**Course Name: Turbulence Modelling**

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**Week - 12**

**Lecture – Lec74**

**74. Numerics in LES and Hybrid models**

So, in the last class we looked at larger dissimulations various SGS models basically right Smagorinsky, dynamic Smagorinsky and scale similarity the three or you can say two class of models. And I am just going to summarize few things that can be useful. And this particular slide tells you about what happens after you compute your LES calculation. You are going to essentially compute resolved velocities and resolved pressure field. So, that means you have access to resolved turbulent fluctuations.

How good it is depends on your resolution. better the mesh, better data that you are going to capture in LES. So, here over bar represents resolved field and angular brackets represents ensemble mean because eventually just like similar to DNS you are going to compute a random field here right. In RANS you are going to compute directly a statistical field mean velocity, mean pressure and mean temperature and so on.

here you are computing everything random in LES just like DNS. So, you have to post process to get statistical field. So, you can apply same idea as Reynolds decomposition and averaging right. So, you have if you want to get your resolved fluctuation there it was $u_i'$ in RANS, here it is $\overline{u}_i$, bar representing it is resolved right. So, this $\overline{u}_i$ is coming from your LES.

So, $\overline{u}_i'$ is essentially $\overline{u}_i$ resolved velocity minus its mean. So, the angular bracket representing the mean here just similar to your RANS decomposition and averaging and this angular bracket representing answerable mean or you can use time mean or whatever mean you are using there. And therefore, once you have access to the resolved fluctuations, the turbulent fluctuations, then you can go on and get all your turbulence data. For example, you can get resolved turbulence kinetic energy. So, the $k_{resolved}$, this will be part of the total turbulence kinetic energy.

There is also a component for model. So, the k is equal to $k_{resolved} + k_{modeled}$. So, the better the resolution most of the resolved kinetic energy will represent your turbulence kinetic energy and that you can compute again using your resolved fluctuations. Essentially, the takeaways use same statistical ideas that you have learnt to average, to separate the mean and the fluctuation and then compute all the single point, two point statistics that is correlations, autocorrelation, two point correlations. or you want to compute Reynolds stresses budgets of Reynolds stresses everything coming from LES you have to decompose the field into mean and fluctuation that is it that is a post processing step after you compute the simulations ok that is it and yeah.

So, coming to the numerical ideas in the LES this is similar to the DNS. So, there we had a Navier Stokes here we are having filtered Navier Stokes equation right And then you are going to use an SGS model. So, you discretize the filtered Navier-Stokes similar to any CFD technique you are using finite volume, finite difference or any method. And if you are using finite volume as I said it is going to be implicit filtering right that is the easiest way grid size becomes filter size. And again we use central schemes here in LES.

Central schemes in CFD it is thought not to be used. ah that is good enough in RANS, but when it comes to DNS and LES central schemes are recommended. This I have highlighted already in the when I talked about the DNS. So, both temporal and spatial schemes are central to be good preferred for LES. Avoid upwind schemes because upwind schemes you know any CFD ah basic knowledge will tell you that upwind schemes introduce numerical dissipation.

But, this is used in a particular idea like there is an idea called miles which is monotonically integrated LDS. So, what this essentially means that in miles they do not use SGS model because what is the use of an SGS model to dissipate the whatever you know the energy that is sent to the SGS scales right. So, the SGS dissipation is the main role of an SGS model. correct and if your upwind scheme is going to do that right. So, then why use an another SGS model is the idea whether that works or not you can go ahead and see in the literature this is an idea and some people use it miles where SGS should not be used then you will have too much dissipation right numerical dissipation from upwind scheme plus SGS ok.

So, MILES does not use upwind schemes. In general, if you are using Smagorinsky, dynamic Smagorinsky, scale similarity and all the models do not use upwind schemes, use central. So, then wall boundary conditions are straightforward here unlike RANS right. So, we have Dirichlet for velocities, Norman for pressure. We are going to compute only for these four fields.

