Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 12

Lecture – Lec70

70. Large Eddy Simulations: Smagorinsky model – I

So, let us get started again. So, in the last class we were looking into one of the SGS models. We started with one sub-grid scale model, which is the Smagorinsky model, right? So, in Smagorinsky model, we essentially used idea similar to Boussinesq, where instead of an eddy viscosity, an SGS viscosity is used, and this SGS viscosity or nu SGS is essentially a function of the filter size, that means mesh size if you use implicit filtering. So, your numerical length scale is directly going into the simulation, the SGS viscosity is directly depending on mesh size. And then there was a model constant C_s which was user dependent that was one of the disadvantage.

So, in SGS or in a Smagorinsky model, we had essentially two unknowns coming out: one is ν_{SGS}, other one was K_{SGS}. This K_{SGS} is introduced only, let us say, if you want to solve one equation for K_{SGS}. So, before that, I would like to talk about this ν_{SGS}. So, if you recall ν_{SGS} in the Smagorinsky, this was $(C_s\Delta)^2$ delta square that is your some idea like a mixing length l square followed by an inverse time scale which is square root 2 Sij, this is filtered strain rates, this is your time scale, inverse time scale.

$$\nu_{SGS} = (C_s\Delta)^2\sqrt{2\overline{S_{ij}}\,\overline{S_{ij}}}$$

With that, you are getting a meter square per second consistent with the dimensions. So, here the one problem you would face here is that close to the wall, you will have large strain rates. This is discussed in eddy viscosity framework very close to the wall the strain rates this is resolved strain rates here, right? $\overline{S_{ij}}$ implies resolved strain rate. That means something you are computing for, and this goes large close to the wall, but ν_{SGS} should account for only SGS fluctuations, not for all the turbulent fluctuations. Therefore, some kind of damping is required, or wall correction is required, right? So, you can take a note here what is called near wall effects.

So, ν_{SGS} near the wall, let us say this becomes very large due to the large value of the resolved strain rate that is $\bar{S}$ ok. So, this is nothing, but your $2\overline{S_{ij}S_{ij}}$ large strain rate. So, one option to solve this is using a damping function. So, this becomes unphysical here. ν_{SGS} this large value, this is unphysical because SGS fluctuation should go to 0.

So, you would need a damping function here. One option is option 1, you can use damping function similar to an idea that we used in eddy viscosity framework, damping function like f_μ equal to 1 minus e raise to minus y plus by 26, where y plus is your near wall distance. Another option is to have a limiter, the limiter on the length scale right. So, the Δ. So, we can put a limiter on that.

$$f_\mu = 1 - e^{-y^+/26}$$

So, that ν_{SGS} reduces. For example, if you look at let us say near wall mesh let us say is like this. If I take a mesh close to the wall but in the other directions, this could be like this is the Δy, but the Δx, let us say, is like this: a large Δx and a small Δy and maybe a large Δz also. Let us say if I am taking in the other direction a large Δz. So, together, it will make the delta V or the delta itself a cube root of delta x delta y delta z right? So, this can be larger.

$$\Delta V = (\Delta x \Delta y \Delta z)^{1/3}$$

So, this can be reset using a limiter using the same idea as this mixing length using von Kármán constant and the near wall distance. So, in option 2, we use a limiter. What we do for this limiter is the delta is set as minimum of the delta V ijk that volume at any location i, j, k index or you take the mixing length kappa is the von Kármán constant y is the near wall distance as before where kappa is 0.41 von Kármán constant, and y is near wall distance. So, obviously, in this situation here the y will be much smaller.

$$\Delta = min\left(\left(\Delta V_{ijk}\right\}^{1/3}, \kappa y\right)$$

So, for example, if ΔV becomes large because you have used a cell stretched cell, usually this happens like you in the wall-normal direction. You would make Δy smaller to capture gradients there, but the horizontal grids that is in the x and z directions, you sometimes have bit longer distance, but ΔV will then take that effect from the other two directions, right? So, resetting that to kappa y will solve this issue. So, this limiter can help with the near-wall correction terms for the ν_{SGS}. So, now the other question is about if you are having K_{SGS} in the computation in the Smagorinsky, then how to get this K_{SGS}? One way out is to solve for a transport equation, which is a popular technique in some of the open-source codes as well as commercial solvers. They use a transport equation for

SGS kinetic energy ok.

So, we will see what it does. So, we will have one equation. one equation K_{SGS} model that is together with the Smagorinsky we can use this to get the K_{SGS} in the formula ok. So, here the transport equation goes similar to your k modelled equation. So, we have dou K_{SGS} by dou t plus dou by dou xj of your filtered velocity and the subgrid-scale kinetic energy that you are computing for equal to the diffusion term dou by dou xj of nu plus nu SGS dou K_{SGS} by dou xj and then a production rate.

