Course Name: Turbulence Modelling

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Week - 9

Lecture – Lec51

51. Correctors for eddy-viscoscity models - II

Now, we will see what will happen to the Pk model term. So, the Pk model you are using Boussinesq, again Boussinesq will cause this problem here. So, Reynolds stresses are replaced by your Boussinesq, which is 2 nu t S ij average minus 2 third k delta ij and the mean strain rate term; this is the model term. Again, if I add it up, so expand this, I get 2 nu t Sij bar, or you can write this as half of dou ui bar by dou xj plus dou uj bar by dou xi dou ui bar by dou xj minus 2 third k delta ij dou ui bar by dou xj. This last term goes away because of continuity and I have essentially these two terms.

$$P_{k_{model}} = \left[2\nu_t \bar{S}_{ij} - \tfrac{2}{3}k\delta_{ij}\right]\frac{\partial \bar{u}_i}{\partial x_j} = \cancel{2}\nu_t \cancel{\tfrac{1}{2}}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\frac{\partial \bar{u}_i}{\partial x_j} - \cancel{\tfrac{2}{3}k\delta_{ij}\frac{\partial \bar{u}_i}{\partial x_j}} \quad \text{continuity}$$

So, if I expand this I get I have nu t dou ui bar by dou xj dou ui bar by dou xj plus nu t dou ui bar by dou xj dou uj bar by dou xi. This is sum of 9 plus 9, 18 terms. So, let us expand it and see what happens to that.

So, if I expand this, I get nu t of dou u1 bar by write it here first nu t of dou u1 bar by dou xj, dou u1 bar by dou xj term plus dou u2 bar by dou xj, dou u2 bar by dou xj plus dou u3 bar by dou xj, dou u3 bar by dou xj term plus nu t of dou u1 bar by dou xj dou uj bar by dou x1 plus dou u2 bar by dou xj dou uj bar by dou x2 plus dou u3 bar by dou xj dou uj bar by dou x3.

$$\nu_t \frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial \bar{u}_i}{\partial x_j} + \nu_t \frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_i} = \nu_t\left(\frac{\partial \bar{u}_1}{\partial x_j}\frac{\partial \bar{u}_1}{\partial x_j} + \frac{\partial \bar{u}_2}{\partial x_j}\frac{\partial \bar{u}_2}{\partial x_j} + \frac{\partial \bar{u}_3}{\partial x_j}\frac{\partial \bar{u}_3}{\partial x_j}\right) + \nu_t\left(\frac{\partial \bar{u}_1}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_1} + \frac{\partial \bar{u}_2}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_2} + \frac{\partial \bar{u}_3}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_3}\right)$$

If I now sum over j, I would get square of this plus dou u1 bar by dou x2 square plus dou u1 bar by dou x3 square plus dou u2 bar by dou x1 square plus dou u2 bar by dou x2 square plus dou u2 bar by dou x3 square and then the third term here which is dou u3 bar by dou x1 plus dou u3 bar by dou x2 plus dou u3 bar by dou x3 ok. Sum of 9 terms for this. And again, the sum of another 9 terms for the cross terms here, which is dou u1 bar by dou x1 dou u1 bar by dou x1 plus dou u1 bar by dou x2 dou u2 bar by dou x1 plus dou u1 bar by dou x3 dou u3 bar by dou x1. The second term on the right-hand side part which is dou u2 bar by dou x1 dou u1 bar by dou x2 plus dou u2 bar by dou x2 dou u2 bar by dou x2 plus dou u2 bar by dou x3 dou u3 bar by dou x2, right? And then the last term, which is dou u3 bar by dou x1 dou u1 bar by dou x3 plus dou u3 bar by dou x2 dou u2 bar by dou x3 plus dou u3 bar by dou x3 dou u3 bar by dou x3, 9 plus 9, 18 terms.

