Computational Fluid Dynamics
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Lecture - 39
Turbulent Flows: Features and Simulation Strategies

Welcome to module 9 on Numerical Simulation of Turbulent Flows. This is one of the most important applications of CFD which we would discuss in this module. So outline of the module, we will have a look at the basic features of turbulent flows.

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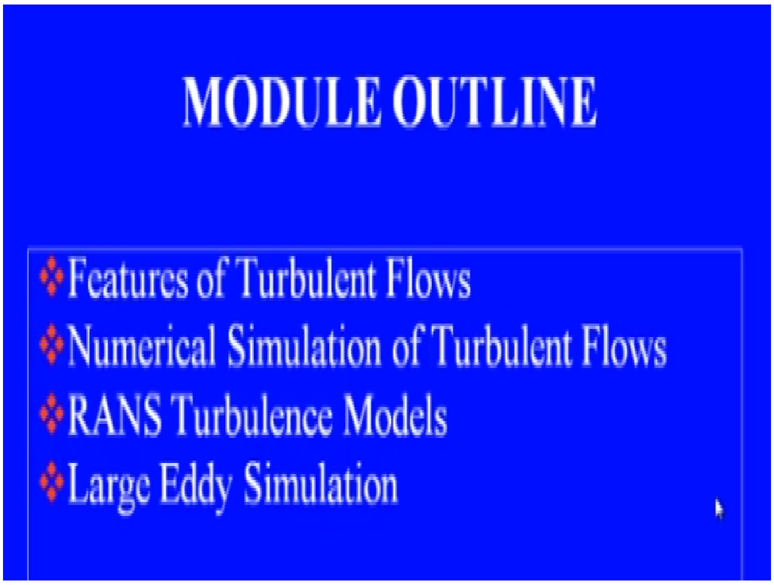

And then we will talk about the numerical simulation strategies which are implied for numerical simulation of these flows. We will look at 2 or 3 strategies in particular what we call RANS simulation. Which is Reynolds-Averaged Navier–Stokes simulation where in certain parts or modeled and they are very turbulence flows which are used in industrial CFD simulation. We will discuss some of those models in this module.

We will then take up one of the most promising simulation strategies specifically with increase in the computational power available to us, what we call Large Eddy Simulation. So, now let us come to the first lecture in this module wherein we will have a look at the basic features of the turbulent flows. Why we need to worry about the turbulent flows and four other simulation strategies which are the ones which are employed in industrial CFD stimulations.

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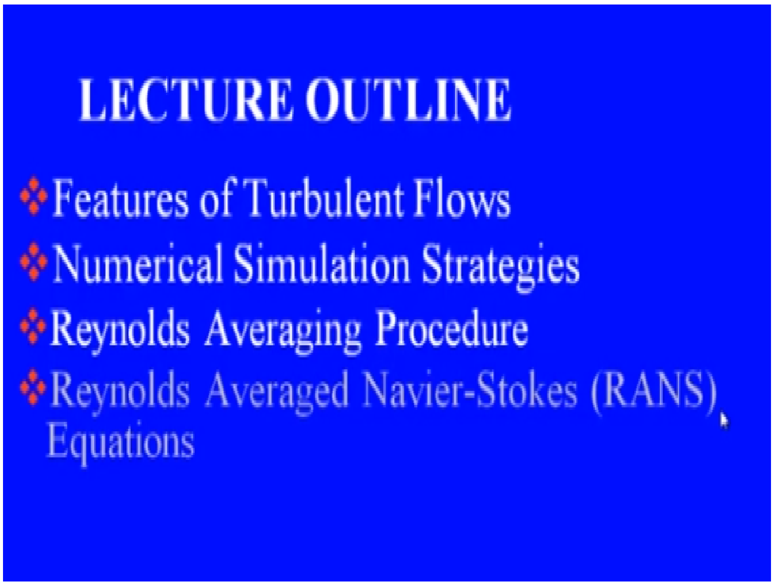

So, outline of the lecture we will have a look at the features of the Turbulent Flows, Numerical Simulation Strategies and then what is referred to us Reynolds Averaging Procedure. And if time permits in today's lecture we will also have a look at Reynolds Averaged Navier- Stokes Equations which are used in our RANS simulations. Now let us see why do we bother about turbulent flows.

That is one thing we have learned about many techniques of CFD simulation but why in particular turbulent flows. What is so special about these flows?

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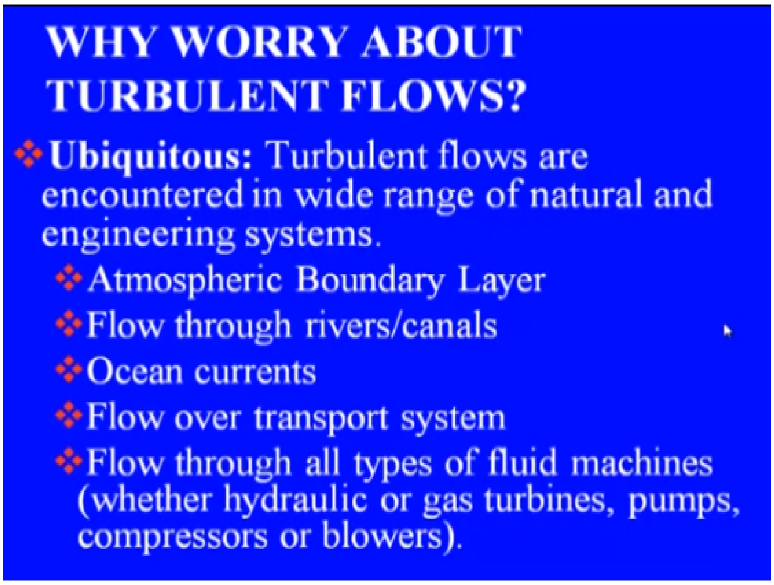

And the answer is very simple. It can be summed up in one word Ubiquitous. You look anywhere all around you, wherever you encounter fluid flow, mass transport or energy transport. The underlying flow most of the time is what is termed as turbulent. So, this is why we will say the turbulent flows are encountered in wide range of natural and enduring systems.

For instance we are live in the atmospheric bounded layer, wherein the air flow is almost always turbulent. And this turbulent air flow has to be simulated taking care of the inputs obtained from different weather stations to have what we call long range as well as short range with a focus. Similarly, the flow through rivers can also go to any river side, you will see the flow.

If you just observe the flow the way it is taking place at a particular point or particular station you will find is continuously changing. And there is no pattern of that change what you will find is pattern is entirely random which is one of the most important features of what we call turbulent flows. Similarly, post incurrence they are almost always turbulent.

Flow over the transport system it could be any of land based, sea based or air transport systems flow over them is what we called high speed flows higher the speed of the vehicle more prominent what the turbulence be. So, the flow over our cars or trains, buses, air crafts the almost they are always turbulent. So, if you want to come up with the design, good design of an air craft or a car we have got to deal with the turbulent flow around it.

We should be capable of modeling all its features and the effect it has introducing the drag which we would like to minimize in the case of a transport system. Similarly, flow through all types of fluid machines which we use in industry, whether they are hydraulic or gas turbines, pumps, compressors or blowers. All of these flows are invariably turbulent. We can also look inside us. The blood flow in our large arteries that is also turbulent.

Let us go out of our planetary system, let us go out of or environment of earth, the interstellar space there are some flow is happening which are of interest to us through phases, this also turbulent. The sun flares, what we called solar flares which affect our life, which affect our communication systems and our energy supplies system in a big way so solar flares are always turbulent.

So, that is why we would say this turbulent flows are found everywhere and we should be able at least get some useable estimate of such flows using numerical simulation or theoretical means or whatever means we have got we should be able to get at these flows.

