Course Name: Turbulence Modelling

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Lecture – Lec60

60. Introduction to Eddy Resolved Models - I

So, let us get started again. So, we managed to complete the Reynolds stress modeling and I also gave you a little bit of hint on what is an algebraic stress model. And so, from now on we will move to the next class of models which is eddy resolved models. or eddy resolved methods. There may be some technique which is which does not require any modeling, but still comes under the class of eddy resolved techniques.

But before that one must understand certain concepts. So, today we will talk about what is called Kolmogorov hypothesis ok. So, it is important to understand this because this is used by turbulence modeling community and you should know the assumptions made in Kolmogorov hypothesis. So, let us revisit ah what we already learnt about dissipation right, dissipation rate of turbulence kinetic energy as well as dissipation rates of individual stresses.

So, the question I am asking you all is that well the answer is already there, but that is my ah my personal take, but you should also reflect on this question is turbulence anisotropic at all scales ok. I am saying yes. But did not we do something there already right when we modeled the dissipation rates in the Reynolds stress modeling ε_{11}, ε_{22}, ε_{33}. Did not we there that you know we can just recall we said that it is statistically isotropic at small scales at these dissipative scales. I told you to assume and we will revisit right.

So, we set ε_{11} equal to ε_{22} and equal to ε_{33}. Eventually we modeled it based on dissipation rate of turbulence kinetic energy just ε. So, but turbulence is anisotropic at all scales that is my take on this. But you should also remember that we did this assumption that at small scales the turbulence is statistically isotropic. This forms the basis for what we learn in today's class, Kolmogorov hypothesis.

Statistically isotropic, this is the assumption we made right. So, I am just showing you the ah the budgets of the four stress components in a channel flow, a canonical flow like plane couette flow or a plane channel flow, pressure driven channel flow. This data comes from this reference Mohin, Kim, Moser ah. So, we did say that ε_{11} is equal to ε_{22} equal to ε_{33}. This was our assumption while modeling Reynolds stress model.

Now, let us see the ε_{11} here. This is the budget of, this is the budget, budget of $\overline{u_1^{'}u_1^{'}}$ stress component. So, where P_{11} is the production rate and so on. So, look at ε_{11}. Do not worry about the trend, trend also you can see, trend shows you also the qualitative nature of the anisotropy.

Look at the magnitude, it is reaching about let us say close to 0.3 and it is maximum on the wall. Now look at ε_{22}, it is going to 0 on the wall and look at the magnitude here. orders of magnitude smaller right ε_{11} is about 0.3 and this is easily orders of magnitude smaller value correct.

So, already you are seeing that this assumption is questionable, but there we assume that at small scales, we introduce scales in that assumption that at small scales it is statistically isotropic that was the assumption. But we have not discussed anything about scales of turbulence so far. In the equation we did not introduce scales. So, here for example, let us take somewhere here where the production rates are maximum. So, let us say there is an eddy here.

a turbulent eddy and where it has it is showing that there is a maximum production rate at the same eddy location if I come down. So, this will be the approximate location at there it has dissipation rates also and at neighboring location. So, each eddy is able to dissipate as well as produce. So, it is perhaps you know this assumption is at a scale which is called small scale. So, what is small scale we need to define and understand here because we do not know what is the scale that is available.

This is purely a statistical nature that we have looking into from the graph. So, we do not know at what scales this is actually true or if it is true. At least if I look into the trend it is showing that. this is not equal this orders of magnitude different ε 2 2 2 and ε 1 1 1 ok. Now, this is ε_{33} also I am showing this is also see orders of magnitude different than ε_{11} ok.

So, very different and we also assumed ε_{12} ε_{13} ε_{23} 0. right in the Reynolds stress modeling. So, it is very small, not really 0, ε_{12} is like it is over here. quite small, but not really is 0 ok. So, in this aspect it is ok.

So, now we will focus on this aspect. This is the data I am showing, but the Kolmogorov hypothesis is coming from much older times than this data coming from 1988 ok. So, we will see what this theory tells and how we can take advantage of this hypothesis for modeling, very important for eddy resolved techniques. Also it is used in RSM already without mentioning that. So, let us talk about scales of turbulence.

So, which scale produces and dissipate turbulence is a question because we said at small scales ah just with respect to dissipation rate we said at small scales it is statistically isotropic then what about large scales we did not talk about. So, the question is are there scales which are only producing turbulence and are there scales which are only dissipating turbulence or is every ah turbulent scale that is every turbulent eddy perfectly capable of producing and dissipating that is the question ok. I believe every eddy is perfectly capable of producing and dissipating turbulence. It may be disproportionately different. Maybe there is a large eddy which is producing lot of energy and dissipating less turbulence and maybe there is a small eddy which is producing little turbulence and dissipating lot of turbulence.

