Vibration Control

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Module - 1
Review of Basics of Mechanical Vibrations
Lecture - 1
Basics of Vibrations for Simple Mechanical Systems

Hi, this is Dr. S P Harsha associate professor in Mechanical and Industrial Department IIT, Roorkee. I am mainly teaching the design related course is like solid mechanics, like machine design and for the higher studies you know like I am mainly teaching the dynamics of mechanical systems, mechanical vibrations and you know like the other courses like synthesis of mechanism and all. This course which is you know like developed under the National programmes on technology enhanced learning NPTEL mainly dealing with the vibrations and the control strategies.

We know that mechanical vibration is one of the key issue, because whenever we are running any machine or even when we are traveling through the train or any vehicle or even when we are dealing with the human body, we know that the dynamic phenomena is always being there. And when we are trying to analyze these dynamic phenomena's, we know that until unless if we are not able analyze the physics involved in those kind of excitations. Means the vibration related issues then we cannot analyze seriously what kind of you know like the signal generator generations are their or out kind of you know like the, these responses are there.

So, in this course mainly we are going to deal with the vibration related problems, that you see why what are the basic causes of the vibrations in any of the machine. And then you see if we know that the vibrations are there even if it is a new machine or the robust design is there, then how do we control. So, what are the parameters which we can control you know like which are executing the vibrations in the machine and what are the you know like the limitation of those things through, which you see here.

Say you see here we know that when we are traveling through machine, traveling through a train or a bus or even in the airplane; we know that there are certain excitations which are always creating some kind of you know like the discomfort to human being. What are those you know like the excitations, why these excitations are coming, even see when we are travelling through a local train or when we are traveling into say in Indian conditions we have you know like the Rajadhani the vibration levels are different.

So, why this is, so what exactly the design features that you see the vibrations are being dammed out or even you see they are controlling. So, this may this course mainly dividing into four main categories, one we need to understand basic principles involved in the vibrations, means the physical significance of those parameters, through which the excitations are coming. Second you see here what are the control strategies? Here are two main methods broadly classified, one is the passive controls, in which you see are we need to apply some external devises or some external materials, through which we can control like you see various insulating materials are there.

We mainly using the springs are there the dampers are there these are the you know like the key elements or even sometimes you see when we are keeping the mass is to suppress the vibrations. So, these are the key elements, so which we can reduce the vibration amplitude or we can suppress the vibration rather. So, first is the passive vibration control, second part is the active vibration control sometimes you see it is not feasible say you see the vibration is coming from the internal feature at the microns levels or you see any of the part of the machine.

And at which you see here it is not feasible to apply these you know like a vibration controlling devices then how do we control. So, there are various you know like the parts, which we are you know like putting under the active vibration controls, so this the third one. And the forth one you see here when we are doing these things what are instrumentation involved in that, how do you first measure those things.

And then once we measure these say a basic elements of vibrations, then how do we you know like what are the instruments involved in that, say you see if we are on the track side or if we are on any of the side you see here and we want to measure that. So, what are instrumentation involved, what exactly the principle involved in measuring those vibrations. So, these issues which we are going to deal here in this particular course.

So, let us begin with first the basics of vibrations and in this basics of vibrations, you see here the first that how we can apply this vibration concept or the dynamics concept to the simple mechanical systems. So, we need to you know like if you want understand the basics of the vibration we need to go to the basic physics in that. So, if we are talking about that, first we need to see what exactly the science is saying about that. So, whenever we are saying, that something is vibrating they are also inducing some kind of sound.

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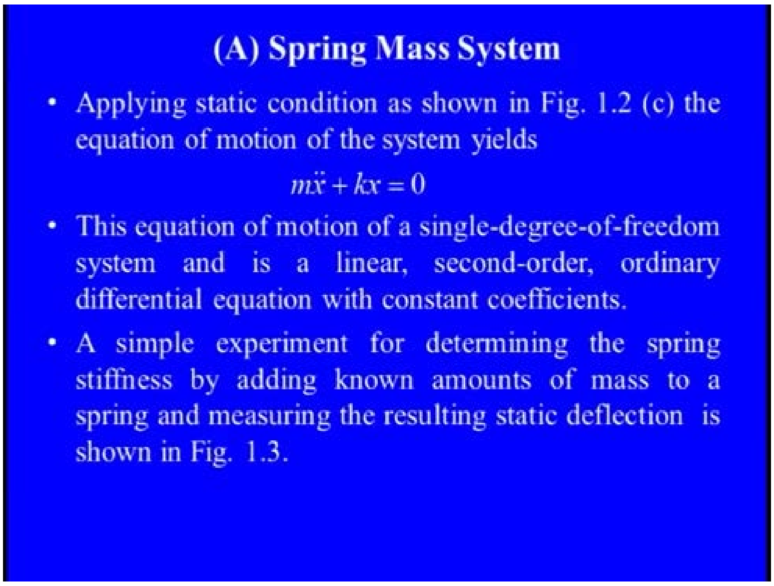

So, sound and vibrations are pretty you know like the common terms which we always use and we need to apply the concept, that the you see why a machine or anything which creates sound is vibrating or why a vibrating system is creating sound. So, the science of sound and vibration is a part of the applied mechanics. So, you see here, if we are talking about the applied mechanics, then we know that one has to study the behavior of solid, liquid or gaseous media, sometimes we know that when we are talking about the solid media or to rigid bodies they are creating, so much sound.

And the impact at the contact mechanics which we are talking during the impact they have a huge noise and there is a clear vibration which is being transmitted through the molecules of that solid body. So, when we are talking about that in the solid or the liquid or gaseous media, they are absolutely involving you see the deformations, the way propagation you see. Like you see any when I am speaking we know that the at the vocal cord there is a clear vibration, because you see the air which is coming out it is always being suppressed and my tongue is you know like in somewhere oscillatory media.

So, this wave propagation when it has you know like when it is coming out from the mouth, it has some media to go up to the certain level. So, starting from the deformation the deformation is particular mechanical term in which there is a clear displacement you know like every localized region where the load application is there or any movement or torque application is there. The word propagation is basically related to the sound phenomena and then you see here we are talking about those transom and the fracture they are all you see creating some kind of you know like the mechanics concepts.

And when these medias are subjected to the physical agent of various types, certain procedures you see here you know like maybe some you know like various types of the forces or the due to the temperature variation, we know that the sound or vibrations are basically coming into that domain. So, sound and vibration fields in that sound and vibration waves basically we know that the sound is certainly a energy which is always being propagated in terms of the waves.

Vibration is nothing but you see here that is a molecular phenomena and you see when we are talking about any vibrating system, we know that it requires a basic domain or solid media to transfer. Like you see a basic common example is our mobile, when the mobile is you know like in the vibration part and if you are keeping on say some solid surface, when it has you see this domain immediately it will transfer through the molecules of that solid or if we are just keeping on the wave in the air open we know that the affected area is very small.

So, sound and vibration waves are mechanical elastic waves and thus you see the conditions for their existence are you know like that the medium causes mass and elasticity. So, you see sometimes when we are talking about the deformation it is always being you know like giving some kind of excitation, because of the deformation as we are applying and this deformation mainly dealing with the elastic deformation.

So, in Vibro Acoustics there are two classes of waves one is the longitudinal waves according to the direction and one is the transverse waves. So, you see you know like in this particular if we are broadly speaking about the applied mechanics part we know that there are two feature.

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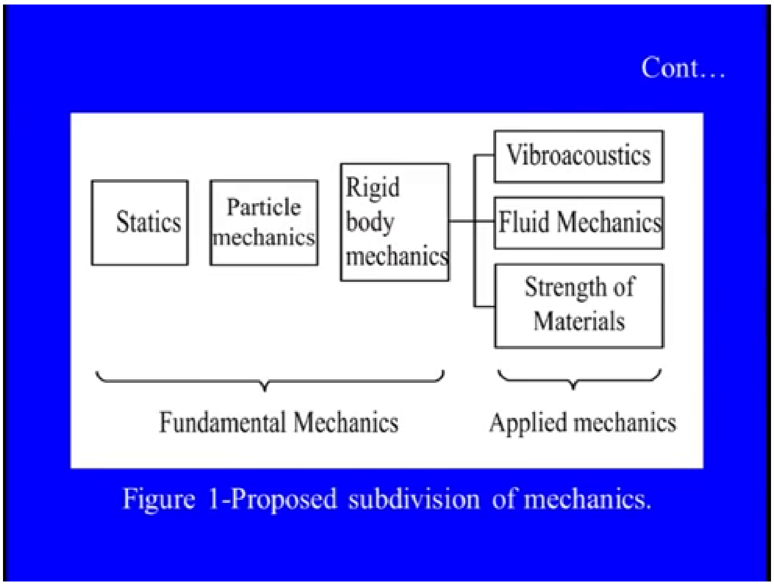

One is the fundamental mechanics, which is related to the statics the particle mechanics and the rigid body mechanics. In that you see here we are mainly dealing with you see you know like that when the bodies in even in the rest there are the forces which are being you know like under the equilibrium position and they are being applied to the system even when the system is the macro or even the micro means the particle part.

So, this rigid body mechanics is always giving that when the two bodies are or when anybody which is being rolling or you know like sliding. Or being the contact with the other body of the surface there is certainly there are deforming conditions and due to the deformation certain kind of excitations are there.

So, when we applied this mechanical concept to any rigid body or the particle part we are under the broad category of applied mechanics in which we have the basic concept of this strength of materials, means what exactly the type of material, what kind of forces are there, how do we resolve these forces you see even under the static or the dynamic part. Then even if we are talking about the fluid mechanics we know that the fluid molecules up always you know like a travel there is no you see here assess the deformation, but there are displacement.

So, whenever you see the fluid particles are moving and the inertia force is due to the density feature, they are always being creating you see the kind of you know like the excitations, whack formations are there various things are there related to the fluid molecules then Vibro Acoustics. In the Vibro Acoustics we know that the things are being coming right from the basic vibration concepts and then it is spreading towards the Acoustical phenomena, Acoustic emissions are there.

