Course Name: Turbulence Modelling

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Week - 11

Lecture – Lec61

61. Introduction to Eddy Resolved Models - II

Epsilon, I told you, you can determine Kolmogorov scales if you know epsilon, but epsilon requires measurements or direct numerical simulations, right. So, in the absence of that, so this thesis became popular because of this particular slide here. tries to quantify how to get epsilon also without doing any measurements or anything just analytical expression. So, how to determine epsilon is the question here to get micro scales right and we also say this epsilon is in dynamic equilibrium whatever the large eddy is transferring to intermediate being transferred to dissipating. So, I should be able to compute epsilon according to the hypothesis.

So, the dissipation rate of this TKE epsilon purely on dimensional grounds is now proportional to $\frac{u^3}{L}$. This we already used when we form wall functions when we you know in many places we actually use this scale epsilon is equal to $\frac{u^3}{L}$. But where we assumed what that L has to be right we use this mixing length kappa y and so on right and u we said it is u star right that was your epsilon wall function was u star cube by kappa y. in many places.

So, now that is an assumption. In hypothesis we have to see what is this $\frac{u^3}{L}$, because this L and u has to be associated with the large eddies right, the energy containing eddies. So, this scale u you can obtain from what is called $\frac{q^2}{3}$ and q square is nothing but your $u_i'u_i'$ it is statistical isotropic right. So, it is a sum total of 3 $(u_1'u_1' + u_2'u_2' + u_3'u_3')/ 3$ that is the isotropic velocity scale of the largest eddy an integral eddy. So, this capital L what I have here.

So, this is what we call an integral length scale that is physical So, if you have a flow, you can actually know what is this capital L. For example, if you have a turbulent jet, I can say the orifice diameter, the nozzle diameter is my integral length scale and I will know what is that. So, this is something physical that you can know, right. If it is a pipe flow, it is a pipe diameter. So, that is something physical.

But the hypothesis talks about something else. So, it talks about this small l as the pseudo integral length scale where it says the pseudo integral length scale of energy containing eddy. So, this eddy contains the maximum energy. So, there is an eddy whose length scale is this small l containing the maximum energy and that l you can obtain using this $\frac{u^3}{\varepsilon}$ and this small l approaches the physical integral length scale in the limit of high Reynolds numbers, the limit Re tends to infinity, L becomes capital L. So, epsilon is now independent of viscosity according to this formula.

I already used this epsilon is $\frac{u^3}{L}$. If I know the velocity turbulent velocity associated with the largest eddy and length scale of the largest eddy. I do not need to know viscosity, that is the idea here, independent of viscosity because it is depending on the large scales. It is depending on u and it is depending on l, large velocity scale, large length scales. So, if you use all this, you can arrive at this particular formula.

So, how did I arrive at this? So, now you can see that now η is ah $(\frac{\nu^3}{\varepsilon})$^ $\frac{1}{4}$that is the formula we used you can go back and see that is the formula here. The Kolmogorov length scale is $(\frac{\nu^3}{\varepsilon})$^ $\frac{1}{4}$. So, Kolmogorov length scale η$(\frac{\nu^3}{\varepsilon})$^ $\frac{1}{4}$. ok. So, now epsilon I know which is $\frac{\nu^3}{\varepsilon}$. So, I can substitute here $\frac{\nu^3}{\varepsilon}$ is $(\frac{u^3}{L})$^ $\frac{1}{4}$. ok.

Now, I would like to make Reynolds number here. So, I have $\frac{\nu^3 L}{u^3}$ I will put L^3 here and (L^3)^(1/4). So, that gives me $(\frac{L^4}{Re})$^$(\frac{1}{4})$. So, η is $(\frac{1}{Re})$^$(\frac{3}{4})$.

Therefore, you have got the formula η by L = $(\frac{1}{Re})$^$(\frac{3}{4})$.ok. So, now I can know the scale separation just by the hypothesis that if I assume the small l to be capital L right because I do not have access to this infinite Reynolds number. Let us I am taking let us say whatever Reynolds number I am taking is very high if I am going to assume then I know its length scale I can assume it as capital L that is my assumption and then η I can compute by simply knowing the Reynolds number right. So, I can compute η by simply knowing the Reynolds number of your given flow and assuming a pseudo length scale. Let us say you can assume it as this is for your own benefit I am telling you and not according to hypothesis.

