Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 8

Lecture – Lec43

43. Introduction to wall-resolved simulations - II

So, let us move to what is called low Reynolds number, that is LRN models. as I mentioned low Reynolds number refers to the locally low Reynolds number in a turbulent boundary layer right. So, near wall region is very important capturing near a wall capturing near wall region is important and that means you need to have a mesh guidelines for this. So, the mesh guidelines will be like this mesh guidelines So, you need to have a mesh say you have a wall. So, you must have grids like the first one should be $y^+ \leq 1$.

The first node or the first node above the wall. So, there will be another node on the wall for boundary condition. or it will be inside the wall if you are using a ghost node implementation that is in the CFD framework. In general let us say you have a node on the wall.

So, this first node above the wall $y^+ \leq 1$ must be there and you must have about 5, 10 nodes 5, 10 nodes must be there in $y^+ \leq 20$. So, within 1 to 20 you must have at least like 5, 10 nodes. Depending on your exact problem how challenging it is you can vary this ok. More the better and depends on how how many nodes you can actually account or accommodate for depending on your computational resources. So, this is the mesh guidelines or grid grid guidelines.

So, the objective now I will come to what we have to do it. for introducing a damping function. So, the objective of LRN model is that the exact equation of a turbulence kinetic energy and the modeled equation of the turbulence kinetic energy must behave similar as we approach the wall ok. So, near wall behavior of both the terms because we have modeled certain terms making some certain approximations. that we have to see whether term by term they are having same behavior, near wall behavior.

So, the modelled equations of let us say k ε, ω whatever you have should behave the same way as exact equations. As we approach wall that is as y tends to 0, the behavior of these two should be similar, the exact modeled equations, model terms and the exact terms must have the same. Otherwise you need to introduce this damping function or some corrections so that the behavior is same as we approach close to the wall and we see that this is not the case. So, in general not in general the eddy viscosity models do not account for this. So, the model terms are different than the exact terms that we see.

So, but this will not this will not be the case since models assume turbulent condition even in the sub layers. that is your linear sublayer. So, when we whenever we are using eddy viscosity model we are computing eddy viscosity everywhere in the flow buffer layer linear inertial sublayer as well as in the linear sublayer right and we do not have access to Reynolds stresses which actually goes to 0 on the wall we have replaced Reynolds stresses with a Boussinesq which depends on velocity gradient right. So, the behaviours are very different of Reynolds stresses and its model counterpart which is strain rate. So, that we will see.

So, basically what we want here that is model term and exact terms having same behavior will not be there by default because this will not be the case since models assume turbulent condition even the sub layer. So, now we see what these two terms should look like. So now let us take a look at each of these fluctuations as we approach the wall. For that we will do a Taylor series expansion. So a Taylor expansion of fluctuations near the wall.

So that means let us say u’. at some y very close to the wall will depend on $u^{'}|_{y} = u^{1}|_{y=0}y^{0} + \frac{du^{'}}{dy}y^{'} + \frac{1}{2!}\frac{d^{2}u^{'}}{dy^{2}}y^{2} + \ldots$ So, here I can take this u’ $\frac{du^{'}}{dy}$ and all this derivative terms as some kind of a coefficients which are functions of time and space. So, let us call this I would like to call this as a 0.

a_1 a_2 and so on ok. So, note here that note that a_0 a_1 a_2 etc are functions of space and time that is your velocity fluctuation and its derivatives higher order derivative terms ok. So, now I can write the equation for u’ as basically $a_0 y_0 + a_1 y_1 + a_2 y^2$ and so on. Similarly, v’I will use $b_0 y_0 + b_1 y_1 + b_2 y^2$ higher order terms. Similarly, w’ the wall parallel fluctuation which is $c_0 y_0 + c_1 y_1 + c_2 y^2$ in the higher order terms ok.

