Course Name: Turbulence Modelling

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Week - 11

Lecture – Lec65

65. Introduction to Large Eddy Simulations (LES) Filtering operation and SGS stresses - II

Okay, here I have some data. So, on the left I have the DNS data on a mesh like this example, just a illustration. So, let us say I have a extremely tiny mesh on which I have computed this DNS. Now, I take the LES filtering operation. on the DNS data that means now my mesh is like this, this is the filter, this is the filter size, this is a DNS mesh ok.

So, now on a larger filter size or on a mesh you can think of I am filtering the DNS data. So, then this is how the data looks like. Obviously it is smeared out. I told you filtering is nothing but low pass filtering we are talking about.

So what is low pass filtering? The low frequency signatures of your turbulent eddies are passed. The tiny eddies have high frequency signatures. Those are filtered out. Low pass means allow the low pass signatures, filter out the high frequency. So, the high frequency is tiny.

So, you can see that here. If you compare DNS figure and the DNS filter, obviously it is smeared out in the DNS filter, right. Tiny information is gone here because that volume averaging has occurred, the low pass filtering has occurred. But now if I use the same LES, this DNS filter size, if I use the LES calculation, this is filtered, this is resolved. You can see the difference here.

Now, since the residue is not exact, it is modeled. So, model effect has come in LES, right. So, compare the figure between these two. You see that it is even more smeared out. Some more information is lost that is because of the modeling.

The modeling error has been introduced in the LES case. Same mesh. Same mesh, one is DNS filtered, the other one is LES resolved. This gives the clear picture what is the difference between filtering and resolving. This is clear.

So, now we will mathematically see what we mean by this resolving, filtering and modeling. For that we apply this concept to the governing equations. Yes, yes. So, the mesh size is same. So, this LES mesh or the filter size it is able to capture whatever it could that means eddies that are larger the tinier one has not been captured by a LES.

So, they are modeled. So, that model error has resulted in some loss of information in this graph in the LES. But in DNS same mesh is used to volume average the data. and then since the residue is exact, some information that modeling error is not there, there is still no loss of that information. This is clear.

So now we will see a little bit more in depth what we mean by this low pass filtering, volume averaging and so on. So I can give an illustration here. So let us say I have a signal a 1D signal. So, let us say I have Φ is my random signal. Let us say it looks like this along x direction.

I look at a 1D signature of a random component Φ. If I do low pass filtering, so there is high frequency noise here and of course there is some low frequency turbulent signature also. So if I am going to do that, if I do, if I set a filter size, let us say I set a filter size like this. Let us say this is my Δ. Δ is the filter size let us say.

Then, if I perform low-pass filtering or volume averaging, the signal will look like this. So, this is $\overline{\phi}$. It is still random; $\overline{\phi}$ is the filtered component obtained by volume averaging the data with a filter size of Δ. I can say that this is an arrow mark based on the fact that if you change the filter size, the signal will look different. Additionally, the cutoff frequency in low-pass filtering will make your signal appear smoother.

The more frequencies you cut off, the smoother it looks; it may even resemble an actual sine wave. However, that depends on your decision. In LES, the user has a greater dependency on choosing the mesh and determining how much should be captured. In principle, you must capture all the large eddies, except for the dissipative ones; you must capture everything else. If you don't, your model will not be capable of producing good results because what you are modeling is only the dissipative eddies.

Their role is very clear: they are meant to capture only the effects of the SGS models, which are designed to account for the dissipative eddies when your mesh is very coarse in LES. If you are asking your SGS model to account for even inertial-range eddies, the LES results will obviously be poor. Your simulation results will not look good because the model is incapable of achieving that. So, this filter size is something you need to select. Anyway, now regarding the context of subtracting the mean: if I eliminate the mean here, you will obtain the fluctuation ϕ'.

Then the ϕ' would look like this, let us say, and if I filter the ϕ', it would appear like this. This is my $\overline{\phi'}$. Now, over-bar filtering is used instead of ensemble averaging in Large Eddy Simulation (LES). So, ϕ' and $\overline{\phi'}$. So, these are the filtered fluctuations.

The sentence is indeed grammatically correct as it stands. No corrections are needed. I am doing something similar to decomposition here. I have the instantaneous signal and the fluctuations, ϕ', and I am filtering both. Say the DNS data consists of this black Φ and ϕ'.

When I apply low-pass filtering or volume averaging, I obtain $\overline{\phi}$ (the red) as well as the fluctuations, $\overline{\phi'}$. This represents the instantaneous value, and these are the fluctuations. Thus, $\overline{\phi'}$ is your filtered fluctuation. The $\overline{\phi}$ is, as I told you before, just a filtered random component here. Is this clear in the context of the illustration? (The original sentence is already grammatically correct.

) So, this is a 1D filter that we have applied. So, you could say that we can define what we mean by filtering here. You already told me that it is a low-pass filter. (The original sentence is already grammatically correct.) Filtering is nothing more than filtering or volume averaging.

So, for a 1D case, if I define this, it looks like this: Φ bar is given by. So, I have $\overline{\phi}$ (x, t) in one-dimensional space. Let me explain it here in 1D: $\overline{\phi}$ (x, t) will be equal to $\frac{1}{\Delta x}$. The filter size in the x-direction is the integral from (x - Δ x) to (x + Δ x) of your Φ dx. This filtering, or volume averaging, that you are performing over Δ x is the filter size in the x direction.

