (i\omega) Y_{plate}}$$

$$= \left\{ \begin{array}{l} 1 / i\omega m = Y_m \\ i\omega / 4\kappa = Y_I \end{array} \right\} = \frac{Y_m}{Y_m + Y_I + Y_{plate}}$$

Then certainly you see here we need to straight way keep in this part that how the  $Y_I m$  means when  $1 / i\omega$  is there its clearly showing the mobility of the machine. And  $i\omega / 4\kappa$  this is absolutely with the isolator it is clearly showing the isolator



mobility. So,  $4 F$  with isolator divided by  $F$  excitation is nothing but equals to the mobility of the machine  $Y_m$  divided by  $Y_m$  plus  $Y_I$  plus  $Y_{plate}$ . So, in this you see here we are considering the mobility not only in the machine and isolator, but also the plate because the plate is acted here as the flexible feature.

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Without isolators, the force on the foundation can be determined by excluding the second of the equations from the system given above, and setting  $x_1$  equal to  $x_2$ . The system then has the solution

$$\frac{4F_1^{\text{without}}}{F_{\text{exc}}} = \frac{1/i\omega m}{1/i\omega m + Y_{\text{plate}}} = \frac{Y_m}{Y_m + Y_{\text{plate}}}$$

The insertion loss is therefore

$$D_{\text{IL}} = 20 \cdot \log \left| \frac{Y_m + Y_I + Y_{\text{plate}}}{Y_m + Y_{\text{plate}}} \right|$$

The above formula can be even apply to the shielding isolation case.

So, without oscillator if we are just saying the forces on the foundation can be determined by excluding the second term in the equation from the system given equation. So, that we can simply say it again the  $4 F$  without divided by  $x$  excitation is nothing but equals to  $1 / i \omega m$  divided by  $1 / i \omega m$  plus  $Y_{plate}$  or else we can say that  $Y_m$  divided by  $Y_m$  plus  $Y_{plate}$   $Y_I$  is now not present. So, the insertion losses are quite significant. In that case it is nothing but equals to  $20 \log Y_m$  plus  $Y_I$  plus  $Y_{plate}$  divided by  $Y_m$  and  $Y_{plate}$  in the case.

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The equation of motion, Hooke's law, and the mobility of a plate yield the following system of equations:

$$m \frac{d^2 x_1}{dt^2} = F_{exc} - 4 F_1$$

$$F_1 = \kappa (x_1 - x_2)$$

$$x_2 = (i\omega)^{-1} Y_{plate} 4 F_1$$

Eliminate  $x_1$  and  $x_2$ ,

$$\frac{4F_1^{with}}{F_{exc}} = \frac{-4\kappa / m\omega^2}{1 - 4\kappa / m\omega^2 + 4\kappa / (i\omega) Y_{plate}}$$

$$= \left\{ \begin{array}{l} 1/i\omega m = Y_m \\ i\omega / 4\kappa = Y_l \end{array} \right\} = \frac{Y_m}{Y_m + Y_l + Y_{plate}}$$

So, if we divide this then we have the insertion losses, which can also be applied to shielding isolation is nothing but equals to the previous case, it was 4 F with divided by 4 excitation is Y m divided by Y m plus Y I plus Y m Y plate.

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Without isolators, the force on the foundation can be determined by excluding the second of the equations from the system given above, and setting  $x_1$  equal to  $x_2$ . The system then has the solution

$$\frac{4F_1^{without}}{F_{exc}} = \frac{1/i\omega m}{1/i\omega m + Y_{plate}} = \frac{Y_m}{Y_m + Y_{plate}}$$

The insertion loss is therefore

$$D_{IL} = 20 \cdot \log \left| \frac{Y_m + Y_l + Y_{plate}}{Y_m + Y_{plate}} \right|$$

The above formula can be even apply to the shielding isolation case.

And here without 4 F divided by 4 F without divide by F excitation is Y m by Y m plus Y plate. So, you see here the insertion losses are of these two parts, so we can simply find that 20 log Y m plus Y I plus Y plate divided by Y m plus Y plate for that. So, in that it is pretty simple to calculate those.

