Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 8

Lecture – Lec45

45. Damping functions for LRN – II

The other features here, and therefore, we must make sure that it has a correct near-wall behaviour. It has to go back to y^4 behaviour in the near wall zone. So, what do we do is simply we introduce a damping function, ok? So, since the modelling behaviour, since the near wall behaviour is different, note here the near wall behaviour of exact and modelled term or the term that we used is modelled turbulent diffusion, exact and modelled turbulent diffusion, the near wall behaviour of exact and modelled turbulent diffusion terms are different. So, the suggestion is to introduce a damping function which will have 1/y behaviour. So, that it goes back to y^4 behaviour in the equation.

So, where do we introduce this damping function? We essentially introduce it in the eddy viscosity term because that is what is coming in the modelled equation here: $\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}$, right? That is replacing this entire triple velocity correlation term. So, we introduce a damping function for the eddy viscosity term as something called f_μ is your damping function which will have a y raise to or 1/y behaviour. You should note here the damping function should be non-dimensional. Because every term in the modelled equation is consistent dimensionally, right? So, this $\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}$ is its dimensions is same as in the exact term.

So, the damping function should be non-dimensional, and yet it should recover they^4 behaviour in the near wall zone ok. So, we introduce this. However, f_μ should be non-dimensional so that the modelled equation remains dimensionally consistent with the exact equation. So, we do not change the dimensions that will become incorrect. So, one option of introducing a damping function is to relate it to what is called you know using this again what is available to us which is k and epsilon using that we can introduce a function.

There are many damping functions available. One option is to use f_μ can be exponential of minus 3.4 divided by 1 plus RT by 50. This is squared, is one of the damping functions that is used in Launder and Sharma, where RT is computed as k square by nu epsilon. So, it is dimensionally, this is the only term that is introducing a parameter.

$$f_\mu = \exp\left(\frac{-3.4}{\left[1+\frac{R_T}{50}\right]^2}\right), \quad \text{where} \quad R_T = \frac{k^2}{\nu\varepsilon}$$

So, it will be non-dimensional. RT is non-dimensional; easy to see this is k is a meter square by a second square. So, the square of that meter rise to 4 by second rise to 4, nu is meter square per second epsilon is meter square by second cube. So, this is non-dimensional. So, f_μ is this particular reference for this is reference is Launder and Sharma article ok.

So, this particular thing can be introduced and where do we introduce in the eddy viscosity. So, what does that mean is that you need to have something called ν_tLRN, eddy viscosity ν_tLRN must be introduced which is $f_\mu C_\mu k^2/$. So, ν_tLRN will replace ν_t eddy viscosity throughout wherever eddy viscosity is occurring this is introduced the damping will become y^4 or it will not y raise to 4 it will just reduce one extra y dependency in the model equation right. It essentially f_μ has 1/y dependency. Now, we have seen only the equation for the turbulent diffusion right.

So, there is another term that we have not seen here. So, this particular term is taken care of this one is done right. So, there is another model another term that is modelled which is the your Reynolds stresses here in the production rate and the in the model equation using Boussinesq. So, we must see whether these two have same behaviour or different behaviour and whether it requires ν_tLRN ok. To our luck, ν_t is appearing here.

So, if the behaviour is different, the same ν_tLRN will work. I do not need to do anything. Simply replacing ν_t by ν_tLRN will work also for this. So, before that let us go and see what is the behaviour that I get for these two production terms right. So, this is the diffusion is done now let me just go and see here.

Production rate, so the Pk, so the Pk exact is you have minus ui prime uj prime over bar here and of course you have this dou ui bar by dou xj term is also appearing. So, dou ui bar by dou xj. So, this will give me sum of 9 terms here, but dou by dou x1 and dou by dou x3 terms are again 0. So, this essentially reduces to minus, so, j has to be 2, so it is ui prime u2 prime dou u1 bar by dou x, sorry dou ui bar by dou x2. Since dou by dou x1 of the statistically average term and dou by dou x3 of any average terms are 0.

statistically homogeneous in x1 and x3. So, only x2 term survives here. Dou by dou x2 of ui bar, which, if I expand, I get a sum of three terms, which is minus u1 prime u2 prime dou u1 bar by dou x2 minus u2 prime u2 prime dou u2 bar by dou x2 minus u3 prime u2 prime dou u3 bar by dou x2. Again, u2 bar and u3 bar, these two mean velocities are 0 for a fully developed flow fully developed So, only one term is surviving. So, I can write this particular term consistently with your x, y, z formulation.

If I use it, I get minus u prime v prime dou u bar by dou y. This is your Pk exact reduces to this particular term now. And what does this give? What is u prime? u prime has y square. Sorry, u prime has let us go back and see u prime and v prime, u prime has y behaviour v prime has y square behaviour, ok? So, this will give me y cube behaviour u prime v prime average dou u bar by dou y u bar or u instantaneous, or u prime will have same y behaviour as u prime. We have not done a Taylor series expansion for mean velocities, but if I do it I get the same thing I expand apply the boundary condition I get the same near wall behaviour, so u bar and u prime will have y behaviour only y dependency right so that means dou u bar by dou y will give me So that means I have only y cube behaviour for the Pk exact.

$$\frac{\partial \overline{\ }}{\partial x_1} = \frac{\partial \overline{\ }}{\partial x_3} = 0$$

$$P_{k_{exact}} = -\overline{u_i'u_j'}\frac{\partial \bar{u}_i}{\partial x_j} = -\overline{u_i'u_2'}\frac{\partial \bar{u}_i}{\partial x_2}$$

$$= -\overline{u_1'u_2'}\frac{\partial \bar{u}_1}{\partial x_2} - \overline{u_2'u_2'}\frac{\partial \cancel{\bar{u}_2}}{\partial x_2} - \overline{u_3'u_2'}\frac{\partial \cancel{\bar{u}_3}}{\partial x_2} \quad \text{fully developed}$$

$$P_{k_{exact}} = -\overline{u'v'}\frac{\partial \bar{u}}{\partial y} \Rightarrow O(y^3)$$

So, now let us see what is Pk model. So, Pk model, we used the Boussinesq, which is 2 nu t Sij, which is dou ui bar by dou xj plus dou uj bar by dou xi 2 nu t Sij minus 2 third k delta ij dou ui bar by dou xj. So, I essentially get this particular term. We have already expanded this once long back when we did the model constants. So, if we quickly recall it is essentially going to reduce to a very small term.