$$
\begin{aligned}
= \nu_t \Bigg[& \left(\frac{\partial \bar{u}_1}{\partial x_1}\right)^2 + \left(\frac{\partial \bar{u}_1}{\partial x_2}\right)^2 + \left(\frac{\partial \bar{u}_1}{\partial x_3}\right)^2 + \left(\frac{\partial \bar{u}_2}{\partial x_1}\right)^2 + \left(\frac{\partial \bar{u}_2}{\partial x_2}\right)^2 + \left(\frac{\partial \bar{u}_2}{\partial x_3}\right)^2 + \left(\frac{\partial \bar{u}_3}{\partial x_1}\right)^2 + \left(\frac{\partial \bar{u}_3}{\partial x_2}\right)^2 + \left(\frac{\partial \bar{u}_3}{\partial x_3}\right)^2 \Bigg] \\
+ \nu_t \Bigg[& \frac{\partial \bar{u}_1}{\partial x_1}\frac{\partial \bar{u}_1}{\partial x_1} + \frac{\partial \bar{u}_1}{\partial x_2}\frac{\partial \bar{u}_2}{\partial x_1} + \frac{\partial \bar{u}_1}{\partial x_3}\frac{\partial \bar{u}_3}{\partial x_1} + \frac{\partial \bar{u}_2}{\partial x_1}\frac{\partial \bar{u}_1}{\partial x_2} + \frac{\partial \bar{u}_2}{\partial x_2}\frac{\partial \bar{u}_2}{\partial x_2} + \frac{\partial \bar{u}_2}{\partial x_3}\frac{\partial \bar{u}_3}{\partial x_2} + \frac{\partial \bar{u}_3}{\partial x_1}\frac{\partial \bar{u}_1}{\partial x_3} + \frac{\partial \bar{u}_3}{\partial x_2}\frac{\partial \bar{u}_2}{\partial x_3} + \frac{\partial \bar{u}_3}{\partial x_3}\frac{\partial \bar{u}_3}{\partial x_3} \Bigg]
\end{aligned}
$$

Luckily, most will go away. We have to expand it to get rid of it in the stagnation zone. But all these terms exist in a regular flow where you do not have such considerations. So what terms go away here? We have all the rotational strain terms. This one, all the rotational strain u2 x3, u3 x1, u3 x2 and then this, this, this, this otherwise you have this, that is small rotational strain.

$$
\begin{aligned}
= \nu_t \Bigg[& \left(\frac{\partial \bar{u}_1}{\partial x_1}\right)^2 + \cancel{\left(\frac{\partial \bar{u}_1}{\partial x_2}\right)^2} + \cancel{\left(\frac{\partial \bar{u}_1}{\partial x_3}\right)^2} + \cancel{\left(\frac{\partial \bar{u}_2}{\partial x_1}\right)^2} + \left(\frac{\partial \bar{u}_2}{\partial x_2}\right)^2 + \cancel{\left(\frac{\partial \bar{u}_2}{\partial x_3}\right)^2} + \cancel{\left(\frac{\partial \bar{u}_3}{\partial x_1}\right)^2} + \cancel{\left(\frac{\partial \bar{u}_3}{\partial x_2}\right)^2} + \left(\frac{\partial \bar{u}_3}{\partial x_3}\right)^2 \Bigg] \\
+ \nu_t \Bigg[& \frac{\partial \bar{u}_1}{\partial x_1}\frac{\partial \bar{u}_1}{\partial x_1} + \cancel{\frac{\partial \bar{u}_1}{\partial x_2}\frac{\partial \bar{u}_2}{\partial x_1}} + \cancel{\frac{\partial \bar{u}_1}{\partial x_3}\frac{\partial \bar{u}_3}{\partial x_1}} + \cancel{\frac{\partial \bar{u}_2}{\partial x_1}\frac{\partial \bar{u}_1}{\partial x_2}} + \frac{\partial \bar{u}_2}{\partial x_2}\frac{\partial \bar{u}_2}{\partial x_2} + \cancel{\frac{\partial \bar{u}_2}{\partial x_3}\frac{\partial \bar{u}_3}{\partial x_2}} + \cancel{\frac{\partial \bar{u}_3}{\partial x_1}\frac{\partial \bar{u}_1}{\partial x_3}} + \cancel{\frac{\partial \bar{u}_3}{\partial x_2}\frac{\partial \bar{u}_2}{\partial x_3}} + \frac{\partial \bar{u}_3}{\partial x_3}\frac{\partial \bar{u}_3}{\partial x_3} \Bigg]
\end{aligned}
$$

Small rotational Strain

So that means I get finally Pk model is equal to, so this was Pk exact, Pk exact was almost 0. Now we have the Pk model equal to nu t of, so I have 3 plus 3, 6 terms, which is essentially 2 nu t if I take because this dou u1 bar by dou x1 is repeated here. So, I get 2 nu t of dou u1 bar by dou x1 square plus I have dou u2 bar by dou x2 term. So, which is dou u2 bar by dou x2 square plus dou u3 bar by dou x3 square. And is this 0 in a stagnation zone? What is it? Extremely large.

$$
P_{K_{model}} = 2\nu_t \left[\left(\frac{\partial \bar{u}_1}{\partial x_1}\right)^2 + \left(\frac{\partial \bar{u}_2}{\partial x_2}\right)^2 + \left(\frac{\partial \bar{u}_3}{\partial x_3}\right)^2 \right] \approx \text{VERY LARGE}
$$

So, this is like super large. So, this is approximately equal to very large much bigger than the font size here. So, this value becomes super large in the stagnation zone. So, what is your model is telling? It is completely opposite to what it should be in reality. So, you need to do something about it otherwise you will get unphysical, unrealizable results at the end.