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The mobility of a 2 cm thick, very large steel plate is calculated from

$$Y_{\text{plate}} \frac{\sqrt{3(1-\nu^2)}}{4h^2\sqrt{\rho E}} = \left\{ \begin{array}{l} \rho = 7800 \text{ kg/m}^3 \quad \nu = 0.3 \\ E = 2.0 \cdot 10^{11} \text{ N/m}^2 \quad h = 0.02 \text{ m} \end{array} \right\} \approx 2.61 \cdot 10^{-5} \text{ m/Ns}$$

Putting values into the expression for the insertion loss leads to the following next graph. That figure also shows the corresponding results for a rigid foundation.

Points to be remembered:

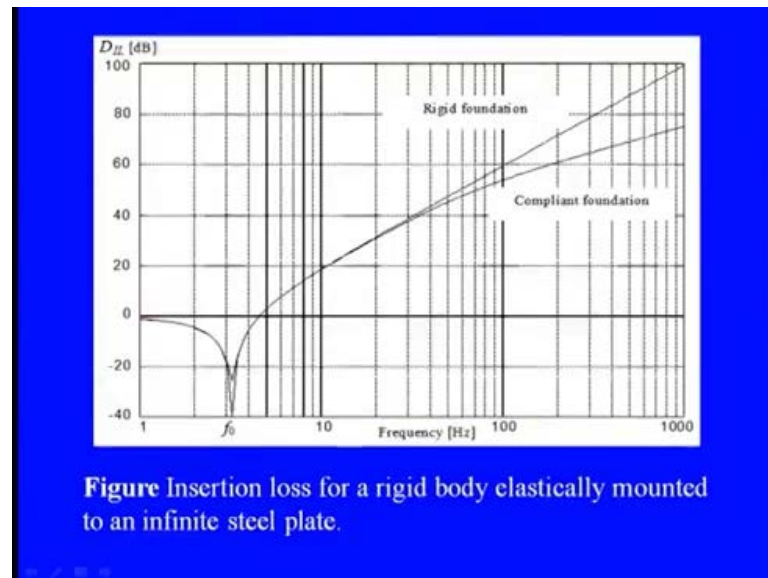
➤ Apparently, the flexible foundation affects the insertion loss in two bands: at the mounting resonance; and, at frequencies over about 50 Hz.

Now, the second point is there the mobility of 2 centimeter thick a very large steel plate, so certainly you see here, we need to go with the Poisson's ratio. And we need to check it out that how you see we can simply adopt, you know like the density, elastic young's modulus and Y plate as well. So, Y plate into square root of 3 1 minus mu square mu is the Poisson ratio here divided by 4 h square rho E, where the rho density certainly we can calculate that is 7800 kilogram per meter cube, the Poisson's ratio is 0.3 which is pretty common for all the metals.

Young's modulus it is a steel plate, so it is 2 into 10 raise power 11 Newton per meter square, and then we have h which is nothing but you see here whatever you know like the thickness of that is 0.02 meter. So, when we are keeping this we can find the mobility here is 261 into 10 raise power minus 5 meter per Newton second, so here when we are getting those things the insertion loss leads to the various features in that, and which we are going to show the next figure.

But, here few of the points which are to be remembered 1, the flexible foundation when we are adopting that, straight way affecting the insertion losses in two bands, one the mounting resonances, and second the frequencies which are more than the 50 hertz as the system excitation is at 50 hertz. So, you see here when we have the flexible moutation in other words it is affecting the low frequency and higher frequency, with respect to the natural frequency of the system.

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So, we can see that this is what the insertion losses are there, when we have the rigid foundation as far as you see you know like the lower part is concerned, you can see what we have, we have a clear feature of that and it is just going with this. So, we have you see the rigid foundations towards that, but when we have the you know like the elastically mounted the infinite steel plate. Then the things are somewhat more complaint foundations, and you see here at the higher harmonics the things are becomes you see showing the less insertion losses at these points. So, this is what you see with the basic criteria for designing the isolator with the rigid or the flexible foundation.

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- At the mounting resonance frequency, the insertion loss increases, due to the ability of the infinite plate to act as an energy sink. Above 50 Hz, the flexible foundation provides a significantly lower insertion loss than the rigid foundation.
- Here, the isolation obtained is largely determined by the ratio of the isolator mobility to the mobility of the plate.
- At high frequencies, the insertion loss now asymptotically approaches a 20 dB per decade rate of increase, instead of the 40 dB per decade obtained earlier.

