Computational Fluid Dynamics
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Lecture - 40
Reynolds Averaging and RANS Simulation Models

Welcome to the second lecture on module 9 on numerical simulation of turbulent flow. In this module, we have discussed the features of turbulent flow, what are the basic elements which differentiate the turbulent flows from laminar flows and we also discussed briefly the numerical simulation strategies of turbulent flow and how different length scales and time scales, which are present in turbulent flow make our life pretty difficult.

So we had a brief look at the main simulation strategies and we had also had a look at the time cost of simulation if you just want to solve simulate all length and time scales in a turbulent flow using the so called DNS or direct numerical simulation strategy.

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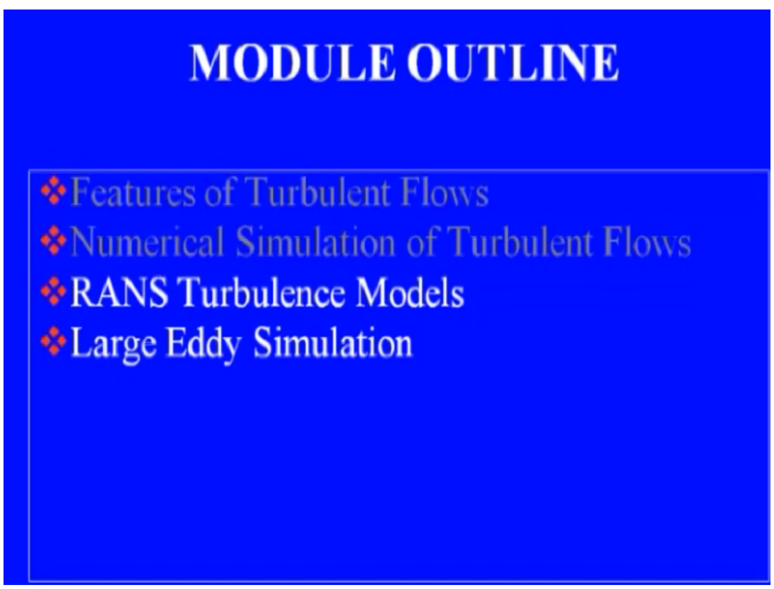

Now next we are going to have a look at what we call a RANS turbulence model and in the next lecture we would cover what we call large area simulation. So let us have a recap of the previous lecture. We discussed the basic features of turbulent flow, its dissipative nature, random fluctuations, which form integral part of turbulent flow, its unsteadiness three-dimensionality, diffusive nature and its dissipative nature.

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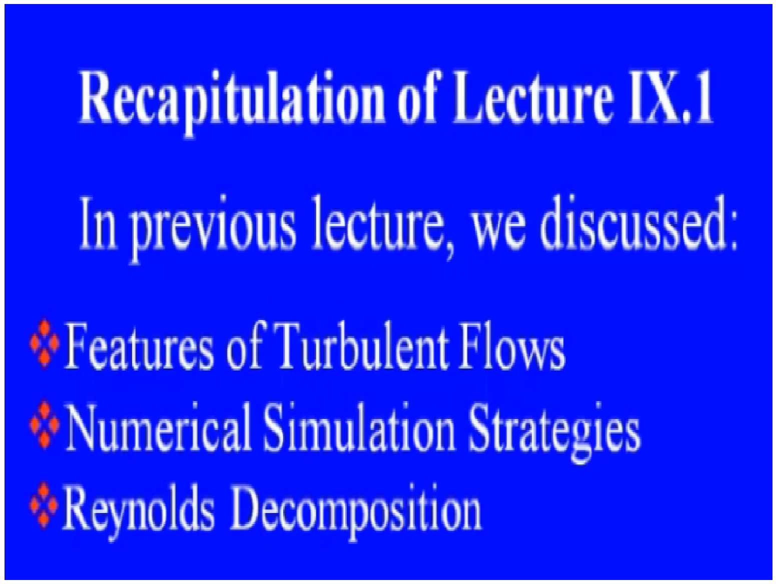

We also discussed the various other features, the length scales which are present, their rough estimation, and the numerical simulation strategies, namely, the direct numerical simulation, which would involve the resolution of all length and time scales requiring very fine grid, which makes DNS impractical for industrial flow simulations and we also looked at the time scales. We can use DNS primarily for research purposes to understand the basics of fluid dynamics, the turbulent flows and to refine what we call the turbulence model used in RANS and large area simulation.

We also briefly discussed the Reynolds proposition of what we call Reynolds decomposition and this way, we are going to pickup form.

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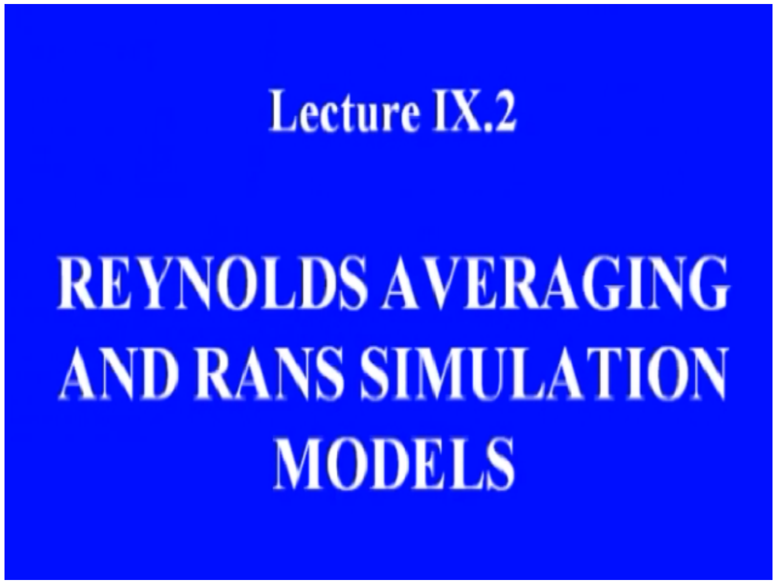

We will start off our lecture on Reynolds averaging and RANS simulation model. RANS stands for Reynolds Averaged Navier-Stokes simulation and RANS simulation models are the ones, which are today the practical tool for industrial scale simulations. Only recently, with the advent of very high performance computers available with big corporations that people have been trying to use large area simulation for final design comparisons.

For the initial design and design a traditional cycle, RANS is still what we can practically employ. So we will have a look at some of the RANS models in this lecture. So the outline lecture, we would first have a look at the so called Reynolds averaging procedure, which is used in Reynolds decomposition.

