Course Name: Turbulence Modelling

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Week - 2

Lecture - Lec09

9. RANS equations I (II)

So, now I can add 0 to equation 6, it does not change anything to that equation right, it is essentially 0. So, I can add this term to equation 6 ok. So, let us go to the next page. I would like to copy this before that. Let me just copy equation.

So, we have equation 6. Now we are going to add the 0 term which is I am going to make this or subtract add or subtract. So, I am going to make minus $u_i' \frac{\partial u_j'}{\partial x_j}$ ok. So, the 0 term 0 term is added.

So now I can write the right-hand side. This is minus if I use the chain rule for products here. I can push the fluctuation inside the differential operator, right? So, I can write this as simply $\frac{\partial}{\partial x_j}\left(\overline{u_i' u_j'}\right)$. ok using chain rule or product rule. Now, this statistical analysis chapter is useful, right? You saw something that we discussed.

So, what is this last term now? So, now this equation is what is the number we achieved 7 right ok. So, let us call this equation 7. So, in equation 7 this particular term we have this $\overline{u_i' u_j'}$. So, this if I expand for different i and j values. So, I would get 9 terms.

So, this is a tensor. second rank tensor 9 terms and it is a symmetric tensor whether it is $\overline{u_j' u_i'}$ or $\overline{u_i' u_j'}$ is the same if you expand it you would get 9 terms. So, the off diagonal terms are same. So, it is a symmetric tensor therefore, I get 6 unique terms here which $u_1' u_1'$, $u_2' u_2'$, $u_3' u_3'$ the diagonal components, and the off diagonal components $u_1' u_2'$, $u_2' u_3'$and $u_3' u_1'$ this we gave a name covariances, cross covariances in a statistical analysis chapter right. one varies together because it is a square term the top three are square terms they are always positive in a turbulent flow if it is 0 there is no turbulence.

So, the top three terms covariances or the diagonal components has to be positive and this can take positive or negative correlation or anti correlation. They can vary together, that is, your u_1' and u_2', 2 fluctuating velocities can vary together or vary in the opposite direction. If one is increasing, the other one can decrease at a given point single point statistics because we are essentially evaluating momentum at every location in your spatial domain x y z right at a single point, and we are looking into how these fluctuating vectors velocity vectors are correlated with each other at the same point, ok. So, that is why you are now appreciating the statistical analysis. Move to the equation part, ok? So, we already had this discussed this is the so called covariances and the correlation terms the six of them.

So, again this is the only term which knows about turbulence the last term here right the only term which knows about turbulence term here, rest are all mean and statistical. These are all statistical part here, the rest of them are completely statistical. Yeah, So before we go ahead and look at how this can be solved, first, let us see whether these equations are closed, ok. So, now how many equations we have and how many unknowns? So, we have in total four equations here; even if you use the mean pressure of the Poisson equation or using the continuity like before, we will get. So, basically, you have $\overline{u_1}$.

Sorry, we have $\overline{u_1}$, $\overline{u_2}$, $\overline{u_3}$ three mean velocities and mean pressure. You can get a Poisson equation for it or you can use the continuity equation, $\bar{p}$ or using the continuity equation. It depends on the way you want to solve. And the unknowns again, I have $\overline{u_1}$, $\overline{u_2}$, $\overline{u_3}$, and $\overline{p}$, then these 6 extra terms, which are $u_2'u_2'$, $u_3'u_3'$, and then I have $u_1'u_2'$, the correlation terms or cross covariances, 6 extra unknowns. So, I have four equations here, and I have ten unknowns.

Again, there is a closure problem, but here we call this turbulence closure problem ok. Therefore, number of equations are less than number of unknowns. This is what is called a turbulence closure problem. Now one can argue what is the big deal here? We had the same problem in while looking into the Navier-Stokes equations, right? It was a momentum equation, the momentum equation became Navier-Stokes only upon adding Newton's law of viscosity. Otherwise a general momentum equation when it was $\frac{\partial \tau_{ij}}{\partial x_j}$ it is a general momentum equation applicable for any type of flow.

We consider a Newtonian fluid and we use that model Newton's law of viscosity and the equations got closed. So, one can also argue here why not why do not we do the same thing here. we can just assume turbulence is a viscosity term, but turbulence is not viscosity that is the problem right. So, there you had help. We had this molecular viscosity or a property of the fluid that was helpful.

You can consider air, water or some particular Newtonian fluid. You know the viscosity value, and therefore, you could close it. The mu was known, right? You essentially related that six unknowns τ_{ij} to a molecular viscosity μ and the velocity gradient, which is uvw, which is known anyway. So, the only new information required was molecular viscosity which is known to you. So, therefore, the equations got closed, but here if you use the same concept there is nothing called turbulence viscosity.