So, at the mounting resonances with these whatever the frequencies are there, the insertion losses increase, due to the ability of infinite plate to be act as a more sink energy. That means, you see here whatever the exciting frequencies are there of the mounting resonances, the insertion losses are always becomes more significant, but above 50 hertz the flexible foundation is always provide a significant effect, with the lower insertion losses.

Because, of we know that you see when are going beyond that, the absorption feature is quite significant in that case as compared to the rigid foundation. Hence the isolation obtained is mainly determined by the ratio of isolator mobility to the mobility of the plate, that how you see you know like this particular factor in which the mobility  $Y_m$  or  $Y_{I2}$  the  $Y$  plate is varied in this case.

But, at the higher frequencies the insertion losses are almost, you know like reaches asymptotically means the curvature feature at 20 d B per decade per whatever you see you know like the decade rate is there and it is being increasing feature as we can see. But, here when we had the rigid foundation this was happened at the 40 d B, so this you see this approach is now drastically reduced to 20 d B only, so this is one of the unique feature when we are adopting the flexible foundation.

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- Compared to an ideal, rigid foundation, the amplification peak at the mounting resonance frequency is reduced and the rate of increase of the insertion loss falls off.
- That latter effect is due to the diminished mobility or impedance gap between the isolator and the foundation.

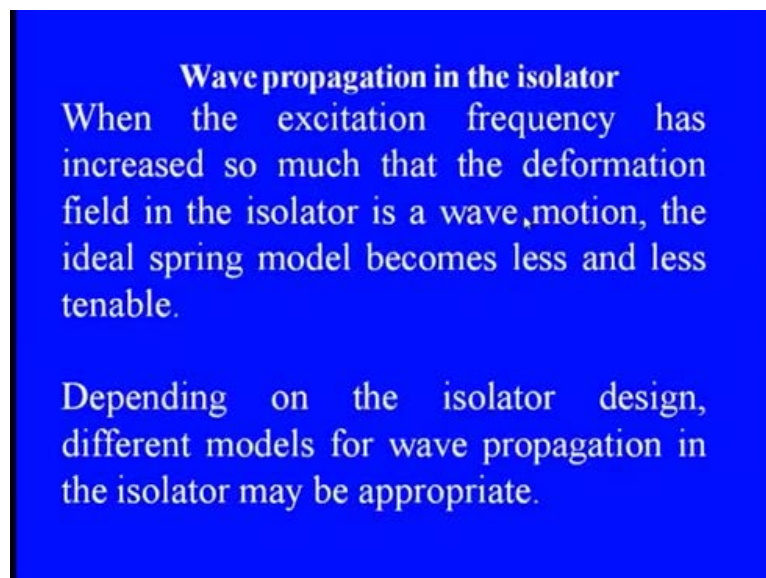
So, compared to an ideal, rigid foundation and amplification peak at the mounting resonant frequency is drastically reduced and this rate of increase is also with the same



insertion losses, and this is one of the great advantage of this. So, when you see the insertion losses are being you know like reduced, it is mainly due to the diminishing the mobility or we can say the impedance gap between the isolator and the foundation. So, when we are saying that you know like the Y I by Y plate is being you know like reducing certainly it has a clear impact on the insertion losses.

So, this is you see you know like one of the great criteria for adopting, the rigid foundation or the flexible foundations according to what the exciting frequencies are there. And how the insertion losses are being varied with the compliant support from the isolators, now we are just going into the another section that is the wave propagation in isolator.

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**Wave propagation in the isolator**  
When the excitation frequency has increased so much that the deformation field in the isolator is a wave motion, the ideal spring model becomes less and less tenable.

Depending on the isolator design, different models for wave propagation in the isolator may be appropriate.

So, when we know that we define the isolator we design accordingly, but how the wave propagations are being occurred, in the isolator itself. And when the exciting frequency is increasing and this is increasing, so much that the deformation field in the isolator is a wave motion, the ideal spring which we are adopting is not at all compatible. So, depending upon the isolator design the various models for these wave propagations in which the isolator is more compatible is always choosing.



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**Example 3 :** Consider the machine mounting situation illustrated in example 2. Assume that the isolator is a circular cylindrical bar undergoing primarily axial deformations. For axial deformation, the motion in the isolator is built up of longitudinal waves. In order to permit a direct comparison between the examples, every isolator is assumed to have a length of 0.05 m and a cross-sectional area of 0.005 m<sup>2</sup>.