So, we essentially get nu t dou ui bar by dou xj square. So, I get square of this plus nu t dou ui bar by dou xj dou uj bar by dou xi the cross term here, and then I have minus 2 third k delta ij dou ui bar by dou xj. This particular term goes to 0 when delta ij is 1 that means i equal to j because of continuity this will go away. Due to continuity this term goes away only these two survives and if you expand all these terms and apply this statistically homogeneous as well as the fully developed condition you will see that this all these terms are vanishing. We have done this before to quickly do it.

$$P_{k\,model} = \left[\frac{\cancel{2}}{\cancel{2}}\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)-\frac{2}{3}k\delta_{ij}\right]\frac{\partial \bar{u}_i}{\partial x_j} = \nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}\right)^2+\nu_t\frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_i}-\frac{2}{3}k\delta_{ij}\cancel{\frac{\partial \bar{u}_i}{\partial x_j}} \quad \text{continuity}$$

It is dou, I am going to take only dou by dou x2 term because dou by dou x1 dou by dou x3 term I drop dou by dou x2 and then I will have this is square term, right? So, i and i are repeated this particular term. So, I get ui bar dou ui bar by dou x2 only this particular term is surviving here plus here I have nu t of same holds good too, so I have dou ui bar by dou x2 dou u2 bar by dou xi so u2 bar is 0 so this is 0 fully developed. So, this entire term goes to 0, the last one is 0. So, I get this is now equal to nu t of dou u1 bar by dou x2 whole square plus nu t dou u2 bar by dou x2 whole square plus nu t dou u3 bar by dou x3 whole square. Again u2, u3 fully developed condition these two goes away making this only one term remain here.

So, I get Pk model is equal to nu t dou u bar by dou y whole square. So, this is what I am getting now. So, what is the dependency here? So, if I look at this, nu t is C mu k square by epsilon dou u bar by dou y square. So, this will give me k square y raise to 4 y square y raise to 4 behaviour epsilon has no y dependency and dou u bar by dou y will have no y dependency again squared. So, that is essentially giving me y 4 behaviour.

$$P_{k\,model} = \left[\frac{\cancel{2}}{\cancel{2}}\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}+\frac{\partial \bar{u}_j}{\partial x_i}\right)-\frac{2}{3}k\delta_{ij}\right]\frac{\partial \bar{u}_i}{\partial x_j} = \nu_t\left(\frac{\partial \bar{u}_i}{\partial x_j}\right)^2+\nu_t\frac{\partial \bar{u}_i}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_i}-\frac{2}{3}k\delta_{ij}\cancel{\frac{\partial \bar{u}_i}{\partial x_j}} \quad \text{continuity}$$

$$= \nu_t\left(\frac{\partial \bar{u}_i}{\partial x_2}\frac{\partial \bar{u}_i}{\partial x_2}\right)+\nu_t\left(\frac{\partial \bar{u}_i}{\partial x_2}\cancel{\frac{\partial \bar{u}_2}{\partial x_i}}^{\,0}\right) = \nu_t\left(\frac{\partial \bar{u}_1}{\partial x_2}\right)^2+\nu_t\left(\cancel{\frac{\partial \bar{u}_2}{\partial x_2}}^{\,0}\right)^2+\nu_t\left(\cancel{\frac{\partial \bar{u}_3}{\partial x_3}}^{\,0}\right)^2 \quad \text{fully developed}$$

$$\boxed{P_{k\,model}} = \nu_t\left(\frac{\partial \bar{u}}{\partial y}\right)^2 \Rightarrow C_\mu\frac{k^2}{\varepsilon}\left(\frac{\partial \bar{u}}{\partial y}\right)^2 \Rightarrow \frac{O(y^2)^2}{O(y^0)}\,O(y^0)^2 \Rightarrow \boxed{O(y^4)}$$

So, let us compare the exact and the model terms. The exact term has y^3 behaviour, the modelled term has y^4, and there is an eddy viscosity term, luckily. So, ν_tLRN will have 1/y behaviour making it y^3 behaviour. So, I can simply introduce ν_tLRN, the ν_tLRN can help here, you can have ν_tLRN will correct the near wall behaviour. and make it y cube like the exact.

So, epsilon, I do not have to do anything here in the previous. If I compare equations 1 and 2, these two are the only model terms rest are all similar to epsilon this is the exact expression for epsilon we have just solved any transport equation for it. We did not model that. So, ν_tLRN will help you give you correct near wall behaviour. So, in a low Reynolds number model, use a damping function, and you need to also use the correct mesh guidelines.

So, by merely having lot of grid points close to the wall does not mean you will get the correct model behaviour. You must use the damping function also. Just because you are capturing the near-wall data does not mean that it should be good. It may give a wrong data unless you damp it correctly to account for two component limit. Otherwise the prediction will be much different than what it should be.