Course Name: Turbulence Modelling

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Week - 10

Lecture – Lec55

55. Dissipation rate and Pressure-Strain rate modelling for RSM - II

So, what we have left with is the fourth term which is the pressure strain rate term. So what did we call this? I think we used either Φ_{ij} or π_{ij} term. In literature you will see both the symbols being used. So the π_{ij} exact we have a formulation $\overline{\frac{p'}{\rho}\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}}$.

So this is something that requires to be modeled because this is a redistribution term accounting for anisotropic nature in the flow or at least I would not call it anisotropic at least it it handles the two component limit and it also tries to drive the flow towards isotropy because it is redistributing. Without this term anisotropy is large Let us say the production is only occurring in one direction, then isotropy is much more large. This term tries to drive the flow, drive the turbulence towards isotropy. It removes turbulence in one direction and distributes to the other direction, tries to make the flow as isotropic as possible.

It is still not isotropic because there is production rate, there is dissipation rate which is driving the flow still to anisotropic state. But this one is working against that. So now entire modeling idea here. So modeling anything this is not applicable to only this term. So modeling implies relating unknowns to knowns of course using some arguments that makes sense of course.

So what is the unknown here? Unknown is this particular term. What are the knowns that we have right? So π_{ij} is the unknown. So what is known? What do we have access while solving Reynolds stress model? Reynolds stresses we have access, so knowns are ui prime, uj prime average have access to this. What do I have access to? In addition to that I am solving seventh transport equation, so I have dissipation rate. In addition to that I am solving the mean momentum equation, so I have the mean strain rate also I have access to $\frac{\partial u_i}{\partial x_j}$.

So I essentially have access to this this is the knowns I have. So I somehow have to establish a relationship between π_{ij} and this. So you can see the contrast here. It is the fluctuation of, so it is a gradient of velocity fluctuation and its correlation with pressure fluctuation. and how it is related to Reynolds stresses or dissipation rate or mean strain rate we will see.

Somehow we will come closer to it okay. So to establish that what is that equation which relates pressure to velocity? Is there an equation that you have learned? Poisson equation right. So this you would have looked into in your fluid mechanics classes or at least in the CFD classes it has been thought. So let us look at that one. So we need a pressure fluctuation P' right.

So what I need is I need a Poisson equation for P'. you would have seen a Poisson equation for pressure instantaneous pressure itself. Now I need it for a pressure fluctuation because that is there and I need to slowly move towards its correlation term also with the velocity fluctuations. So to get this the procedure to get this is essentially similar to you derive your Poisson equation. You have to take divergence of your Navier Stokes equation minus the divergence of your RANS equation.

Just like the way we derived equation for fluctuating momentum right we derived an equation for u_i'. for that we said we take Navier Stokes equation decompose Reynolds decompose and then from that minus of we took the mean momentum equation to get transport equation for u_i' right. So, so we will do that also for to get the p' equation. So, how we will just start with this part. So, first we take the this part the first one.

So, it is $\frac{\partial}{\partial x_i}$ of the left hand side of your Navier Stokes equation which is $\frac{\partial u_i}{\partial t}$ plus or I am adding this divergence term everywhere. So, $\frac{\partial}{\partial x_j}\left(\frac{\partial u_i u_j}{\partial x_j}\right)$ your advection term. I am taking the conservative form here okay equal to the $\frac{\partial}{\partial x_i}\left(-\frac{1}{\rho}\frac{\partial p}{\partial x_i}\right)$ okay minus 1 by rho okay the densities I left it $\left(-\frac{1}{\rho}\frac{\partial p}{\partial x_i}\right)$ and I have the viscous part which is $\nu\frac{\partial}{\partial x_i}\left(\frac{\partial^2 u_i}{\partial x_j \partial x_j}\right)$ ok. So, now I readily see that this I can interchange here.

I can first differentiate with respect to xi and with respect to t correct. So, I can differentiate this with respect to I can do this differentiate first with respect to the xi and then the time. So that makes it this go away right. So goes away due to continuity. This term survives and then this term obviously survives.

