Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 9

Lecture – Lec48

48. Realizability constraints in eddy-viscoscity models – I

So, let us get started. So, we were looking into the initial conditions and inflow conditions, how to generate them in a RANS framework. And we gave some formulas, we also looked into how do I generate like you guess turbulence intensity, you guess the turbulence length scale from that you can get k and epsilon values, some values like that. And then I also said that the consequences of this. So, this is all user dependent.

So, we have to be little bit careful giving these values. So, the initial guess or initial condition of this turbulence intensity and length scale. So, the initial condition or the value that you are going to guess of, let us say, the turbulence intensity I or the u_{rms} or turbulence intensity that you are going to guess. Essentially, it is u_{rms} only you are guessing, but in the form of turbulence intensity u_{rms}/ u_{∞} or turbulence intensity and the length scale turbulent length scale.

So, this one first thing to know here is that your guess or initial condition that you are giving for this, this will have this will always have an implication on the convergence of your code ok. So, this influences convergence, numerical convergence in all cases. Initial conditions always influences your convergence. This is in generic only, this is nothing to do with turbulent flows. Initial conditions can also, in some cases, affect the converged solution.

This convergence is only the route to the final solution, how fast or slow the solution you get ok. So, influences convergence means the rate at which the solution is converging faster or slower, but it also influences converged solutions in special cases. So, example can look at for example, if your flow has hydrodynamic instabilities, thermal instabilities on top of turbulent flows hydrodynamic or thermal instabilities, you have transient or unsteady flows. So, this may influence I would not say influences, there is a good chance that you have to be careful that your initial condition that you give can influence your final solution. That means, you can change initial condition and your final solution is different.

You have to be careful in those problems right, but in general, it just influences the numerical convergence and you can have an example for l, the turbulence intensity is a bit straightforward to assume as already said, 1 percent to 10 percent something you can guess and give it, but l is a bit complicated because l has no physical meaning. This turbulent length scale has no physical meaning whatsoever. l is coming purely from your using k and epsilon, right? l is $\frac{k^{3/2}}{\varepsilon}$. That was a formula we used to get l. There is also square root of C_μ and all these things are there.

But it is essentially l is $\frac{k^{3/2}}{\varepsilon}$. So, it is like a ratio of k and epsilon. No physical meaning. So, example of l that you can use it is not a guideline, but something you can use. In the absence of knowing what is a turbulent length scale in your flow, which eddy to choose, you can maybe go ahead and choose something which is related to the physical geometry of the problem.

Not really a turbulent length scale, but some geometric length scale. So, search for a geometric length scale. For example, in internal flows, may be hydraulic diameter. I am putting a question mark here it is up to your wisdom to choose whether you want to use it ok. This is all geometric, and in a wake flow, you can use like the chord length or the size of your bluff body.

The size of the bluff body can be used again a question mark because this determines the largest, perhaps the largest, eddy that you can have. If you have a like an aerofoil at an angle of attack or a bluff body this can determine perhaps the how large the eddy can be in the flow. Whether you want to base the dissipation rate on the largest eddy that you have is your choice, right? That is something you have to remember. And then like in jet flows of course, you have the orifice size right orifice diameter. So, all these are like geometric, geometry-based length scales that you have to remember.

So, l is not a physical or I can say l has no physical meaning, purely numerical scale, right? So, l is essentially coming as your $\frac{k^{3/2}}{\varepsilon}$. It is looking into the ratio of these two k and epsilon in a given flow. So, I can show you some data how it looks like in one of the problem. So, you would understand this better. So, here I am plotting the symbols are all DNS data and the lines are all eddy viscosity models.

Focus on only this red symbol red circles that is a rib channel or a rough channel flow from our own DNS data and then I have these three lines red solid line and in a green dashed line and dotted blue line these are three k epsilon models that is standard k epsilon RNG k epsilon and realizable k epsilon three types of k epsilon models and you see the turbulence kinetic energy the trend is ok the values are not matching perfectly with the DNS data, but the trend is ok, right. And the trend is also ok in the dissipation rate also the figure B where this again the trend is somewhat following the DNS data, right. So, in the k epsilon are fine even the eddy viscosity model is reasonably qualitatively it is ok, but look at the length scale here. This length scale as I said it is l is $\frac{k^{3/2}}{\varepsilon}$, maybe there was a square root of μ, C_μ and so on. So, l is looking into the ratio of turbulence kinetic energy to the dissipation rate of turbulence kinetic energy.

So, it is essentially looking into a ratio and if I plot the same thing the red symbol here, here k and epsilon data is coming from DNS. So, you see that l is just rising in a channel. So, what this channel implies is this is the wall here. So, the x axis is the wall normal distance and 0 means wall. So, this is the wall here.

So, I have plotted a channel like this. So, if I have a channel. So, I am plotting data from here to here, this is up to the mid. So, this is y equal to 0 and this is y equal to 1. that zone is what you are seeing, this is at the center of the channel, channel center.

So, the length scale is just growing, you know as I said it has no meaning here, it is just purely a ratio of the k and epsilon even in the DNS data. It is even worse in the at the viscosity models. At the center of the channel, the turbulence length scale is maximum. What does that mean? It has no physical meaning. We see that the turbulence is has its peak close to the wall right.