If you assume that this is same as your jet diameter or pipe diameter whatever, I can now compute what should be the η. And if you want to study turbulence, you need to capture all scales of turbulence. So, at least now I know what should be the smallest scale. So, you should know what should be the mesh size. So, in your calculation, you will know the mesh size or if you are doing experiments, you should know what should be the resolution of your experimental apparatus to capture η.

You must capture the eddy of size η, both in numerically as well as in the experiments. if you are able to do that ok. So, the Kolmogorov hypothesis gives you this ratio η/ L which goes like this. This we later see that actually creates a big challenge when we actually do eddy resolved simulations. Because if you take very large Reynolds numbers the separation is larger that is the difference between L and η becomes bigger and bigger as Reynolds number is increasing.

And that means, you need to have a very tiny and tinier mesh to capture higher and higher Reynolds number flow which becomes complicated and becomes very difficult for making a computation. You require very large computational resource for that one. Is this clear so far? The three statements and its assumptions and how using those three statements we can arrive at this formula where we can compute the Kolmogorov length scale η by simply knowing the Reynolds number of this flow as well as the pseudo length scale. The length scale can you can assume it as the integral length scale for your own benefit. So, now what about the reality this is again a general inference.

So, we have to always remember the basic assumptions here that it is fully turbulent the statement, but Kolmogorov changed the hypothesis in the 1963 where it considers what is called intermittency. So, intermittency means that let us say you have a turbulent jet right. And then let us say you have a turbulent jet like this. So, if you put a probe here, this will show fully turbulent signal. And if you put a probe here, it shows a laminar signal ambient let us say assuming it is laminar.

But somewhere if you put it here, let us say, so at this time it is like this, at the next time instant. because turbulence is three dimensional and temporal unsteady nature. So, the probe sometimes hits turbulence, sometimes it hits laminar flow. So, that signature at this particular probe here ok, if I plot it, it will be like some burst of energy and so on. But the probe here this will show only signal like this ok.

So, this particular probe will show what is called this intermittency, turbulence is intermittent. So, that intermittency nature that means this fully turbulent has been corrected in K 63 hypothesis that it also takes into account the intermittency effects. But this question of high Reynolds number is still open for discussion how high Reynolds number I need to go to actually prove or disprove Kolmogorov hypothesis. to because it has certain universal nature in the assumptions or the consequences are it is saying universality is there right. It is statistical isotropic independent of flows.

So, those things you want to prove you need to go at very high Reynolds numbers. So, it is still open for research and also the cascade. So, if we today look into DNS PIV data, we do see the breakup of eddy, some large eddy breaking into smaller and so on. We do see that, but that progression where a large eddy breaking into a smaller one and then a smaller one and then we do not see that. There is a burst of eddy, one leddy breaks up suddenly into smaller eddies, those things happen.

Sometimes small eddies also can amalgamate to form a slightly bigger eddy. Those things we observe in both in the DNS data as well as PIV implying that experiments particle image velocimetry. It uses a Doppler effect to measure experiments that is anyway. So, I can say this is just experiments both in DNS and in experiments. we do not actually see this kind of linear progression of break up of eddy from large to small like this.

So, it is still I can put a question mark on this and this energy transfer definitely occurs between large and small scales that does happen that we have seen from data. But theoretically there are three possibilities. One is that there can be a direct transfer of energy from large to small scale. I mean your parents can directly give pocket money to you. It need not be to your brother, elder brother or sister and then tell them to give it.

That can lead to some dissipation. So, they can directly transfer money to you. That is a possibility. But there is a the Kolmogorov hypothesis goes in this progressive transfer from large to small. So, your parents will give it to your elder siblings and they will give it to slightly smaller siblings and so on.