This production rate is for SGS fluctuations $P_{k_{SGS}}$ and the dissipation rate, but the dissipation rate is not for the SGS because, remember, the idea of an SGS model itself is to dissipate all the energy that is in the system ok. So, that will be the true rate of dissipation rate. but these are unknowns. I am computing for K_{SGS}, which is an unknown, but in that equation, now I need to figure it out what is $P_{k_{SGS}}$, how to get epsilon right and also whether ν_{SGS} has to be same as ν_{SGS} in the Smagorinsky or not. So, in the equation what we do is ν_{SGS} is calculated slightly differently here.

$$\frac{\partial k_{SGS}}{\partial t} + \frac{\partial \bar{u}_j k_{SGS}}{\partial x_j} = \frac{\partial}{\partial x_j}\left\{ (\nu + \nu_{SGS}) \frac{\partial k_{SGS}}{\partial x_j} \right\} + P_{k_{SGS}} - \varepsilon$$

u_j you do not have to worry because resolved velocities are available here. So, this u_j this is the filtered or resolved right? Resolved velocity. This is known. You do not have to worry about it, but the unknown part, the first one, is, of course, ν_{SGS}. This is slightly computed differently than in the Smagorinsky in the one when you use one equation model.

So, what we do is there is a model constant C_k and then Δ the length scale. So, that makes it meter here. So, you still need a velocity scale, right? So, the velocity scale can come easily from K_{SGS} itself because you are computing for the K_{SGS}. So, you can use square root of K_{SGS}. So that makes this meter meters per second meter square per second.

$$\nu_{SGS} = C_k \Delta \sqrt{K_{SGS}}$$

So, here of course, the C_k value again C_k value changes from 0.05 to 0.1 in different codes while implementing or flow dependent and delta is same as before it is the filter size. But the other two things you need to figure it out. One is $P_{k_{SGS}}$ other one is ε.

So, now what should be the production rate of K_{SGS}? If you look into an energy spectrum, let us say I have an energy spectrum $E(k)$; k is the wave number here, ok. So, this is the wave number spectrum let us call this k is the wave number. So, we already seen this kind of a graph ok. So, in this plot, let us say somewhere here is the inertial sub-range, right? So, if this is the cut-off size here and this is the absolute dissipation rate is let us say ε is here, up to here is where you are resolving, you are resolving all the scales up to here, and the last part is modelled. So, what will be the dissipation rate up to the resolution? What should that be called? Is not that the SGS dissipation rate, this is nothing but ε_{SGS} ok.

This is what is being resolved in your LES, right? So, the last part is modelled here, This one. So, if ε_{SGS} is at the cut-off, this is the ε_{SGS}, and we learnt from Kolmogorov hypothesis that there is a dynamic equilibrium here. Shouldn't that be the $P_{k_{SGS}}$? Whatever is being drained out of the resolved data should be the production rate of the SGS, right? So, this essentially becomes your $P_{k_{SGS}}$. So, $P_{k_{SGS}}$ is set equal to ε_{SGS}, and ε_{SGS} can come easily from the resolved data, as I said. So, that means, I can use 2 ν_{SGS} and the resolved strain rates Sij Sij bar.

$$P_{k_{SGS}} = \varepsilon_{SGS} = 2\nu_{SGS}\overline{S_{\iota J}S_{\iota J}}$$

This is not same as ε because ε was your you had ν as well as $\overline{S'_{\iota J}S'_{\iota J}}$ implying ensemble averaging here this is filtered strain rates here ok right? So, S_{ij} is the filtered strain rate. $\overline{S_{\iota J}}$ is, let us say, resolved strain rates, ν_{SGS} is known. So, $P_{k_{SGS}}$ is set equal to ε_{SGS}here and finally, epsilon itself has to be given the last sink term in the this one equation k SGS model. So, that comes from so finally, the epsilon here is given empirically. So, we take C_ε a model constant and then K_{SGS}, you have access to you are computing for it.

So, 3 half of K_{SGS} is divided by, of course, the whole thing divided by delta, which is your length scale, right? So, this essentially gives you, this is nothing but your meter cube by second cube divided by meter which gives you meter square by second cube, dimensionally correct. So, this empirical formula is used to get epsilon here. And of course, C epsilon is 1 here, here C epsilon is 1. So, this will complete the Smagorinsky model itself with the combination of one equation K_{SGS} model. So now, despite this, we still have a ν_{SGS} computed where the model constant is varying.

$$\varepsilon = C_\varepsilon \frac{(K_{SGS})^{3/2}}{\Delta}$$

You have to choose either this constant or. So, there is a user-dependent part in choosing the model constant here, right? So, one way of getting rid of that is actually go to another type of SGS model, which is called the dynamic subgrid-scale model. So, or a dynamic Smagorinsky model where dynamically the constant is computed. Now, the constant is fixed for a given case by user.

So, it is static. Dynamic implies on the fly, that means when you are running the computation, you are going to compute that constant at each time step at each iteration you are going to compute this ok. Any doubts on this before we move to a dynamic Smagorinsky model? No. Okay.