Course Name: Turbulence Modelling

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Lecture – Lec50

50. Correctors for eddy-viscoscity models – I

So, let us get started again. So, we were looking into essentially the realizability constraint. There are, of course, some models which are by default made it to be realizable. Those kind of options also one can look at, but here we are essentially considering models by default which are not realizable and how do you actually fix it? So, it is a bit of like, let us say you already implemented a model which is giving you some unphysical results, then what is that bit of thing that you can implement in the code which can help it to make realizable, ok? So, in that way you do not have to implement a new model right. So, we saw that there are many realizable conditions and one thing was this normal stresses staying positive, right? Normal stresses staying positive.

This, of course, would make the turbulence kinetic energy stay positive everywhere, right? And one issue we saw was that there you can actually get into certain flows where locally this normal stresses can go negative. That is one example was we took this stagnation zone, stagnation zone or stagnation flow. Impinging flows like jet impinging on a plate, and so on. So, here, in such types of flows, we saw that there will be large irrotational strain, either large positive or large negative irrotational strain will be there which is causing issues, right? And then, we also found that when such a thing exists, we need to fix it.

So, today, we will discuss one such solution for this. So, here, one of the realizability conditions, realizability constraint, that you can implement this particular realizability constraint comes from Paul Durbin 1996. This constraint is very elegant because, as I said, you can implement this into your existing code, right? And it would make it realizable. So, the issue we have seen that in large irrotational strain, in this kind of situations you will have the normal stresses going at least one of the normal stresses going it could be your $u' u'$ or $v' v'$ or $w' w'$ these things going negative. That is unphysical.

So, to do away with it or you need to have a solution for this. So, what he does is he will take your original definition from Boussinesq that is the stress part here ui prime ui prime over bar. So, this will be 2 third k minus 2 nu t instead of Sij bar he calls it lambda i here. Earlier, it was 2 third k minus 2 nu t Sij bar or Sii bar because we contracted the indices we set i equal to j for normal stresses instead of Sii bar, it became lambda i where this lambda i is set equal to square root of two-third Sij bar Sij bar alright. But of course, here lambda i is a vector right.

$$\overline{u_i'^2} = \frac{2}{3}k - 2\nu_t \lambda_i \quad ; \quad \text{where } \lambda_i = \sqrt{\frac{2}{3}\bar{S}_{ij}\bar{S}_{ij}}$$

So, you this you need to set i equal to a certain value when you expand this one, j is summed over. So, this will give you lambda 1 will particularly give you what to say. For example, if you are looking into the stress for normal stress, let us say i equal to 1, setting i equal to 1, I get u1 prime u1 prime average will be 2 third k minus 2 nu t of this lambda i which is square root of 2 third s1j bar s1j bar. This will be 2 third k minus 2 nu t square root of 2 third. So, this will sum over j which will be your s11 bar s11 bar plus s12 bar s12 bar plus s13 bar s13 bar.

$$\text{E.g. } i=1 \Rightarrow \overline{u_1' u_1'} = \frac{2}{3}k - 2\nu_t \sqrt{\frac{2}{3}\bar{S}_{1j}\bar{S}_{1j}} = \frac{2}{3}k - 2\nu_t \sqrt{\frac{2}{3}\left(\bar{S}_{11}\bar{S}_{11} + \bar{S}_{12}\bar{S}_{12} + \bar{S}_{13}\bar{S}_{13}\right)}$$

So, there is no summation over i here, i is a free index, i is a free index here, sum over j. So, anyway, so, he takes this particular part here, and now. If you want normal stresses to stay positive, the condition should be that this 2 nu t lambda I should be smaller than 2 third k. So, for normal stresses for normal stress to stay positive, this 2 nu t lambda i should be less than or equal to 2 third k. We are only deriving the constraint for this now.

$$\text{For normal Stress to Stay positive} \Rightarrow \cancel{2}\nu_t \lambda_i \le \frac{\cancel{2}}{3}k$$

So, here now if I take out this part, I essentially get a condition which is ν_t should be less than or equal to k over 3 lambda i.

$$\boxed{\nu_t \le \frac{k}{3\lambda_i}}$$

So, we can easily implement this constraint for eddy viscosity. Eddy viscosity taking this value, but you are also computing eddy viscosity in your model. I told you this is a generic condition that you can implement; it is a generic constraint for eddy viscosity that you can implement for any k epsilon, k omega, k omega SST or any type of model. So, you are actually computing eddy viscosity using that formulation and then you have this.