Let us say you have temperature or any other scalar quantity you can compute and give an appropriate boundary condition. Domain size exactly similar to DNS. We need to find out how big size you need because you are going to compute turbulent structures turbulent eddies are being captured in LES. you are not computing mean field like in RANS. So, therefore, in LES you need to make sure you must have two large eddies being captured within your numerical domain.

So, make sure two point correlations goes to 0 in all three directions and then take twice the size of what two point correlation is telling minimum twice you can if you can afford you can take bigger and bigger box. And how to start turbulence in LES, same idea as DNS, you need some kind of synthetic turbulence, some forcing, some tricks are required to start turbulence in LES also. So, lot of things are similar to the DNS except this miles idea. So, where upwind scheme is used as an SGS model. So, general guidelines for LES resolution, this is generic applicable to any type of flow problem.

So, the spatial grid size delta also filter width in point volume method. So, this should not be set equal to Kolmogorov length scale, one because LES is made actually so that it is computationally cheaper than DNS. So, that it is affordable. So, no point resolving all the scales using LES and even if you are doing you are adding an SGS contribution also on top of it.

So, it is not good. So, no need to resolve all the scales and you get this ballpark figure Kolmogorov length scale from the theory itself. So, you would know what is η from your Reynolds number. So, you definitely know what resolution you should not use. usually use let us say at least one or two orders of magnitude, at least one order of magnitude larger even bigger than η right. If η is like let us say coming to be 10 micron use like maybe 100 micron or 1 millimeter or even larger grid.

Its idea is to use a coarser grid and still compute good information is that is the idea behind LES. So, number of grid points in an LES that is n LES, n is n cube should be much smaller than n DNS of course, and DNS mesh we already know Re raise to 2.25 right very expensive. So, your n cube should be much smaller in LES. And delta is often orders of magnitude larger than η that I told you, it is good to have that.

And you should remember grid independent solution is not viable in LES. You cannot get grid independent solution unlike RANS or DNS. DNS we do not usually check for a grid independency, we check for resolving all the scales, proving that we have computed all the scales is more important there. So, in LES what you do is you have to check for grid sensitivity, you need to vary the grid and see if the results are converging to some state ok, whether the trend is similar that is what you can do best in LES check for grid convergence or grid sensitivity. And there are many techniques to do that one idea is to look into let us say you can test filter on your data to see what is the difference between the grid and the test filter data.

right to see how much is the kind of loss if you go to the test filter. So, if you are going to refine to the next level that gives an idea how much will be your loss. So, it gives an idea about sensitivity what is the extra information that you are capturing by going one level below or above the mesh. There are other techniques also to look into this the quality of an LES. So, the if you have wall turbulence of course, you have the advantage of the viscous length scale that is your $\frac{\nu}{u^*}$ right.

So, that viscous length scale can help you to determine what you need and depending on whether you are resolving the wall or not we have what is called WR LES WM LES. So, essentially WR is wall resolved that means very similar to your low Reynolds number formulation that means you are going to capture all the way to the wall right that means your first grid y should be $y^+$ less than or equal to 1. So, you are capturing all the way from linear sub layer and above. If you are doing that then it is called wall resolved LES boundary conditions are straightforward here, but in some problems this will be very expensive. Actually in LES we see that the moment wall comes in a boundary layer comes in it becomes very expensive because your grid guidelines depends on this the physics of a turbulent boundary layer it does not depend on whether doing LES or DNS it does not matter.

because it wants you to have your first grid at y plus less than or equal to 1 independent of you know whether it is a RAN's LES or DNS. So, wall actually poses a channel challenge here right. So, for that in some cases if you can afford to not resolve the wall. So, if wall effects are not so important to you, you can use a wall function that is called a wall modeled LES. I am just giving a schematic representation here these are not exact data just showing you the mesh how it would perhaps look like in a wall resolved LES and a wall modeled LES.