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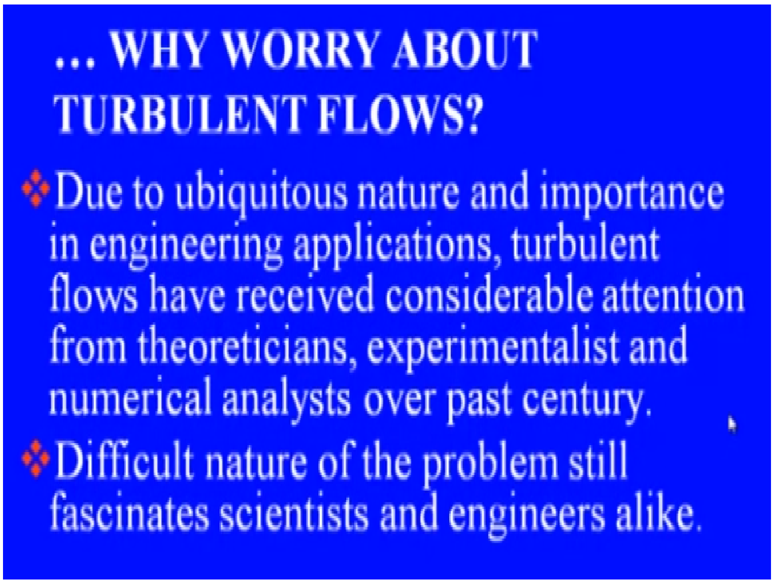

So, it is due to this ubiquitous nature and importance in engineering applications and projects analysis. These turbulent flows have received what we called considerable attention from theoreticians, mathematicians, physicist, engineering scientist you name any experimentalist of all kind and numerical analyst over past century outside other centuries. The first study of turbulent flows takes back to many many centuries ago.

And there has been fewer risk effort in understanding the features of turbulent flow. But sadly it is still one of the what we call unsolved problems of physics. We have not yet been able to solve it in it is entirely. So, the difficult nature of the problem it is still fascinates scientist and engineers alike. So, this is still considerable amount of work to be done using theoretical tools, experimental tools and CFD tools to understand the turbulent flows around us.

So, that is the region why we would like to take up as one of the most important applications of CFD in the numerical simulation of turbulent flows and that is the subject of this module. Now let us have a look at what are the basic features of turbulent flows. In a fluid mechanics class, you might have heard this classification of laminar flow and turbulent flow. Laminar flows are always very nice layers of fluids sliding past each other in a very smooth fashion.

And you might have also heard about the famous experiment Osborne Reynolds through flow through a pipe wherein we steadily increase the rate of the flow mass flow through the tube and the pattern of the flow changes from being what we call a beautiful laminar flow in the beginning when the flow rate is small.

To the appearance of the undulations or unsteadiness in the flow which might die a little later to the full blown random flow patterns and the cube depending on the mass flow rate which we later link to a parameter which we called Reynolds number and what was (()) (07:53) flow that there is certain critical Reynolds number below which the flow would always remain laminar irrespective of the disturbance is up straight.

Though these turbulences will die down and will give us a very smoothly flying laminar flow. And there are certain range beyond which the flow will always contain any disturbance in the flow which come from upstream they get magnified and we get a chaotic flow pattern.

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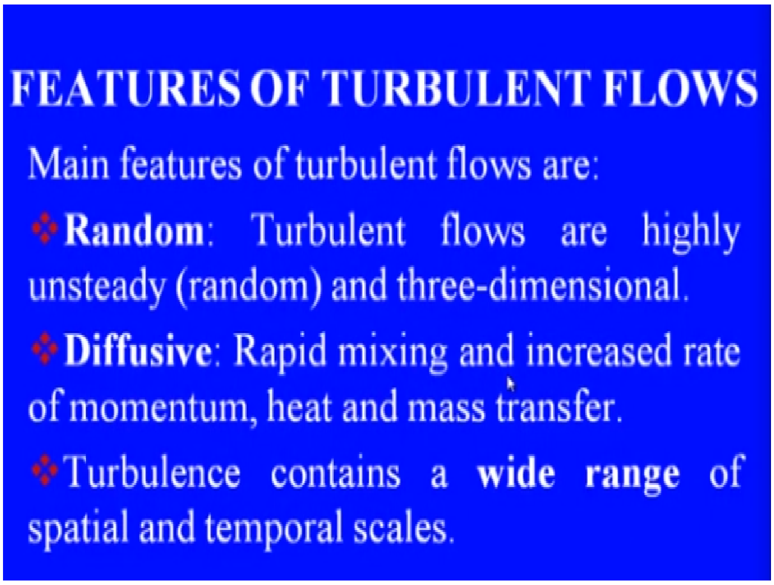

So what are these basic things. One thing which we observed, which we learned in a fluid mechanics classes, it is the Reynolds number which is a strong indicator of the nature of the flow. So higher the Reynolds number, higher is the likelihood of the flow becoming turbulent. But what are other features of turbulent flow? So let us have a look at some of the prominent features of turbulent flow.

The first one, first would which comes to our mind is Random. Turbulent flow is a highly unsteady. In fact they cannot be predicted in entirety. That is why we use the term Random. A simplest example could be that you just go on sit-down near a river stream. Fix your attention at a particular point, we are sitting and just say the flow pattern. The flow pattern will change in an unimaginable ways which is unpredictable.

Go the same time next day you will find a different flow pattern. Though average flow through the river or the canal would be the same as it was in the previous day or rather forget about days just few minutes back or few seconds back. So the turbulent flow, they are highly and a steady or random and there always 3 dimensional. It is nothing like one dimensional or two dimensional turbulent flow.

The second most important characteristic is what we call diffusive. This is a rapid mixing in turbulent flows. This rapid mixing is of great important to us in our chemical process where we require thorough mixing. So, we have in turbulent flows rapid mixing which leaves to increased rate of momentum, heat and mass transfer. In many of the chemical industries it would be one of the most desirable features.

And when we dealing with the transport industry there it would one of the features which we would like to hate or rather you would try to minimize its effects because it also leads to increased drag. So, if you are designing a car, you are designing air craft we would like to reduce this diffusive effect so that the drag is reduced on our transport machine. And if you look carefully in turbulent flow for example go and observe have a look at the way the flow is taking place in a river.

What you will find then a wide range of structures. These structures, some structures are very small some of it is are very large. Some last for a very long time. They remain almost similar say they retain their shape over a fairly good period of time. And the some we just they arise and vanish. So, that is why we would say that turbulent flows or turbulence contains a wide range of spatial as well as temporal lens of scales.

We will have some structures, vertical structures which are fairly large and they would retain their shape for a fairly large time. So, they have got large lens of scale or vertical spatial scale as well as large time scale. There were some smaller areas which arise and then they are rapidly vanished. So, we will say they those areas have got a very small length in time scales.

And if you want to have a complete grasp or understanding for turbulent flow we should be able to understand what is happening at all these length and time scales for all these spatial and temporal the scales of the same time and what is the effect of their interaction.

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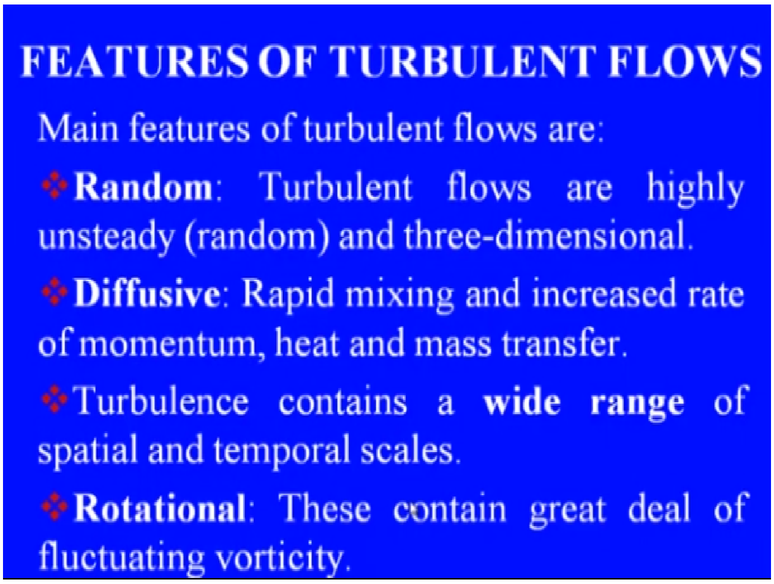

The next characteristic is what we called rotational. The turbulent flows they always contain a great deal of fluctuating vorticity. You go to a smaller length of scales and there fluctuation becomes even more random and vigorous. So, this is in contrast to what we used to make assumptions which you used to make in our fluid mechanics class that if you want to solve a problem away from the solved boundary we will invoke the irrotationality assumption that flow could be treated as irrotational.

And we came up with this scalar potential which we can solve for and thereby we can obtain the solution of a flow problem we can get the velocities and use Bernoulli's theorem to get the pressure distribution and so on. But the same thing we cannot do with the turbulent flows because turbulent flows are never ever irrotational. They contain lots and lots of vorticity at different length and times scales.