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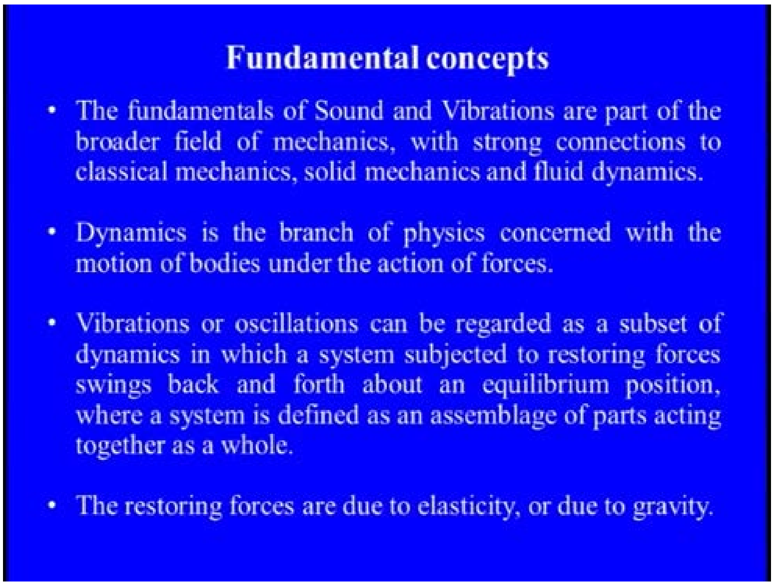

So, you see here when we are dealing such things we know that the fundamentals of sound and vibrations are the part of the broader field of mechanics, with the strong connection to the basic classical mechanics, which is related to the solid or fluid dynamics. And since we are talking about any movement feature or the motion the dynamic says you know like one of the basic branch of the physics, which is mainly concerned with the motion and the force.

When we are talking about the motion feature we know that we are all talking about the kinematics part and when you know like the forces are being involved along with you see the kind of motion, then certainly you see the dynamics coming. So, you see here when we are in the domain of dynamics the first thing is coming how the body is behaving under the motion.

So, the vibration or oscillation these are the two common movement is there of the body can be regarded as the subset of dynamics in which the system is subjected to certain restoring forces like swing back or forth, because two and you know like the flow motion is there. So, how these restoring forces are really controlling that kind of to and fro motion. And then you see you know like when we are talking about that this is my you know like the periodic excitation, the periodic vibration or you see the pendulum oscillation, then we know that there is an equilibrium position or there is you know like the position where you see the body is moving to and fro.

So, we can say the system is nothing but the vibrating system is nothing but you see you know like the assemblies of the parts acting together as a whole from the to and fro motion. And these restoring forces they are coming due to the elasticity or due to the gravity, which are the two common elements which are always being available when any system is executing it is motion or the dynamic phenomena in the real nature.

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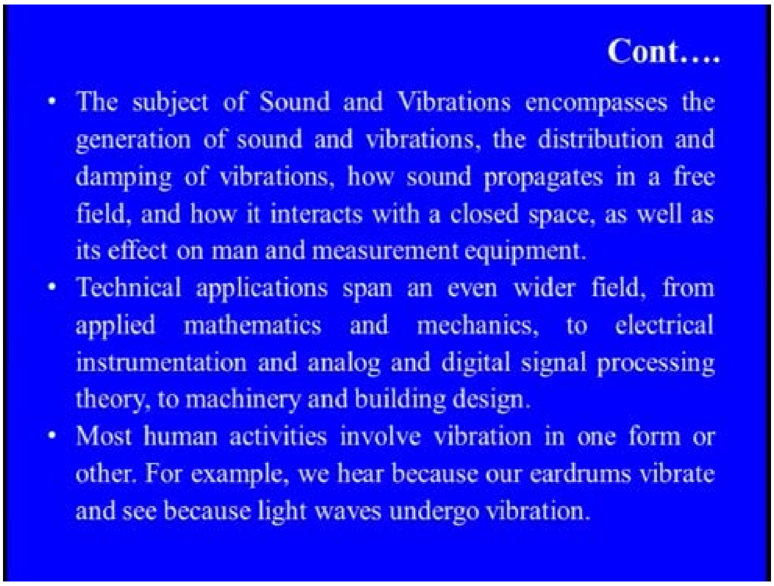

And when this subject, when we are talking about the sound and vibration it is especially encompasses the generation of the sound and vibration, the distribution and the damping of vibrations that, the how these you know like the sound is distributed. How the vibrations are being damped out what are the key features you know like through which we can damped out, the oscillation or the vibration amplitude and then the distribution.

And then how the sound is you know like the propagating in free field and how it interacts with the closed space, we know that you see here when we are putting you know like any sounds source and you see when if it has you know like the source or it has you know like the media it will propagate immediately at the very faster rate. So, you see here you know like the physics say that it absolutely the molecules absolutely requires whatever you know like that it is propagating the sound or the vibration they need the basic domain the basic structure.

The solid structure which you know like consist of the various molecules as well as you see it is effect on the main and measurement equipment. Because ultimately you see here we know that when we are in the you know like the noisy environment, what exactly the impact of the common performances on the man. And how do we measure those things, so what are the measuring equipments are there. So, technical application you know like span you know like even in the wider field from applied mathematics and mechanics to electrical instrumentation.

And you know like in which you see we have analog or digital processing theory in which you would like whatever you know like the signal processing tools are there to a machinery and the building design. So, this sound and vibration is not a specially we can say restricted our self towards the mechanical application only it can you know like if we know that some buildings we want to design the building we need to put you see various kind of you know like the sound controlling features vibration controlling features.

Because you see you know like various things are there and you know like which can transmit which can generate the vibration and the acoustic feature. And then you see the like most human activities when we are coming just towards the human most, human activities involve the vibration in one or other form you see here they are always be you know like analyzed based on the dynamic phenomena like you see the blood movement is there from the heart. We know that you see when we are running or when we are in the idle situation the blood motion is different and through E G G we can simply find out that where is the problem.

So, this is nothing but a vibration signature only the movement even you see when we are talking about the brain structure, we know that the neurons are all the time they are just you know like taking the information and spreading all around. So, this is a dynamic phenomena and we are using the E E G to see the irregularities of these motion. So, I mean to say even when we are dealing with the human part the E C G, E E G they are nothing but the signal processing you know like we can say tools through which we can analyze the motion of blood or the neurons.

So, you see you know like or else you see as we can say that when we hear our endromes you see vibrate and you see the light waves are absolutely you know like a expanded under the vibrating conditions and then neurons are getting entire things. Another very common example for the human being is the breathing.

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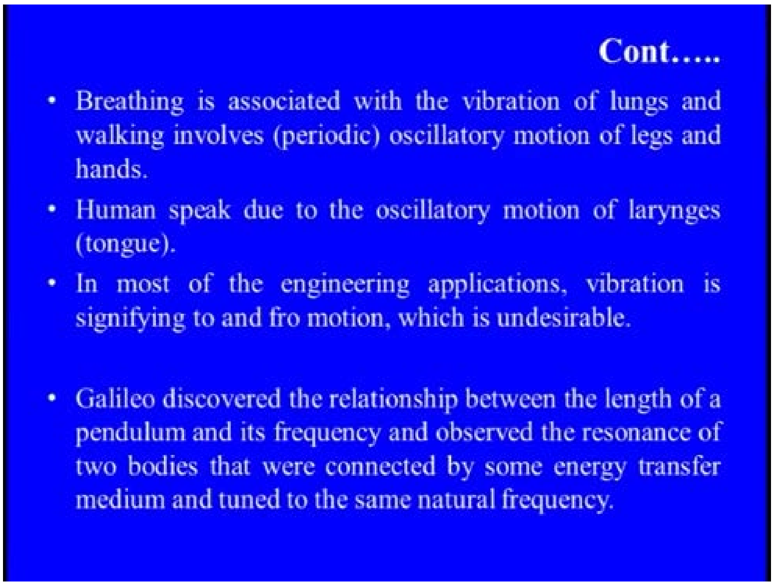

Breathing is associated with the vibration of lungs and you see you know like the walking involves some times the periodic we can say oscillatory motions are there of the legs and the hands. So, this is a great combination of two things the periodic motion of your legs and hands and the same time you see here your lungs are also you know like creating some kind of periodic motions. Or else when we are speaking say you see I delivering the lecture I know that you see here you know like the sound waves are being generated this is the energy which is coming out from the throat.

But, the same time you see here this you know like giving a particular direction or whatever the words articulation the modulation all the things are coming out due to the various oscillatory motion of our this larynges or we can say the this over tongue. So, in most of the engineering applications right from you see the machinery to the human being the vibration is simply we can say analyzed based on the to and fro motion, which is pretty undesirable.

So, you see you know like if you are just going back to that then we know that the Galileo you know like who discovered the relationship between the length of pendulum.

And it is frequency that you see how many number of times it repeats it is entire motion in a particular time, then he observed that at a particular frequency you know like when it is just related to its natural frequency, the resonance will happen.

The resonance is nothing but you see here it has a use amplitude of the we can say the vibration due to the maximum energy involved there. Here you see here you know like the maximum energy is being transferred towards the system, so that it can simply you know like have the greater motion at an infinite feature. So, you see you know like way we can say natural frequency is nothing but it is inherent feature of the system because sometimes we are talking about the natural frequency of different you know like any different system.

The different systems have different natural frequencies because you see here it is a inherent property, how the system is reacting towards the applied force within it is own structure and accordingly that is why we are saying that the natural frequency is always being coming out due to the free vibration condition. So, the vibration may result in a failure of machine or their critical components, because you see here when the amplitude you know like amplitude of vibration is increases from it is limit or the range certainly you see it can damage, it can even put some kind of plastic deformation some kind of permanent deviation there.

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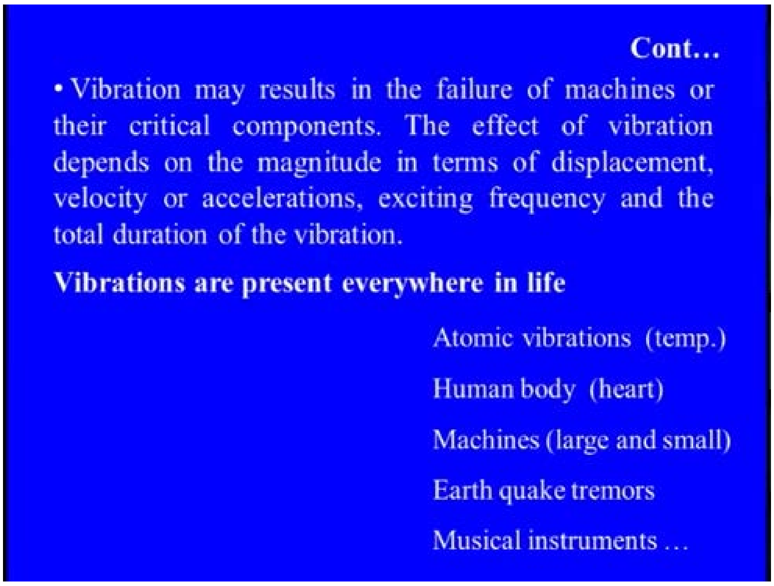

So, the effect of vibration absolutely depending on the magnitude in terms of the displacement velocity and acceleration, these are the three common we can say dynamic parameters through which we can clearly classified the basics of vibrations. And you see here since we know that the amplitude of these either the displacement or the velocity or acceleration are clearly reflecting the vibration significance we can simply find out that at what frequency this amplitude is there.