And in wall parallel directions literature tells that you can go with the resolutions like this $\Delta$ x + $\Delta$ z plus going like 100 or 30 respectively according to some reference articles. So, this wall modeled LES becomes important in some of the problems like atmospheric flow because you cannot resolve all the way to the ground of a planetary boundary layer. There are many complicated objects trees, canopy, forest all these things. So, in certain situations you have to use a wall function and model the wall effects. So, this is the resolution some kind of an estimate not a guideline.

So, in wall modeled LES as I told you meteorology or if you are computing planetary boundary layer, atmospheric flow, oceanic flows, then all the topological effects are difficult to resolve on a mesh. The geometry is complicated like buildings, cities all these things. So, wall functions are usually used also in combustion. So, the wall effects are usually wall is essentially going to quench a flame. So, they would also be sometimes comfortable using a wall function in combustion.

But if you are doing aerodynamics then you need to compute drag then obviously, you need to resolve up to the wall also heat transfer right on the your surface. So, those kind of problems you have to do wall resolved LES and wall resolved LES becomes very complicated very what to say expensive. And, that you can see here this is just a estimate this is not telling that we have computed all these flows. This is just an estimate given because we can have the formulas to estimate here. So, if Reynolds numbers goes like this from 1 million to let us say billion Reynolds number typical industrial flows.

right then we have this ah ah wall modelled ah LES the n cube coming from wall modelled LES is ah goes like this ok. So, typically in a ah wall modelled LES that is n LES or n cube LES scales like Reynolds number right. So, that is almost true here you can see it is about 10 times 10 power of 9 has 9 10 10 raise to 9 and so on. So, you can almost see that it is scaling the wall model LES demand is scaling like Reynolds number more or less and for the wall result it scales like Re raise to 1.85. and for DNS you see that it is 2.25. So, the wall resolved LES is becoming quite expensive you know approaching DNS demands and if I want 1 billion Reynolds number a typical industrial or an atmospheric flow you can see that this is the grid requirement. This is beyond the reach of current supercomputers as of let us say speaking 2024. So, wall modelled LES is the solution wherever you can afford such a flow. Otherwise you have to live with wall resolved LES. But this is something you can use the rough estimates like n LES for wall modelling is scaling like Reynolds number, n LES for wall resolved is Re raise to 1.85. So, this is the grid guidelines and some estimates for LES and you need to do some sensitivity study, you need to figure it out right. And there was one problem that I discussed with LES which is wall. Whenever a wall is coming in and a boundary layer is getting developed, it is posing a channel for computational demand. You need to resolve the wall in aerodynamics and heat transfer and such problems, then the it is computationally very expensive then the whole point of using LES is actually being defeated becoming very expensive and not being affordable also in many cases. So, then the idea came like how about mixing RANS and LES.

RANS is affordable and there is lot of developments has happened for many decades. So, and it is stable and so you can use it, but it does not give you turbulent data, turbulent fluctuations are not computed. LES computes, but wall resolved LES is expensive. So, to counter the wall resolved LES problems instead of modeling LES that is using instead of using a wall function they thought it is better to mix RANS and LES to our advantage and that is what is called an hybrid turbulence modeling. I will just give a brief introduction here what it is about.

So, if you just look at the governing equations in RANS that we have done and the LES, both looks very similar. The angular brackets here indicates its ensemble averaged. So, look at the left hand side of the RANS equation and the left hand side of the LES. These are filtered velocities, these are you know ensemble averaged velocities. And on the right hand side we have the mean pressure gradient here, here the filtered pressure gradient and we have viscous and the modeled terms here.

So, if I right hand side equation alone if I rewrite it I can get these two and you can see that both the rands and alias looks lot similar except that these two terms right. In RANS you are using eddy viscosity, in LES you are using SGS viscosity. So, there is actually an idea that if you have a RANS solver you can convert it to an LES solver and vice versa by just simply switching from eddy viscosity to SGS viscosity. But the question is where do you switch? Can I switch at whichever grid I want? Is it like grid by grid or zone by zone? this demarcates two different sub techniques in hybrid models whether I should have a zone like near wall zone where I will do runs and far away I will do LES that is one idea. The other idea is I have no clue where I can switch I will actually find out some kind of a automatic switching function where the code itself will switch from LES to runs as it demands that is an idea.