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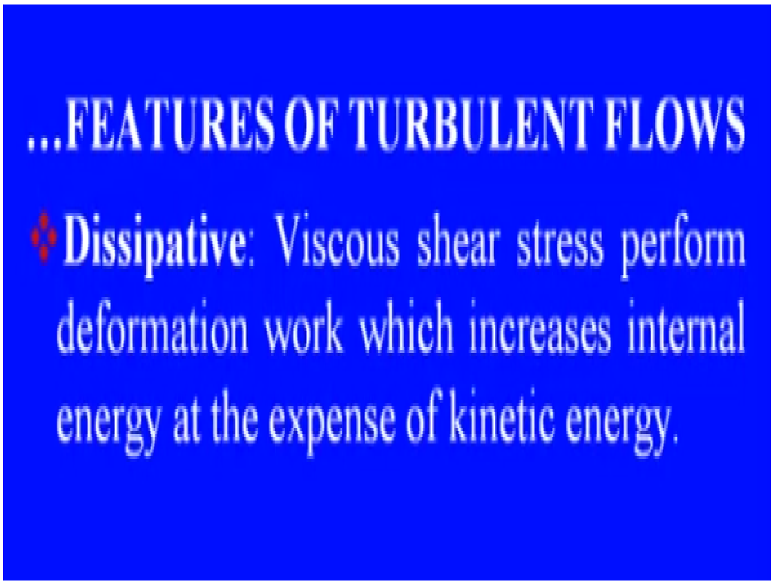

The next most important feature is dissipation. Turbulent flows are always dissipative and this dissipative comes up because of our viscous action. And there would this is sort of dichotomy here**.** We would say the turbulence occurs at higher Reynolds number. At high Reynolds number viscous effects are fairly small compared to the initial effects. But the losses in the turbulent flows or dissipation loss of energy occurs because of the viscous shear stress.

In fact, this dissipative is entirely due to the action of viscosity. So, this viscous shear stress they perform deformation work which increases internal energy at the expenses of the kinetic energy of the fluid. And to maintain the turbulence basic features of this random flow turbulence needs a continue supply of energy to make up for the viscous losses. The energy which is being lost at what we called dissipative scale by action of molecular viscosity.

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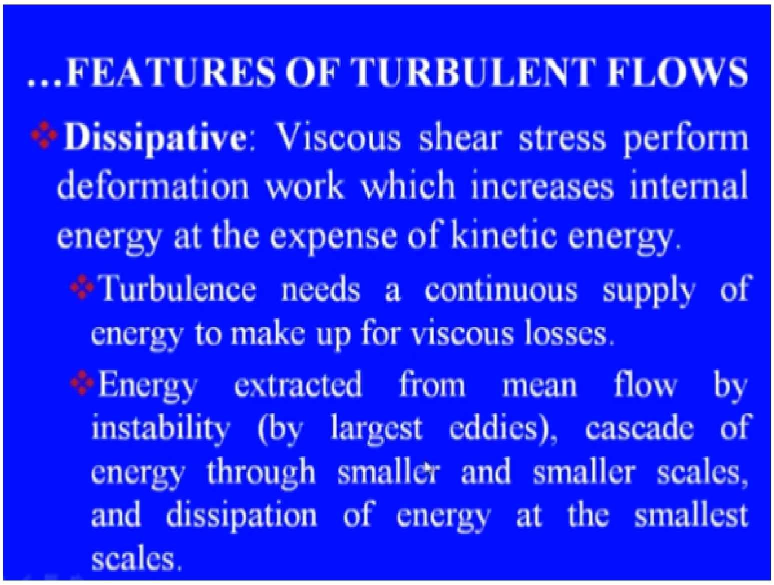

And from where do we get this energy? This energy is extracted from the mean flow by what we call instabilities and one of the ways in which it is characterized is what we call eddies or largest eddies in the flow. They are the once which extract the energy from the mean flow and then that energy is cascade it or what we call pass through in a hierarchy of areas.

So we have got let us say, pretty large area which whose length of scale might be equal to the dimensions of the flow of interest and then we have got slightly small eddies which is smaller than that and smaller than that and so on. So, why we have got what call a cascade of eddies and through this hierarchy this cascade of energy through its smaller and smaller scale eddies.

And ultimately there is a dissipation of energy at the smallest scale which we call dissipation scale. We will discuss these scales in bit more detail little later.

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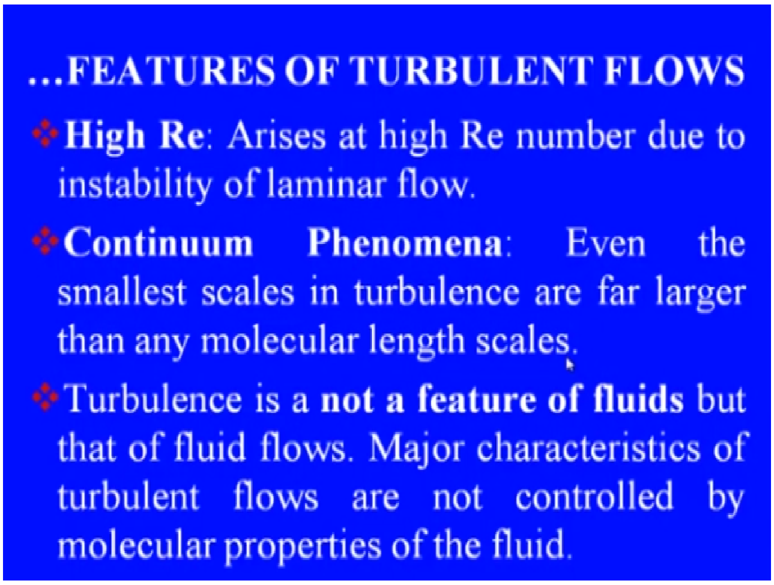

Now I noted earlier turbulent flows always occur at high Reynolds number due to instability of the laminar flow and please remember this though the turbulent flow is random we talked about various lengths of scales of wide lead differing magnitude. We talked about some very large length scales which we, which are linked to the largest eddies and very small length of scales where the energy is dissipative due to molecular action.

Nevertheless, the turbulence is still what we call a continuum phenomenon. So, with the smallest scales which we encounter in turbulent flow they are far larger than any molecular length scales. So, we are not in the realm of dealing with each molecule separately. We are still talking about the effect at a gross level what we call a continuum level. And also remember the turbulence is a not a feature of a fluid.

It is nothing like a turbulent fluid but it is a feature of the fluid flow. And major characteristic turbulent flow they are not controlled by the molecular properties of the fluid. So, you might have seen the water flowing through different channels and one channel depending on the dimension of the channel and the flow velocity the flow might be turbulent. In other ones it might be laminar.

So, the turbulence does not depend on the properties of the fluid but it depends on the other features of the flow. Now let us talk about the scales which we encounter in turbulent flows. We talked about that we encounter wide range of length and time scales in turbulent flows. So, we will have a qualitative description of some of these scales the way they have been proposed in the literature.

We will talk about these scales and we also talk about the rate of dissipation of kinetic energy and we will talk about characteristic flow velocity and lens scales and then we talked about what we call energy cascade which was proposed by Richardson way back in 1920's. So, what is this energy cascades? So, some of these features we are going to discuss in detail and what you mean by large areas and small areas, okay.

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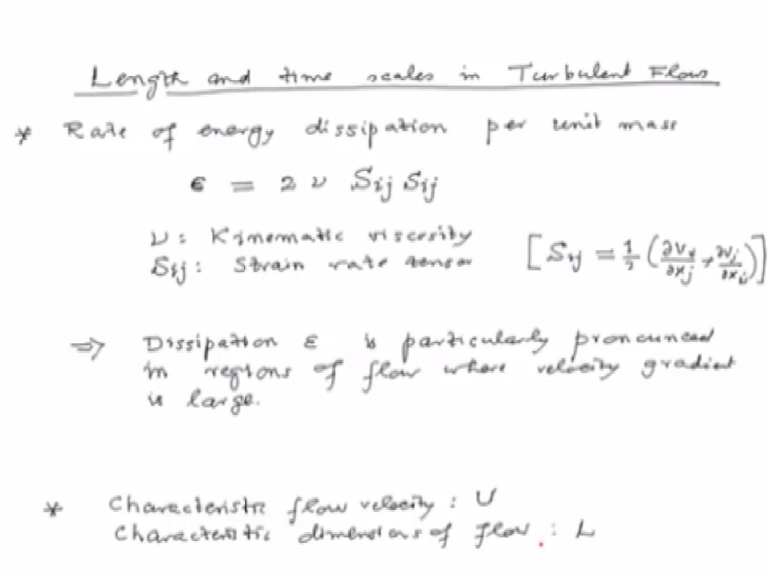

To understand these let us go back to about, so length and time scales in turbulence. Before we deals with these scales this one particular quantity which is very important in turbulent flows which we call rate of dissipation of kinetic energy. That is the energy which is dissipated at the smallest scales of the motion in turbulent flows. So, can we get a major of that dissipative energy.