And what is the duration of this much the vibration; that means, in terms of how much time is there at which you see this frequency is coming and this much amplitude is coming out from the system. So, this is whole you see, if you want to characterize the vibration now there are two main features in that, one is the exciting frequency and one is the amplitude in terms of displacement, in terms of velocity or in terms of acceleration. So, if you broadly classified the vibration we know that every day we are dealing the vibrations right from the atomic vibration, means the temperature variations.

When you see, when we are in the common room you see the room temperature which we are dealing with it has a common feature all along the room length or the dimensions. But, when you know like switch on the fan or AC or something you know that now the temperature variations are there. It is mainly due to the molecular motions they are interacting and when they are interacting they are exchanging the energy and this temperature variation is a basic cause of this molecular dynamics.

Second the human body the heart or even you know like as we discussed about the breathing or even we are just talking in which the lungs motion is there or if we are talking about you see you know like the, whatever the speak things are coming out from the human or else the neuron motion. All these are basically the feature of the vibrations, then broadly speaking the machines even a small machine which say you see we have the fan in our C P U of the computer it is also creating, so much vibration even it has a light part.

Even you see when we are just going to any of our thermal power plant the huge turbo generator machines are there the generators are there they are creating huge noise and huge vibrations. So, even the small machine to bigger machine they are all inducing the vibration and this vibration is transmitted through any structural media then the earth quake tremors this is one of the good example of our random vibrations.

So, you see here right now we are not classifying the vibrations, but you see the vibration can be classified in the discrete level, in the continuous level or even at the random or the non-linear features. Then the musical instruments all these specific tune, which is coming out you see from the any of the musical instruments is absolutely related to the vibrations or the material property of that string.

So, you see here you know like when we are dealing say if we are driving a motor cycle we know that huge you know like vibrations are being coming out, due to the interaction of the tyre tube the surfaces and these are being absorbed by various things and then it is coming to the human part. So, you know like the vibration we are dealing every day. So, now, if you are thinking from that point we know that there is a vibration, which is there in the system; now first we are starting from the linear systems.

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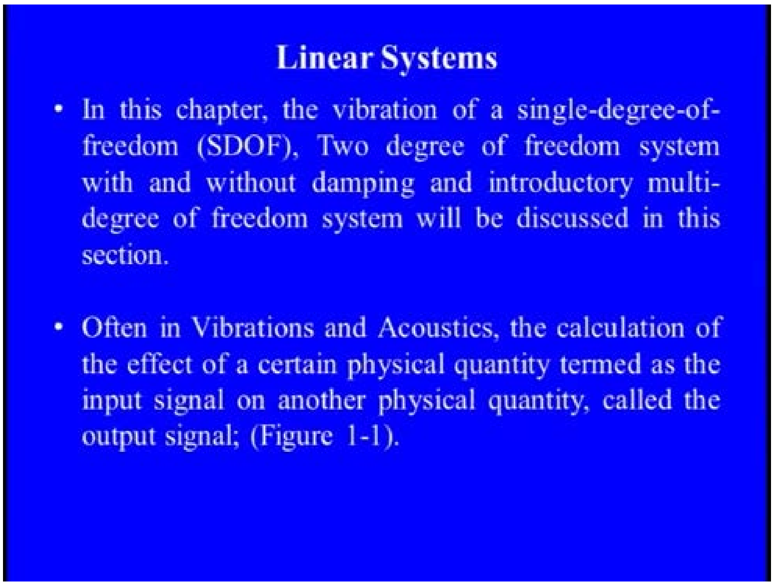

The linear system is a well behaved reason is the systems are always you see you know like behaving in a very proper way. So, this chapter is mainly related to the linear system in which the linear inputs are there and the linear outputs are there, well behaved systems are there. The vibration of any single degree, 2 degree or even the multi degree systems with and without damping they are absolutely you know like related to this linear system.

So, we can say that if you want to really understand the basic principle involved in the linear systems, we can start from the single degree of freedom system, in which you see sometimes the damping is there sometimes the damping is not there. And then we can further go towards you see here the 2 degree of freedom system means you know like what exactly the coordinates are available in which we can relate the force or the energy terms in that domain.

This provides us the degree of freedom means what exactly the constraints which we are trying to keep to see the force equilibrium feature. And then even in the multi body multi degree of freedom system like you see the human body is there the robotic features are there or any system in which you see here like the various you know like the parts of the machine are you know like moving together, then how do we analyze the system in the linear part.

So, often in the vibration and the acoustics in the calculation of the effect of you know like certain physical quantities termed as the input signal, certainly you see say you know like we are giving electric supply. So, certainly you see now the rotation starts, so this is what our input feature is this is a well you know like we can say defined controlled input systems are. And when the system is you know like moving then they are just you know like executing the linear output what exactly the output signals are they are coming, in terms of say, if we are talking about the mechanical system the output may be in terms of displacement, may be in terms of velocity or acceleration.

But, again you see there are different devices to record these things by a single devise say I have accelerometer or I have you see the velocity probe or the displacement probe I cannot measure by you know like all three quantities by a single device, because there are various other depending parameters are there in that. So, now if I am coming back to the you know like the linear system.

You can see that you have a figure on your screen in which you see we have the input signal. The input signal can be in form of the dynamic force F which is the function of t we may have you see the velocity as a input, we may have the pressure as a input or we may have the velocity as you see again the input. So, you see in any of the combination right from force to velocity pressure to velocity these are all you see you know like the key input parameters.

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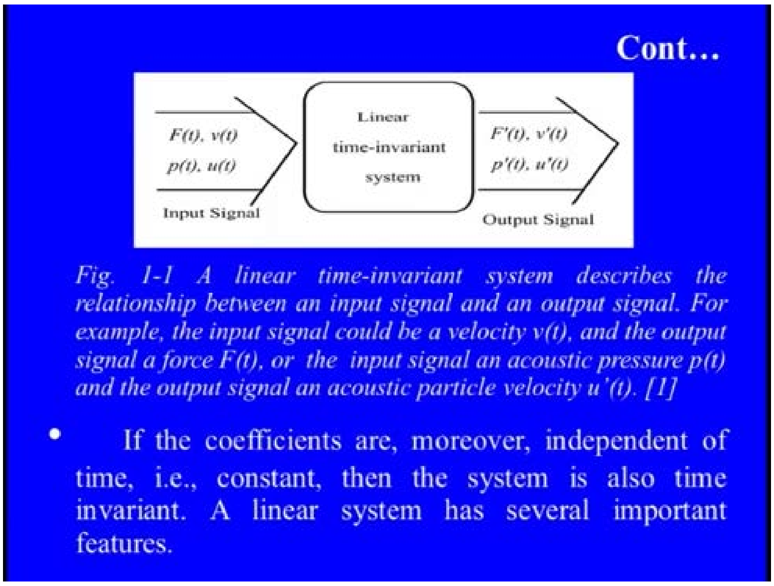

And if the system is linearly time invariant, then certainly we can say that you know like the system is not inducing any kind of you know like the nonlinearity in the system it is a well behaved reason. So, we may expect certain output from that say you see even in terms of force of velocity in terms of pressure or velocity in that. So, linear time invariant system simply describe the relationship between the input signal to the output signal.

And then you see here since as I told you that no nonlinearities involved in that then we can say that you see since it is a single degree of freedom system we can say the force input force output can be easily related even the velocity input velocity output can be easily related.

So, if the coefficients are moreover you see independent of time, certainly you see here we are not trying to relate with the time itself then we can say you know like the system is also the time invariant. And a linear system has a several important features in this because you see here it is a well behaved part like you see when we are talking about the elastic deformation where the Hooke's law is valid. We know that whatever the parameters are there may be force with deformation stress with strain or anything or under the you know like we can say the Newtonian mechanics. When we are saying that the Newton's law are applicable we can simply say that they are absolutely valid towards you see you know like the elastic feature where the linear, propagations are there the linear you know like we can say the systems are there, let us focus on one example.

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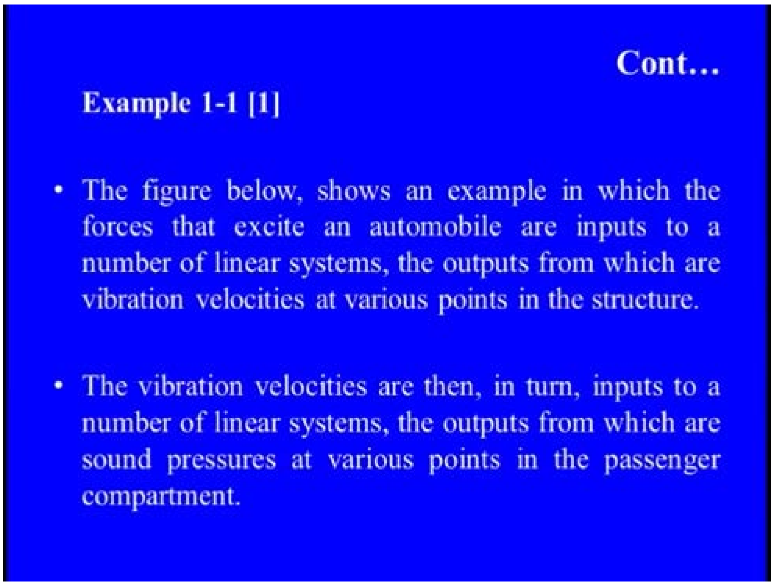

Now, I am going to show you the you know like one of the basic feature of our you know like the automobile systems, where number of inputs are there. So, does not matter you see that you always have to have you see you know like the one kind of input and one kind of output there may be more than one inputs are there, may be more than one outputs are there under the linear actuation of the system.