The isolator material is assumed to have a density of 2500 kg/m<sup>3</sup> and a complex E-modulus  $0.1(1 + i 0.01)$  MPa. For these input values, the isolator stiffness at low frequencies matches that used in examples 1 and 2 above.

So, now we how we can go that now we are going to the another numerical problem with this in which we are considering the machine foundation situation, as we have shown in the previous two cases. And now we are assuming that the isolator is a circular cylindrical bar, which is undergoing primarily the axial deformation, and for this axial deformation motion in isolator is build up of longitudinal waves. That how you see you know like the wave propagations are there, what is the nature of the waves.

And in order to permit, the direct comparison between this what we are going to see we are going to take the same, isolator design and every isolator is assumed to have the length of 0.05 meter, and the cross sectional area is 0.005 meter square. And the material which we can adopt here having the density is 2500 kilogram per meter square, and you see the young's modulus of this which has the complex nature because of you know like the variation and it is 0.01 into 1 plus 0.01 iota mega Pascal.

Now, for these input values the isolator stiffness at the lower frequencies are just you know like matches, as we have just chosen in the previous case. So, with this now we would like to solve the problem and we would like to find out that how you see the losses are being occurred in that and how it is effective.

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**Solution**

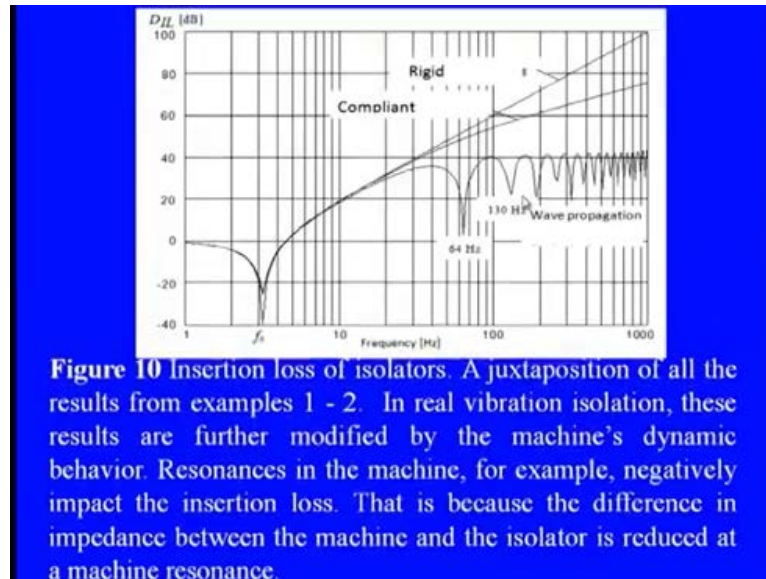
Steps involved:

- Block one end of the isolator.
- By calculating the ratio of the force on the blocked end to the excited displacement response at the free end, a frequency-dependent dynamic isolator stiffness can be calculated.
- If the stiffness in the result from example 2 is replaced with this *dynamic stiffness*, an insertion loss accounting for longitudinal wave propagation is obtained for the isolator.
- The result can be directly compared to those obtained earlier; see figure.

So, first of all we need to go with various steps in which we need to make first the block one end of the isolator, that you see you know like whatever the first end of isolator is there we need to make a proper block means just blocking that, and by calculating the ratio of force on the blocked end, which we are just blocking to the excited displacement response at free end. The frequency depending the dynamic isolator stiffness is simply being carried out, and it is simply giving that how the frequency dependent parameters are being varied. And if this stiffness in the result you see you know like is just taken with the dynamic stiffness, as the another parameter the insertion loss can be you know like find out for longitudinal wave propagation, in which you see you know like the isolator is just feeling like that. Now, you see we can compare those things, so here the insertion loss of the isolator is just shown here.

And in that you see here, we can simply see that whatever the previous two cases in which the stiffness and everything which considered. In the real vibration isolation we can simply see the insertion losses, varying with the low frequency, the mounting resonance frequency and the higher frequencies are there. So, the resonance in the machine for example, say whatever the things are being there, it has you know like negatively impact on the insertion losses.

