Course Name: Turbulence Modelling

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Week - 8

Lecture – Lec46

46. Boundary and Initial conditions for RANS simulations - I

Ok, let us get started. So, in the last class we looked at the low Reynolds number formulation and we see that the near wall dependency should be consistent between the model terms and the exact terms. and for that we introduce what is called a damping function into the eddy viscosity formulation. So, that is present in the model terms and therefore, that will become consistent. So, the damping rates will become consistent between modeled and exact.

Today we will see what boundary conditions we have to apply when we use low Reynolds number formulations. If you recall wall functions the boundary node or the not the boundary node the first node that is above the wall was placed far far above the wall that is above y plus 30 and it can go up to let us say y^+ 200 or 100 and there we gave a boundary condition ok. So, the wall effect was displaced away from the wall. But in the low Reynolds number formulation as I mentioned you need to resolve the wall that means you need to have enough mesh points so that you capture all the near wall effects right.

So, you will have many points closer to the wall so that you capture all the effects ok. And I did mention the first this is the boundary node then the let us call this p node here right. So, this is the south this is the north. So, the p node is the first node above the wall ok first node above the wall and this must be as I said y plus is less than or equal to 1 for the P node. So, it has to be in the linear sub layer and many nodes in the in the linear and buffer layer.

So, we will see what conditions we have to give when we resolve such a condition ok. So, the velocities are all straightforward right on the wall at at y equal to 0 if this was your y direction y equal to 0 that is on the wall, you can specify your velocities to be 0 coming from your no slip and kinematic boundary condition. And what do you do for pressure? Pressure boundary condition on the wall, this must have been thought in the CFD course. Is pressure known to you? No. Then what do you do? Yes you set a

Neumann boundary condition right.

So, the velocity you have a Dirichlet condition right for the pressure it is Neumann. So, this is your Dirichlet condition right. For pressure you give $\frac{dP}{dy}$ here if you have a curved surface then it is $\frac{dP}{d\eta}$ where η is the wall normal direction equal to 0. Neumann condition. So, this will update the pressure on the wall right.

So, the let the flow compute and it will get updated this is fine. So, now you are computing not just the flow a laminar flow this is fine. velocities and pressure. So, now this is mean velocity and mean pressure gradient here that means there must be a fluctuating or turbulent component that we are computing using k and epsilon. So, we need a boundary condition for k epsilon or if you are computing omega and so on.

So, you need a boundary condition for these things as you resolve the wall. So let us see what they have to be. So k on the wall is straightforward also. So this is not a problem. So k what should k be on the wall? k has to be 0, k is 0 on the wall right.

It is made up of fluctuating velocity components. Velocity goes to 0, k goes to 0. Only epsilon and omega are something you have to think about it. Let us go back and see what we can do for epsilon boundary condition. So, wall boundary condition for epsilon.

So, if you recall from yesterday's class epsilon had no y dependency ok, no y dependency in the near wall region this of course, to first order right to first order all the terms that we discussed yesterday were first order we dropped the higher order terms. So, to first order we have epsilon having no y dependency in the near wall zone, but we have seen from the budget if you recall I did mention what is called a budget where we saw the data for let us say the turbulence the production rate P□ was going something like this. and the dissipation rate were going something like this. The production rate of turbulence kinetic energy and the dissipation rate of turbulence kinetic energy this is how it was looking like ok, this is your wall normal let us say this is the y direction here. So, on the wall at y = 0.

epsilon will be epsilon maximum in a wall bounded flow ok. So, dissipation rate becomes maximum on the wall ok. So, this is your typical budget that we looked at right, budget of your turbulence kinetic energy if you look at. So, now we say that to first order there is no y dependency in the near wall zone, but then we see that epsilon becomes maximum on the wall. So, how do I specify a boundary condition? Therefore, how to specify epsilon wall boundary condition? What we do now is that we take the help from k equation.

because turbulence kinetic energy has both exact equation and model counterpart. So, we take help from k to see whether epsilon boundary condition can be given ok. So, to proceed further we take help from turbulence kinetic energy equation because both model and exact terms exist for TKE. For epsilon we only have model terms the exact equation exists, but we did not model that right. So, therefore, we start to take help from the turbulence kinetic energy equation ok.

So, as y tends to 0 what will happen to your k model equation. So, consider k model equation ok, which is $\frac{\partial k}{\partial t} + \overline{u}_j \frac{\partial k}{\partial x_j}$ equal to I have the diffusion term $\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}$ + P_k - ε. P_k also I can expand it if you want. So, P_k term is $\left(2\nu_T \overline{S}_{ij} - \frac{2}{3}k\delta_{ij}\right)\frac{\partial \overline{u}_i}{\partial x_j}$ - ε. This is your production rate diffusion rate.

So, now in this we are going to consider again the sub layer right as as y tends to 0 that is in the sub layer zone. We are looking at a zone which is very close to the wall. So, in this and then considering the same conditions as before a statistically stationary homogeneous right same conditions as before. statistically stationary homogeneous homogeneous means statistically homogeneous statistically homogeneous along (x,z) same conditions fully developed also fully developed same flow conditions we are using. for a turbulent boundary layer in a very thin zone close to the wall.