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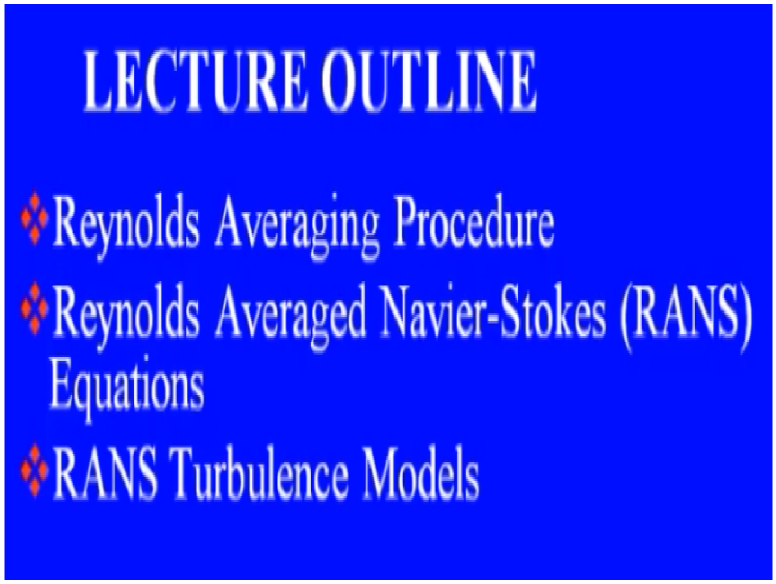

Then we would derive so called Reynolds Averaged Navier-Stokes equations. In short, they are called RANS equations and we will look at few major turbulence models. Why do we need these models that would become apparent when we have a look at Reynolds Averaged Navier-Stokes equations, which have additional terms, which must be modeled and this turbulence models they try to come up with some formulae for those unknown additional terms in terms of the primary quantities.

Now this is what we had discussed in the last lecture, the Reynolds decomposition. (()) (04:12) suggested that we have flow variable phi in a turbulent flow field. It can be broken into two parts, which can be attributed to the turbulent fluctuations, which are random in nature and if a flow were statistically stationary, other part could be called time averaged value, so that is to say that for any variable phi.

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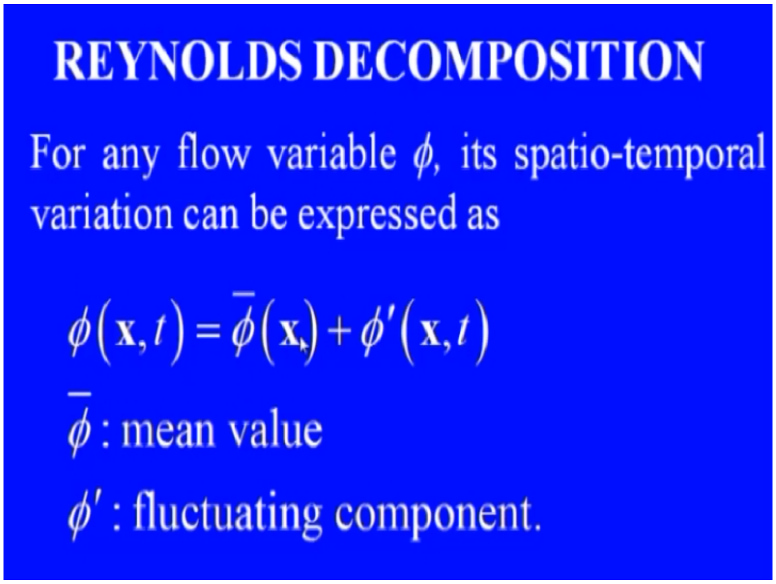

We can write phi at a special location x and time instant t. it can be written as sum of 2 parts phi bar x, now this particular part the average part or mean value part, it is independent of time. It only depends on the spatial location + high prime x, t. So phi prime x, t that represents randomly varying fluctuating components of the scalar phi. Now this phi could be one of the velocity components. It could represent density in compressible flows and pressure or temperature.

So any flow variable, which we encounter in turbulent flows might be decomposed using this simple strategy in terms of its mean value and fluctuating components. Now this is specific reason why we wanted. In industrial flow simulations, we are interested in time averaged quantity. We are not interested in what happens to the flow variables at every special occasion at each instant of time, doing that would be next impossible for large scale industrial simulation.

We are interested in average quantities like we want to know the forces drag left, the pressure drops occurring across a length of a pipe or piping material and so on. So we are primarily interested in what we call the gross quantities or average quantities and it does not make much sense for us to try and resolve all the length and time scales in our simulation, that is what we would attempt in direct numerical simulation that would be a waste of time.

So we would try to find out these time averaged quantities through our simulations. In terms of these quantities, we can calculate the values of engineering interest. That is to say the force is acting on the surfaces that in terms of what we call as a net drag force or lift force or the pressure drop energy loss and so on. Now we have to use this word phi or average, how do you refine these average terms.

So what Reynolds suggest is that if you are dealing with statistically steady turbulent flows that is to say the flow rate remains constant with respect to time. if you take the average over a fairly large time period. So in that case for such flows, which are formally called statistically steady.

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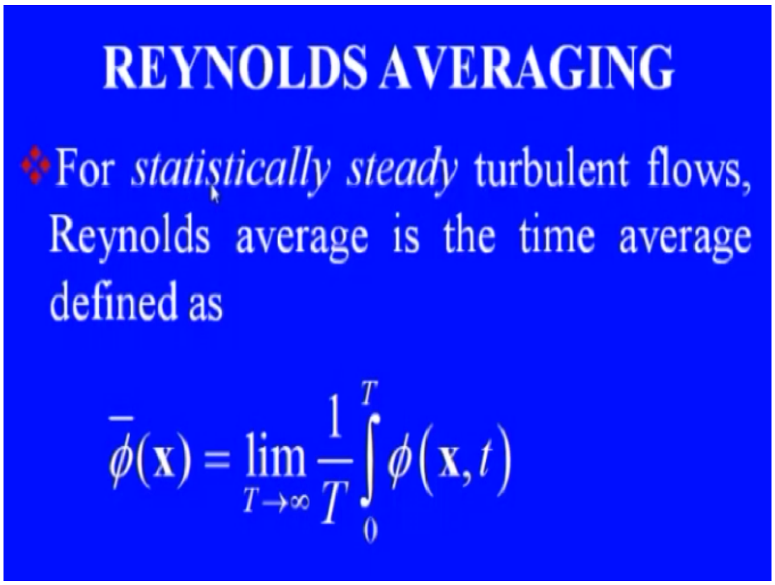

Reynolds averaged the time average and we would define it as phi bar x=limit T tending to infinity 1/T 0-T phi xt dt. Now this capital T represents that averaging interval which must be large compared to the typical time scales of fluctuations. Otherwise, we would not get statistically averaged value of the scalar field phi at special location x.

