Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 11

Lecture – Lec64

64. Introduction to Large Eddy Simulations (LES) Filtering operation and SGS stresses – I

So, let us get started with another chapter large eddy simulations, also called LES ok. So, now here, first we have to recall, as I told you this Kolmogorov hypothesis has a lot of advantages for the turbulence modelling community. One of the big advantages we see it in the so called LES modeling. So, what is this Kolmogorov hypothesis if we quickly recollect you recall the Kolmogorov hypothesis of the so-called K41 theory? So, we had this very energetic eddies right, the large integral scale and then we have the tinier ones let us say the intermediate scales. and then finally some dissipative eddies.

So, this is the cascade image that we also discussed that is the energy cascade. So, what we discussed there is that in the Kolmogorov hypothesis, it is considered that the large eddies, the integral eddies or the places where energy injection occurs, these are the energy containing eddies right. So, the energy injection occurs at these can say energetic eddies. the injection of energy occurs at this large scale according to the cascade process or the Kolmogorov theory, and we do have these dissipative eddies and, of course, the intermediate ones, which is the inertial subrange right, the inertial range where it is purely inertia driven.

So, the main advantage of this Kolmogorov hypothesis in the context of LES is we discussed that there exists this tiny scales where it is statistically isotropic right. So, local isotropy or statistically isotropic at small scales. This has a profound impact in LES. So, the idea here is thatas against direct numerical simulations where you need to capture all the scales of turbulence right. In DNS, all scales of turbulence must be captured all scales of turbulence must be captured, computed using your temporal and spatial grid.

Now, in LES, the idea is that if this so called tiny scales, tiny eddies or dissipative eddies, if there is a local isotropy at these dissipative eddies, why not take advantage of that? You know, if there is local isotropy at these scales and it is not defined what kind of flows there right at every high Reynolds number flows in a fully turbulent flow, there exist small scales where dissipation is occurring, and it is universal according to Kolmogorov hypothesis. So, I can make a universal model right. So, one challenge in the DNS was that the scale separation right. So, the eta was so tiny you had to resolve the here. You had to capture the Kolmogorov microscales.

You have to capture ok, you have to capture Kolmogorov microscales. That means, essentially, you are capturing these dissipative eddies. But if it is locally isotropic, then I can introduce a model so that the mesh size that I am going to use is not eta but much larger than eta, maybe 10 times 100 times than eta, right? So then I will resolve not up to here but for example if I say this will be the entire spectrum Like this will be the entire DNA spectrum here, which is going to capture from the largest to the tiniest. Right? So everything is being captured. In LES, we don't do that.

What we do in LES is that we only look at, we will have a mesh. This is resolved. and the rest part here is modeled ok. So, in LES, this is what we call resolved or captured using here mesh computed right you can say computed, and this is modelled. So the tiniest scales are modelled since it is statistically isotropic universal in behaviour according to Kolmogorov hypothesis.

We take advantage of that. So this is the general idea behind LES. Of course, in RANS you are not capturing any of this. Everything is modeled here. No eddy was captured using your numerical mesh there.

It is statistical. You are only computing the mean flow. Here, as you see, even though I have written it, it has come out to be a little bit random. If you see the large eddy or any of this, it is slightly looking different, right? So, that random component is being captured here using LES or DNS. It is just that DNS would capture all the scales in LES depending on your mesh size you would capture mostly the large eddies and that is how the name came here the large eddy.

because you are capturing the large scales by modeling the small scales ok. So, in LES, large scales are computed ok, the small scales are that is dissipative eddies, and small scales are modelled, taking advantage of this local isotropy assumption. So, no need to resolve all the way to Kolmogorov. So, we will come to grid guidelines later what we do at LES. So, now we have to compare between RANS and LES to proceed ok.

A quick comparison here is that, let us say so, comparing RANS and LES. So, in RANS what we did is Reynolds decomposition and averaging right. So, in RANS, we did Reynolds decomposition and ensemble averaging. So, what does that mean symbolically is if I have ϕ, a random component, I am going to decompose it into $\bar{\phi} + \phi'$. Here, the ϕ

is the random component, a random signal, the ϕ' is random, and the $\bar{\phi}$ is statistical, right? So, the randomness has been separated, split out and then statistical part is what you are focusing and in RANS of course, you modeled this part.

This is modeled, this is computed, statistical part is computed. In LES this is slightly different. in LES if I have the same components here, let us say ϕ I write this as $\bar{\phi} + \phi''$. I am going to use double prime here ok; it has a different meaning. So, here all three are random ok.

All the three components are random. Why? Then what is this meaning of $\bar{\phi}$ here? Here over bar ok, here over bar this symbol represents volume averaging or low pass filtering. Here, over bar means low pass filtering or volume averaging. This is different compared to the ensemble averaging right. So, this $\bar{\phi}$ here means ensemble average, right? So, here over bar means over bar means ensemble average this part here, in RANS.