But the equation offers very similar between RANS and LES as long as you are able to switch between eddy viscosity to SGS viscosity. So, these two main approaches one is this segregated LES RANS that means you are using separate mesh for each. So, there is a RANS mesh and there is an LES mesh. completely segregated right and an example of this method is this hybrid LES RANS. So, whenever we talk about hybrid LES RANS it means that we have separate mesh for LES and RANS and it has sub names like zonal LES that means one zone is LES one zone is RANS or embedded LES right LES zone is embedded inside a bigger RANS zone.

Let us say the whole you know your computing RANS for the entire city let us say, but maybe around a certain building you would like to have better data. So, maybe you embed LES just around that particular building. rest of the city is RANS like that. So, we can do this embedded. So, at least there are two different mesh forests and they have to somehow interact.

So, one easy example is this one where I told you have this wall and then walls offer high demand for resolution. So, you can actually think of doing a RANS region here. This URANS is essentially nothing but RANS. The tradition many decades ago people started to call RANS as URANS and steady RANS as RANS.

But RANS equation always has an unsteady term. So, URANS and RANS are same. So, basically you are computing near wall as RANS zone and then somehow you need to switch to an LES above into the outer layer you can say. ok and that is of course, a challenge because there is always a mesh here right where that switch must happen it cannot just switch by becoming an statistical to an eddy resolving ok. So, it poses some challenge the idea looks simple on a schematic level ok and then you have this unified approach where use the same mesh right here there is zone where there are problems where you know like I cannot make a separate zone for LES separate zone for RANS. I let the flow decide if the flow wants an LES calculation certain places let it do it.

So, I am going to use the same mesh that is an unified approach and one popular technique is called this Detached Ready Simulation DES which has become incredibly popular now. So, and then this DDS is just a variation of DS, the DD refers to delayed detached simulation. Here of course, a switch is required between from going from LES to RANS. ok because you are you are going to use the same mesh shear so which one you are going to use a Eddy viscosity or an SGS viscosity or a turbulent length scale coming from a Eddy viscosity model or the filter size this which is required essentially you have two ideas now in the hybrid And when I come to hybrid LES model that is zonal LES or embedded LES, another important thing to know is that in hybrid LES RANS if you recollect turbulent boundary layer has many layers. So, there is an inner layer and an outer layer broadly and there are many sub layers.

So, the inner layer or you can say the viscous sub layer that is modeled with RANS and the is modeled with LES in the hybrid LES RANS. So, hybrid LES RANS is little bit demanding also only the inner layer has to be modeled outer layer is LES only the inner layer is RANS. So, we can use this equation where this $\nu_T$ what it means is that it switches from $\nu_t$ that is eddy viscosity this is your eddy viscosity. for RANS and it switches to SGS viscosity in the LES zone. But the inner layer has to be modeled by RANS here in a hybrid LES RANS.

So, when you use this hybrid LES RANS, you would need as I said a switch between this RANS to LES or URANS to LES at the interface is an issue because you are doing a zone. right one zone is LES one zone is RANS. So, you can use synthetic fluctuations stochastic forcing backscatter model there are many ideas how it can switch between one zone to the other zone because one side of the zone you have one node and then you have an another node on the other side. So, these two must you know exchange information right you are essentially using two different mesh here, but at the interface they must communicate.

right it does not have the idea of what is on the other side. So, the like a boundary condition it must interact and the way you can do this is this is from going from RANS to LES. Let us say from near wall to outer layer let us say you are going it then RANS you know that it computes only mean velocity. right but the neighbour LES demands filtered random component. So, you have to introduce the filtering there filtered component or randomness must be introduced for that of course, you take help from Reynolds stresses and you take this synthetic fluctuations or filtered fluctuations this vi star using this you will get your LES or the filtered velocity for the on the other side of the interface. So, this star indicates filtered fluctuation or say the synthetic fluctuations and angular brackets as I said is RANS data, ensemble mean data here.