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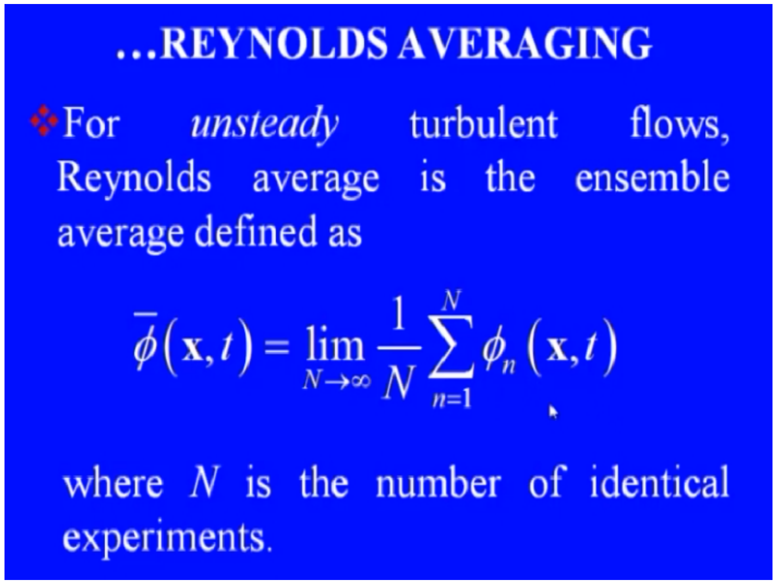

But in case of the flow were inherently unsteady, that is to say the above fluctuations, but even the average flow rate is varying with time. Suppose, we have taken pipe flow and we are steadily increasing the flow rate through the pipe network. So if the flow rate even in average sense that is varying or changing with time. In such situations, we cannot use a time average. So we have to define a different averaging procedure.

And now in this case our phi bar would not be just a function of this spatial location x, it will also depend on time and we would define it using what we call unsummable average. Unsummable average is defined in terms of what we call identical experiments if we had had an opportunity to perform the experiments in identical conditions and each time, we measure our variable pi at different locations in time.

So at a given location, at a given time, let us take out N number of measurements and let us take average of those N number of measurements. So each phi N xt that represents one measurement, we have taken N number of measurements, take their average and this N should be fairly large number to give us what we call unsummable average and this phi bar xt would be now the unsteady average term and the related arrays of equations what we call, those are called unsteady arrays.

But most of the time, you would be dealing with time averaging and if you look at the 2 definitions, which we had in the previous slide, which is defined in terms of integral of the quantity phi xt over time or in terms of this summation, both of them are basically linear operations. So there are certain consequences of this linearity.

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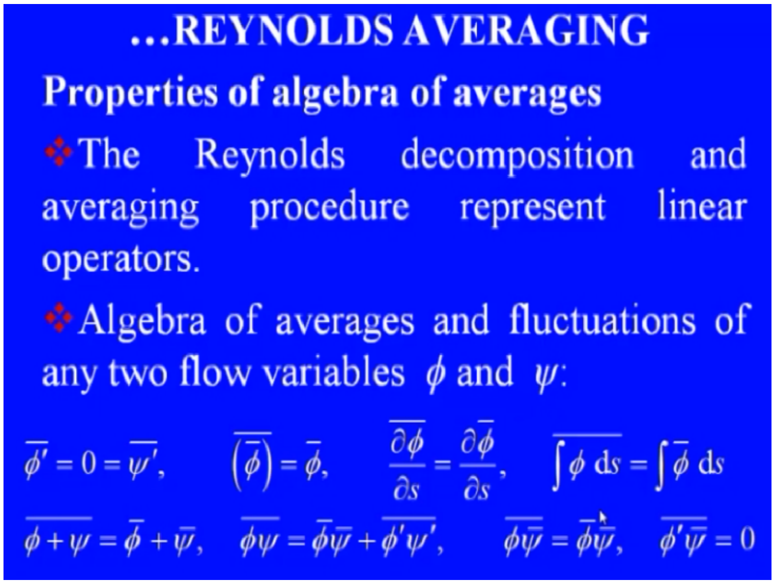

There are decomposition which we had that was defined as sum of 2 quantities that is a linear operator. Similarly, the averages which we refined those were defined in terms of linear operators. Integral again is essentially it can be represented as summation, which is a linear operator, similarly an unsummable average we had some summation, so that is again a linear operator and there is certain algebraic rules, which we can formulate for algebra of averages and fluctuations of any two flow variable phi and psi.

The way we had defined our time averaged values, so if you take Reynolds decomposition, if you want to find out average of the fluctuation, what do we get. Now let us briefly have a look at whether this particular thing are we satisfied with this phi or average would be 0.

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Algebra of Reynolds Averaging

$$\phi(\vec{x},t) = \bar{\phi}(\vec{x}) + \phi'(\vec{x},t) \qquad (1)$$

$$\bar{\phi}(x) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \phi(x,t)\,dt \qquad (2)$$

Apply averaging process to both sides of Eq(1)

$$\overline{\phi}(x) = \bar{\phi}(x) + \overline{\phi'(x,t)}$$

$$\Rightarrow \boxed{\overline{\phi'} = 0}$$

So remember we have defined our decomposition phi xt, this was defined as phi bar x+phi prime xt. For the time being, for the sake of simplicity, I am going to drop the arrow operators from x and we would presume that where we write x, that represents the 3 dimensional coordinates of a point. Now how did you find our average operator. This phi bar x, this was defined as limit T tending to infinity 0-T phi xt dt 1/T.

We would say that wherever we use the over bar that represents this particular Reynolds averaging operator or Reynolds averaging procedure. Now let us apply this procedure to our decomposition. So let us apply this integration process or this averaging process on both the sides of equation 1. Apply this averaging process to both sides of equation 1. Then what we get. On the left hand side, we will get phi bar and with this what we will get after averaging.

If you average this phi bar xt, we get simply phi bar, what would be the time average of an average quantity. Remember this phi bar does not depend on time. So if you want to take its average over the long time interval, it will remain phi bar so the first on the right hand side what we get, that remains as phi bar x. The next term which we had was this phi prime bar, t, so what do we get from here. It is obvious enough that average of the fluctuations =0.