Disproportionate behavior can be there, but I believe it is perfectly capable of every eddy is perfectly capable of dissipating and producing energy. So, let us recall this Richardson's poem where he said big worlds have little worlds which feed on their velocity and little worlds have lesser worlds and so on to viscosity. So, it is giving a qualitative picture of what is called an energy cascade that is there are this large eddies here. like this and these are breaking into smaller and smaller and smaller eventually becoming very tiny and these tiny scales is what we assumed. Assume at small scales it is statistically isotropic ok not at the bigger scales.

So, he gave this picture big walls have little one little walls and so on right they break down and finally, he says this important thing that it is viscous effects are actually playing an effect at the small scales. for dissipation. That we have also seen when we have the dissipation rate of turbulence kinetic energy, we have seen that it is a viscous driven phenomena, right. The formula for ε was $\nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$, right. A viscous driven phenomena, viscosity diffuses that means transports as well as viscosity dissipates.

So, he has already kind of qualitative argument has been placed for that one. And this particular idea is what is called an energy cascade. And it talks about this particular idea called scale separation. That means there are eddies which are large. So, now we are going into some kind of an assumption that there are eddies which are large which are producing energy and there are eddies here, the tiny tiny ones.

which are actually so tiny that they are actually dissipating the energy. So, there is roles for each eddies kind of defined. Large eddies produce energy, small small eddies dissipate energy that scale separation idea has come right that is in the hypothesis or in the idea. And Kolmogorov hypothesis is 1941 paper this actually quantifies this qualitative poem from Richardson. So, we will see what the Kolmogorov hypothesis says ok.

So, this Kolmogorov hypothesis or the what is called a K41 hypothesis essentially it has three important statements ok which is very useful for turbulence modeling community, but you should also remember that this is a hypothesis. So, now according to the hypothesis, it says you have to consider a flow turbulent of course, and it should be fully turbulent. That means, you should not have a laminar turbulent mix or a transition turbulent mix. So, in your flow the whatever is control system that you have taken the entire flow has to be fully turbulent everywhere ok. It is considered a fully turbulent flow in a system and it should be high Reynolds numbers ok.

That means, high Reynolds number is based on the integral length scales where this u_L these are all integral length scales or sorry integral scales velocity and the velocity under the velocity this is the length some integral scales. So, integral Reynolds number is very high, but we do not know what we mean what he means by high. That is a question to explore or to actually prove this hypothesis is true or not. We need data and at what Reynolds number? We do not know.

It says at high Reynolds numbers. What is high is open for discussion. So, how high Reynolds number is a question here. But anyway, consider that there is a high Reynolds number fully turbulent flow. In that, he makes three important statements.

One is called this local isotropy. that we already used while modeling Reynolds stress right. So, this local isotropy statement says at high Reynolds numbers small scale turbulent motions are statistically isotropic ok. So, this is the assumption that we made which is actually coming from Kolmogorov hypothesis that at high Reynolds numbers where the flow is fully turbulent the small scale turbulent motions that is he is introducing what is called small scales. the small scale turbulent motions are statistically isotropic ok. That means, whatever statistics that is associated with those tiny eddies, it should show statistically isotropic behavior that is one rms quantity should be enough to define it.

I do not need q_{rms}, v_{rms}, w_{rms} at those tiny scales one rms should define its behavior ok. And therefore, ε 1, 1, 2, 2, 3, 3 are same there. There is no anisotropic nature when it is statistically isotropic. So, what this implies? So, this is my inference here. So, this is, these are all my inferences.

The statement is given here, the local isotropy statement that is according to Kolmogorov hypothesis. So, my inference is this implies statistics of the small scale motions are universal. This may not be visible from the statement he has made. What it implies is that you are now considering fully turbulent flow, high Reynolds number flow. Does it say what type of flow? Does it say it is a jet? Does it say it is a boundary layer? Does it say it is a combustion flow? Or does it say it is an atmospheric turbulence? No.

It does not matter for the hypothesis that you just take any high Reynolds number turbulent flow. This should be valid according to the hypothesis right. So, you take a turbulent jet or you take a turbulent boundary layer the small scales and its statistics should be isotropic right. So, that is statistics of the small scale motions are universal that is similar in every high Reynolds number flow. I am only telling you the hypothesis and the general inference that you can make from those statements.

Then of course, if you define what is a small scale here, he also makes an effort to define it, quantify it. What is that small scale? This is useful for both turbulence modeling community as well as those who study turbulence. So, it quantifies small scale. So, again the statement says in every flow at high Reynolds numbers, statistics of small scale motion is determined by dissipation rate and kinematic viscosity alone. So, if you want to know about this scale, the small scale that is the time scale, velocity scale and length scale of this tiny eddy.

which is statistically isotropic at high Reynolds numbers fully turbulent flow, you can determine it just by knowing the dissipation rate and kinematic viscosity. We will see how to get dissipation rate right that is something we have to measure. Now, we will see from hypothesis itself whether we can get it. So, kinematic viscosity we have access that we know from a given flow problem right. So, in every flow at high Reynolds numbers statistics of the small scale motion is determined by dissipation rate of turbulence kinetic energy ε and kinematic viscosity nu.