And the other way that is going from RANS to LES and the next one is going from LES to RANS. the feedback coming from LES to RANS. In LES you are computing the resolved turbulence kinetic energy and in RANS I need full turbulent kinetic energy. So, you essentially this is lot simpler you use a constant here C is some constant. then you switch back from resolved turbulence kinetic energy, you take k and that goes into the RANS calculations at the interface.

These are broad ideas I am telling you about. Each one requires its own deeper knowledge when you go into implementations. But some ideas are lot simpler to implement, it is not as complicated ok. That I will come to the next idea is much more easier. So, this ds which is part of what is called a unified approach right. So, the same mesh is used this is lot easier to implement let us say you already have a RANS code easier to make it into a DES or you have an LES code easy to make it into a ds because you are using the same mesh essentially the same equation you just change from eddy viscosity to sgs viscosity.

So, this is similar to hybrid LES RANS the detached eddy simulation. So, what is different is here the entire boundary layer is modeled with RANS. So, in hybrid LES RANS only the inner layer right outer turbulent layer is still LES only the inner turbulent layer is RANS, but in DAS this is much more economical. The entire turbulent boundary layer, inner and outer layer is RANS and only the outer, not the outer turbulent boundary layer, the outer flow that is what is called detached So, this all the eddies that is above the boundary layer that part is called detached eddies and they are modeled by LES and this is a much more economical and today it is being used in industry and there is actually a project called DECIDER. You can go and see this project it was a project from European project about 10 years back I think. where they looked into is DES a viable technique for many aerospace applications and they found that this is a very good technique.

So, it is being today used and it is also available in your commercial codes DES. So, instead of worrying about should I use RANS, should I use LES, one option is to use just DES. So, it is a good tool. good method I would say and it is computationally cheaper than hybrid LES RANS. So, often if I give a ballpark figure let us say the RANS is taking x amount of time ah then LES can be orders of magnitude larger ok ah let us say one order of magnitude or more DNS can be many orders of magnitude.

Then DS is usually like let us say 2 or 3 or 4 times than the RANS. If your RANS calculation is taking let us say 1 day, maybe the DES takes 4 days you know and you can afford 4 days let us say and you can do a DES. LES can maybe take weeks, DNS probably months or impossible right in many cases. So, that is why the DS became popular available in commercial codes, open source codes. So, again challenges here is that you need a switch between the RANS alias here, but here of course this is a unified approach. So, it is node to node rather than zone to zone and there are ideas to handle that.

And if one issue that comes in DES is that since you are going from grid to grid same mesh is used for both. So, if wall parallel grid is of the order of boundary layer thickness then LES can sometimes switch on in the range zone right. Let us say you have you are computing a boundary layer then you have wall normal grids are but wall parallel grids are let us say 10 times or even 5 times bigger than your wall normal usually let us say. Then LES can actually switch on in the RAN zone you do not want that to happen because this is the trick to switch is often this the trick go you know goes with the mesh size there are there are many ways to implement it if you are going to use like a grid size or filter size here. then sometimes LES can switch in a ramp zone which is undesirable and that is addressed in this so called delayed detached D simulation.

So, I will give a reference article for DS you can go and see all the other thing it is just a basic introduction to this, this itself can be thought like half a course or something. But you are now empowered actually to understand on your own because you know very well I believe the RANS and LES. So, one can go ahead and see because this is more on the implementation level not on the modeling right. Models are already there for both. How we are going to implement it node to node or zone to zone? How are you switching from statistical to eddy resolved, eddy resolved to statistical? That if you can figure it out.

or there are techniques you can just read and see how it is done. Modeling part is all same that you already know about it.