So, you have to choose one of this at every location node by node at every time step by time step. So, how you do that? So, this is the realizability constraint ok, realizability constraint. if eddy viscosity will be always less than or equal to this $k/3\lambda_i$ where λ_i is this, then the model becomes realizable physical. So, what you do now is that you can do a generic realizability constraint. And what that looks like is essentially, you have ν_t should be equal to this is now at the implementation level in your code you can do you should be minimum of you choose one of this either your ν_t from your model, this $k/3\lambda_i$.

Generic realizability constraint $\Rightarrow$ $\boxed{\nu_t = \min\left(\nu_{t_{\text{model}}}, \frac{k}{3\lambda_i}\right)}$

Okay, so you compute eddy viscosity from your model. Let us say for standard k epsilon, it is $C_\mu k^2/\epsilon$, and then you also have this $k/3\lambda_i$. So easy to implement just one line implementation can help you make your model give realizable or physically correct results okay. So this is one constraint that people can use. So, I said nu t model here, here nu t model depends on the EVM.

So, for a standard example for a standard k epsilon. You would implement this as ν_t is equal to minimum of C mu k square by epsilon comma k by 3 lambda i or for a k omega, it is k by omega comma k by 3 lambda i and so on. So, implementation is straight forward for this. So, this makes at least it makes sure that normal Reynolds stresses stay positive in your flow. We are not talking about accuracy.

$$\nu_t = \min\left(C_\mu \frac{k^2}{\varepsilon}, \frac{k}{3\lambda_i}\right)$$

Accuracy is another issue because you already closed the Reynolds stresses using a Boussinesq approximation. Those things are another issue, but at least you will not get unphysical results using this constraint. And there could be another problem coming in this so called stagnation zones. So, we look into that what other problems appears in stagnation zone and what you should do to make the models realizable. So, again let us take up consider stagnation zone.

So, here in every stagnation zone or stagnation flow, you will have in that local zone you will have as we already seen large irrotational strain, right? And that means this irrotational strains are much larger than rotational strain components. In addition to that, we will also have a local isotropy in this stagnation zone. So, in this type of flows, it is characterized by first thing is that you will have large irrotational strain, which implies your dou u1 bar by dou x1 dou u2 bar by dou x2. u 3 bar by dou x 3 and so on, not so on. These three are the irrotational strain.

These are much, much larger than your rotational strain components, which are dou u1 bar by dou x2, dou u2 bar by dou x3, dou u3 bar by dou x1 and so on. So, irrotational strain or volumetric strain, this is the rotational strain ok. So, your irrotational strains are much much larger than rotational strain in the stagnation zone that is one thing.

(a) large irrotational strain $\Rightarrow \dfrac{\partial \bar{u}_1}{\partial x_1}, \dfrac{\partial \bar{u}_2}{\partial x_2}, \dfrac{\partial \bar{u}_3}{\partial x_3} >> \dfrac{\partial \bar{u}_1}{\partial x_2}, \dfrac{\partial \bar{u}_2}{\partial x_3}, \dfrac{\partial \bar{u}_3}{\partial x_1} ---$

(irrotational strain) >> (rotational strain)

Another thing is you will also get locally isotropy, a statistical isotropy appears in stagnation zones. You will get statistically local isotropy, which implies your u1 prime square average more or less equal to your or equal to the other two components, your three RMS components are nearly equal, a local statistically local in that particular zone only, not everywhere and this occurs what I am talking about is inflows these two conditions occur.

(b) Statistically local isotropy $\Rightarrow \overline{u_1'^2} \simeq \overline{u_2'^2} \simeq \overline{u_3'^2}$

You will get large irrotational strains compared to rotational strain and you will also get local isotropy in those kind of zones like as already discussed if you have flow impingement this kind of zones. So, this is the this kind of area here, right, the stagnation zone. So, you get this issue. So, if I have these two now, what we have to do is to look at this is what is being considered actually in the physics. So, whether our models work, we have to first see the modelled counterpart of the production rate of turbulence kinetic energy and its exact term.

So, we look at Pk exact and Pk model to see whether both behave with these considerations. What will the Pk exact and Pk model look like? So, let us look at Pk exact in a stagnation zone. So, consider Pk exact in a stagnation zone. What would Pk exact be? So, Reynolds stresses you have ui prime uj prime average dou ui bar by dou xj right minus of it, of course. This is your Pk exact term.