So, here over bar, whenever I write over bar in the LES chapter, it means low pass filtering or volume averaging, not your ensemble averaging, ok? So, now let us see what this actually means. This $\bar{\phi}$ and ϕ'' them, ok? So, now I have this $\bar{\phi}$ component this is usually called filtered, filtered component, sometimes also referred to as resolved or resolved component even though these two has a different meaning but interchangeably people use it colloquially they use filtered or resolved, but it has a different meaning I'll explain why it is different and the phi double bar sorry p double prime p prime prime so this means residual component or sub grid scale component, sub grid scale SGS component. Again here, these two are used interchangeably. ok, but they have a different meaning: filtered versus resolved, residual versus sub-grid scale has a different meaning to it. Ok and over bar, as I said, always means filtering operation.

So, to understand it let us take an example of what does it come to your head when I talk about filter? Coffee. Coffee right ok. or tea whatever you are going to do or juice since it is summer. So, let us take up this example of let us say you have a coffee filter ok. So, let us say I have let us say I have this coffee filter here it has mesh right to capture whatever you want to capture.

So this is your coffee filter mesh right and this is just for your understanding. So now you are going to pour your coffee or whatever the coffee beans powder and hot water and everything. So you do get certain things here collected. Some things are getting collected and some things come out of it. What is residue here in the context of coffee? What you have captured inside the mesh, this red symbols that I have drawn, right? That is your residue that is useless.

You are not going to eat that, right? You are going to drink this green dots. That is the coffee droplets. It is opposite here. Filtered means what you are capturing by your filter.

Let's have a numerical mesh. What can it capture? It can capture eddies, vortices that are larger than the numerical mesh. If your numerical mesh is, let's say, 1 millimeter cube, you cannot calculate or capture vertices or eddies, which is, let's say, 10 micrometer cube. So obviously, the eddies that are larger than the mesh size is only you can capture. So, what is useful here for us is what is being captured. So this red is your filtered component in the context of LES.

So this red part here is filtered component. The eddies that you are capturing. So, turbulent eddies being captured by mesh, the LES mesh that you have used and this so called green droplets that is coming out of the mesh that is a residue. This residue is what is being not captured in LES, ok? So this is your residue what is coming out of it ok. So now, if you have access to these two things ok the filtered as well as residue.

Then both are exact. Let us say I have a DNS data and I use an LES filter. I will tell you what the filtering operation is. But let us assume that you have a DNS data and then you are doing filtering operation alone. So let us say you have, as I said, 10 micrometer mesh. You calculate DNS on that and now you move to 1 millimeter mesh.

On that new mesh, you are going to filter the DNS data. So, then you have the residue that is coming out that means you have this the ϕ in LES is $\bar{\phi} + \phi^{\prime\prime}$ right. So, if ϕ is the DNS data and I am doing a filtering operation $\bar{\phi}$, then $\phi^{\prime\prime}$ the residue is exact. You have access to this. You can subtract the DNS data minus filtered DNS data to get the residue right.

So, for example, if this is ϕ DNS this is $\bar{\phi}$ DNS filter. or you can say filtered DNS data. Then this is the exact residue. $\phi^{\prime\prime}$ you have access to it. In LES as I told you this residue that is, the tiny eddies that is not being captured by your mesh, is modelled you have no just like RANS, you have no access to the tiny eddies this green vortices no access to that you are modeling them so obviously $\phi^{\prime\prime}$ is unknown in LES you have access only to the $\bar{\phi}$ in LES which is LES resolved.

That is the name between the difference between filtered and resolved. So, in LES you will have right here ϕ will be $\bar{\phi} + \phi^{\prime\prime}$. So, then this particular component will be your LES resolved captured by your LES mesh, ok? an LES mesh is used here. This is what it is able to capture, and the phi bar is unknown.

So, this is called sub grid scale. So, this is your unknown that is the name sub-grid itself explains the vortices that are sub-grid scale, scale smaller than the grid size, right? Subgrid scale this is to be modeled in LES. So, ϕ'' is a modelled component, but ϕ'' if you do filtering operation on a DNS data is exact. So, these two are different that is why the name filtered and resolved are different, but people do use it interchangeably filtering and resolving, you understand, right? You have a DNS data you can filter it out to get exact residue, but if you use the same filter size as your mesh size and run LES calculation then you would get a filtered or you can say sorry not filtered, you will get a resolved data whatever you are resolving is capturing on that particular filter size or mesh. then the received you have no access it will become unknowns and you need to model this ok. So, I will show you an example, then it becomes little more clear.