So, this rate of energy dissipation per unit mass and it is very popular and later it has used symbol epsilon to denote this energy loss. So, epsilon is gained by 2 nu Sij Sij. So, Sij into Sij we have already been through this Sij, it is a scalar product of these second order tension. So here nu is our kinematic viscosity and Sij is our strain rate tensor which if you can recall we have defined it in terms of the velocity gradient tensor.

So, this Sij was defined as half of del Vi del xj + del Vj over del xi. So, you can clearly say this strain rate in a fluid anywhere. That is proportional to the velocity gradients and this epsilon it depends on the product of the strain rate tensor with itself. So, one consequence is very clear that higher the velocity gradients higher would be our dissipation right.

So, dissipation is pronounced is particularly pronounced in those regions, in regions of flow where velocity gradient is large. Now we have introduced this term that rate of energy dissipation because we one of the length scales this smaller length of scales we would define in terms of this energy dissipation per unit mass. So, this is our first definition. Secondly, we are dealing with the flow.

Now, whenever you talk about a flow that happens in certain situation or certain surroundings. So, there would be a characteristic velocity sort of you can think of an average flow velocity. So, we will talk about the characteristic flow velocity. Let us denote it by symbol capital U. Similarly, we can also talk about the dimensions or what we call characteristic dimensions of the flow region let us denoted by symbol L.

Both of these you can easily correlate with what happens in the case pipe flow. In the case of flow through a pipe U could be the average velocity and L could be identifying as a diameter of the tube.

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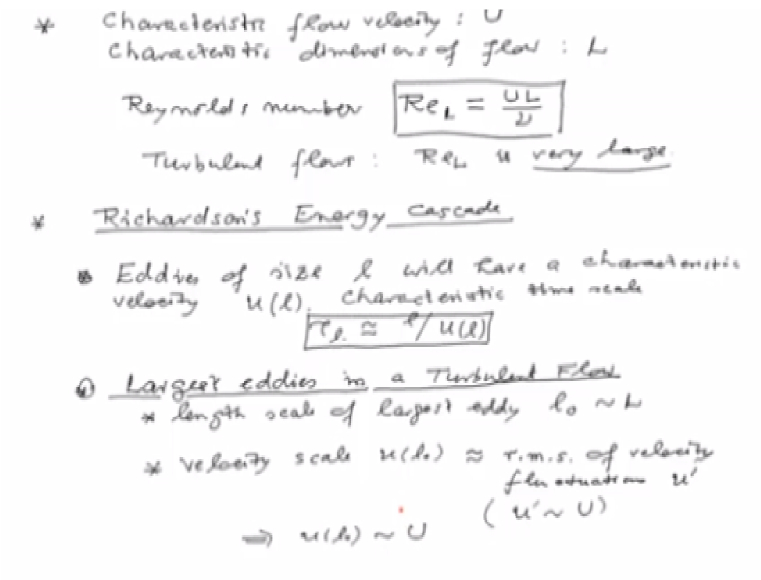

Now based on these 2 characteristic scales that velocity and length scale we can define a Reynolds number or Re let us put a subscript L to indicate that it is based on our characteristic length scale which is, which characterize is our flow domain. It could be diameter of the tube; it could be let us say width of which or depth of a channel or it could be the span wing span of an aircraft depending on weather flow is taking place what is the flow of interest to us.

Re L is U into L by nu. In turbulent flows we are particularly interested in what we call large Reynolds number flows. So, turbulent flows this would happen when this Re L is very large. So, what is this very large that is again it is dependent on the problem and definition of the large thus in different situation we have got different limits on what we call critical Reynolds number when transition from laminated to turbulence takes place, okay.

Now based on, now these are few defining things for us we have got Reynolds number and we have got our characteristics velocity and length's scales. And then next we are going to talk about what we call eddies in our Richardson energy cascades. So what happens in this energy cascades? Richardson talks about eddies which are vertical features in a flow. So, eddies, an eddy would be identified by its length of scale of size let us call it l will have a characteristic velocity.

Let us identified as U (l). We know the length's scale, we know the velocity. So what will be characteristic time scale? Let us call it tau subscript l. This would be l divide by U (l). So, far so good. Now how do we identify what we call large eddies and how do we differentiating what we say what would be the small eddies. So, what would be the largest eddy?

Can we come up with an estimate of the size of the largest eddy in the flow in a turbulent flow. Now there are 2 things which we can easily identify specifically feel recall the flow through a channel or follow through the rivers. We might find the vertical structures which occupy almost the whole width or depth of the flow. So, as far as the length of this largest eddies is denoted by L0, L0 they would be of the same order as that of the characteristic dimension of the flow.

So, the length of scale of largest eddy let us denote it by l 0. l 0 would be of the same order as our characteristic length scale of the problem capital L. How about velocity or what we called characteristic velocity of these large eddies or largest eddies? So, velocity scale which we will call as U (l 0) can be obtained estimate of this velocity. For that we have to look into 2 parts of the turbulent flow.

The flow velocity can put as or could be decomposed later on we are going to formally define that Reynolds decomposition as an average component and a fluctuating component. This fluctuating part is the one which is a feature of the turbulent flow and we can get its root - square value. So, the velocity is scale of this eddies U (l 0). This is identified as that r.m.s of velocity fluctuations.

And very often in higher Reynolds number flows this r.m.s value is of the same order of magnitude as our characteristic flow velocity. So, if we call this r.m.s fluctuation velocity as U prime. This U prime is of the same order as Capital U. So, this implies that our U (l 0) is of the same order as capital U. Now how about Reynolds number link to the largest eddy?

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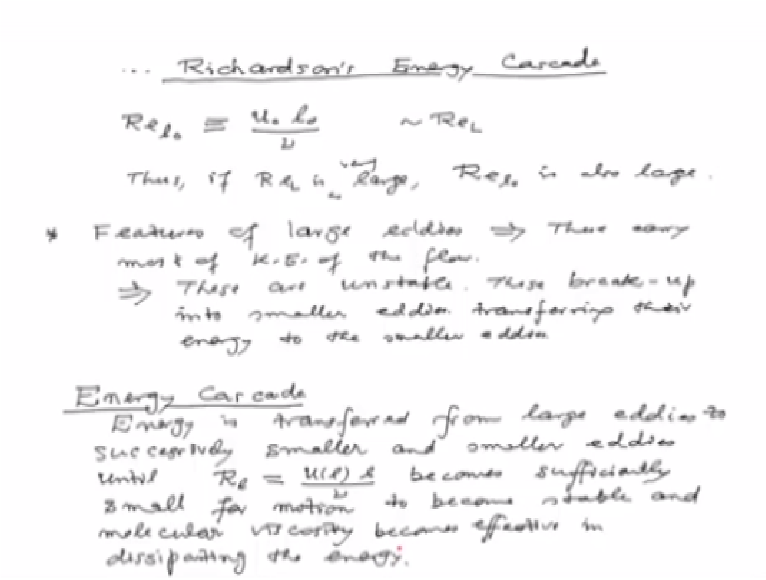

If we call it as Re l 0 this is defined as U 0 l 0 by nu hence we saw that U 0 is of the same order that of capital U shows the characteristic velocity. l 0 was the same order as our characteristic dimensions. So, this U 0, l 0 divide by nu it would be roughly of the similar order as our Re L. So, if Re L is very large, so thus if Re l is large or very large this Re l 0 is also large, rather we have already seen it is of the same order of magnitude is Re l 0.