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And that is because the difference in the impedance between the machine and isolator is always reduced with the machine resonances. So, you can see that we have the same at the lower frequencies, the insertion losses are being coming down at the exciting frequencies you see here. And then you see it is being you know like at the higher harmonic frequencies it is being increasingly, you know like showing the insertion losses.

So, in that we have the rigid foundation we have the compliant foundations this one, and then you see here in this when the wave is propagating you can see the wave propagation at two different frequencies, the 64 hertz and 130 hertz for this. And this wave propagation, since it has you see the longitudinal feature, since you see you know like this rigid foundation is providing the good support they are you know like speedy, and you see the nature is just like that.

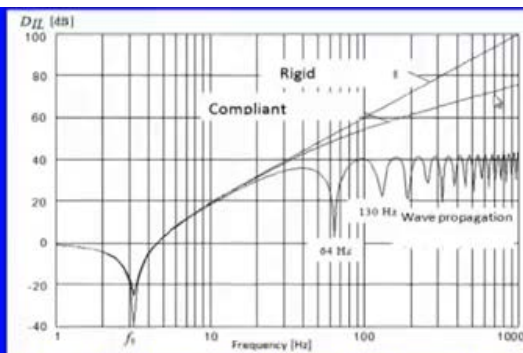
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### Deformable machine:

- In examples 1 to 3, it was assumed that the machine moves along a coordinate direction as a rigid point mass.
- The insertion loss is then very low at excitation frequencies near the mounting frequency.
- If several of the machine's six rigid body degrees-of-freedom are taken into account by the model, several mounting resonance frequencies are then exhibited.

So, you see here you know like this which we seen in the three examples the rigid, the flexible base both you see here or else even you see here.

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**Figure 10** Insertion loss of isolators. A juxtaposition of all the results from examples 1 - 2. In real vibration isolation, these results are further modified by the machine's dynamic behavior. Resonances in the machine, for example, negatively impact the insertion loss. That is because the difference in impedance between the machine and the isolator is reduced at a machine resonance.

In the previous diagram when we see that you see how the wave propagations are there this part which we discussed. Now, here if there is you see the nature of machine is of deformable nature.

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**Deformable machine:**

- In examples 1 to 3, it was assumed that the machine moves along a coordinate direction as a rigid point mass.
- The insertion loss is then very low at excitation frequencies near the mounting frequency.
- If several of the machine's six rigid body degrees-of-freedom are taken into account by the model, several mounting resonance frequencies are then exhibited.

And if it is, so then we can see that how the coordinate directions are being considered with the masses. Because, you see till now we considered the point mass of the motor itself, and now we also would like to see the insertion loss how it is being you know like varied. So, the insertion loss is very low because the rigid point mass at the exciting frequency just near to the mounting frequencies.

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**Deformable machine:**

- In the most general case, we therefore have critical frequencies at six different mounting resonances.
- In fact, every real machine also exhibits internal resonances at certain frequencies. Typically, the first resonance frequency of a compact machine with a 100-kg mass, e.g., a small internal combustion engine, falls in the 100 Hz - 500 Hz range.

And if the several you know like the coordinates thus we can say the 6 degrees of freedom the three translational and three rotational coordinates are being considered here



or are being taken, then you see we can see that what are the mounting resonant frequencies, and how they are being exhibited here.

So, in general case since the six critical frequencies are being considered, so we have the six different mounting resonances, and every real machine also exhibit the internal resonances at certain frequency. So, if we are saying that the first frequency, the first resonance frequency of the machine with say 100 kilogram as we have considered in the previous diagram. If it is you know like the entire IC engine is falling down from 100 hertz to 500 hertz range, then you see how exactly you know like the deformable machine is playing.

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- If the machine is composed of flexibly attached sections, the first resonance can of course lie at even lower frequencies.
- The possibility of *wave propagation in the machine* also effects the isolation insertion loss obtained from elastic mounting.
- That depends, after all, on the relative stiffnesses of the machine and the isolator. If the machine stiffness varies significantly, due to resonances and antiresonances, then even the insertion loss will vary.

Since the machine is composed of the flexibility attached to the various section, the first resonance can of course, you see lie at even lower frequencies. Because, of the flexibility provided by the machine itself, and the possibility of this wave propagation in the machine can also effect the isolation insertion loss even with the elastic mountings. But, this is all you see you know like depending on the relative stiffness variation of the machine to isolator.