So this is first rule of our Reynolds averaging procedure. Now what will happen if you want. Let us have a look at the next few rules on this algebraic process. This we have already clarified that average of the average, that will remain phi bar. The next thing is differentiation and integration. They commute with this averaging process. A differentiation also represents a linear operation. So that is what we say that differentiation and averaging process they commute.

That is why we get the del phi/del s over bar, that is average of del phi/del s is same as the derivative of del/del s of phi bar. Similarly if you had an integral phi ds, s average is same as the average of phi bar ds. So in nutshell, what it means is our two linear operations, for instance in this case the differentiation and averaging, they commute. Similarly integration and averaging, they also commute, similar.

If you have sum of 2 fluctuating quantities, phi and psi, the sum of that 2, that would represent the third quantity, which is fluctuating with time. How do we get the average of that quantity. That is simply given by the average of 2 quantities. So phi + phi bar=phi bar+psi bar. Now let us verify these terms. But when we have a product, now remember this product is no more a linear operation. So product of phi*psi, its average would be given by these 2 terms.

That is phi bar + phi bar*psi bar+average of the product of the fluctuating components phi prime and psi prime.

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... Reynolds averaging process

$$\overline{\phi+\psi} = \lim_{T\to\infty}\frac{1}{T}\int_0^T[\phi(x,t)+\psi(x,t)]\,dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\left[\int_0^T\phi(x,t)\,dt+\int_0^T\psi(x,t)\,dt\right]$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_0^T\phi(x,t)\,dt+\lim_{T\to\infty}\int_0^T\psi(x,t)\,dt$$

$$\Rightarrow \boxed{\overline{\phi+\psi} = \bar{\phi}+\bar{\psi}}$$

Suppose you want to find out average of phi+psi, how do you work it out. By definition this is limit T tending to 0 1/T integral 0-T phi xt+psi xt dt. By the basic rule of algebra, we know this integral can now be broken into 2 parts. Integral of the first function+integral of the second function, so we get (0-T phi xt dt+0-T psi xt dt), limit again, the limit of 2 quantities or the sum of 2 quantities=the limit of each individual quantity.

So limit T tending to 0 1/T 0-T phi xt dt+limit T tending to 0 0-T psi xt dt and you can easily recognize these 2 elements, they are nothing but the respective averages phi bar+psi bar. Now let us have a look at what happens if you want to find out the average of the product of the 2 functions. So we want to find out what will happen to phi to psi. Now let us use our Reynolds decomposition.

So phi can be expressed in terms of phi bar+phi prime and psi can be expressed as psi bar+psi prime and we want to apply this over averaging operator to both of it. Now let us multiply an open product, so we phi bar psi bar+phi bar psi prime+psi bar phi prime + phi prime psi prime. So here we want to take average of these quantities, which are being separated by additional operator. So this overall summation can be broken by the average of the separate quantities.

So we get this phi bar psi bar average+phi bar psi prime average+psi bar phi prime average+phi prime psi prime average. Now remember the way we have defined this average of phi bar and average of psi bar is independent of the time, so this would essentially be the product of phi bar psi bar.

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$$\Rightarrow \boxed{\overline{\phi+\psi} = \bar{\phi} + \bar{\psi}}$$

$$\overline{\phi\psi} = \overline{(\bar{\phi}+\phi')(\bar{\psi}+\psi')}$$

$$= \overline{\bar{\phi}\bar{\psi} + \bar{\phi}\psi' + \bar{\psi}\phi' + \phi'\psi'}$$

$$= \overline{\bar{\phi}\bar{\psi}} + \overline{\bar{\phi}\psi'} + \overline{\bar{\psi}\phi'} + \overline{\phi'\psi'}$$

$$= \bar{\phi}\bar{\psi} + \bar{\phi}\cancelto{0}{\overline{\psi'}} + \bar{\psi}\cancelto{0}{\overline{\phi'}} + \overline{\phi'\psi'}$$

$$\Rightarrow \boxed{\overline{\phi\psi} = \bar{\phi}\bar{\psi} + \overline{\phi'\psi'}}$$

Similarly, from the next term phi bar comes out and we get psi prime bar+psi bar and phi prime bar+phi bar psi prime whole over bar. We have just learnt earlier that the average of the fluctuations that is 0. So these terms which contain the average of fluctuating terms only. This becomes 0 so we get the average of the product of 2 functions phi psi that is given as the product of the individual averages + average of the product of fluctuations.

So look carefully, the average of product of the fluctuations that will not be 0. That will depend on how these 2 quantities are correlated. Now we can use these rules and apply these rules to our Navier-Stokes equations to obtain the so called Reynolds Averaged Navier-Stokes equations. So the summary of the equations on the slide and now let us see how do we actually obtain these equations.

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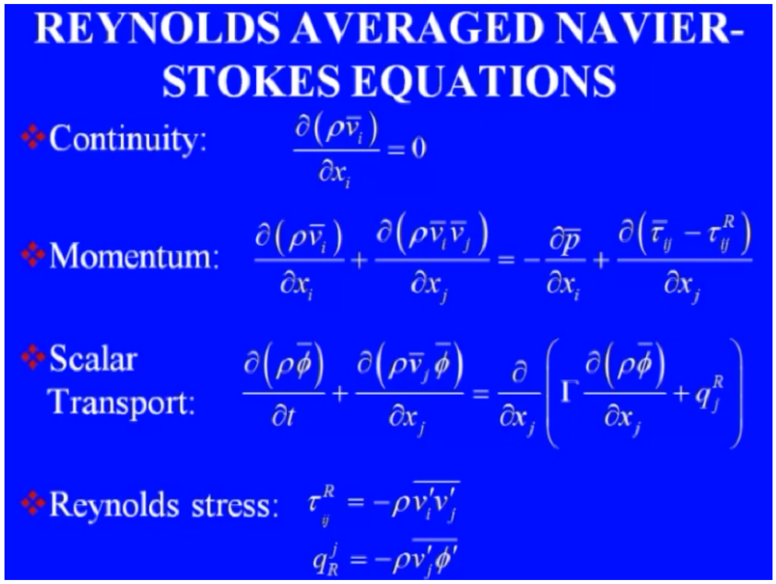

So first one we would like to find out, we would apply Reynolds averaging procedure to our continuity equation.

