Course Name: Turbulence Modelling

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Lecture – Lec49

49. Realizability constraints in eddy-viscoscity models – II

Let us look at an issue. So, before that let us frame the question. Are eddy viscosity models realizable? That means physical? So, we will take an example and see what it looks like. So, one issue is coming from Boussinesq itself. That was the origin; the closure for your Reynolds stresses came from Boussinesq.

So, we look at a limitation of Boussinesq. This issue is also called irrotational strain issue. So, we will see what happens here. So, if you recall Boussinesq, it is essentially ui prime uj prime average minus of this is equal to 2 nu t Sij bar minus 2 third k delta ij.

$$-\overline{u_i' u_j'} = 2\nu_t \bar{S}_{ij} - \frac{2}{3} k \delta_{ij}$$

This is the turbulence closure using Boussinesq. Now, I would like to take one of the normal stresses because I already said normal stresses must stay positive. It is a square term. So, I would like to take one normal stress. Let us say let i equal to j equal to 1.

That will give me $\overline{u_1' u_1'}$. So, this must stay positive. So, this I can write it as 2 third k delta 1 1 of course, which is 1 minus 2 nu t S 1 1 average ok. So, what does this give me actually here? This gives me 2 third k minus 2 nu t, this is nothing but half of dou u1 bar by dou x1 plus dou u1 dou u1 bar by dou x1 dou i bar by dou x j plus dou u j bar by dou x i, i and j both are 1. So, this essentially becomes, so this is equal to your u 1 prime, u 1 prime is therefore equal to 2 third k minus 2 nu t dou u 1 bar by dou x 1.

$$\overline{u_1'u_1'} = \frac{2}{3}k\delta_{11} - 2\nu_t\overline{S_{11}} = \frac{2}{3}k - \cancel{2}\nu_t\left(\frac{1}{\cancel{2}}\right)\left(\frac{\partial\bar{u}_1}{\partial x_1} + \frac{\partial\bar{u}_1}{\partial x_1}\right)$$

$$\therefore\ \overline{u_1'u_1'} = \frac{2}{3}k - 2\nu_t\frac{\partial\bar{u}_1}{\partial x_1}$$

Now, is there a chance that this Reynolds stresses this Reynolds particular Reynolds stress go negative? There is a chance here right. So, it must always stay positive then only it is realizable or physical. Now, according to Boussinesq there is a chance $\overline{u_1'u_1'}$ can become negative and when does that happen? When this particular value takes large value. if this becomes a very large value that means large strain, irrotational strain. It is a dou u1 bar by dou x1 is an irrotational strain, it is not a rotational part, volumetric strain.

So, large irrotational strain. So, turbulent is vertical flow. One of the definitions I told you is that turbulence is characterized by three-dimensional vorticity fluctuations. So, it is rotation dominant flow, not this volumetric strain, ok, but here we have a situation where there can be a large irrotational strain that is $\frac{\partial\overline{u_1}}{\partial x_1}$ becoming sorry becoming very, very large. if this happens then you are ok, you can say for a large irrotational strain this $\overline{u_1'u_1'}$ can become negative right, normal stress becoming negative according to Boussinesq in a given flow where such a thing can exist where you have a large irrotational strain and therefore, this is unrealizable or unphysical.

So, by default, the model can give wrong result. Similarly, this can happen for the other two normal stresses $\overline{u_2'u_2'}$, $\overline{u_3'u_3'}$, ok. So, this is one situation where you can have a large irrotational strain large positive value here of course, large positive value right. This can lead to normal stress going negative. We can also have an example for this where something like this can happen and which is commonly occurring in many flows.

I take an example; I can consider another situation or an example here, an unrealizable flow example, or I can say in a stagnation zone. So, this kind of a flow you would have seen, or we will encounter is flow. Let us say you have a flow over a bluff body. Let us say I have a bluff body and then you have flow over it. So, this is your stagnation zone, right? The stagnation point here such a type of a flow or you can have get a region like this.

Let us say you have a plate and then you have flow impingement flow impinging on a plate. This kind of situations can occur right. So, we can have the coordinate system for this as let us say x1, x2. This is x1, x2 same coordinate system. Again, you have a stagnation zone here.

So, around that you will have locally a stagnation zone flow is stagnating. So, in that kind of a situation what will happen in this stagnation zone? What will happen to the irrotational strain? This. So, in such stagnation zones, what would happen to let us say, dou u 1 bar by dou x 1? What will this be $\partial\overline{u_1}$? The $\overline{u_1}$ on the wall here, nearly 0 around it and little bit away from it, it is very large. So, this is a large negative value, right? So, $\partial\overline{u_1}$ here $\frac{\partial\overline{u_1}}{\partial x_1}$ is a large negative value that is large negative value irrotational again. So, if I have this a very negative $\frac{\partial\overline{u_1}}{\partial x_1}$.