Because, ultimately you see you know like the entire things are being varied through this connectors or through these you know like the contact regions. So, how the isolator is being varied with this, and if the machine is stiffness varies significantly due to any



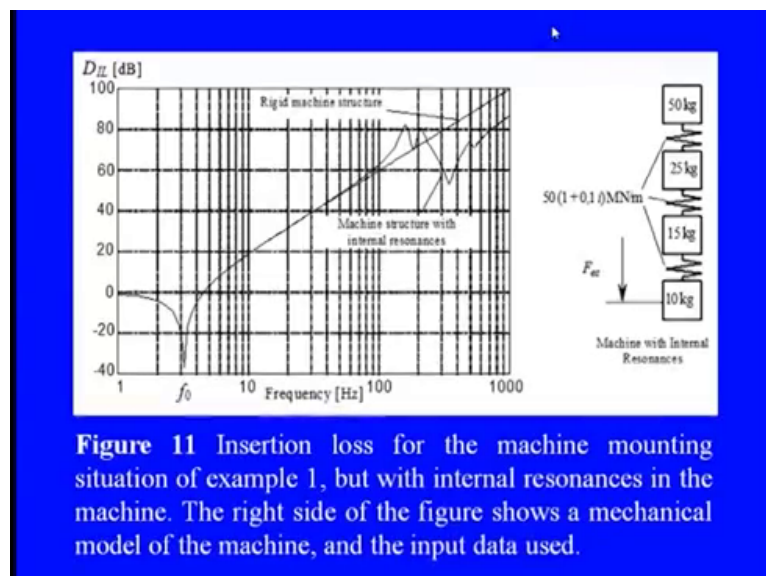
reason, may be resonances may be anti-resonances then even the its insertion loss will also vary accordingly and it plays a critical role in designing of these isolators.

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- On average, the isolation performance is degraded above the machine's first resonance frequency.
- Figure shows the insertion loss of a simple system consisting of a machine with internal resonances. The foundation is rigid and the isolator is same as in example 1.

But, on an average we can say that the isolation performance is degraded with that because you see here. When you have the flexible machine then certainly you see here, whatever the performance features which are being coming out due to this isolator having different deformation rate and insertion losses also varied with that. So, certainly you see the performance is degraded with the first resonance itself.

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So, the insertion loss for these you know like flexible machine can also be seen in that particular diagram with the internal resonances, and even when the foundation is rigid and the isolator is you see the same as we adopted that. So, this diagram is clearly showing that how the insertion losses are being varied for the machine, which is being mounted on the even the elastic mountings. But, with this you see the internal insertion in the machine, there are you see you know like the greater affected area at the lower frequency. And on the right hand side you see we can see that how the arrangements are there.

So, if you look at the insertion losses pick this figure, we will find that the insertion losses at the lower end is somewhat showing the internal you know like the resonances. And when it is approaching to the mounting resonances certainly we have this feature, and when you see we are going towards higher order, the higher order frequencies there are two things the rigid machine structure, which is again showing the similar kind of insertion losses and it has a linear propagation.

But, when we have the internal you know like we can say the resonances with machine structure, you can see that after even the 100 hertz or even after the 50 hertz. The nature of the insertion losses are being varied in a drastic way why because we know that when the system is exciting at the higher harmonics level. And since our machine is also, so in the flexibility in those cases, then certainly you see here we cannot effectively control the insertion losses and the variation will be of drastic level.

So, we can see that how the variations are there with the internal resonances, so in this cases you see here, when we are just simply you know like showing these things even on the right hand side you can see that, we have a clear arrangement of the masses. The different masses say we have 15, 20 kilogram 15 kilogram even 25 kilogram and the 50 kilogram. We know that the stiffness of the spring which is being required here is 50 into 1 plus 0.1 iota Newton per meter.

And when the excitation is there, you see this is what my machines are and when excitations is there on that, the internal resonances of the entire feature you see this feature is critically playing in the design of the isolator. And you see if the appropriate isolator is not being chosen, certainly we have a different a kind of insertion losses, and you see this mechanical model of machine is clearly showing that how the transmissions

are there. Through this spring's or through these connectors and we need to choose appropriately the stiffness of these isolators are.