If I do this then this equation here let us call this 1, equation 1 reduces to a simple form. So, the entire left hand side is 0. So, to this particular equation would result in I would get So, as you approach wall ok. So, this is approaching. So, I would use approximate here to first order.

ν_T here what will happen to ν_T on the wall eddy viscosity on the wall 0 right $\frac{k^2}{\varepsilon}$ goes to 0 eddy viscosity has to go to 0 right as y tends to 0 eddy viscosity will be 0 here. ν_T also approaches 0. So, that means to first order this turbulent diffusion term is removed $\frac{\nu_T}{\sigma_k}\frac{\partial k}{\partial x_j}$. I will essentially have equation which is $\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_3}$ are also gone because of the homogeneity only $\frac{\partial}{\partial x_2}$ term which is $\frac{\partial}{\partial y}$ right. So, I will get $\frac{\partial}{\partial y}\left(\nu\frac{\partial k}{\partial y_j}\right)$ this term exists the viscous diffusion rate will be dominant in the linear sub layer and little bit above that zone right in a sub layer.

And then what will happen to the production rate then is going ν_T to 0. So, the last term here this particular term $\delta_{ij}\frac{\partial \overline{u}_i}{\partial x_j}$ by continuity it is 0 right when i = j $\frac{\partial \overline{u}_i}{\partial x_i}$. So, it becomes 0 because of continuity. So, I have essentially $\left(2\nu \overline{S}_{ij}\right)\frac{\partial \overline{u_i}}{\partial x_j}$ and ν approaches 0. So, to first order it is dropped.

So, P is 0. production rate and we also see from data. Very thin zone, very close production rate is nearly 0, epsilon goes maximum. The viscous diffusion rate is also large. If I am going to write viscous diffusion rate, it will be large. Viscous diffusion rate will go like this ok.

This is your viscous diffusion rate which is ν or this term $\nu\frac{\partial^2 k}{\partial y^2}$ the viscous diffusion rate that will be large. So, this term survives and the dissipation rate survives. So, to first order I am going to get this simplified equation here ok in the sub layer as y tends to 0 and therefore, I can set a boundary condition here epsilon wall will be equal to $\nu\frac{\partial^2 k}{\partial y^2}$. I can set this as the boundary condition. This boundary condition has one problem.

Anything that you will notice here any the CFD experts should know what will be the problem here implementing this boundary condition. This is one option ok. You can say option 1 here, boundary condition option 1 for epsilon wall right. So, so what is that you think? What will be the problem implementing this? Yes, there is a second order term is sitting here ok. The order of this term is it is not $\frac{\partial \overline{k}}{\partial y}$ it is $\frac{\partial^2 k}{\partial y^2}$.

So, this will be numerically unstable if you implement it in your assignments, you can see that it will be unstable very difficult to get convergence ok. So, this is numerically unstable unstable because you have second derivative on the wall. This condition can be implemented, but it will be numerically unstable. The Neumann condition as ah Nirupam suggested that is also possible, but it will not give So, we need to make to make it go maximum it may not go maximum if you specify the code may work but you will not get good results. So, then that means we need to search for a condition which is numerically stable and also gives reasonably good result.

For that we go and take help from the epsilon exact term because there is if you ignore the transport equation for the epsilon model the epsilon itself has an exact term. in the turbulence kinetic energy equation right. So, we can take help from that. So, an option 2 is say boundary condition option 2 is we take help from the epsilon exact, epsilon exact term is $\nu\underline{\frac{\partial \overline{u_i'}}{\partial x_j}\frac{\partial \overline{u_i'}}{\partial x_j}}$. This if I expand it is a sum of 9 terms.

So, I get $\nu\left(\overline{\frac{\partial u_1'}{\partial x_j}\frac{\partial u_1'}{\partial x_j}} + \overline{\frac{\partial u_2'}{\partial x_j}\frac{\partial u_2'}{\partial x_j}} + \overline{\frac{\partial u_3'}{\partial x_j}\frac{\partial u_3'}{\partial x_j}}\right)$ and then j is also repeated. So, Einstein summation again. So, I would get sum of 9 terms. So, which will be if I modify in the same equation or let me write it down . So, 1 1 let us say then I would get the other terms $\nu\,\overline{\frac{\partial u_1'}{\partial x_2}\frac{\partial u_1'}{\partial x_2}} + \nu\,\overline{\frac{\partial u_2'}{\partial x_2}\frac{\partial u_2'}{\partial x_2}} + \nu\,\overline{\frac{\partial u_3'}{\partial x_2}\frac{\partial u_3'}{\partial x_2}}$.

sum over j and then of course, extra 3 terms for summing over the $u_1' u_1'$ x 3 x 3 plus nu x 3 x 3 plus nu u 3 prime. dou x 3 dou x 3 sum of 9 terms . Now, we consider again in the near wall zone ok. So, when you go to near wall that is as y tends to 0 ok. We consider let $\frac{\partial}{\partial x_2}$ term will be much much larger than $\frac{\partial}{\partial x_1}$ or $\frac{\partial}{\partial x_3}$terms.