Because turbulence has nothing to do with viscosity. Turbulence we already characterized. It does dissipation. It does diffusion that is transport, ok. Viscosity also does transport.

Viscosity can also do dissipation. But turbulence does more than that. It has many other characteristics. So it does not act only like it is dissipative or just diffusive. And, therefore, it cannot be considered as a viscosity term.

So, it is not straightforward here. The model is not straightforward and not as good as the model that we achieved in the Navier-Stokes equations. That is a better equation closed form completely coming from first principles. Here, the equation is fine, but it is it remains unclosed, ok. So, the idea here, note the idea of closing these 6 unknowns using an, I will use code like a turbulence viscosity, okay the idea of closing these six unknowns using a turbulence-like viscosity does not help because turbulent flows are just turbulent flows it's a flow unlike the viscosity which was a property of the fluid here whatever you want to call it even if you want to call it a turbulence viscosity it is a property of the flow, not the fluid, ok.

So, then, it will be a problem that does not help since the so-called turbulence viscosity will be a property of the flow. Not the fluid that is unlike your Newton's law of viscosity. So, that kind of an idea, an idea we can use it definitely, but it is not as straightforward as before. and one must be ready to accept some error coming out of this assumption. So, we will come to that class of models the so called RANS models then we will see what it does there ok.

One more important part I want you to grasp is that what do we call this term? This term has a special name ok. So, this so called $u_i'u_j'$and take another note here. So, this $u_i'u_j'$ this has a name called Reynolds stresses since we did Reynolds decomposition and averaging. So, this is called Reynolds stresses. So, why did we or anybody called it Reynolds stresses? The question is is it a stress here? If you go back Is it really a stress or is it something else? See this term has its origin from the nonlinear term.

It is just a correlation. Its physical meaning is it's a correlation. It's a nonlinear correlation term. And it's the origin of turbulence. It knows about turbulence. It's nonlinear and it's a correlation.

It is not a stress. So then why is it called stress? Not to solve. its dimensions are like stress. So, if you take this particular equation here, let us rewrite that stress term only here. So, before that, write it down here. Reynolds stress is not a stress term.

but a non-linear interaction term or simply you can say correlation term . So, why did we call it a stress is because, if you rewrite the viscous term and the ah Reynolds stress term, that is, these two terms: the $\mu\frac{\partial^2\overline{u_i}}{\partial x_j^2}$ and this term, if I rewrite this, you would get. So, rewriting the last two terms in equation 7, right, equation number 7, yes. I can write this as $\frac{\partial}{\partial x_j}\left(\frac{\mu}{\rho}\frac{\partial\overline{u_i}}{\partial x_j}\right)$ right this particular term, and this one is both minus yes plus I have sorry $\frac{\partial}{x_j}\left(\overline{u_i'u_j'}\right)$,mu by Rho, yes.

So, now, if I multiply throughout by density, I can rewrite this simply as. if I take this down then I would get this last term as so this will be simply $\mu\frac{\partial\overline{u_i}}{\partial x_j}$ right here. So, this will $\mu\frac{\partial\overline{u_i}}{\partial x_j}$ this is your viscous stress then this term will be rho dou by dou xj of wait wait wait did I make a mistake yes there is a mistake. So, this differential operator does not exist right.

am taking it out here. So, dou by dou xj is the one differential operator I am taking it out. So, the viscous term survives because of the second-order differential term, but here, this coming out, only Reynolds stress remains, correct? So, only this term is surviving here. So, only so it is $\overline{u_i'u_j'}$ ok. So, if I multiply throughout by density, I would get this, and here I would get $\rho u_i'u_j'$ ok, and this is your viscous stress tensor, and since you are able to write two terms together, one is stress the other one has the units of stress. So, if you look into this, is Newton per meter square.

Now, this is your viscous stress. Since this term has units of stress, therefore, this term has units of stress and can be written together with the viscous stress term. Thereby, the name came; thereby, it is called Reynolds stresses. But you must remember that it is not a stress, not a stress term. Its true meaning is. Basically, It has a non-linear interaction term that represents, and you can think of it as a correlation term, a non-linear correlation term, not really a stress, but popularly called Reynolds stresses.

So, if you can close Reynolds stresses the equation 7 for mean momentum gets closed and then you can solve for mean velocities solve for mean pressure and get whatever engineering data you want to achieve. So, the takeaway here is that by Reynolds decomposing and averaging you basically introduce new unknowns into the equation and therefore there is a turbulence closure problem. The equations are not possible to solve in a simultaneous fashion because you have 4 equations 10 unknowns you cannot solve this. and this extra 6 terms has the name Reynolds stresses but not really a stress but takes the unit of the stress.