Now what was the observations of Richardson? His proposal was that look these largest eddies or the ones which carry the most of the energy in the flow and what are the features of these largest eddies? So, these carry most of the kinetic energy of the flow, this one important feature. Next feature is these are unstable. Then unstable is what drives a turbulent flows.

So, these break up into smaller eddies to which they transfer the energy. Transferring the energy to the smaller eddies. And in fact it does not stop at one level, it continues. So, energy is transfer from one large eddy to the next smaller one in which breaks up to this, further smaller one to further smaller ones and so on. So, there is what we call a cascade of energy. So, that is what Richardson termed as energy cascade.

And formally we can define as energy transfer. Energy is transferred from the largest eddies to successively smaller and smaller eddies until this Re l which we had defined as Ul times l by nu becomes sufficiently small, small for viscosity to take over. So, that our motion becomes, from motion to become stable and molecular viscosity becomes effective in dissipating the energy.

So what do we do, we can think of a mental picture, let us say we have got very big or a very large scale eddies.

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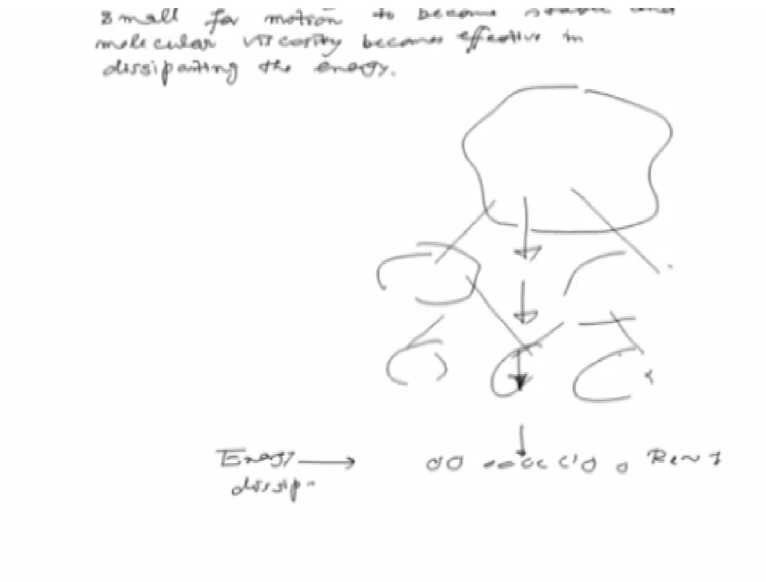

Now these eddies they break into its smaller ones. The smaller ones will then break into further smaller pieces and so on. And at all these case there is a transfer of energy from one to another. Until we have reach at this smallest level where the Reynolds number now here it becomes of the order 1 and at these smallest levels there is no further breakup the eddies. Eddies are fairly stable.

Now the viscosity takes over and energy is dissipated, okay and this is what was termed by Richardson as the energy cascade that they are largest eddies. Now these largest eddies they draw the energy from the mean flow and dissipate it to successively its smaller eddies. Now one thing which very clear in those pictures that energy dissipation right will depend on how much energy has been extracted from the flow where the largest eddies, okay.

And whatever energy is extracted that would be dissipated at the smallest scales. So, the largest eddy is of course that will depend on our Reynolds number. Now let us define a few more length scales. We have already find what is, what we call a large eddies scale or largest eddies lengths scale which is very similar to or of the same order as our characteristic scale of the problem.

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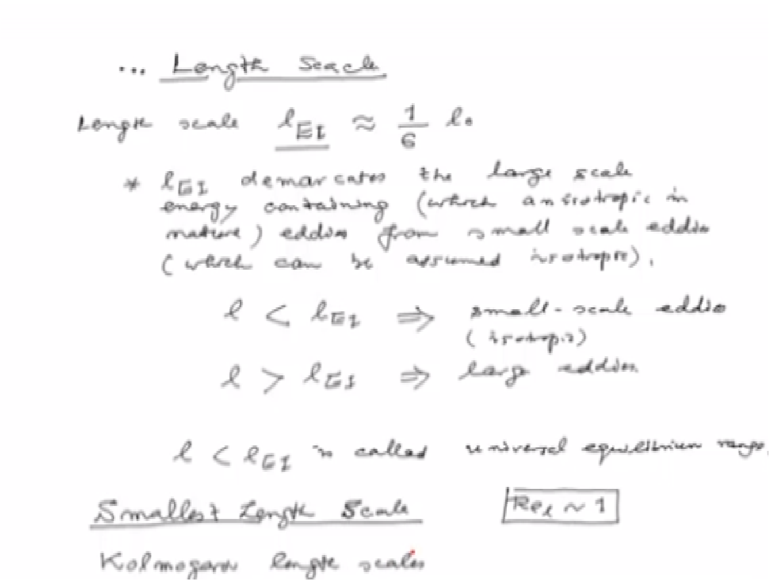

Let us introduce it and the length scale. This denoted by symbol lEi and we will say it is approximately 1 by 6 of l 0. So, what is the purpose of this lens scale? It is to demarcate this particular lengths of scale which we will call as equilibrium length of scale and this lEi demarcates the, rather it provides a demarcating its large scale energy containing eddies which are anisotropic in nature from small scale eddies which can be assumed isotropic.

So, there is this one small of very important purpose which length scale shows, it says they look, it provides sort of a demarcation that if l is less than lEi. That is the length scale we will say these are our small scale eddies which can be treated as isotropic and l greater than lEi we will call them as large eddies. These are the ones which would be primarily extracting the energy from the mean flow, okay.

Now this l less than lEi we will have a specific length scale range. This is referred to as universal equilibrium range. Now let us come down to the smallest length of scale. So, how can we figure out what would this smallest lengths of scale be? The smallest lengths scale is the one where we say that our eddies become relatively stable for the viscosity to become dominant at that level.

The viscosity to become dominant we should have that the Reynolds number linked to the eddies should be roughly if the order 1. So, that is what we are looking at the smallest level. Now, Kolmogorov came up with the estimate of these lengths of scale. So these are also that is why they are called Kolmogorov length scales and they are defined in terms of our Kolmogorov scales. We are defined in terms of our energy dissipation.

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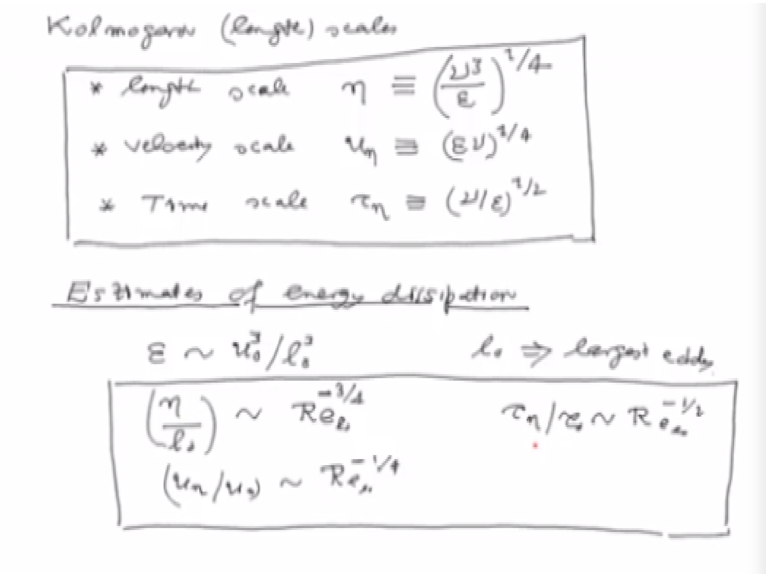

So, length scale we would use the symbol eta. Eta is defined as nu cube divide by epsilon to the power 1 by 4. Now this estimates were based on the dimensional arguments. Similarly, Kolmogorov defined a velocity scale. Let us denote it by U subscript eta this was defined to be epsilon nu to the power 1 by 4 and the time scale linked to this smallest eddy tau eta, this was defined as nu by epsilon to the power half.

And you can do a simple algebraic calculations and find out what would be the Reynolds number based on this length scale and time scale. What is the order of that Reynolds number? Now, before proceeding further let us have a look at few more things. The dissipation somehow estimates of energy dissipation. This epsilon is estimated to be order of U 0 to cube divide by l 0 cube where of course our U 0 and l 0, l 0 represents our largest eddy.