So, you see here you know like we have just I am going to show you that the automobile which has various inputs the number of inputs are there of the linear system and the outputs are there from the vibration velocities. And these vibration velocities are then again we are trying to feed as the input of the various linear systems and we can find the outputs as a pressure sources. So, you see that this one in which it is very clear that you see you know like this is what my forces.

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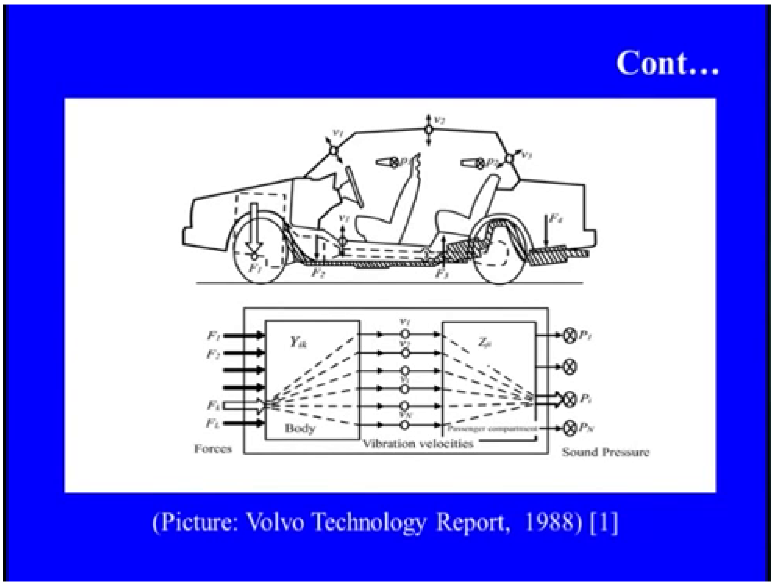

The input forces are there and when the forces you see on a vehicle there are various forces right from the engine to the air drag the lift forces various you see you know like the dynamic forces are there, even the forces which can be generated due to the excitation you see at this. So, you can see that this we have various force reasons are there which are coming out due to we can say drag forces, these forces, so this is what the all input forces.

When they are coming to that you see here at this particular feature now we have various vibrations velocities, means you see here there is an excitation you know like at this particular part due to the motion of the vehicle. And when you see you know like this part when it is being you know like coming to you know like the human being this is basically you can see that a kind of input to the human being as a vibration velocity input. And then you see we can see that you see how much you know like the pressures are being there at you know like we can say in terms of sound pressure or in terms of anything.

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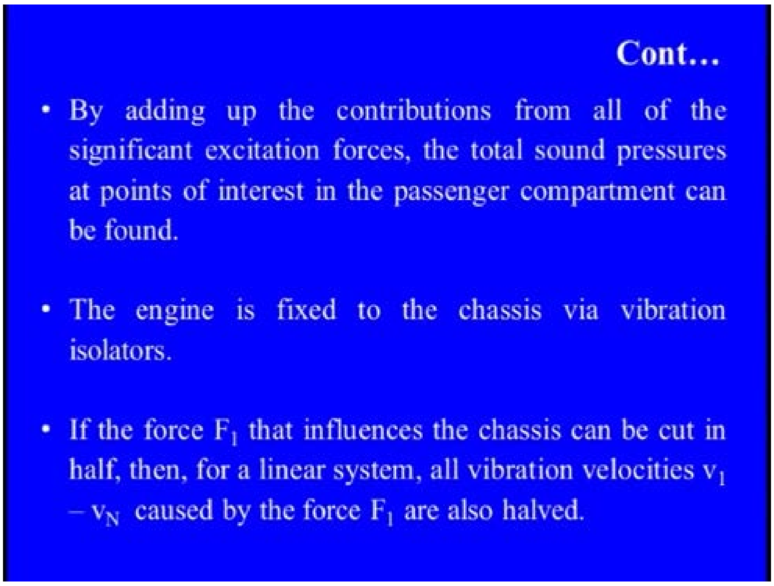

So, by adding up the various contributions from all the significant excitation forces, the total sound pressure at the point of at the point of interest is nothing but the passenger's compartment. And we can just see that what exactly the comfort levels are being there due to the excitation of various forces, which are being termed into the vibration velocity and then into the sound pressure. The engine is also you know like we can say fixed and you see you know like through these chassis this is basically the media which is being like transmitted that and through which you see we can apply the vibration isolators as well.

So, what we can say that you see various forces which we have shown in the previous diagram right from the F 1 force which is influences through the chassis 1 we can put any linear actuation feature and the velocity difference will certainly give you the kind of vibration excitation. So, a linear oscillation in the mechanical system can be considered here and whatever the oscillations are coming into the system is absolutely giving us the linear relationship between the exciting forces and the resulting motion.

And since there is a linear feature, so there is a pretty well defined output is there as the you know like as we can describe in terms of the basic three dynamic properties, the displacement velocity and the acceleration. And then you see even in that we can further add these contributions which are just coming out in terms of you see the vibrations or in terms of vibration velocities from these forces, then it can be simply converted into the velocity component and then it can give simply this one. So, when we are talking about the you know like the two main features.

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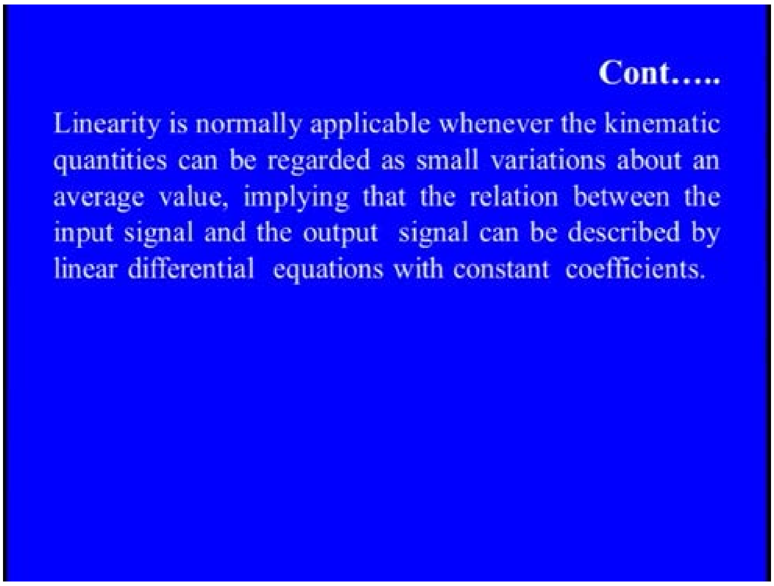

We know that the linearity is a basic property of the system through which you see here we can simply propagate we can make a proper relation of you know like the force or the velocity or sound pressure in that way. And we can apply the various you know like the coefficient which can relate like you see in the Hooke's law we have the young's modulus like a Newtonian's law we have the Newtonian's coefficient. You see it is like the viscosity or in any of the term these you know like the linear variant features can be strait away relate using some of the coefficient.

Now, we are going towards one of our basic case that is the single degree of freedom system. So, in that the single degree of freedom system is very, very essential in such a way that we just want to study, you know like the information about how the system characteristics are really you know like the influenced by the various different quantities, because if you are just taking the various other parameters then sometimes we are not in a position to analyze the dominant feature of that. May be you see sometimes you know like the lower amplitude of some exciting part is more important.

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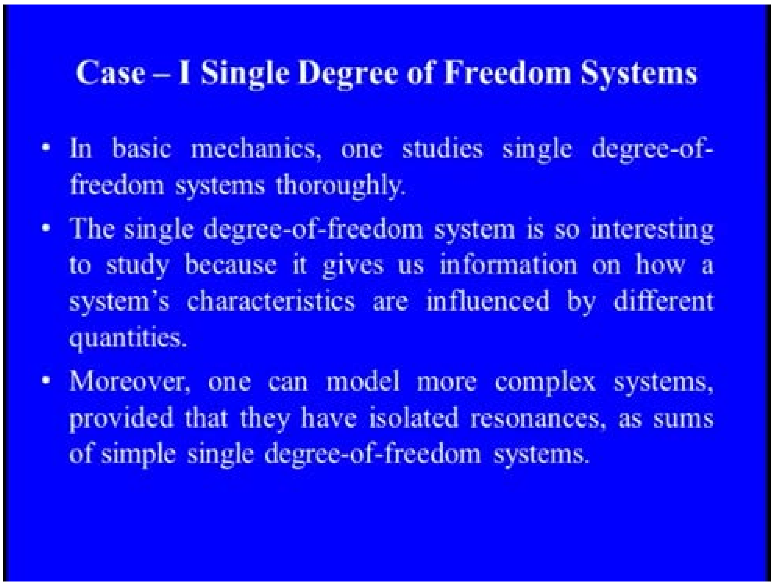

So, that is why you see sometimes we need to constraint the other feature and just to see a single degree of freedom system analyses. So, moreover one can model even the more complex system provided that they are you know like they have isolated resonances as sum of the single degree of freedom systems. So, that is why as I told you that the other features can also be you know like influence the parametric excitations. So, we need to just see that what exactly the dominant features are…

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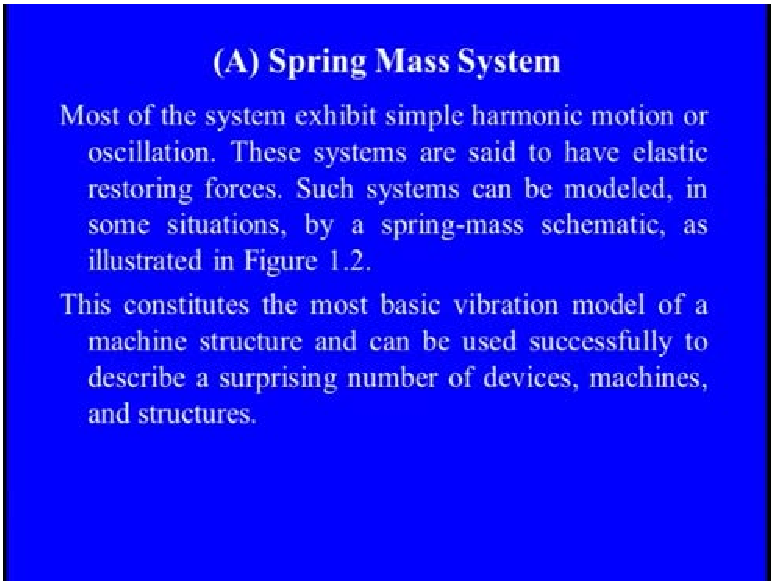

Most of system exhibit simple harmonic motion or the oscillation the periodic oscillation. So, these systems are you know like said to have the elastic restoring forces in which you see you know like, the restoring forces are nothing but you see some kind of you know like the constraint again the motion like you see the spring by spring forces are always coming under the restoring forces. The gravitational forces these forces are always termed as restoring forces and such system can be easily modeled as I am going to show you a simple spring mass system.