This is where your buffer layer or the viscous sub layer is sitting here. This is where the turbulence kinetic energy is speaking and dissipation rate is has its peak on the wall, but length scale is growing like this has no physical meaning, purely a numerical scale here. Similarly, eddy viscosity this also I told you this is nothing to do with viscosity and at all right, it is just a numerical argument here. So, eddy viscosity non-dimensional by molecular viscosity again you see something is growing even in the DNS data it grows up and then comes down here has no physical meaning for an eddy viscosity and in a eddy viscosity models the three lines it is even worse. It is achieving some peak value at the centre.

You have to remember this l is something that you are choosing, and eddy viscosity has also no physical meaning, right this you have to remember. So, an alternative to the sometimes you do not want to use a length scale based on this k and epsilon these are two turbulent quantities. Instead of that sometimes it is good to fall back on to a flow time scale and we already discussed this a flow time scale you can get it from the strain rate right. So, there is one option for that and this particular length scale is called Von Kármán length scale. This Von Kármán length scale is used in what is called this SAS formulation.

There are some models called scale adaptive simulation or SAS models. In that they use this Von Kármán length scale where κ is the Von Kármán constant right this one. Von Karman constant and S and U'' these are your strain rate magnitudes of the strain rate tensor and the velocity field Laplacian that is given here S is $\sqrt{2S_{ij}S_{ij}}$ and $U''$is your velocity Laplacian ok. So, this length scale is also used in sometimes it can be beneficial to use this one in your calculations, which is falling back onto flow parameters right. So, these two are flow parameters, not turbulent.

$$L_{\nu k} \equiv \kappa \frac{S}{U''}$$

$$S = \sqrt{2 \cdot S_{ij} \cdot S_{ij}}$$

$$U'' = \sqrt{\sum_{i=1}^{3} \sum_{j,k=1}^{3} \frac{\partial^2 u_i}{\partial x_j^2} \frac{\partial^2 u_i}{\partial x_k^2}}$$

One argument is the flow parameters are resolved in an eddy viscosity model. You are modeling turbulence right, you are closing the Reynolds stresses using Boussinesq, but the momentums are calculated. And therefore, an argument is why not use the flow time scale instead of a turbulent you know a turbulent length time and all these scales ok. So, this is also something you can be using it to look at it. So, any doubts on this before we move to another topic? So, another topic to look at which is very important is realisability.

This I mentioned in some of the lectures that eddy viscosity models can be unrealizable or unphysical. So, one has to look at what we mean by this. So, there are numerous conditions. We will not go ahead and deal with every of this. Some of this we have already seen, some we will see today, numerous conditions.

One example is we have already seen that is the two-component limit, two component limit. So, that is something we see in the physical sense. There is a physics behind this, and your model must accommodate this; models must try to capture the physics right. So, in a two-component limit that is your v prime square average is smaller than the u prime square average or w prime square average ($\overline{v'^2} < \overline{u'^2}, \overline{w'^2}$). That is in the absence of Coriolis force or stratification effects, wall normal stress is the smallest one compared to the other two.

This is already been discussed, right? We have already looked at it. One can have a low

Reynolds number modeling or other techniques are also there using a V2F model or solving a Reynolds stress model. Now, there is one more condition to look at, which makes the model realizable right. So, this is normal stresses must stay positive normal stresses stay or must must stay positive. This is a no-brainer: normal stress is u prime u prime average v prime v prime average w prime w prime average ($\overline{u'u'}, \overline{v'v'}, \overline{w'w'} \geq 0$).

It is a square term. So, it has to stay positive it cannot take negative values right. So, what this implies is you are your $\overline{u_i' \ u_i'}$ must stay positive. So, what does this mean? All the three stresses right. So, we are that is your $u'u'$ or $v'v'$and so on. All this must stay positive. If this three term stays positive, what will happen to turbulence kinetic energy? This also should stay positive.

Your turbulence kinetic energy is nothing but $1/2\,\overline{u_i' \ u_i'}$. So, turbulence kinetic energy must also stay positive ok. So, we have this condition here. Two component limit is already addressed. How to make this how to make your eddy viscosity model capture it.

We will see today how to make your eddy viscosity model make sure that this normal stresses are staying positive. First we have to discuss whether eddy viscosity model is unrealizable by default the models that we discussed and what can be the solution for that one ok. So, there is one more there are many as I said you can take another example is called the shear stress correlation coefficient. Shear stress correlation coefficient should not exceed 1.

It is a coefficient value. So, how we define this is essentially your u i prime u j prime average that is either this is a shear stress. So, I should not be equal to j; that is your u prime v prime v prime w prime u prime w prime the shear stresses, of course, the correlation coefficient that I mentioned. So, it has to be normalized by its rms correct. So, we have the square root of ui prime square average square root of uj prime square average. This value should be less than or equal to 1 minus 1.

$$-1 \leq \frac{\overline{u_i' u_j'}}{\sqrt{\overline{u_i'^2}}\,\sqrt{\overline{u_j'^2}}} \leq 1$$

So, the value should be in the range. The shear stress can take negative values also, u prime, v prime, v prime, w prime and therefore, value should be within plus or minus 1. This particular condition is also called Schwarz inequality. So, here and this, i is not equal to j, no summation here ok, no summation, no summation over i comma j because you are essentially looking into a one particular shear stress when i equal to 1, j equal to 2 or i equal to 1, j equal to 3 and so on. The three shear stresses, shear stress correlation coefficient should not exceed 1. So, we will see whether at the viscosity models are realizable or unrealizable and see what can be done for it in the context of this particular second one that is what I will address in this particular course ok.

Normal stresses staying positive is very important. Otherwise k goes negative you will have divergence your code will most likely crash ok. So,