And we can easily see that eta by l 0 this is of the order of Re l 0 to the power – 3 by 4 and U eta by U 0 that is the ratio of the velocity length of scale for this smallest eddies to the largest eddy it is up the order of Re l 0 to the power – 1 by 4. And similarly, your time scale tau eta by tau 0 this is the order of Re l 0 to the power – half. Now these scales or these relations are pertinent very pertinent in the simulation numerical simulation of the turbulent flows.

We will see in one particular situation what we call direct numerical simulation wherein we would like to resolve of flow at all length and time scales. So, if our lengths scale of the problem is of the order l 0 we can easily see our requirements that how small or how fine this grid size should be. So, this eta gives us an estimate of the grid size which we must use if you want to resolve all scales of motions.

Similarly, in time integration we have got to choose the time step which should be of the same order as tau eta which we can estimate from our Re l 0, remember it is of the same order RE l. So, if you know Re L which is very easy to estimate or Re L. Based on that we can obtain an estimate of the time step and the grids size which we should use in our numerical simulation if you want to resolve all scales of motion.

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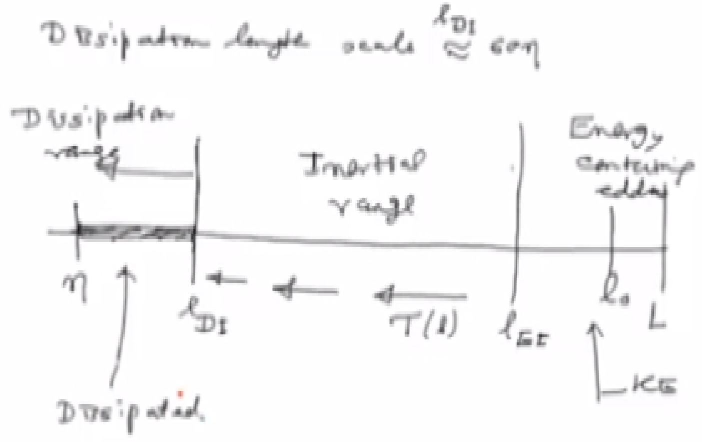

So to summarize there is discussions which we have let us draw a diagram. Suppose this was well in scale. Capital L over, these are energetic eddies or large eddies l 0, l e i we took as 1 by 6th of l 0. So, this is the one which demarcates. This energy containing eddies from this small scale eddies and the small scale eddies they are the ones which first transfer energy from the large eddies to the small to small and in the end they reach the smallest lengths scale which we call eta.

Now dissipation is start much before eta. So, let us call let us define it another lengths scale. Let us call it as dissipation lengths scale. It is roughly defined to be of the order of 60 eta and we use the symbol L Di for it. So, this is our L Di. So, between L Ei and L Di this particular range is referred to us inertial range. And beyond this L Di the length scale is more than L Di. This is called as our dissipation arrange.

So, what happens here in that energy containing eddies at this lengths scale the eddies will get the energy from the mean flow KE goes to the eddies. In inertial range there is a transfer of energy from one lengths scale to another lengths scale. So, there is a transfer until we reach this dissipation range and it is in this dissipation range where the energy is dissipated.

And it also gives some idea about the numerical strategies which we should adopt and we will have a look at few of them that what are numerical simulation strategies which are used for turbulent flows.

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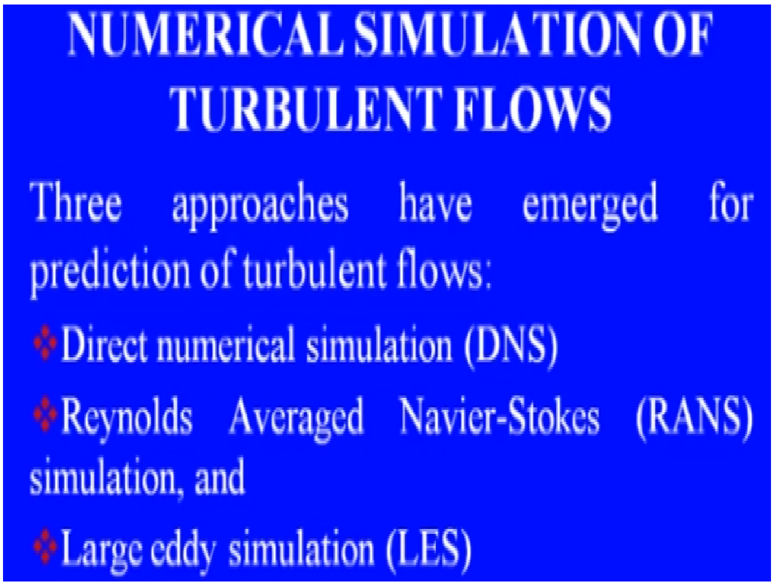

So, there are 3 approaches which have emerged for prediction of turbulent flows. The first one is what we called direct numerical simulation. Direct numerical simulation something which is the simplest one we said look we are going to take the grid which is fine enough a grid size would be of the same order as that of our Kolmogorov lengths scale eta, time is step-up with the same order it is Kolmogorov time scale tau eta.

So, that is what we call out direct numerical simulation. The next category is what is known as Reynolds-averaged Navier–Stokes simulation and the third one is called large eddy simulation. Now let us have a brief look at these 3 strategies based on our graph which we have done earlier.

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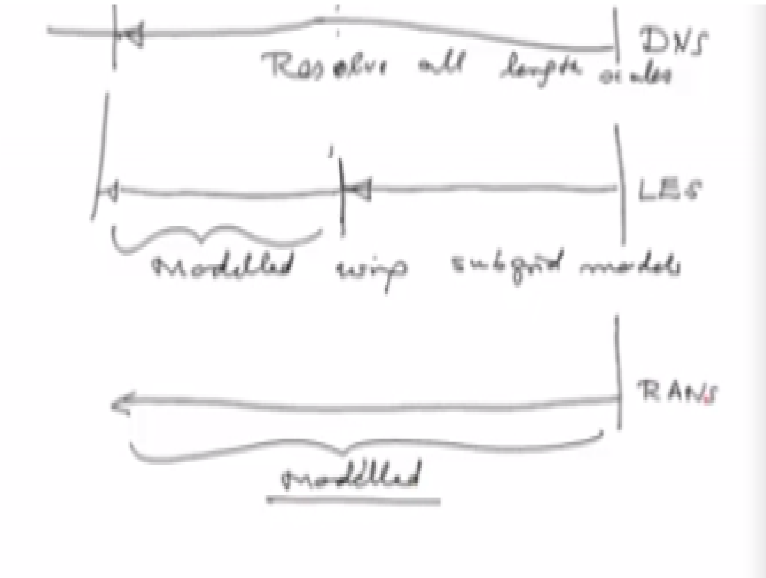

In terms of the lengths scales simulation strategies we had our problem lengths scale capital L. Now here is our l 0, l Ei, l Di and eta. So, let us note down our ranges. This is the range of energetic eddies. This is our initial range and this was our dissipation range. So, what in effect we said in DNS we are going to take up all the length scale up to this up to eta.
So, here resolve all lengths scale.

So for our grid size if it is of the order of eta, yes we have been able to resolve all the lengths of scales is what we call direct numerical simulation. What happens in the case of large eddy simulation? Large eddy simulations look we are going to resolve all these large eddies or energetic eddies. So, in fact most of these will be resolved. We will take some our grid size which is somewhere in between l Ei and l Di.

So that we are absolutely sure that all large scale eddies which contain most of the energy the flow they have been resolved fully. So, up to this point ever, flow all the lengths fully resolved below this up to our eta. This part in large eddy simulation is what we called modeled using sub grid models. And in contrast to these 2 in our so called RANS simulation what do we do that everything is modeled, okay.

No attempt is made to resolve any of the eddy structures. So, this how the based on this length scale that is the division of these 3 simulation strategies.