Where you see there is no damping through which you see here or the damper through which we can damp out the oscillation amplitude of the vibration amplitude. This constitute the most of the basic vibration model of the machine or any structure you see here and can be successfully described the any you know like various number of surprising number of the devices machines or any kind of the structures we can straightly apply this. So, now you can look at that you see we have a simple spring and the mass system and when you see here we are trying to disturb the mass certainly it will you know like deviate from it is position or we can say it has a clear displacement from it is position.

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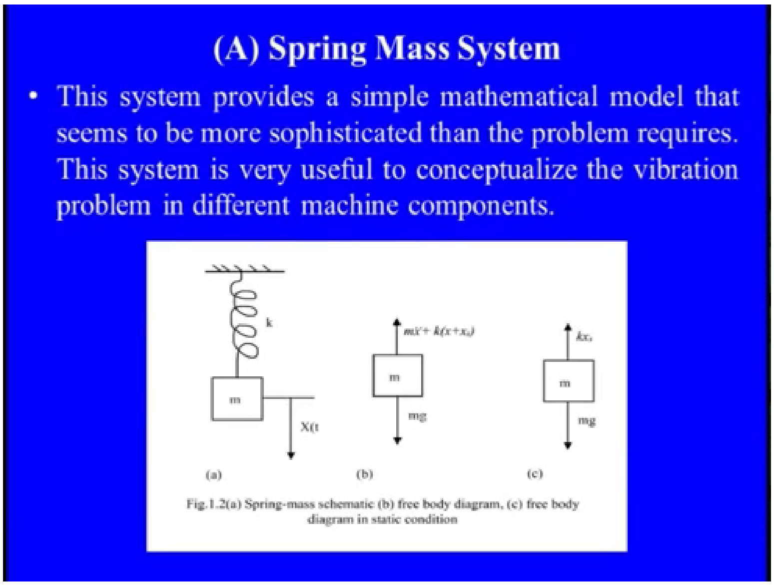

And through which you see the various forces are being generated and if you are saying that it is under the equilibrium position; that means, these forces are well balanced. So, this system provides a simple mechanical model that seems to be more sophisticated than the you know like we can say any problem requirements. So, this system is very useful to conceptualize the vibration problem in different machine components.

So, you can see that what we have, we have a spring system and the spring is always being characterized using the stiffness. The stiffness is you know like giving you the kind of again the material property also it is giving that how much force it can sustain and how much deformation can be generated according to the applied force. And then this is the you see the mass is hanging from the you know like the mass is hanging over the spring.

And when there is a displacement say x of t when the displacement is being given or the disturbance is given to the mass there are the two forces which are being clearly generated here one the inertia force as the movement of mass. So, m x double dot is there this one and then you see on top of that, because you see here this is under equilibrium position.

So, you see on top of that we have you know like the restoring forces and when they are you know like we can say if you are just putting the free body diagram, we know that both are just balancing each other. And when they are balancing each other we can simply say that the equation of motion is nothing but equals to the sum of inertia force and restoring forces are equals to zero because of the equilibrium systems.

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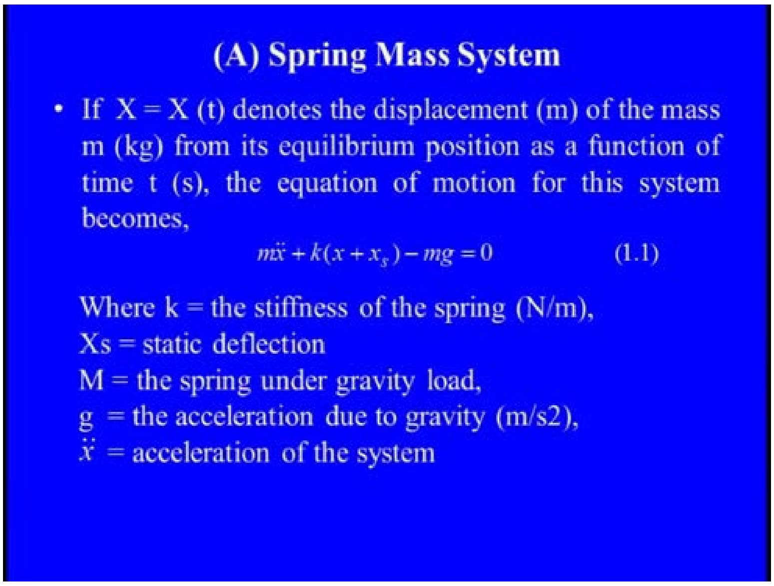

So, say that you see here, if we have the displacement X equals to X of t and then the mass which is just you know like moving periodically, you know like under the excitation of this feature. Then we can say the equation is m x double dot plus this you know like k x plus x x and this m g this weight is also sometimes adding one of the additional feature in terms of force.

So, K is a stiffness which is a material property of this spring you see here which can be you know like using like Newton parameter, x s is the static deflection we disturb and leave it like that. The mass is nothing but you see the in kilo gram the g is the gravitational acceleration and you see the acceleration is this 1 x double dot.

So, the inertia force is because the mass is there which is moving at the acceleration x double dot we have m x double dot as the inertia forces. So, when we apply you know like the static conditions on that then certainly you see we are ending up the equation m x double dot plus k x equal to 0. And this is one of the basic equation for single degree of freedom system and you see here, we know that it is a linear equation.

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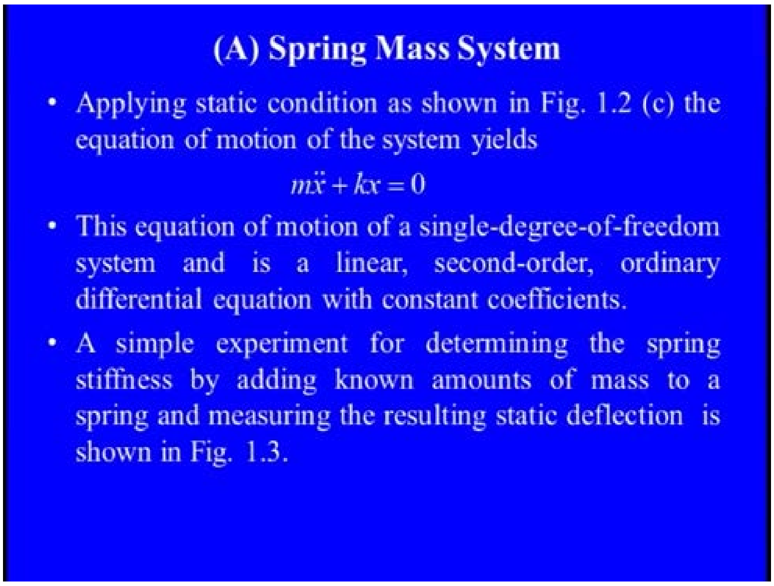

Because all the term x double dot and the x they are simply posing means acceleration or the displacement they are just giving the linear actuation to the system and it is a second order, ordinary differential equation because it is a discrete system. So, we have ordinary differential equation with the constant coefficients. So, simple experiment we can simply perform to find out the spring stiffness, because we know that the spring stiffness is coming under the elastic deformation. So, we can get you see here with the force deformation relations or you see with the stressed trend relations as well and you see how we can get you see.

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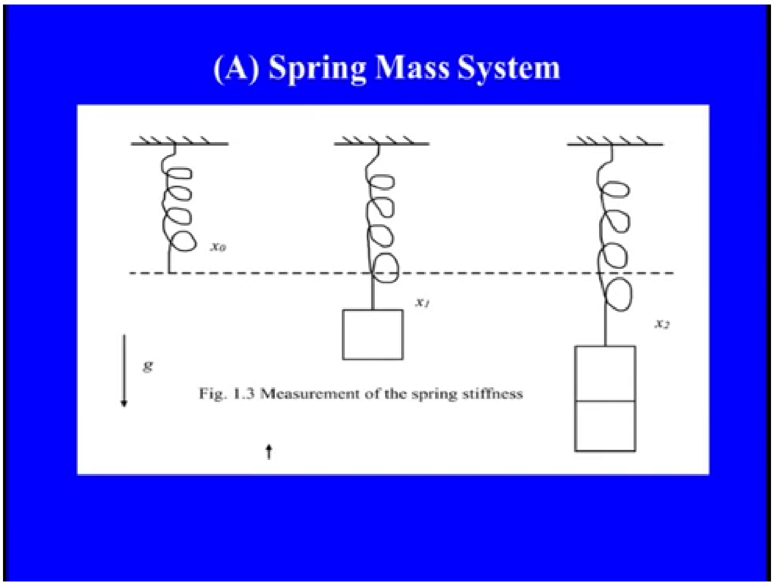

Now, we are going to see, this is what the experiment is we have a simple spring, which has the initial displacement x 0 without having any additional feature of the mass in the first diagram. Then you see here when we are adding up the mass we know that this is known value of mass and with that you see here this is what my the dotted line is showing my static you know like the equilibrium position. And then we add any known mass the certain amount of deformation is being there within the spring itself.

So, you see here when it is just coming a down their again we are adding the another same amount of mass you see here there is you know like the deformation is x 2. So, what we have we have x 0, we have x 1 when the one mass is being added and you see the x 2 when the two masses are being added. So, you see here you know like, this is a linear integration of the mass and corresponding deformations are being there the elastic deformations are being there in this spring.

So, as a result of this static experiment means there is no you see here the dynamic feature of any of the things. Then you see we can simply plot what the forces this is what my inertia force even you see, because it is coming down verses you see the slope means the value of the displacement. So, if you can look at that you see here after certain part you see we have the linear and the or nonlinearity feature is being added, this is what the realistic nature of the spring is; up to even though elastic deformation which is you know like the Hooke's law is saying that after certain point there are you see you know like the sloppy feature has come.