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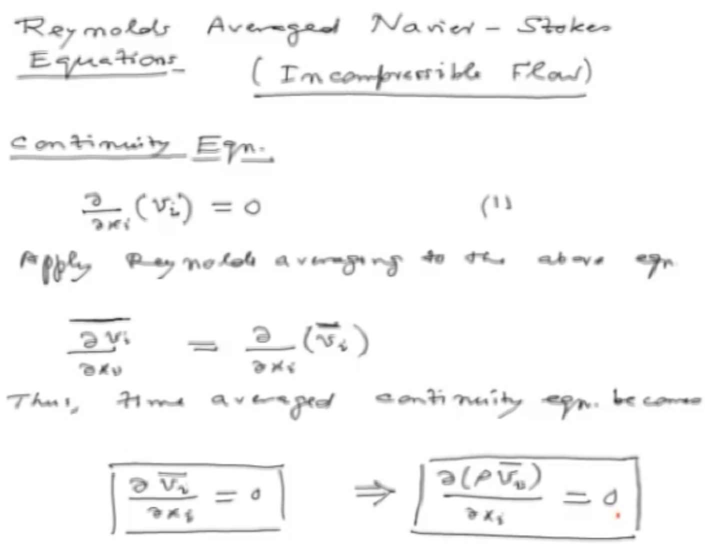

So Reynolds Averaged Navier-Stokes equations and for the time being, we will discuss the case of incompressible flows for which density is constant. This is slight difference procedure for obtaining this Reynolds averaging process, which is referred to forward averaging for compressible flows or variable density flows, that is something I would leave you to explore from the literature.

For the time being, let us focus on the continuity equation for our incompressible force. This will be del rho/del t term that vanishes so we get del/del xi=0, vi is our velocity vector. So apply averaging process to it. So apply Reynolds averaging to the above equation. Now remember the rule which we had learnt earlier that differentiation and averaging process they commute. So this will be simply del/del xi of vi bar.

So thus our time averaged continuity equation becomes del vi bar/del xi=0. Now sometimes in our calculations, we keep densities together. Densities are constant so that it does not really matter if you want to put it inside the brackets. So del vi bar/xi. So these are 2 forms, which are frequently used in numerical simulations. Both of them are basically identical for incompressible flows. Next let us work on our Navier-Stokes equations and we will look at each term separately and find out its average.

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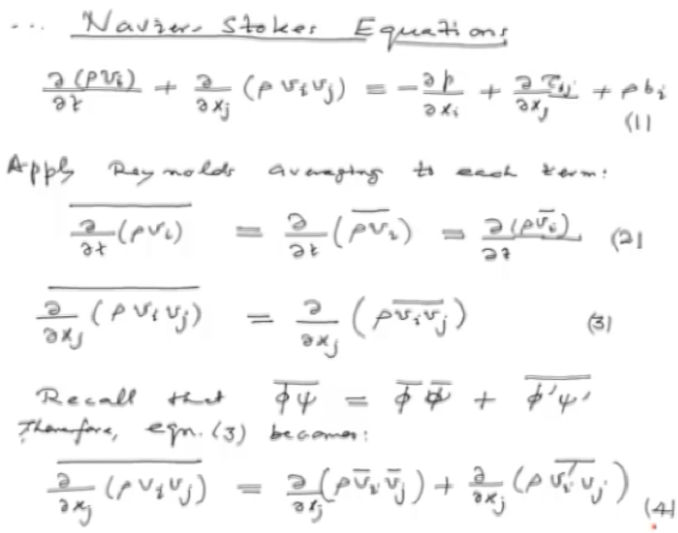

So Navier-Stokes equations were given by del/del t of rho vi+del/del xj of rho vi vj=-del p/del xi+del/del xj of tau ij+rho vi. This was our time dependent Navier-Stokes equation. So now let us apply averaging process to each terms. So apply Reynolds averaging to each term. The first term contains a time derivative. So del/del t of rho vi, we want to find out the average of it. Remember again that averaging and differentiation that commute, so del/del t of rho vi average.

Rho was basically a constant for incompressible flows. We can also write this as del/del t of rho vi bar. So this is our first step. The tricky part is the next term, which is convective term del/del xj of rho vi vj over bar. So the averaging process that will commute with our differentiation operator, so this is del/del xj of rho vi vj over bar. Now remember what we have learnt earlier, recall that formula.

Recall that the product of phi and psi, their average is given by the product of phi bar psi bar+phi prime*psi prime average that is average of the product of fluctuating components. So use it in equation 3 so therefore, equation 3 becomes del/del xj of rho vi vj over bar=del/del xj of rho vi bar vj bar+del/del xj of rho vi prime vj prime over bar. Similarly, the two terms on the right hand side they are linear operators. So we can straight away apply the averaging process.

Bi is a source term or what we call body force term which would actually not depend on the turbulence. It will be fixed quantity.

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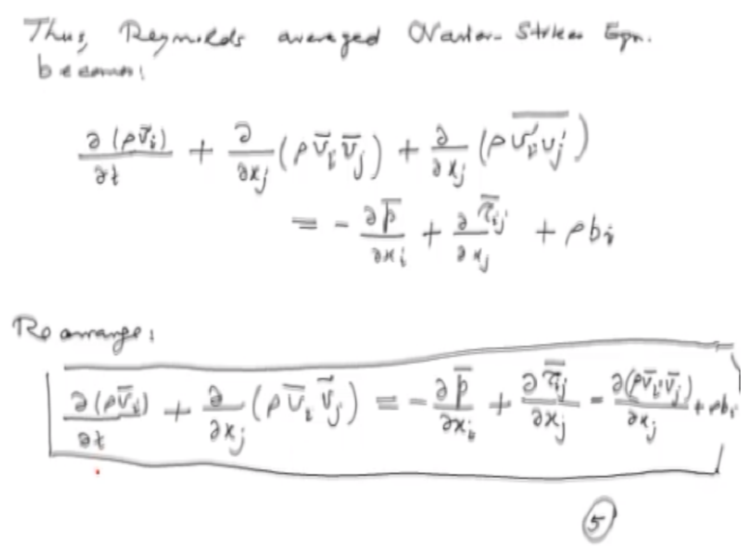

So thus Reynolds averaged Navier-Stokes equation becomes del/del t of rho vi+del/del xj of rho vi bar vj bar+del/del xj of rho vi prime vj prime over bar=-del p bar/del xi+del/del xj of tau ij bar+rho vi. Now what we normally do is, transfer this term, which involves fluctuations to the right hand side and club it with our stress term. So rearrange del/del t of rho vi bar+del/del xj of rho vi bar vj bar=-del p bar/del xi+del tau ij bar del xj-del rho vi bar vj bar del xj+rho bi.