So, there are two directions to fix this particular part. One is an explicit method, the other is an implicit method. Is this clear so far before we go into a solution? So, basically, we looked into a stagnation zone where there is a local isotropy and also the rotational strains are small compared to irrotational strains that is what that is what occurs in a stagnation zone. And we see that model is giving predicting very large values because you are essentially going after strain. In the Boussinesq, you essentially related your stress to the strain and this is the consequence of that.

So, there are two methods to have a solution here, one modelling option is to go after a limiter for the production rate of turbulence kinetic energy, a Pk limiter and this modelling option you can say or call it explicit method, explicit method not to be confused with any CFD techniques here, explicit implicit methods. But what I mean by explicit is explicit implying, explicit implies explicitly controlling Pk in the k equation. You directly go after what is called a Pk limiter. So, what we look at is called a Pk limiter. Because Pk is going very large, you need to enforce a limiter for that one ok and these are explicit because you directly use it in the k equation.

So, there are many production limiters, Pk limiters for a k epsilon. For example, for a k epsilon model you can get the Pk is set as that is a Pk model will be set as a minimum of whatever you are computing from the model. So, you just have to add an extra line in your code comma you have a C_{P_k} epsilon. So, your production rates are becoming very large because of large irrotational strain, but I will transfer this to the dissipation rate. So that there is a you know the balance between the Pk and epsilon.

$$P_{k\,\text{limiter}} \Rightarrow k\text{-}\varepsilon \Rightarrow P_k = \min\left(P_{k\,\text{model}},\ C_{P_k}\varepsilon\right).$$

So I am going to take the minimum of these two Pk being set, let us say some coefficient times epsilon, but the C_{P_k} requires some optimization. So, user-dependent value that is a downside here, ok? So, the downside is that C_{P_k} is user-dependent or, you can say flow, flow-dependent. So, some optimization is required here, flow-dependent, not flow-dependent. I would not call it flow-dependent here. You can say some optimization is required.

So, the Pk limiter this could be one option you can play with it to see whether you get a good value. In a K omega SST model, Mentor suggested a limiter for it, which is Pk equal to a minimum of again the Pk model value, or you take 10 times the beta star K omega value. So, this is coming from your, the reference for this is 1993, 1994. So, this is you take minimum of whatever is computed or 10 beta star, beta star if you remember is also a constant there k omega value here and k epsilon also got a correction from the original group which made standard k epsilon at Manchester. So, this is called Kato-Launder correction.

$$K\text{-}\omega \text{ SST} \Rightarrow P_K = \min\left(P_{K\,model},\ 10\beta^* K\omega\right)$$

So, for a k epsilon also, you can do another option, which is the Pk model itself. You are computing not taking this mean function, but look at using the rotation rate here. So, the strain rate caused a problem. So, instead of that, they want to use rotation rate into the Pk computation. So, nu t square root of 2 Sij Sij and then you look at 2 omega ij, omega ij.

Omega ij is the rotation rate tensor if you recall what it is. So, omega ij is the mean rotation rate tensor. So, including this will cause Pk to be better. So, this particular correction is called the Kato-Launder correction. So, this is Kato-Launder.

It is 1993. That is a reference for this. So these are explicitly controlling you put a main function, or you directly compute the Pk model using another formula. So basically, you are attacking the k equation, then it is explicit. If you are not if you are going to if you are going after epsilon equation then it becomes implicit.

k is untouched. You correct everything through epsilon implicitly. Even though the unphysical behaviour is in the k equation, you can control it using epsilon or the second term. So, the implicit method is modelling option 2 or realizability constraints only. The implicit technique implicit method implying controlling through controlling excessive Pk through epsilon equation. So, this particular option is basically you would have a time scale limiter here.

If you look at time scale, you have a k by epsilon ratio. So, in implicit techniques you usually use a time scale limiter or a turbulent time scale, turbulence time scale limiter. So, for a k epsilon model, for example, you can look at this tau can be for a k epsilon model, you can implement after all you compute k and epsilon, you can set this time scale as take a minimum of k by epsilon, or you take 1 by 3 C mu lambda i. lambda i you know from the this Durbin's Reliability Constraint right, so that you can use it here lambda i ok.