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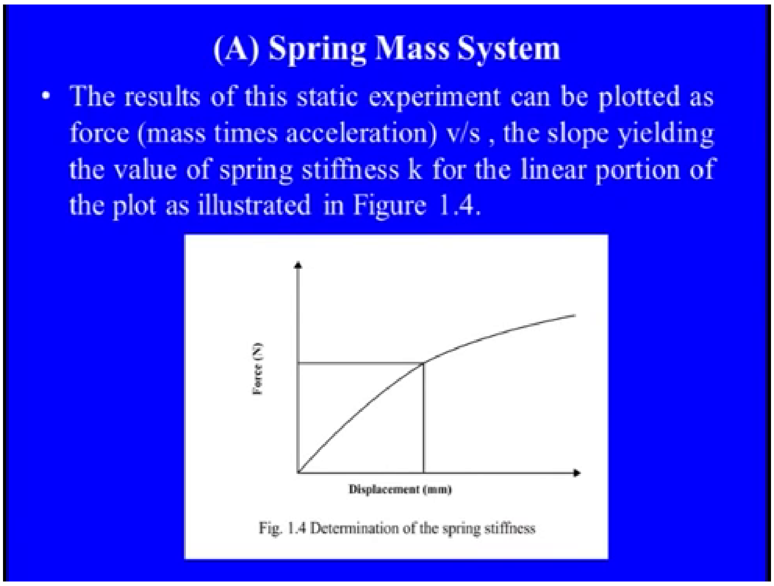

So, up to the linear propagation now you see when we are simply draping say this is what my displacement and the force we can get you see here, you now like the spring the spring stiffness by force per unit deformation under the elastic feature, means this straight line. So, once m and k the mass is known to us, the k is known to us, once we get those things we can solve the equation, because you say this is what the simple harmonic motion. So, we can apply a simple you know like the solution feature.

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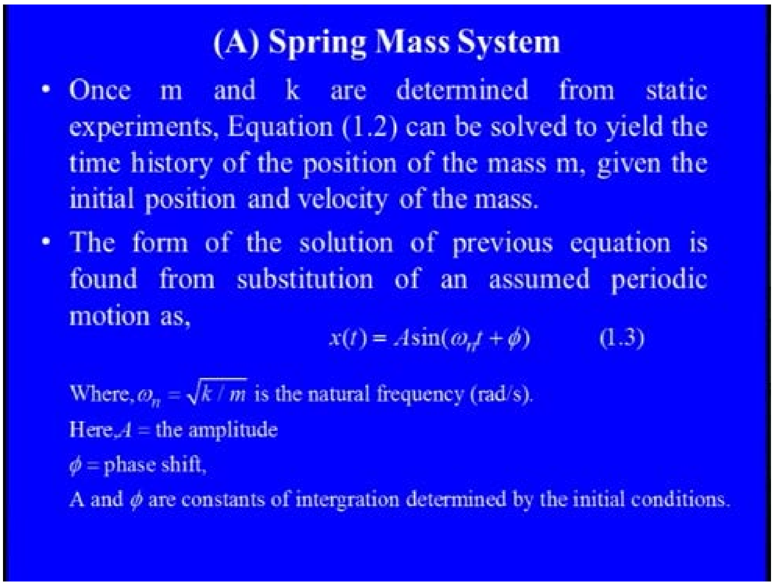

As we are applying to any ordinary differential equation with the time history of the position of mass with the initial position means if we have an initial position and initial velocity. You know like of the mass we can directly give as the input x 0 and you see here whatever the v 0 is there. And the form of solution and the previous equation is found from the substitution of the you know like the of all the these parameter as we are assuming that motion of this spring, of the mass which is constraint by this spring is of the periodic motion.

So, you can look at that you see x of t this is what my displacement outcome is nothing but equals to A sine omega t plus phi, A is the amplitude, omega n is my you know like the natural frequency. And as I told you the natural frequency is the system's inherent property you know like, because you see through that the system is exciting at the free vibration level. So, you can see that it is nothing but equals to square root of k by m, k is your spring stiffness and m is the mass distribution of this system itself.

And then you see you have sine because it is a harmonic feature, so you have sinusoidal feature of omega and t plus phi, phi is the phase shift. Then you see a and phi you know like we can simply find out you know like the constant you know like through the integration by applying all the boundary condition or the initial conditions. So, if x 0 is the you know like we can say is specified initial displacement the known one parameter from the equilibrium and mass m and v 0 which is again you see the known parameters to us.

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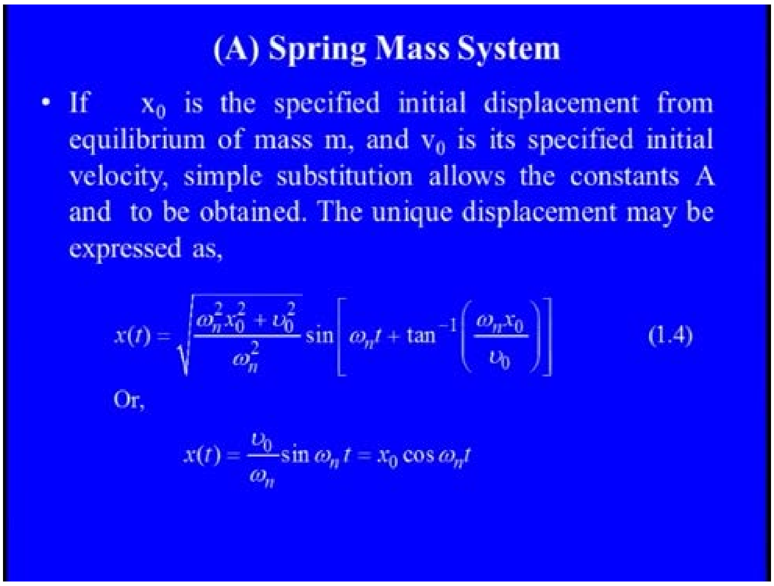

We can simply get the displacement x of t is nothing but equals to square root of omega n square x 0 square plus you see here u square by omega square sine of these things this is what my generalized solution. As we know that in the ordinary differential equation there are two main parts of the ordinary differential equation solution one is the particular integral and one is the complimentary function. The complimentary function is mainly responsible for free vibration concept and particular integral is mainly responsible for the forced vibration. So, through that you see here you have both the thing x of t is nothing but equals to v zero omega zero sine omega n t plus x 0 cos omega n t. So, you see both things have been you know like included there.

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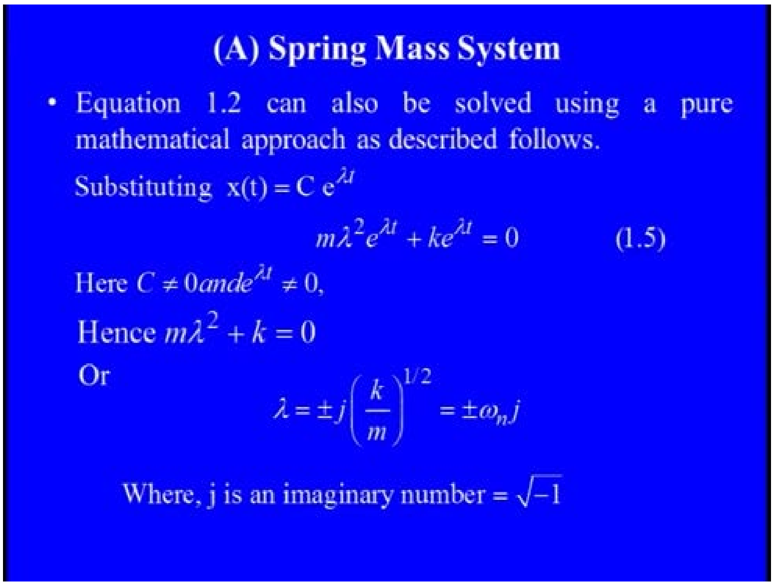

And when we are now substituting the displacement as a periodic input is c e into e to the power lambda t you can find that you see here, we can simply get the equation 1.5 as m lambda square e to the power lambda t plus k into e to the power lambda t equals to 0. We know that you see you know like this cannot be the whatever the coefficient this c which is the amplitude feature cannot be zero certainly we have the e to the power lambda t equals to 0.

That means, you see here finally, we are ending up with the equation of m lambda square m into lambda square plus k equals to 0 or the lambda is nothing but equals to plus minus j which is the unit factor is square root of k by m. This square root of k by m is the system's property through which you see we are getting the Eigen value or we can say the characteristic root of the equation we are referred that as you know like the natural frequency.

So, sometimes when we are trying to map this into the mathematical description we know that the this natural frequency is nothing but the Eigen value of any characteristic equation or it is nothing but the characteristic root of the basic equation of the system. So, we can say that this is you know like the j is nothing but the imaginary feature and it is always reflecting from plus minus j you see here.

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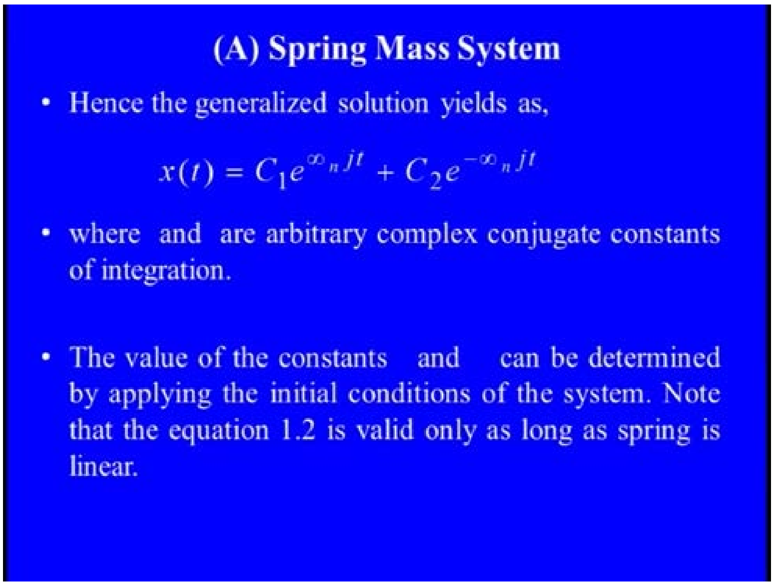

So, when we are trying to put this one we know that we have a generalized solution for this is x of t is equals to C 1 into e to the power omega n j t plus C 2 e to the power minus omega and j t. Where you see we have both you know like this e, e to the power omega n C 1 and C 2 basically are you know like which is related to the e to the power omega n j t and minus term are arbitrary complex conjugate. You know like terms which can be easily get you know like can be get using the integrants.