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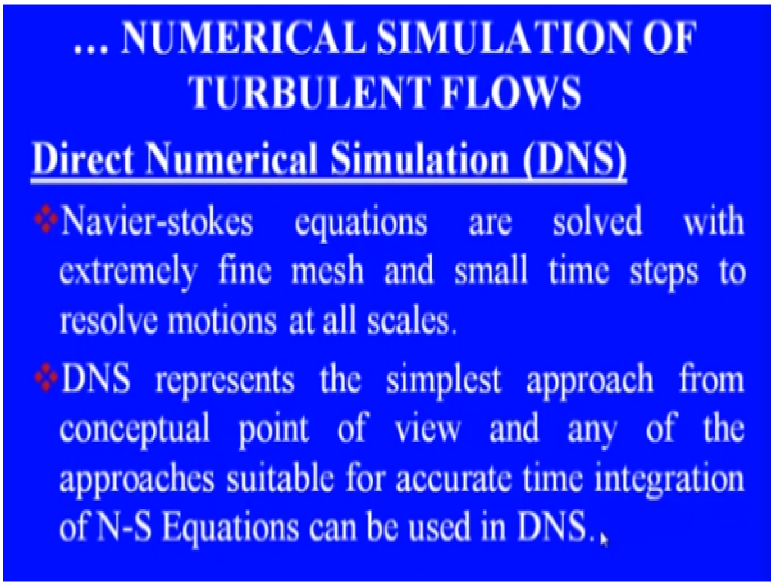

Let us have a brief look at some of the features of the simulation strategies. First direct numerical simulation, so here we are going to solve Navier–Stokes equations with extremely fine mesh. So that our grid size is basically of the order of eta and in small times steps so that time is of the order of tau eta to resolve motions at all scales.

So, in terms of programming effort in terms of using a particular computational methodology DNS represents the simplest approach from the conceptual point of view and any of the approaches suitable for accurate time integration of Navier–Stokes equations can be used in direct numerical simulation.

So, if you want to write a code in fact the code which we would write for laminar flow that is what can be used for a direct numerical simulation turbulent flow as well as long as we take care of the mess size and small time step requirement. So, nothing is specially to be done in this case. Then what are the down sides here? Will have a detailed look at the down side a little later.

Reynolds average simulation here we would later on see a time averaging procedure. So, we will have, we will not solve the Navier–Stokes equation per say but we are going to solve what we call averaged equations.

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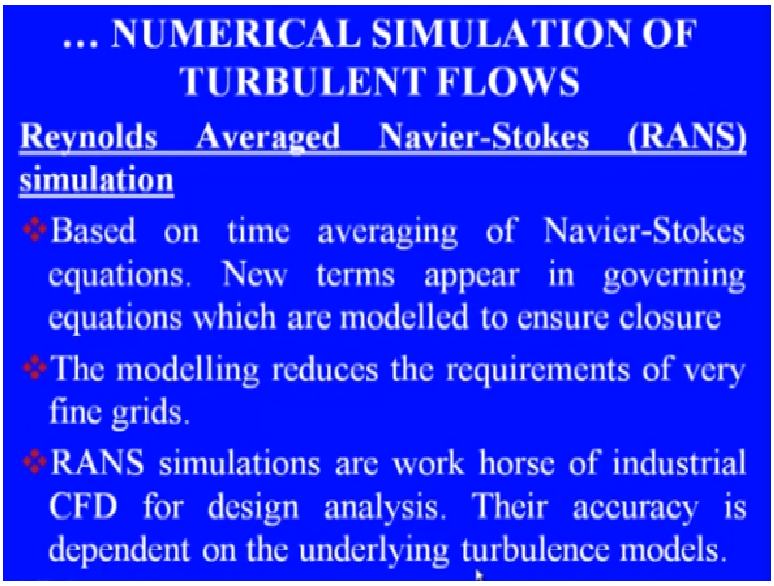

There are new terms to going to appear in governing equations which are modeled to ensure closure. That is why we saw in the previous diagram that the lots of modeling which is used modeling at of all the eddy's scales.

And the modeling reduces the requirements of fine grids. Now RANS simulations or what we call work horse of industrial CFD for design analysis. They are the only ones which are being used from past few years for design analysis purposes in industry. And their accuracy of course would dependent on the turbulence model which have been used to ensure closure.

And the last one is for the large eddy simulation. So, here LES resolves the largest scales of motion that is the most energetic eddy is of fully resolved while modeling this small scale sub motion which we saw in the previous diagram.

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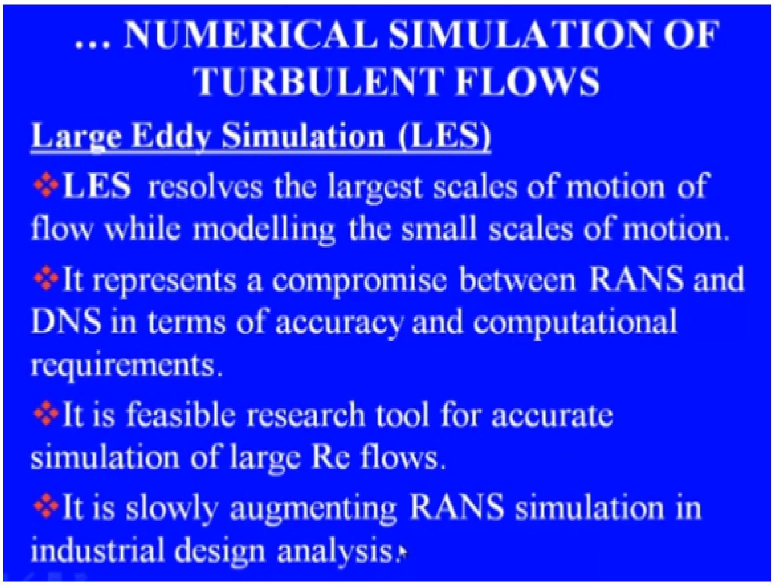

So, it represents a compromise between Reynolds-averaged Navier–Stokes simulation and DNS in terms of accuracy and computational requirements. So large eddy simulation results or less accurate compared our DNS but their computational requirements are again much much less compared to DNS. And they are far more accurate than RANS. And it is feasible research tool for accurate simulation of large Reynolds number flows at the moment.

And that is why it is slowly augmenting Reynolds-averaged numerical simulations in industrial design analysis by those companies which can effort to have a very large scale parallel cluster. They are now using this large eddy simulation in the final design stages.
For the initial design analysis RANS is still what is being used. It is only in the final iterations we will go for large eddy simulation.

Let us have a bit more detailed look at DNS, features of DNS.

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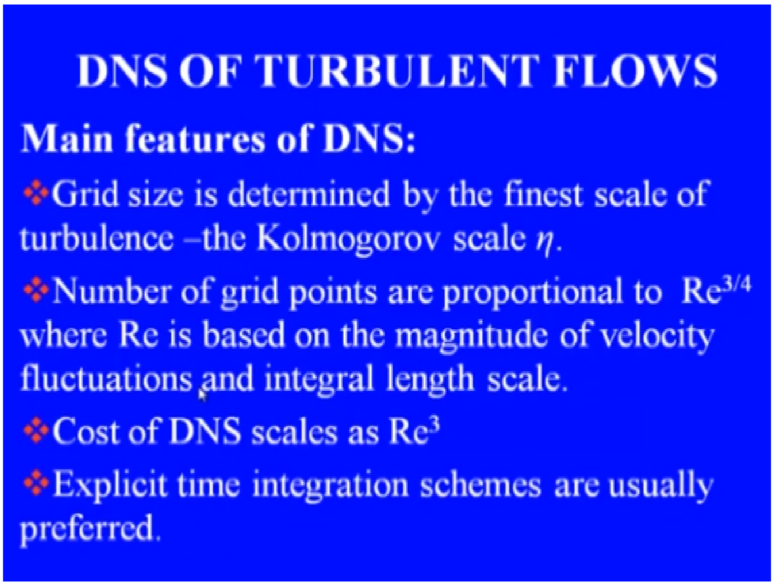

We said we are going to resolve all these scales the grid size is determined by the finest scale of turbulence as our Kolmogorov scale eta which was defined earlier and you can easily work out based on the definition of eta that number of grids points are proportional to Reynolds number to the power 3 by 4. Where your Reynolds number we have defined its Re l 0 that is to say it is based in the magnitude of velocity fluctuations and the integral length scale largest eddies.

And the cost of DNS scales as Reynolds number to cube. So that is why any increase in the Reynolds number would lead to a tremendous increase in the cost of performing a direct numerical simulation. We require indirect numerical simulation very accurate time history and that is the reason why we can overcome this stability requirement rather easily they are met automatically.

So, explicit time integration schemes are usually preferred in direct numerical simulation.