The last term again the same thing I can first differentiate this with respect to xi. So, if I rewrite this actually this particular term is nothing but it is $\frac{\partial}{\partial x_j}(\frac{\partial u_i}{\partial x_j})$correct $\frac{\partial^2 u_i}{\partial x_j \partial x_j}$. So, now I can rewrite this as $\frac{\partial}{\partial x_j}(\frac{\partial u_i}{\partial x_i})$. First differentiate with respect to xi and then the j, then again going to 0 continuity. This equation you would have derived before how to get a Poisson equation for instantaneous pressure.

So, I have these two a Laplacian for pressure and a complicated source term. So, we have Laplacian here, Laplacian term for the pressure and this is a complex source term. Why do I call it complex? non-linear term is there here okay. So you have non-linear this makes life difficult for those who are working with incompressible flows. If you are working with compressible flows you have an equation of state relating your pressure to density and temperature for an ideal gas or you can take you can use thermodynamics to get different for a different types of gas property you can still get an equation of state for a dense phase gas and so on.

But for incompressible flow this is the only way you can relate your pressure to the velocity computation here and you have a non-linear source term. So just put your hat as a CFD expert and then you will see the complexity here. You have to solve this for pressure numerically by discretizing this equation where there is a nonlinear source term. And this those who have experience solving this know that this is the most computationally expensive part in your Navier-Stokes calculations. That means the velocity calculations for u v w would not take much computational effort compared to this.

It is probably one or two orders of magnitude larger. Let us say you would require 100 iterations to get pressure converged this Poisson equation. But you probably need just one iteration to get your velocities converged. So you are just waiting for this to done. So it requires massive parallelization techniques.

So the Poisson equation is there for this particular component. So now I can decompose this, apply Reynolds decomposition. So I can say apply Reynolds decomposition. So I get essentially $\frac{\partial}{\partial x_i \partial x_j}\left(\left(\overline{u}_i + u_i^{'}\right)\left(\overline{u}_j + u_j^{'}\right)\right)$ equal to this term which is $-\frac{1}{\rho}\frac{\partial^2 \overline{p}}{\partial x_i^2}$

and then I also get this term which is $\frac{\partial^2 \rho^{'}}{\partial x_i^2}$ decomposed into a mean pressure and fluctuating pressure term. So I get here 4 different terms here right. So I get $\frac{\partial \overline{u}_i \overline{u}_j}{\partial x_i \partial x_j} + \frac{\partial \overline{u}_i u'_j}{\partial x_i \partial x_j} + \frac{\partial \overline{u}_j u'_i}{\partial x_i \partial x_j} + \frac{\partial u'_i u'_j}{\partial x_i \partial x_j}$ equal to the right hand side part. So, of course now we have to do the other part which is the divergence of the RANS equation. So, this is one equation now here.

So, I can take just taking this. copy and then paste this ok. So, I call this equation 1. Now I need to do the second part which is the Rans part here. So, again I would get $\frac{\partial}{\partial x_i}\left(\frac{\partial \overline{u}_i}{\partial t}\right) + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_i \partial x_j}$ equal to the right hand part which is $\frac{\partial}{\partial x_i}\left(\frac{-1}{\rho}\frac{\partial \overline{p}}{\partial x_i}\right)$.

plus I have the two stress terms here, one is the viscous, one is the turbulent stress term. So, I would get $\nu \frac{\partial}{\partial x_i}\left(\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j}\right)$ and then I have this - $- \frac{\partial}{\partial x_1}\left(\frac{\partial \overline{u'_i u'_j}}{\partial x_j}\right)$. So, this gives me the equation 2 here. Of course, I can get rid of some terms before that.

Again this interchanging interchanging here again I get $\frac{\partial t}{\partial x_i}$. So, this term goes away continuity again this term survives the non-linear source pressure Laplacian survives the viscous goes away in again right. So, I can say this is dou xi here and this is dou xj. again this goes to 0 same as previous argument continuity.

So, 3 terms survive here. So, this is my equation 2. Now I need to subtract 1 from 2 and do some more rearrangements to get an equation for P'. We will see this in the next class.