We will perform a generic 3D filtering later, just to understand the 1D filtering that I have just shown you in the schematic diagram here with oscillations. This is how a filtering operation would look, similar to volume averaging in one dimension. We have examined the differences between RANS and LES, specifically concerning 1D averaging or filtering. In this context, we will also examine some additional differences between RANS and LES before discussing the governing equations, okay? Again, it's RANS versus LES. In RANS, we know that we perform ensemble averaging.

So, that means the $\overline{\phi'}$ here is the ensemble average, correct? So, what is the ensemble average component of φ' ? Zero is correct. The ensemble average of a fluctuation is zero. There is no need to prove this; it has already been done in class. But in LES, ϕ'' is our residue, and the overbar represents filtering and volume averaging. The sentence is already grammatically correct.

However, if you're looking for variations, you could also say, "What will this be?" or "What is this going to be?" Is it zero or nonzero? It is generally non-zero except in spectral filters. When I discuss the types of filters, I will cover everything except for spectral filters. So, in general, ϕ'', or the volume averaging or filtering of your residue, is not going to yield a value of 0. So, that means $\overline{\phi}$ is not the same as $\overline{\overline{\phi}}$, okay? So, this is filtering; this is double filtering, even with the same filter size. This is not the ensemble average in the RANS; it was the bar that represented the ensemble average.

So, ensemble averaging an ensemble-averaged quantity does not change anything; it has achieved its true mean. However, here the bar represents filtering in LES. So, this is the primary difference here, even for the same thing. Filter size Δ. (Note: The original phrase is already grammatically correct as a concise expression.

If you need it in a different context or format, please provide more details.) We will discuss this in detail later when we cover the types of filters, the exceptions, and how we can say that double filtering is not the same as filtering once. Any number of filtering operations you perform will slightly change the data. Okay, we also discussed this in the context of DNS filtering and LES; I just forgot to mention it here. Yes, here are the Φ DNS and the LES parts.

Here, I am saying that only if your LES model is good will this φ double prime be the same as the exact φ double prime. If I look, I am looking at the black symbol here: Φ DNS is equal to this part. So, if the exact residue of ϕ'' here is equal to ϕ'' in the LES, that means your LES model is so good that it can capture it. However, it usually is not; in fact, let's just say it will not be that way. Therefore, these two are not the same—just a note.

We can briefly go back to the equations to see how we should proceed. Are there any doubts about this before we move on to the governing equations? "Why are we filtering ϕ'?. ϕ' represents the turbulent fluctuation.

I want to filter the signals. Let’s say that I have the DNS fluctuation data. I want to average the volume to see what it looks like because I want to compare how the fluctuations captured in LES appear.

So, LES is an eddy-resolved technique that I mentioned, similar to DNS; however, there is a modeling component here, unlike in DNS, as some turbulence modeling must be done for the dissipative eddies. Therefore, you are interested in capturing the random component in the LES. It is not a statistical approach you are using; you are actually computing the actual turbulent flow as it appears. Some modeling errors are present.

So, obviously, you have to perform Reynolds decomposition to obtain the mean and the fluctuations. In RANS, the mean is directly accessible when you are computing any quantity, specifically the mean pressure and mean velocity. So, the turbulent component to which you do not have access is fully modeled; however, when you run LES, by default, you are using random velocities and random pressure quantities. If you want to convey that meaning, you have to do post-processing. You need to perform an averaging operation—whether a time average or any other method—to obtain a mean velocity and mean pressure while separating the fluctuations.

From the separated fluctuations, you can obtain the Reynolds stresses, right? That is the whole point. So, the overbar has a different meaning in this context. So, when we apply this concept to the governing equations, you will understand what we mean by it. That means you are filtering it again, as I showed you in the graph, right? So, I have the DNS data.

Let's say I filter it using a 10 LES filter size. If I perform that operation again with the same filter size, the data will change. If the filter size is changing, then yes, if the Δ becomes 2 Δ, the results will obviously change; with 3 Δ and 4 Δ, it looks more smeared out, right? I agree, but this occurs even with the same filter size, except for spectral filters.

Each time you perform a volume-averaging operation, this will occur. This is more like, let us say, you have a room divided into sections with temperature probes placed every meter. You are obtaining some 10 by 10 measurements, which totals 100 temperature readings from thermocouples. Let us say that if I divide it into only 2 by 2 now, you will average whatever is in that particular section. You will create 4 squares, and all the thermocouples in that large square will be averaged.

So, some information is lost, right? But here you are assuming that you have 10 by 10 thermocouple data and that you are averaging over a volume. But what if you are asked to compute only these 2 by 2 components of temperature? Obviously, it will not be the same as averaging the data you have from the 10 by 2 filter to the 2 by 2. How is the large eddy becoming a small eddy? I do not know why it is becoming what it is.

Perhaps I can explain why it is changing. The vertices or eddies do break up; there is shear, and it eventually leads to turbulent flows, which are also shear flows. So, there is shear if you take two counter-rotating eddies; there is shear between them, and we are observing high momentum or high inertial effects, where the Reynolds numbers are very high.

So, inertial effects dominate viscous effects. So, you have shear and nonlinear interactions. The vertices can break apart; there are many factors involved. They can also amalgamate and form a bigger eddy, which can break apart again. We have not correctly introduced any scales into the governing equation. The governing equation does not account for the scales; it only considers them when you simulate it by providing a numerical mesh size.

It can only capture vertical structures that are larger than the mesh size. So, we will review the governing equations tomorrow.