So, that the proper transformation can be take place with the flexible machine, so in this case you see here in the entire chapter, we mainly discussed about the three different cases like you see here. If we have first the rigid base means the foundation is rigid, if we have the flexible foundation and now if we have the flexible machine.

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**Points to be remember:**

- In the example(s), the machine has resonances at 185, 245 and 525 Hz, and antiresonances at 160, 205 and 495 Hz.
- Figure(s) shows that, at the resonance frequencies, at which the machine is compliant, there are insertion loss minima, i.e., frequencies at which the isolation is poor.

So, in flexible machine case we can certainly you know like go towards the main conclusion that the machine has the resonant frequencies, as I shown there 150 245 and 525 hertz. And the anti resonances according to the resonant feature is 160 to 205 and 495 hertz, and that figure shows that the resonant frequency exactly the machine is complaint. And there are you see you know like the minimum insertion losses are there; that means, the frequency at which the isolations are just poor. So, when we adopt these things we know that how the isolators can be designed, and what could be the possible you know like the resonance frequencies are there.

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**Points to be remember:**

- On the other hand, extra isolation is obtained at the antiresonances, at which the machine is very stiff.
- For increasing excitation frequency in the region above the first machine internal resonance, the average insertion loss falls off gradually. That effect is due to the ever smaller unsprung mass that takes part in the motions of the point(s) of contact with the isolator(s).

But, in other hand you see if the extra isolation is obtained at anti resonances, we need to see that the machine is not the flexible one, the machine is even more stiff. And for increasing exciting frequency in the region, which is above then the first machine internal resonances like 5 to 10 hertz are all the average insertion loss is drastically falls of the feature. So, we need to check it out that you see here, what is the you know like the machine foundation and you see here the this internal resonances are there, and how the corresponding exciting frequencies are being varied with this.

And this fact which is you know like coming out due to this you know like this machine foundation can also be smaller with the unsprung mass. And that is taking you see you know like more of the care, when you know like the entire motions are there in the just highly or we can say the rapidly feature with this isolators. So, in these you know like for formations now we could easily figure out that when, the machine is acted as the flexible feature exactly just at the contact point or when the foundation is occurred at the flexible feature.

Then certainly there are different insertion losses, and when we are designing the isolators in between these flexible features of machine of foundation. We need to check it out that how the insertion losses are being varied at the lower frequencies or at the higher frequencies or even exactly, when these frequencies are meeting with the internal

resonances. And this is one of the drastic criteria, when the frequencies are meeting with the internal resonances, the insertion losses are always being drastic there.

So, in this lecture you see you know like we discussed mainly about the foundation feature, and appropriate isolators according to that the foundation or the machine rigidity or flexibility of these two features. In the next lecture our main intension would be focused on that how we can check the insertion losses, specially when we are designing the isolators for various different applications. When you have steady state the continuous operations of the forces, means you see the continuous dynamic features are there or the dynamic stiffness are being you know like playing in that.

And also you see here, when the continuous loading is there it is very simple to find out, the frequency range according to the insertion losses and appropriate we can say, the mobility or the impedance can be chosen for that. And we know that when the difference between impedance and mobility is high, the isolator is working in a very perfect way. But, the same time you see we are also going to discuss in the next chapter about the shock loading, when the adrupt energy is coming to the system, then how the isolator is going to work.

Because, ultimately the effectiveness of the isolator is absolutely based on when the energy is coming, how much time will it take to release that energy according to the performance criteria. So, as for these common examples are concerned there is no problem with that once you have chosen the isolator, whatever the energy excitation is coming it is of a continuous nature. Because, of the harmonic excitation and we can certainly find out, the way to release the energy and based on that the performance can be immediately evaluated.

But, when the energy is coming in the shocking feature must a sudden impact forces are there, impulsive nature is coming. Then how we can design the isolator, so that the energy can be restored for maximum time period, and then you see slowly, slowly it can be released, so that we can maintain the steady state feature. Because, till now you see we only discussed in all the numerical's the steady state responses, even the force excitation or the displacement outcome will be taken care.

But, when you see the transient nature of the force is being acted on the system, then how the system is going to respond, how we can effectively measure the performance of

the isolator or even other way, how we can design the isolator based on the insulation losses, the insertion losses can be checked out.

Thank you very much.