So this is what is referred to as Reynolds Averaged Navier-Stokes equation. On the left hand side, you see this equation is basically a transferred equation for time averaged or Reynolds averaged velocity components vi bar. So we have the first term is our temporal term this will vanish if the flow is statistically steady. Next term is convective term in terms of the averaged components vi bar and vj bar.

The first term on the right hand side that is gradient of time Reynolds averaged pressure field, the next term is delta ij bar/del xj bar that is in terms of our Reynolds averaged velocity components as far we would compute del ij bar, but we have got additional term here. That is bi prime, vj prime over bar. We do not know what are these components. These are unknown components and in fact this represents a second order terms, which will have 9 components.

So the averaging process has introduced an unknown tensor quantity, which must be modeled and that reasons of Reynolds stress models, which must be used to ensure what we call the closure of these equations. So similarly if you had a scalar transport equation, we can again apply our averaging procedure. Left hand side remains in the form very similar to our normal equation del/del t of rho phi bar+del/del xj of rho vj bar phi bar=del/del xj gamma of del rho phi/del xj+qj bar.

This is an additional term. We normally use this symbol tau ij superscript r and we call this as Reynolds stress as defined by –rho vi prime vj prime, so this is fluctuating components. They give rise to a stress like term. This was the reason why we call them as Reynolds stress. Similarly this is a turbulent flux, which is in terms of the fluctuating components of velocity would lead to additional scalar transport and thus the reason why it is referred to as turbulent flux.

Let us have a look at the physical significance of these Reynolds stress terms.

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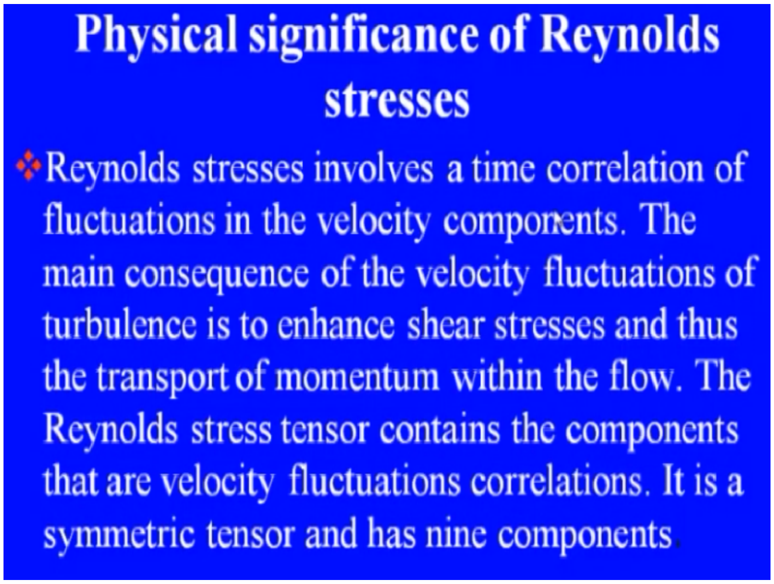

So these stresses the way we saw. They involve time correlation that is why we had this vi prime, vj prime over bar. That is the time correlation of the fluctuating components velocity. The main consequence of this velocity fluctuation turbulence is to enhance shear stresses and thus the transport of momentum within the flow. So the Reynolds stress terms contains components that are velocity fluctuation correlations and you can verify this is a symmetric tensor and has 9 components.

So we will have basically 6 unknown components for this symmetric stress tensor and those have to be somehow modeled in terms of Reynolds averaged velocity field. So this is what majority of turbulence models attempt to ensure closure of Reynolds Averaged Navier-Stoke equations. Similarly, the turbulent fluxes they arise from the convective transport due to turbulent fluctuations, fluctuations of the velocity field as well as fluctuations in scalar component.

So when this equation is time averaged, the influenced fluctuations over averaging time periods include via these additional flux terms, which represent enhanced heat transfer or enhanced mass transport and via result of the Reynolds averaging applied to our conservation equation and their sole purpose is to incorporate the effect of turbulence in enhancement of the transport of the respective queries.

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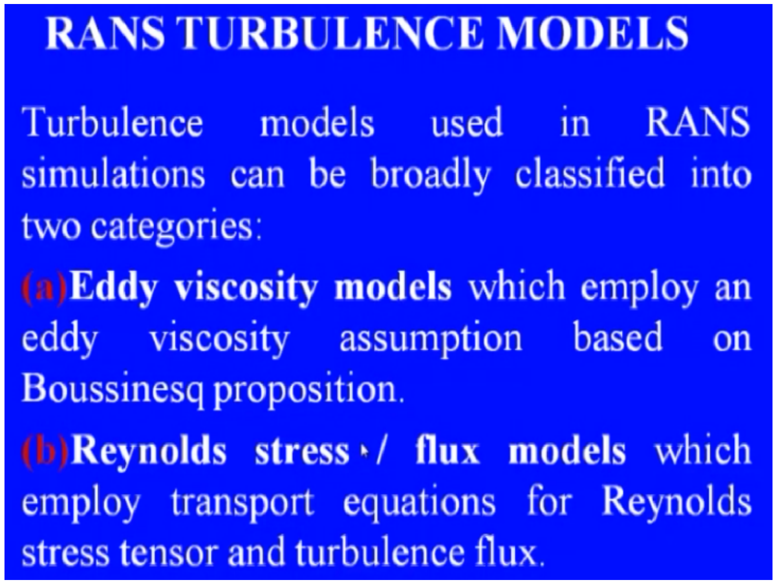

Now let us have a look at some of what we call RANS turbulence model Reynolds Averaged Navier-Stokes turbulence models and we can classify these turbulence models broadly into 2 categories. The first one is what we call Eddy viscosity models and these models, they employ what we call eddy viscosity assumption based on Boussingsq preposition.

The preposition of Boussingsq was that we can incorporate these enhanced effects coming from the fluctuating components via an additional or enhanced viscosity and since that viscosity is basically caused by the eddy's or what we call fluctuating components of the velocity field, so that is why this term eddy viscosity is used here. Other model is what we call Reynolds stress for the velocity field that is for Navier-Stokes equations and Reynolds flux model for our scalar transport equation.

So now these imply additional transport equations for Reynolds stress tensor and turbulence flux.