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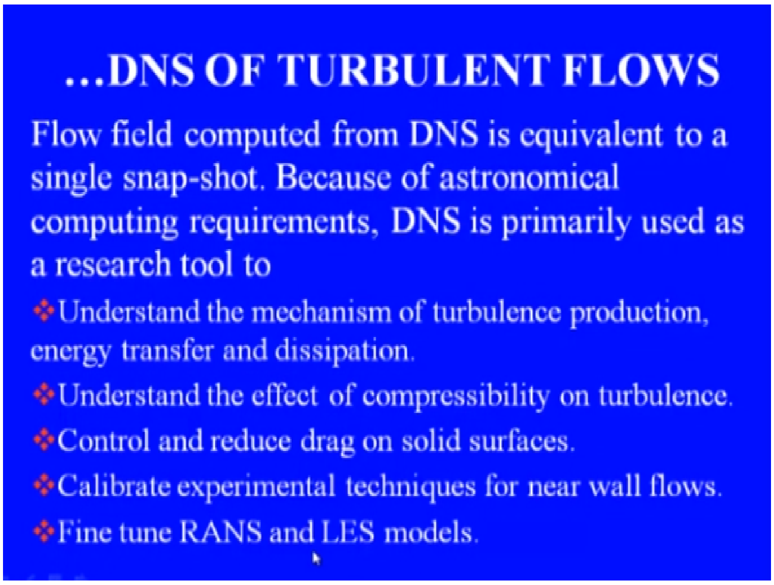

Now please remember this flow fields which we compute from direct numerical simulation is equivalent to a single snap-shot. We are doing the time stepping and at any given time step we can have a dump of the solution that would represent a snap-shot that is what happens to our flow at a given time movement.

The computational requirements are astronomical and that is why the DNS is being used primarily as a research tool to understand the mechanism of turbulence, production, energy transfer and dissipation in very small domains. To understand the effect of compressibility on turbulence to control and reduce drag on solid surfaces wherein we can perform very controlled numerical simulation in very small solution domain.

And calibrate experimental techniques for near wall flows wherein there are lots of areas which are introduced because of the effect of the wall. There also being used to fine tune our Reynolds-averaged Navier–Stokes models and large eddy simulation models. So that is the primary uses of the direct numerical simulation at the moment.

I would also like to give you very brief picture of just a rough estimate of the time requirements of direct numerical simulation.

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DNS Time Cost

(Simulation are performed at good parallel HPC facility)

Re	10^3	10^4	10^5	10^6
No. of grid points	10^6	10^8	10^9	10^{13}
Simulation Time	~ 10 hr	$\sim 10^3$ hrs	6 years	~ 1000 yr

Now remember we have a very rough estimates just to give you a comparative idea and we should presume that we have got you are going to perform your simulations on a very good simulations are performed at a good parallel cluster. A parallel what we called parallel HPC facility. So, if let us have a look at this put these things in a tabular form. Our Reynolds number, the number of grid points and the simulation time.

Now, I once again like to caution you that these are just hypothetical estimates, okay. It is nothing, they give you only an order of magnitude. So, depending on your HPC facility the numbers were sided, they might look quite different. So, let us say if you dealing with Reynolds number of order of 10 to the power 3. Our typical requirements of the grid points would beaten to power 6.

And we can perform this our simulation in let us say the number of hours 10. From 10 to the power 3 suppose we go to the 10 to the power 4 order of our Reynolds number. So, here our grids would be 10 to the power 8 and the time requirements will go to the 10 to the power 3 hours. From 10 to the power 4 let us go to 10 to power 5.

Grid size requirement will be around 10 to the power 9 as number of grid points which you need and we would need you will be surprised to know that we would require not in hours but few years 10 to the power 6. So typical industrial scale Reynolds number we require number of grid points the order of 10 to power 13 and the solution time would go into few thousand years.

Of course, just 1,000 years might depend on they might vary this as well as even if you will say 100 years that tells you about the impossibility of caring out a direct numerical simulation for our industrial flow problems. So, we cannot use direct numerical simulation to assimilate at all the length scales the flow around raising car o flow around an aircraft.

That is next to impossible unless there is a breakthrough in the computing technology and the algorithms which can give us a very very fast numerical simulations.

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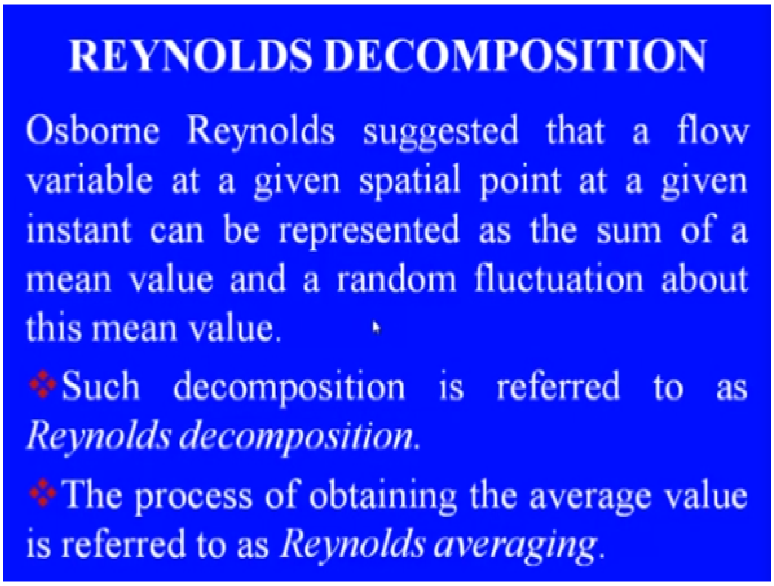

Now we will introduce one particular terminology before we finish today. Which relates to our Reynolds-averaged Navier–Stokes simulations. So, this is what we call Reynolds decomposition. So, Osborne Reynolds suggested that a flow variable at a given spatial point at a given instant can represented as the sum of mean value and random fluctuation about this mean value.

And this decomposition is referred to us Reynolds decomposition and the process of obtaining the average value which is used in this decomposition that is referred to us Reynolds averaging. There are many things which are looked, we are going to have a detailed look at how do we decompose or define our agnostic decomposition for any flow variable.

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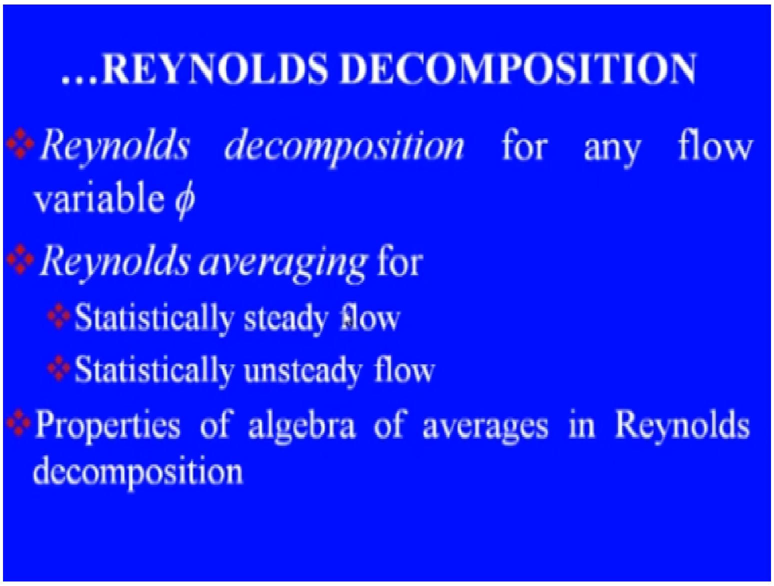

We will have a look at the Reynolds averaging for statistically steady flow and statistically unsteady flow that the slight difference the way we would define our averaging process for the tube cases and we will also have a look at the properties of algebra of averages in Reynolds decomposition. Because these will be useful in obtaining the Reynolds-averaged Navier–Stokes equations for RANS simulations.

But this we are going to take up in next lecture. For this lecture we would stop here. If you are interested there are few references listed here for turbulent flows.

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There are many books the 2 most popular books listed here is Lesieur in 2008 named Turbulence and another one by Pope published in 2000 on Turbulent flows. Numerical simulation on turbulent flows there was CFD books which we have already seen earlier can look the book by Chung Computational Fluid Dynamics or book by Ferziger and Peric and the book by Versteeg and Malalasekera across this book is specifically relevant for the details about different RANS simulation models.