The value of constants either you see can be again you see you know like can be easily achieved using you know like boundary condition and the applied conditions of the feature, it all depends on that. So, this is you see here you know like one of the basic system in which you see we are not applying any kind of decay of the amplitude of the vibration. We are only applying that when the you know like the any system is under excitation then how the system the mass system and the restoring forces means the inertia force and restoring forces are interacted.

How we can get the generalized solution of such kind of systems, when they are just simply forming the equations of motion. Now, we are adding the damper as one of the feature, because we know that even when the system is you know like exciting it does not have you see the infinite you know like oscillatory feature, there is always you see you know like some decaying feature is there and this decay is mainly due to the damping part.

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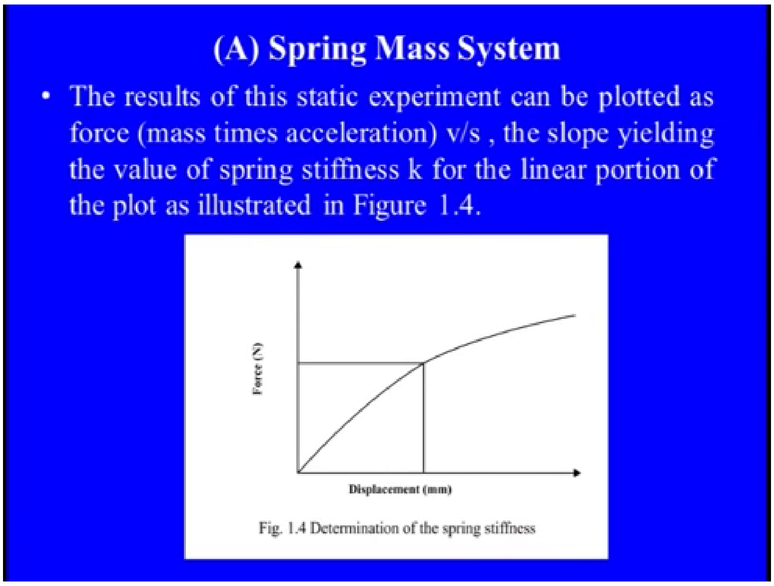

So, most of the system will not oscillate as I told you indefinitely when you know like it is being disturbed either you see it has some kind of you know like the mechanism or through which these oscillations are being dying out; typically the periodic motion damped out after sometime. The easiest way to model this in terms of mathematical way it just put the damping force along with the restoring forces and the inertia forces. So, when we are applying this to the same basic system in which we have the mass is spring and damping.

We can say that there are three forces one when the mass is moving due to the disturbance is always at inertia forces, mass into acceleration when the spring is there certainly the restoring forces are coming out and it is equals to k into displacement. And when the damping is there through damper may be the dampers are of various kind which we are going to you know like discuss in later form, we are right now saying that it is if it the viscous damper. Certainly you see here whatever forces which are being coming out due to this you know like the viscous dampers or viscous forces and they are nothing but equals to viscous coefficients into velocity.

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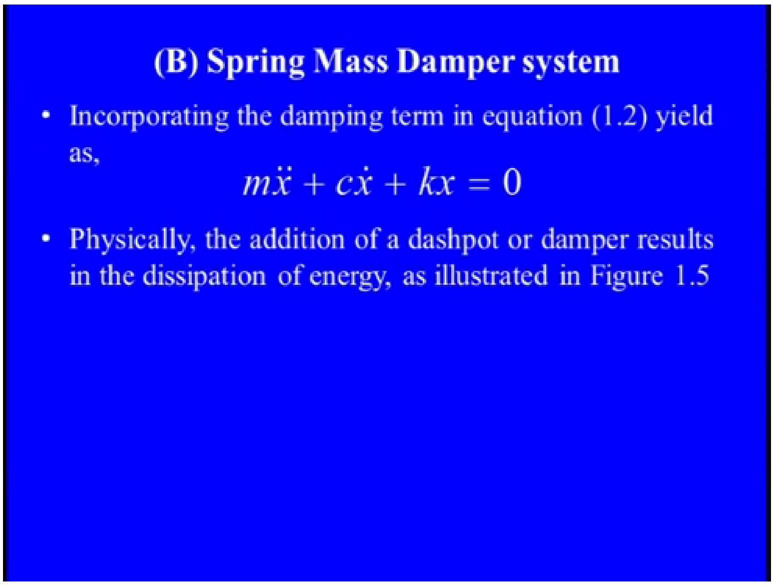

So, and you see the system is under the equilibrium conditions; that means, is here m x double dot plus c x dot plus k x equals to 0. Physically the addition of dashpot or you know like the damper which always results in the dissipation of the energy, it always dissipate the energy and that is how you see we can control the oscillatory movement. So, you see you can see that now this is what our you know like the system is...

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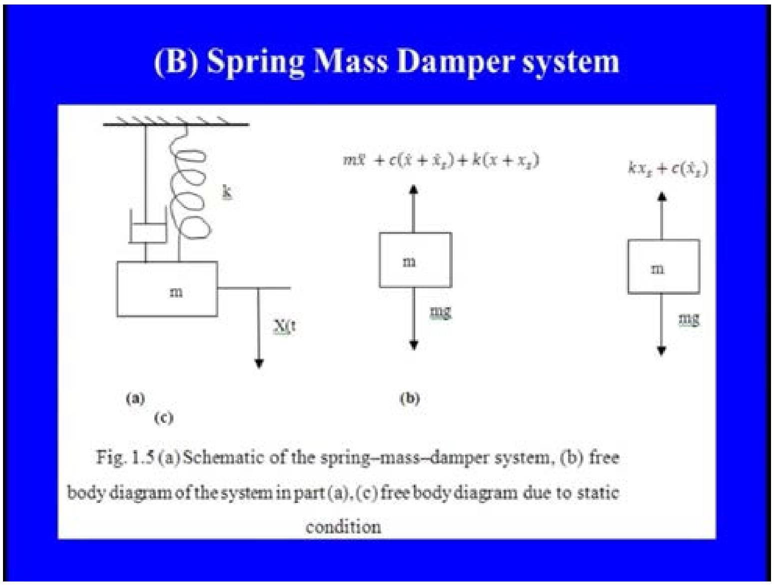

We have the system the mass same the spring and the damper and these are you see you know like all the forces are that you can look at that m x double dot is there plus you see the c the damping coefficient and the interaction of these the velocity of you see x double dot and this one. And then you see here we have you know like the spring coefficient at k and this is you see the x and x 0. And when we are just trying to make balance of these we know that we can get the basic equation, which we discussed already.

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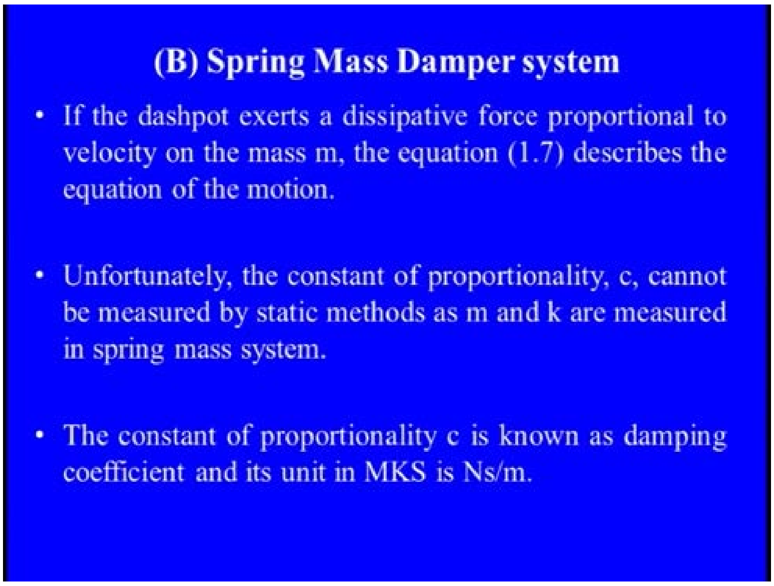

So, if the dashpot exerts a dissipative force proportional to the velocity on the mass certainly we know that whatever the dissipative energy which is you know like being there it can be easily absorbed there. So, unfortunately the constant of the proportionality the c, which is the basic property of the damper cannot be measured by the static method, because in that you see here we have just like you see in the spring it was a simple linear you know like we can say the displacement part, but here we have the velocity term.

And this c which is the property of the velocity the c is nothing but the constant damping constant it is a property of the velocity. And this velocity when we are trying to see it not only depends on the amplitude, but also depends on the frequency feature. So, you see here you know like we can get you know and again you see in the damper if you look at the damper configuration we know that there is a viscosity feature involved in that, what is the oil which is being there in the viscous part and what is the viscosity is there of this viscous damping.

So, now, you see here now if you are just going towards again the same harmonic motion, we know that this you know like this whatever the oscillation is there it is a periodic one and x of t is nothing but equals to a into e to the power i x.

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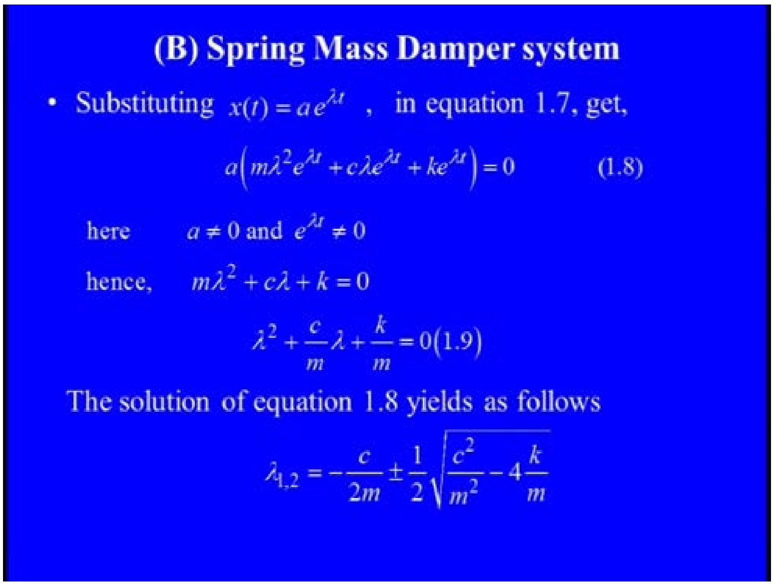

And then we are keeping that you see in the same equation m x double dot plus c x dot plus k x equals to 0. We know that we have a into m lambda square e to the power lambda t plus c lambda e to the power lambda t plus k into e to the power lambda t equals to 0. And here when we are keeping a equals to 0, we know that this cannot be equals to 0 e to the power lambda t. So, then you see we are ending up with the equation n inside of the bracket e into m into lambda square plus c lambda plus a k equals 0.