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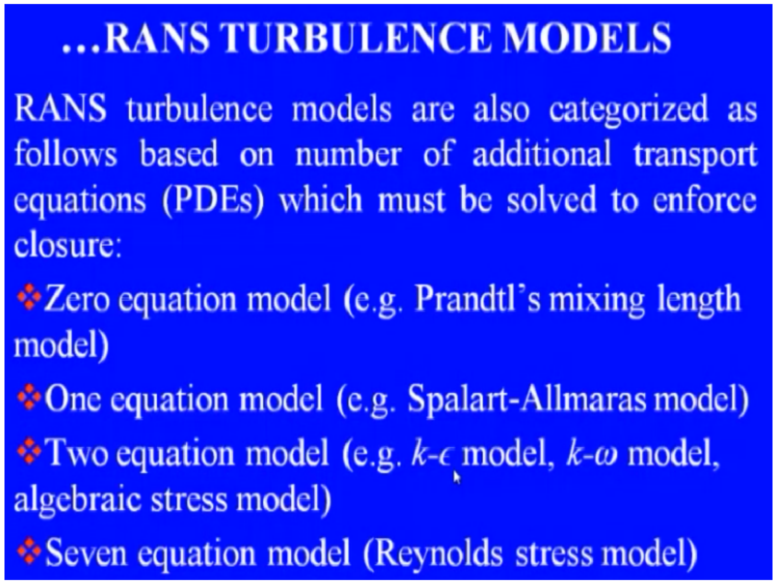

Now let us have a brief look at few other categorization. This turbulence models can also be categorized based on the number of additional transport equations, which we must solve. For instance if you are solving a time dependent problem or incompressible flow problem, we have to solve continuity equation and 3 momentum equations. So we have to solve 4 partial differential equations in laminar flow.

For turbulent flow, to take care of these fluctuating components or what we call RANS components, we have to use additional number of partial differential equations. So how many number of additional transport equations which we need to employ a categorization of RANS models can be based on that as well. So the number of PD additional PDs, which we must need to enforce closure.

The simplest one would be, we do not use any additional partial difference equation. We somehow get an estimate of this eddy viscosity based on the time averaged solution for the velocity field and this popularly referred to as Prandil's mixing length model and since there are no equations, no additional PDs are involved, this is referred to as 0 equation model. Then we can have one additional PD for instance one model known as Spalart-Allmaras model that implies 1 additional partial differential equation to compute the eddy viscosity.

So that is why it is referred to as one equation model, but the most popular in industry are what we call are 2 equation models. For example k-epsilon model and k-omega model are algebraic stress models, which imply 2 additional transport equations, 2 additional partial differential equations. For instance k-epsilon model, you will have 1 equation for this kinetic energy k, and 1 equation for the dissipation term epsilon.

If you go for Reynolds stress model, we have basically 7 equations. So Reynolds stress model is referred to as 7 equation model. So you can just realize that if you want to solve incompressible flow problem, we had four equations, 1 corresponding to continuity equation and 3 for momentum equations and to incorporate these Reynolds stresses, we need to solve for 7 additional equations, which will lead to a tremendous computational burn.

So thus far, these Reynolds stress models they are used rather infrequently in industrial CFD simulations.

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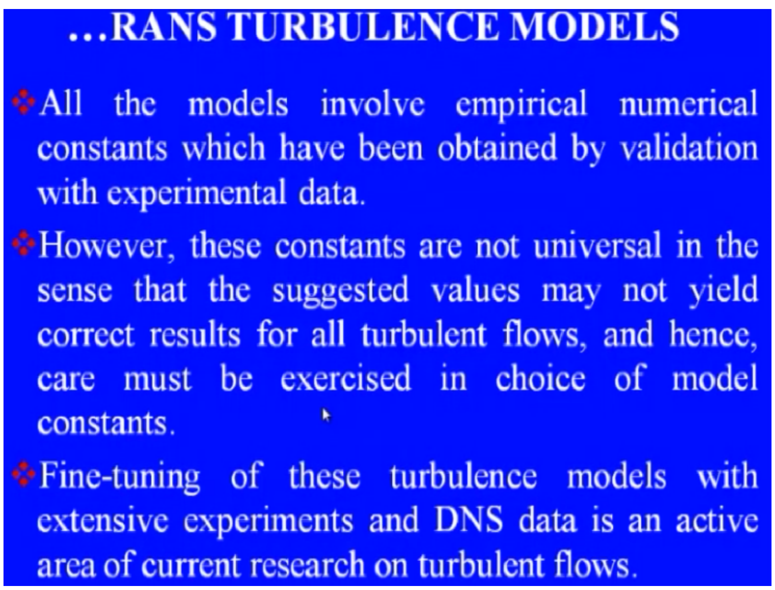

Some more things, which we must remember that all these models they involve empirical numerical constants. They are called models. They have not been arrived from vigorous first principles of continue mechanics, majority of them are based on an extensive experimental data and empirical numerical constants have been obtained based on the validation of an assumed model with experimental data.

These constants which we get from the experimental data of course, those will depend on the circumstances of the problems for which the experiments were performed. So that is why these constants are not universal in the sense that suggested values may not yield correct results for all turbulent flows and hence care must be exercised in the choice of the model constants. We might have one set of model constants.

Let us say in one type of flow and for a different type of flow, we have to go for either a different model or we have to fine tune or we might have to change these model constants. The fine tuning is typically achieved using 2 means that is we can perform extensive control experiments, that is one. Another way is to use direct numerical simulation on a very small computational domain performed very fine grid direct numerical simulation.

That can be used to fine tune the model constants in the RANS turbulence models. So both of these approaches these are very active areas of current research in turbulent flows that is perform direct numerical simulation on certain set of geometrics, certain flow situations and perform a RANS simulation for the same problem for similar type of problem, which involves similar physics and then try to fine tune the model constants used in RANS model.

So that the RANS model gives the average quantities which are fairly close to what we would obtain from direct numerical simulations. So it is still a very active open area of research. Now we will have a look at Boussingsq preposition which we just referred to. Now this Boussingsq preposition is based on 1 simple observation which Boussingsq had that looked if we had a simple laminar flow, viscous stresses they are proportional to what. They are related to velocity gradients.