So, by the time it comes to you, it should be in dynamic equilibrium. So, there should be no loss because whatever is being given, but let us not go into economics, but yes. So, the Kolmogorov hypothesis. will go in a progressive transfer from the large to small that is the cascade. And there is one more idea which came from another person called Kraichnan.

He proposed what is called an inverse cascade where how about transferring energy from small to large scales. Why not the small scale give back energy? This is debatable. I think it is being seen in certain flows not in all types of flows maybe one or two problems you can actually see this what is called an inverse cascade. So, at least it challenges this universal theory from Kolmogorov right this what is called Kraichnan.

inverse cascade. So, it challenges this progressive transfer idea like you know sometimes you feel I mean if the economic situation is not good at home you have little bit of savings in your kiddy bank you give it to your parents that is like that the smaller eddy contributing to the economics of the home right. So, it does happen not all the time right usually you are happy getting pocket money giving it back. Okay. So, that is my take on the Kolmogorov hypothesis. And also this this in inertial sub range I told you right where there is epsilon is in dynamic equilibrium between the largest and the smallest the intermediate scale.

So, there in the inertial sub range if you plot this is a common you could go and see there usually in turbulence research we plot what is called an energy spectrum. People want to see energy spectrum. associated with the scales and it would probably look like this. This is in log-log plot. So logarithmic of energy E and logarithmic of let us say the wave number, k is wave number.

A log-log plot if you plot you usually see this kind of a graph. somewhere here what you can L or L whatever you want to call it and then there should exist a slope here. This particular slope let us say this slope according to Kolmogorov theory the turbulence kinetic energy or the energy associated this slope has to be like this k^(- 5/3). If the slope is not like this that means the hypothesis and your data is showing not matching with each other and that is usually we see today that not many turbulent data matches this slope k^(- 5/3) slope is not matched something else will come even if it matches only a small fraction of the spectrum matches it. We do not see this and of course, then we do not know whether this is high Reynolds number whatever data that you are producing is it high Reynolds number we do not know that question is also there.

So, that slope has to be at this thing. So, at what scales energy is actually dissipated? That is the question we have to ask and my take is at all scales. All scales are perfectly capable of producing dissipating energy, but disproportionately at small scales. The smaller eddies are capable of dissipating more energy than the large one. So, it is disproportionately large at small scales. And another question where is very important is, is η the Kolmogorov length scale the smallest turbulent scale ok? Kolmogorov length scale η is this the smallest? many actually believe that it is a smallest turbulent scale, but it is no.

Because our governing equations do not know anything about scales. Scales is not what we are introducing right, but continuum approximation breaks down below η right. So, and therefore, η is the smallest dynamically significant length scale. It is not the smallest length scale, but dynamically significant for us in the continuum world it is the smallest length scale. So, η must be resolved in your numerical calculations or η must be captured in your experiments if you want the correct data because η is representing the dissipation if your ah simulation or experiments is not capturing the proper dissipation then you may have in the numerical calculation you will have problems because there is accumulation of energy inside your system there is no proper dissipation occurring ok.

So, that is ah that actually rests the Kolmogorov hypothesis very useful ah idea for turbulence modeling especially larger dissimulations and related techniques. this Kolmogorov length scale also we seek to capture this η in direct numerical simulations also. But, but we do see that the η coming from, so then the question comes the η that is coming from this formula and the η actually coming from, so I have this η right. So, now I say η is 1/Re, what was it? ¾ th L So, this is the hypothesis right or can say a theory and I can actually get η $\frac{\nu^3}{\varepsilon}$^(1/4). Let us say I am getting epsilon from my DNS data actual.

I compute now. So, now what is the difference between these two is are they same? We see that this Kolmogorov hypothesis η coming from there is usually conservative. That means, η let us say the actual η coming from DNS let us say is let us say it is saying 10 micron then the theory will say it is 1 micron. So, it is more conservative that is in general that I have experienced since I work on direct numerical simulations. So, I have seen that the theory gives conservative estimates. The η is much smaller than reality, but if you have epsilon you can actually compute from this, but this is this formula is still useful for me because I can estimate my initial mesh size η.

because I would like to capture this in DNS. If I do not capture then no point in saying that it is a direct numerical simulation.