We can make this equation by dividing the mass just to keep the characteristic generalized equation. So, we have lambda square plus e by m into lambda plus k by m equals to 0. And when we just trying to solve using you know like the characteristic equation we have the characteristics roots are there lambda 1 comma 2 which is nothing but equals to minus c by 2 plus minus square root of c square by 2 m square plus minus 2 times of k by m.

Or else you see here when we are just trying to see this we know that the c by m k by m is giving a very specific characteristic of that which we are going to discuss the quantities under the radical is called the discriminant as we know that and the value of this discriminant decides whether the roots are real or complex. So, you see here, now we are going to first define the terms involved in that one is the damping ratio.

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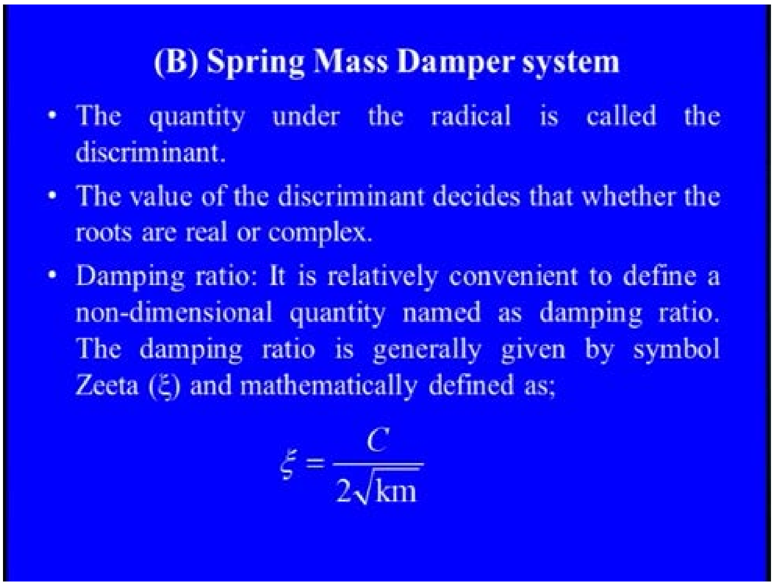

The damping ratio is nothing but equals to as you know like the term itself speaks that the damping there are two different damping are being involved you know like one is the c. The c is the applied damping which we are applying trying to reduce the vibration amplitude. And c c is the critical damping, critical damping is one of the minimum damping you see which should be there to reduce you know like the oscillation at the minimum time.

So, you know like it is relatively convenient to define the non dimensional you see as this one and it is nothing but equals to c by c c. As I already told you and we can simply say that the damping ratio is nothing but shown by the zeta, zeta is equals to c by c c where c c is nothing but equals to 2 times of square root of k m. So, you see when we are keeping this now our equation, when we are trying to keep this k m and c replaced with omega n. And this one we know that omega is nothing but equals to square root of k by m and zeta is nothing but equals to c by c c or c by 2 k m.

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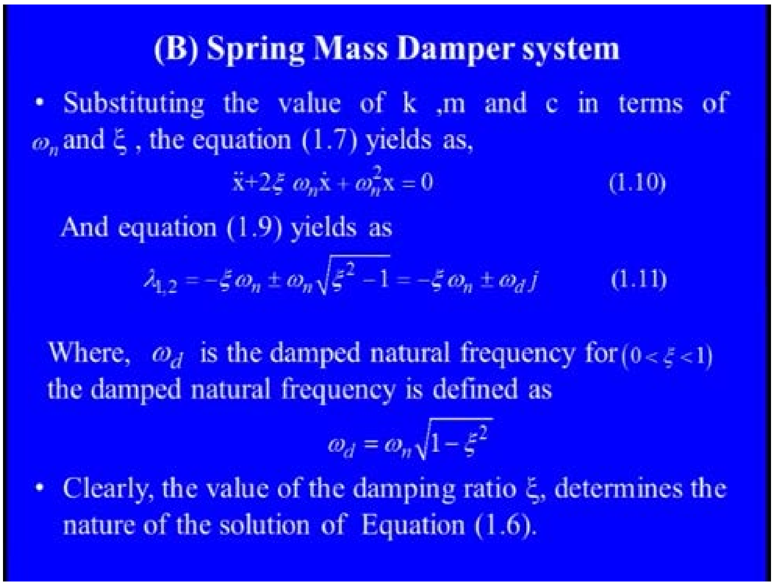

Now we can replace this equation as x double dot plus 2 zeta omega n x dot plus omega n square into x equals to 0. When we are now this is my characteristic equation which is representing a single degree of freedom system with this spring and damping. Now, it has the two characteristic roots lambda 1 comma 2 is nothing but equals to minus zeta omega n plus minus omega n zeta and under square root of zeta square minus 1 or else I can replace with this minus zeta omega n plus minus omega d.

Now, you see my system which has the damper as one of the essential element, now it has two different frequency one is the natural un damped frequency and one is the natural damped frequency. And we can relate both the things that the natural damped frequency is omega d equals to omega n the square root of 1 minus zeta square. And you see here you know like with this particular concept we can easily find out that what is the value of zeta, zeta is nothing but the damping ratio which has a physical you know like we can say relation.

So, it can clearly give that you see here if we are going towards the damping, over damping, critical damping, under damping what exactly the kind of features which we require and accordingly we can give the system properties. So, you know like we can get you see straightaway that how the system will behave what exactly the type of you know like the damping should be provided.

And then how much the excitations will come out from that. So, now, if we apply this concept to the real physical system say this is what you see the basics of vibration, when we are applying to any physical dynamic system say this is my say the two vehicle, 2 wheel vehicle is there.

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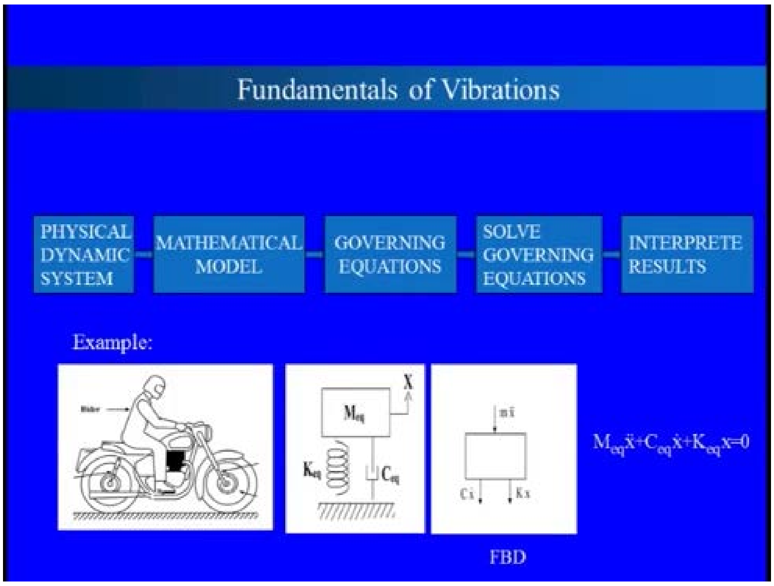

We have the rider, we have you see you know like the vehicle itself this is my physical dynamic system. Now, I just want to go to analyze the motion, so these are the basic steps which I need to follow strictly, one is first I need to map this physical system into the real mathematical features and when I am trying do this I need to see that what I have the input variables which I have right now is, the mass, mass of rider, mass of vehicle.

The two masses are there then what I have in other terms the tires the tires are absolutely in the contact of the road. So, they are in the deformation and this deformation can be straightaway replaced by this spring, other thing when the rider is just sitting on the seat there is a clear interaction between the base part of the rider and the seat part. So, there is a deformation, so I can also replace this as the spring. And then I have you see the struts are there you can see that on the diagram these are the struts. So, these struts have you see the damping features, so I have the dampers. So, this physical system is clearly reflecting the three basic modes mass damper and the spring and now I can relate these things with my three basic dynamic properties displacement, velocity and the acceleration.

So, I can first you see here equate that what exactly the replica in terms of what that is what we discussed about the mass spring and damper. So, we have the equilibrium mass as I told you the rider mass and the vehicle mass, second we have the equivalent stiffness, the stiffness of two tires, the stiffness of even the contact reason between the human body means rider and the vehicle part. And then we have the damper the two struts are there the front wheel and the rear wheel.

So, you see here this is the equivalent feature and then you see when we are trying to map into the other part that is because we want to generate the third part is the equation generation that what exactly the equation of motion is there. So, you see here we need to first port the force balance condition. So, on the mass we have all three forces the inertia forces restoring forces and the damping forces as we can describe here.

And once you have this you have now the equation of motion as we discussed already m x double dot plus c x dot plus k x equals to 0. And once you have these things in terms of the single degree of freedom system we can simply find out this and then we can apply since it is a periodic motion which we are assuming or a simple harmonic motion we can apply the input part x of t is nothing but equals to c into e to the power lambda t. And when we are keeping that part we can have the solution the characteristic routes which are simply giving us the Eigen value or in other terms we are we are having the exciting frequencies of that.

So, in today's lecture we discussed about the basics what is the important of the vibration is, why we are dealing why we are you know like all basic mechanics concept when we are talking about the vibration. And then you see here when we have any physical real system how we can relate the real physical system towards the mathematical formulation what are the basic we can say the terms are there which are always being a governing terms for the exciting vibrations.

And as I told you that remember there are three basic dynamic parameters which are always being responsible for having any kind of excitation in the system displacement, the velocity and the acceleration. And if you want to measure these things we need to have a three different devices for that. So, you see in this particular lecture that that is what we discussed, in the next lecture now we are going to deal with the two degrees of freedom.

As you see that in this particular screen we have the rider and the vehicle and this system is absolutely for you know like right now the single degree as you know that we have a single mass, single you know like the damper. And we can say the k which are being there just with the x and the x is the horizontal or vertical movement just in one direction, but we know that you see here this is not correct, we can say you know like the conversion of this we may have you see a different displacement of say you know like the rider or the vehicle.

We may have you see you know like the different stiffness at the you know like the rider to seat or this tire to this one road or we have you see the different damping from rear or front struts. So, when we know that these things are quite feasible we need to now see towards the other features of that it is the 2 degrees of freedom system without damping with damping; and then we are trying to solve mathematically about the real physical system.

Thank you.