The wall normal gradients will be much much larger than the wall parallel gradient in the sub layer ok in the sub layer. In addition the wall parallel fluctuations will be much much larger than the wall normal fluctuation. We already saw two component limit right v rms is much much smaller than the the u_{rms} or the w_{rms}. Therefore, in addition to this we will have the u_2' or sorry u_1', u_3' will be much much larger than your u_2' in the sub layer zone ok. So, if I use this consideration then this particular equation here now this will be only this $\frac{\partial}{\partial x_2}$ terms the three terms here will be important.

In that the $u_2' u_2'$ will be omitted. So, this approximately becomes only this particular term, this one and this one. So, this will be $\nu\,\overline{\frac{\partial u_1'}{\partial x_2}\frac{\partial u_1'}{\partial x_2}} + \nu\,\overline{\frac{\partial u_3'}{\partial x_2}\frac{\partial u_3'}{\partial x_2}}$ all others are higher order terms here ok. Is this clear? So, we are considering the wall normal fluctuation to be much much smaller than the wall parallel and the wall normal gradient is much much larger than wall parallel gradients. So, only $\frac{\partial}{\partial x_2}$ gradients are much larger and in that u_2' is much smaller than u_1' and u_3'.

So, I get this equation. So, epsilon exact will now therefore, be epsilon exact is now approximately equal to if I write in the x, y, z or u, v, w formulation, I would get essentially $\nu\,\frac{\partial u'}{\partial y}\frac{\partial u'}{\partial y} + \frac{\partial w'}{\partial y}\frac{\partial w'}{\partial y}$. So what is $\frac{\partial u'}{\partial y}$ when we did a Taylor series expansion? $\frac{\partial u'}{\partial y}$ we set it as some a_1 b_1 c_1 something of that order right. So $\frac{\partial u'}{\partial y}$ is the a_1 $\frac{\partial w'}{\partial y}$ is the c_1. So I can write this essentially as so epsilon I would write this as $\nu\left(\overline{a_1 a_1}\right)$ it is a correlation term + $\overline{c_1 c_1}$ or can write this as $\left(\overline{a}_1^2 + \overline{c}_1^2\right)$.

Let us call this equation 1 here ok. So, now, we take turbulence kinetic energy k which is $\frac{1}{2}\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right)$. Again here same condition the v $_{rms}$ has to be much much smaller than the u and the w $_{rms}$ right. So, this is we can say approximately equal to $\frac{1}{2}\left(\overline{u'u'} + \overline{w'w'}\right)$. is a much smaller term right in the near wall near wall zone. So, that implies k now is equal to $\frac{1}{2}u'^2$.

So, this is nothing but again if you look at what is u' it was a_1 y to first order right. So, it is a 1 a_1 y, $\overline{a_1 y}$ which is a_1 y, $\overline{a_1 y}$ + we have c_1 y, $\overline{c_1 y}$ is equal to $\frac{1}{2}\left(\overline{a}_1^2 y^2 + \overline{c}_1^2 y^2\right)$. Or I can write this as $\frac{y^2}{2}\left(\overline{a}_1^2 + \overline{c}_1^2\right)$ equation 2. I need to give boundary condition for epsilon here on the wall. And the k is known to me, k boundary condition is 0 on the wall and everywhere I am computing for k.

So, the epsilon boundary condition here will be if I make use of equation 1 and 2. Equation 1 , 2 gives me essentially epsilon is equal to ν this $\left(\overline{a}_1^2 + \overline{c}_1^2\right)$ that particular term can be replaced by $\frac{2k}{y^2}$ So, you have your boundary condition for the wall. But if you implement it on the wall node then k is 0.

So, you should not implement this on the wall. I did mention in the low Reynolds number the boundary condition P node first node above the wall for all other things $\overline{uv\omega}$ that is for the south node. this is south node you can implement here, south node or the S what I have written here, the wall on the wall this particular node here. So, that you can implement velocities can be go to 0 exactly on the wall node k goes to 0 you implement your Neumann for pressure no problem. but the epsilon boundary condition is for the P node, first node above the wall.

Do not implement epsilon boundary condition on the wall. It should be first node above the wall where it is y+ less than or equal to 1. This is important. That is the wall boundary condition for your epsilon. So, this is epsilon. So, you can write it here epsilon which is the first node above the wall first node above wall.

Then only k will be nonzero.and we have a magic here epsilon had no y dependency, but now suddenly it is $\frac{1}{y^2}$ dependency as you approach wall suddenly you get epsilon very big right. Now you see suddenly it has $\frac{1}{y^2}$ behavior right $\frac{1}{y^2}$ behavior for modeling. the exact one does not have, but for modeling idea it is coming as $\frac{1}{y^2}$. So, as you approach wall epsilon will increase. So, this is one good condition that can be used option 2.