So, on dimensional grounds, he gives the formula for what is called microscales, Kolmogorov microscales. There are other microscales, but this is the one that is generally considered lot useful today. So, the Kolmogorov microscales, there are three. This is the time scale, velocity scale, these are microscales and the length scale.

All are microscales here, microscale. So, the time scale, velocity scale and the length scale you see you can determine it only by knowing dissipation rate of turbulence kinetic energy and the kinematic viscosity. The formulas are given on dimensional grounds that makes the Reynolds number based on the Kolmogorov length scale ok. So, this is your Kolmogorov length scale η.

So, Re_η becomes 1. with this definition ok. So, now the small scale definition is also been done that there is something called a Kolmogorov microscale which you can find out if you have access to ε and the theory also helps to get or get an estimate for ε without actually making a measurement or calculation. So, we will see how it goes. So, now there is another third statement. This third statement comes at talks about the scale separation. So, I talked about large eddy and then a small eddy, but there is something like intermediate for the cascade to the larger structure is breaking down and breaking down to smaller and smaller and eventually become so tiny that we assume that it is statistically isotropic and we can determine its scales using the Kolmogorov microscale.

So, now about the intermediate one where this cascade is occurring, vortices are breaking down. So, it talks about what is called a universal equilibrium range or inertial sub-range. So, what it says is in every flow at high Reynolds numbers, statistics of motions of scale intermediate that is we call this L intermediate. So, this L intermediate scale. or let us use this inertial intermediate also intermediate intermediate or L inertial because we are talking about an inertial sub range.

We will see why the name inertia comes here. So, in every flow of high Reynolds number statistics of motions of scale L inertia or L intermediate that is larger than much larger than the Kolmogorov length scale η, but much smaller than what is called a pseudo length scale. We will define what that is later. So, let us take L as the length scale associated with the integral eddy a very large eddy energy producing eddy. So, somewhere you have this intermediate eddy and its length scale is L inertial.

in the inertial sub range. This has universal form and is independent of viscosity. So, that means this cascade process a large eddy breaking out into smaller and smaller he is saying it is not viscous driven. He is saying it is purely inertia driven. So, it is a purely an irrotational process, nonlinear inertial process he is saying. So, viscosity does not play any role in breakup of this eddies, this according to the hypothesis.

So, what does that mean is the general inference is that cascade occurs due to inertia. Cascade occurs due to inertia alone, non-linear interactions and its viscous independent. Viscosity does not play a role here. This does not mean in this intermediate zone the dissipation rates are 0. We did not see any such scale separation when we looked into the budget of the dissipation rates.

It was a continuous curve. It was showing that some places it is small, some places it is large, but there is nothing like a discontinuity where ε is suddenly going to 0. So, dissipation rates are occurring, but this breakup cascade process breakage of vertices from large to small to small to small is intermediate it according to the hypothesis according to him it says it is inertia driven and it is viscous independent, viscosity has no role. So, this another inference we can draw from is that ε is in dynamic equilibrium with energy transfer rate from both sides. So, now I have this large eddies. a big one and then there is an intermediate and then there is a tiny one.

So, there is a breakup of eddies from large to intermediate to tiny that is inertial the breakup and he is saying ε is in dynamic equilibrium with both the range that means whatever dissipation rate is being transferred from the large to the small is being given to the smaller to the smallest. So, ε that means the dissipation rate that it is occurring at the smallest eddy is actually the energy that is coming from the largest. So, if you know how much energy that is being produced at this large scale. you can actually know that that should be the energy that should be dissipated.

That is the connection this hypothesis making. It is separating the scales, it is giving roles for each of this eddy. It is telling that large eddy you produce energy, small scale you dissipate energy and you can dissipate energy of whatever is being produced by the large scale. So, if I know the energy that is contained in the large eddy, then I will know how much it is getting dissipated. So, it is making this connection by making assumptions, these three primary statements.

So, ε is in dynamic equilibrium. E coming from large to intermediate is just transferred to the small one. So, this idea will be very useful for turbulence modeling. I am not talking about the theory and if you want to study turbulence physics of course, you can go and argue produce data to prove or disprove Kolmogorov that is another direction. But if you just put the hat of a turbulence modeling person this idea is useful. because it is giving some roles which is beneficial to you to modeling right.

So, turbulence modeling takes advantage of this Kolmogorov 41 hypothesis ok. So, now we see