So, let us look at the Boussinesq again. So, I have $\overline{u_i^{'}u_i^{'}}$ is equal to the same formula. So, I have this 2 third k minus 2 third k delta ij minus 2 nu t Sij bar. So, here I take i equal to j equal to 2 now. Let us see what happens to that.

i equal to j equal to 2 gives me u2 prime, u2 prime average is 2 third k minus 2 nu t, minus 2 nu t dou u2 bar by dou x2. just like the equation that we have seen here, 2 third k minus 2 nu t dou u1 bar by dou x1 above, this will be one equation let us say. Similarly, I have i equal to j equal to 3, I get u3 prime, u3 prime average is 2 third k minus 2 nu t dou u3 bar by dou x3. Now, if I add these two.

$$\overline{u_i'u_j'} = \frac{2}{3}k\,\delta_{ij} - 2\nu_t\,\overline{S_{ij}}$$

$$i=j=2 \Rightarrow \overline{u_2'u_2'} = \frac{2}{3}k - 2\nu_t\frac{\partial\bar{u}_2}{\partial x_2} \longrightarrow (1)$$

$$i=j=3 \Rightarrow \overline{u_3'u_3'} = \frac{2}{3}k - 2\nu_t\frac{\partial\bar{u}_3}{\partial x_3} \longrightarrow (2)$$

Adding 1 and 2. So, equation 1 plus 2 will give me u 2 prime u 2 prime average plus u 3 prime u 3 prime average equal to 4 thirds k minus 2 nu t dou u2 bar by dou x2 plus dou u3 bar by dou x3. Now, I can use continuity here. So, this gives me 4 third k minus 2 nu t of minus dou u1 bar by dou x1 using continuity here. Substituting this part by minus dou u1 bar by dou x1. So, the formula here becomes 4 third k plus 2 nu t dou u1 bar by dou x1.

$$\text{Eq}(1)+(2) \Rightarrow \overline{u_2'u_2'} + \overline{u_3'u_3'} = \frac{4}{3}k - 2\nu_t\left(\frac{\partial\bar{u}_2}{\partial x_2} + \frac{\partial\bar{u}_3}{\partial x_3}\right) = \frac{4}{3}k - 2\nu_t\left(-\frac{\partial\bar{u}_1}{\partial x_1}\right)$$

But what is $\frac{\partial \overline{u_1}}{\partial x_1}$? Here, large negative value, extremely large negative value. $\frac{\partial \overline{u_1}}{\partial x_1}$ is large negative. So, this becomes very large negative. So, this becomes 4 third k, k is always positive here. So, note k is positive, k is positive and your $\frac{\partial \overline{u_1}}{\partial x_1}$ is large negative.

sorry not equal to 0, it is just its large negative value right, large negative value $\frac{\partial \overline{u_1}}{\partial x_1}$. So, that means this is positive, and this is a large negative, making the entire equation that is the sum of these two as negative. So, this can imply that this implies $\overline{u_2'u_2'} + \overline{u_3'u_3'}$, the sum can go negative, the sum is going negative. I am looking into the total sum this plus this is equal to the right part. k becoming positive $\frac{\partial \overline{u_1}}{\partial x_1}$ is extremely large negative value and therefore, the sum of the two normal stresses becoming negative.

So, now, this does not mean that both are negative, there is a good chance that the both stresses can be negative or one is very negative compared to the other one either way it is unrealizable or unphysical because these two has to be positive correct. So, this means that one of them must be very large negative or I can write in the next slide. So, this means either $\overline{u_2'u_2'}$ and $\overline{u_3'u_3'}$, both are negative or one of them is very large negative value. Either way, it is unphysical, it is unphysical since normal stresses must stay positive, ok? So, we have seen an example where it can go. Completely unphysical your eddy viscosity model by default whichever model you are using will give you because you are using fundamentally Boussinesq.

Then you are deciding for eddy viscosity, you are deciding k epsilon, k omega, k omega SST so many variations right different types of models 1, 2, 3, 4 equation models, but this is an issue with the Boussinesq itself. So, this will lead to unphysical, unrealizable behaviour. We will see how we can fix this in the next class as well as look into other problems you will get from Boussinesq and also the solutions for that.