Lambda i is the square root of 2 third S ij bar S ij bar or for a k omega it will be minimum of your 1 by omega or 1 by 3 lambda i for a k omega model.

Both of these techniques are used explicit implicit corrections to limit the excessive production rate if you have stagnation type of flows. in general to make it sure. So, this completes the realizability constraints and as I mentioned there are realizable models. One of the examples I have already uploaded in the Moodle. You will see there is a realizable k epsilon model.

One can also implement that if you like that model you can implement it. Or if you already have implemented let us say k omega and I am you are just going to implement this limiters to make sure that the model behaves physically that is also a perfectly good choice. And I will show you some results what can happen. Sometime this realizability constraints can give good results making flow realizable. So, this is an example of Durbin's realizeability constraint.

So, the wake is expected to be unsteady oscillating, but when we just use let us say a k epsilon model without any condition the wake is completely steady no matter what we do it is not oscillating at all, but by implementing that Durbin's constraint then we get this wake starts to be unsteady. So, it gives you even though this flow you see that there is no stagnation zone here, but still it helped. Stagnation zone definitely it will have an impact. In general, it is good to use these constraints just to make sure that the unphysical part is eliminated locally if it is present in the flow. So, these are good and easy to implement just an added extra line in the code.

Ok and in spite of all this of course, this now completes the entire eddy viscosity model. As I said we will not go into every other type of eddy viscosity model there are many, but you would figure it out yourself any other model that is there. I have covered majorly the k epsilon, RNG, realizable and then low Reynolds number, high Reynolds number all this major part. And despite all this sometimes you will see that the results are not good. That is inherently you are using Boussinesq.

You are taking Reynolds stresses, which is a completely random component. The stochastic behaviour of turbulence is modelled as statistical. That is dou u bar by dou x2 terms and so on. You are looking into a mean strain instead of a stochastic component. So that Boussinesq itself will not give you good results many times and I have an example here I can show you.

This is from our own results. So, here, this is a dense phase of carbon dioxide. So, it is heavier than air and it is dispersion. So, I am essentially looking into the concentration profile here. The y-axis shows the CO_2 concentration, the x-axis is the distance. So, you can see the experimental values are this.

open squares, and these three others are standard k epsilon, SKE, RNG k epsilon and realizable k epsilon models. So, this realizable it is a realizable k epsilon model, we did not implement realizability constraint on a standard k epsilon, this is actually a realizable k epsilon model. So, you see, all three are behaving far, far away from the experimental data; this does not mean that this is the behaviour in every class of flow. But, in this particular situation or in general, you will see that there is a limitation of an eddy viscosity model on how far it can do good. Sometimes you just need to switch to a complicated model or even go to an eddy resolved technique.

You have reached the limitation of eddy viscosity modeling regardless of changing k epsilon to k omega or anything it will not yield any great result. with so much assumptions that we have made, sometimes results are not good, and these are like practical cases. It could be like life and death here. I mean, if you believe eddy viscosity model and say that the concentration is low here, and I will go and walk, and you are dead. the actual experimental value is twice that of what the eddy viscosity model has predicted.

So, it is about, I think, 6 percent, 6 percent volume fraction CO_2 is like the life and death value. I think you will be unconscious. Maybe 8 percent you are dead. And this is like 20 percent in the experiments.

This is predicting like 10 percent. So, if you want to take chances, it is an individual choice. So this exists. So that means I am pitching for that there is a need to go into models that are much more complicated and giving much better accurate results. And we see this also in I think I have another site. Yeah, this is even more very industrial problem like atmospheric dispersion.

This is like very large case. This is actually actual topography. We have some hills and mountains, valleys and so on, some buildings. An actual case actually is not hypothetical. It is taken, one can download such a map and reconstruct in CFD and this is a chlorine dispersion. you can have pipelines carrying or some industrial facility where there is a leakage it can happen.

So, again here you know the experimental values are way up compared to all the three standard k epsilon R and G realizable or giving nearly the same behavior only a marginal 5-10 percent difference in their behavior, but the experimental values are so large here. So, that will be there at the end of the day ok. Of course, one should also see the cost benefit, the viscosity models are cheaper, it is faster to run, stable, numerically much stable and you are assured of results rather than getting no results. So, there is always this cost benefit one has to look at. So, in the next class, we will start the Reynolds stress modelling, we will go to another class of models.