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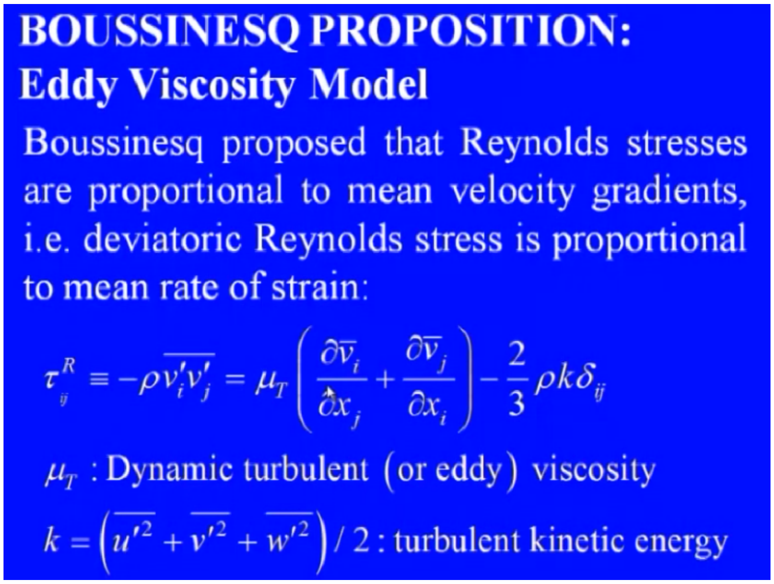

So they are proportional to velocity gradients or what we call as strain rate tensor and normally for new term includes we say that deviatoric components, they are directly proportional to what viscosity times the strain rate tensor. So can we do the same thing. So with tremendous experimental evidence and theoretical evidence to suggest that the rate of mixing due to turbulence or the turbulent ADs themselves, they depend on the velocity gradients in the flow.

So based on those experimental and theoretical observations, Boussingsq proposed that even Reynolds stresses, they should be proportional to the mean velocity gradients especially the deviatoric Reynolds stress, it is proportional to the mean rate of strain. So in terms of the symbols, which of interest tau ijr that is what is our Reynolds stress tensor, which is =-rho vi prime vj prime over.

This can be represented in terms of mu t*velocity gradient or strain rate tensors or mu t times del vi bar/del xj+del vj bar/del xi-2/3 rho k delta ij. So the first term on the right hand side, you can clearly say this is mean velocity gradient. The constant multiply which we have taken, in fact this is not a constant. This would depend on position of the flow fields. It will vary from point to point in flow field and this particular symbol.

Since we have used the same symbol which we used for viscosity. So this is referred to as mu t is called dynamic turbulent or eddy viscosity. The term eddy is often used to correlate formally that look these stresses arise because of the different eddies, which are there in the turbulent flows. So that is why this mu t is called dynamic turbulent or eddy viscosity and the k, which we have used in this relationship that is what is referred to as turbulent kinetic energy.

That is to say we obtained mu prime square over bar+v prime square over bar+w prime square over bar/2. This gives us turbulent kinetic energy per unit mass. So this is the Boussingsq preposition, which is the basis of eddy viscosity based turbulence models.

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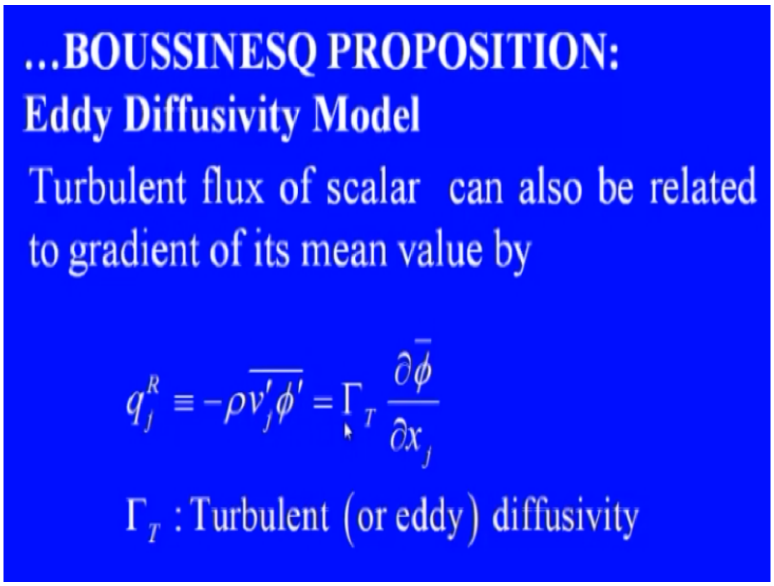

Similarly, the same concept can also be applied to the diffusion of a scalar quantity that is to say that we can say the turbulent fluxes will also be proportional to the gradients of the scalar quantity. So we can say this qjr, which we define as –rho vj prime phi prime over bar=gamma subscript T del phi r/del xj, so this del phi bar/del xj. This is the gradient of the mean value of the scalar field phi and this gamma subscript T, this is called turbulent or eddy diffusively.

Remember this gamma T or mu T which we have introduced earlier, these are not material properties. These depend on the flow and they would vary from point to point. So we have to obtain estimates of these quantities at each point in our numerical simulation and that is precisely what turbulence models attempt to do. So we will take a few of the popular turbulence models in the next lecture. For the time being, we will have just a brief look at the names.

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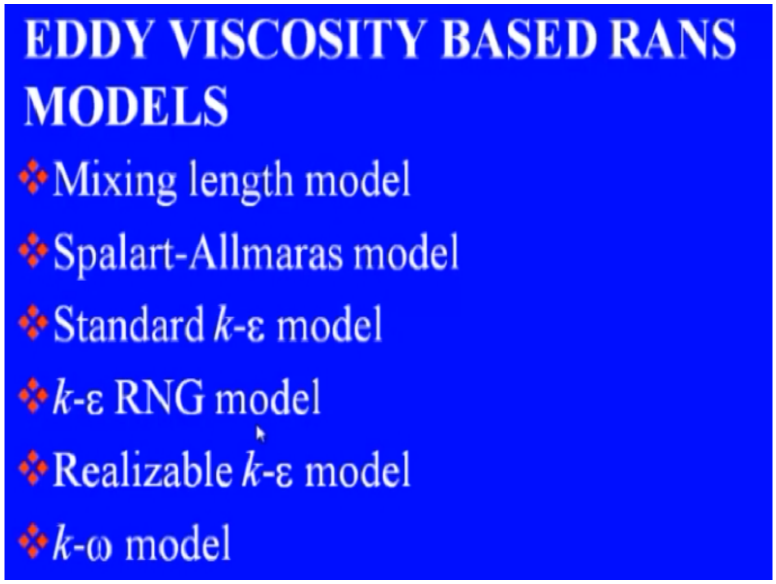

Eddy viscosity based RANS models, we will have a brief look at mixing length model. We will briefly look at Spalart-Allmaras model, then standard k-epsilon model, for the improvement model like k-epsilon RNG model or realizable k-epsilon model or k-omega model, I would refer you to appropriate reference or other textbook. So today we stop here.

In the next lecture, we would have a bit more detailed look at few of these turbulence models and Reynolds stress model and thereafter we will start off with large eddy simulation.