So, if I now expand it, I get Pk exact is equal to, i and j are repeated, so it has to be summed up, so I get minus u1 prime uj prime, I will do it here first, minus u1 prime uj prime average dou u1 bar by dou xj minus I get minus u2 prime uj prime dou u2 bar by dou xj minus u3 prime uj bar dou u3 bar by dou xj. Now if I sum it up also in the other direction, this is the sum of 9 components here. So, I essentially get minus u1 prime u1 prime dou u1 bar by dou x1 minus u1 prime u2 prime dou u1 bar by dou x2 minus u1 prime u3 prime dou u1 bar by dou x3 and the sum of three terms for the second term here which is minus u2 prime u1 prime dou u2 bar by dou x1 minus u2 prime u2 prime dou u2 bar by dou x2 minus u2 prime u3 prime dou u2 bar by dou x3 minus of the last term sum of 3 which is u3 prime u1 prime dou u3 bar by dou x1 minus u3 prime u2 prime dou u3 bar by dou x2 minus u3 prime u3 prime dou u3 bar by dou x3 ok. So, I have a sum of 9 terms here, and we discussed that the normal stresses are statistically isotropic, and we have large irrotational strain compared to rotational strain. So, what are the rotational strains here? This one dou u1 bar by dou x2 right u1 bar by x3 u2 bar by x1 and then u2 bar by x3. So, these are small compared to small rotational strain compared to irrotational terms.

Consider $P_{K_{exact}}$ in a Stagnation Zone $\Rightarrow -\overline{u_i'u_j'}\dfrac{\partial \bar{u}_i}{\partial x_j} = -\overline{u_1'u_j'}\dfrac{\partial \bar{u}_1}{\partial x_j} - \overline{u_2'u_j'}\dfrac{\partial \bar{u}_2}{\partial x_j} - \overline{u_3'u_j'}\dfrac{\partial \bar{u}_3}{\partial x_j}$

$$P_{K_{exact}} = -\overline{u_1'u_1'}\frac{\partial \bar{u}_1}{\partial x_1} - \overline{u_1'u_2'}\cancel{\frac{\partial \bar{u}_1}{\partial x_2}} - \overline{u_1'u_3'}\cancel{\frac{\partial \bar{u}_1}{\partial x_3}} - \overline{u_2'u_1'}\cancel{\frac{\partial \bar{u}_2}{\partial x_1}} - \overline{u_2'u_2'}\frac{\partial \bar{u}_2}{\partial x_2} - \overline{u_2'u_3'}\cancel{\frac{\partial \bar{u}_2}{\partial x_3}} - \overline{u_3'u_1'}\cancel{\frac{\partial \bar{u}_3}{\partial x_1}} - \overline{u_3'u_2'}\cancel{\frac{\partial \bar{u}_3}{\partial x_2}} - \overline{u_3'u_3'}\frac{\partial \bar{u}_3}{\partial x_3}$$

Small rotational Strain term (Compared to irrotational term)

So, I get only the 3 terms surviving here which is minus u1 prime u1 prime average dou u1 bar by dou x1 minus u2 prime u2 prime average dou u2 bar by dou x2 and then minus u3 prime u3 prime average dou u3 bar by dou x3. The second condition that occurs in a stagnation zone is local isotropy. Now, that means the three components here they are statistically isotropic. So, that means this is equal to essentially I can write only one component instead of three, and that will be the sum of dou u1 bar by dou x1 plus dou u2 bar by dou x2 plus dou u3 bar by dou x3.

This is statistically, or I can write here statistically local isotropy in the stagnation zone. What will happen to this bracketed term? 0, right? Continuity. So, that means the Pk exact has to be not exactly 0. I would say approximately 0 because this is small. The rotational strain terms are small, and this is only a local zone that we are looking at is exactly not 0.

$$= -\underbrace{\overline{u_1'u_1'}}\frac{\partial \bar{u}_1}{\partial x_1} - \underbrace{\overline{u_2'u_2'}}\frac{\partial \bar{u}_2}{\partial x_2} - \underbrace{\overline{u_3'u_3'}}\frac{\partial \bar{u}_3}{\partial x_3} = -\overline{u_1'u_1'}\left(\frac{\partial \bar{u}_1}{\partial x_1} + \cancel{\frac{\partial \bar{u}_2}{\partial x_2}} + \frac{\partial \bar{u}_3}{\partial x_3}\right) \simeq 0$$

Continuity

Statistically local isotropy in the Stagnation Zone

They are just small. So it's approximately zero. So the production rates has to be zero means less turbulence has to be there in stagnation zones.