So, now at the wall I can apply a wall boundary condition for this. So, for u’ and w’ wall parallel fluctuation and v’ wall normal fluctuation I have both no slip and kinematic boundary condition. So, if I apply boundary condition applying boundary condition on the wall what do I get? So, I have this no slip comma kinematic boundary condition. So, basically u’ is 0 u’ v’ w’ is 0 on the wall. So, $u'|_{y=0} = v'|_{y=0} = w'|_{y=0} = 0$ is 0 which implies a_0 b_0 c_0 is 0 the first one is going away.

Also on the wall also on the wall the wall parallel gradients that is you have $\frac{\partial u'}{\partial x}|_{y=0} = \frac{\partial w'}{\partial x}|_{y=0} = 0$ The second component where a_1 b_1 c_1 is coming $\frac{\partial u'}{\partial y}$ and the other components. So, this on the wall here this is also 0 here. On the wall I am looking at on the wall right. So, in CFD terminology let us say you have a wall right and then I have nodes if you can think about this as your P node, east node, west node.

Now, look at the gradient $\frac{\partial u'}{\partial x}$ on the wall is 0. Similarly, $\frac{\partial w'}{\partial z}$ only the wall normal gradient exists right $\frac{\partial u'}{\partial y}$ and so on exists, but the wall parallel gradients are 0. So, if this is the case then what happens to the other one by continuity we can get So, by continuity by continuity equation $\frac{\partial v'}{\partial y} = - \frac{\partial u'}{\partial x} - \frac{\partial w'}{\partial z}$ which is equal to 0. So, what does this imply? Where does this $\frac{\partial v'}{\partial y}$ comes? That is $b_1 = 0$. So, I have this implies b_1 equal to 0, b_1 here, b_1 which is $\frac{\partial u'}{\partial y}$.

So, I have b_1 equal to 0. these three equal to 0 a_0 b_0 c_0. So, what will happen in let us call this equation 1. So, equation 1 reduces to $u' = a_1 y^+$ and so on which is $a_1 y^+$ $b_2 y^2$ + higher order terms. So, that means to first order it is behavior is y.

So, u’ behavior as you approach wall depends on y itself right, order of magnitude y behavior as you approach the wall to first order ignoring the higher order terms. So, v’ b_0 is 0, b_1 is also 0. So, it is $b_2 y^2$ + it is showing a y^2 behaviour We do not have even have to look at the data even here we are seeing that the wall normal fluctuation is having a y square behaviour, but the wall parallel one u’ is having a y behaviour. Similarly, w’ is your $c_1 y^+$ $c_2 y^2$ and so on. So, it is of the order y.

So, this is having a different behavior here y^2 behavior indicating a two component limit a different behavior. So y^2 behavior here you can say it is a two component behavior, it is kind of damping as y^2 . as you approach wall the u’ and w’ damps as y, but v’ damps as y^2. So, there is a faster level of damping here even in the equation just by Taylor series expansion ok.

So, now what did I said in the beginning the objective is to look at modeled equation all the term term by term in the modeled equation and the term by term in the exact equation the counterparts. of two equations and we see from these two what are the whether they have the same y behavior or y square y cube what kind of behavior they have between the two terms exact and its counterpart in a modeled equation. For that we need of course to know the behavior we have the behavior of this u' v' w'. So, now we can go ahead and get the behavior of other terms very easily. So, for example, we can look at the stresses.

So, stresses are occurring in the production rate of turbulence kinetic energy in the exact equation you have $\overline{u_{ij} u_i u_j}$. So, all the stresses are occurring. So, let us see the that behavior of u' u' average. So, what is u' u' itself is a_1 y^+ a_2 y^2 and so on. So, this will be $\overline{a}_1^2 y^2$ plus higher order terms and simply multiplying u', u' was a_1 y + 1.

So, it is a_1^2, a_1 if you recall it is nothing but your u' So, it has to have your sorry a_1 is basically $\frac{\partial u'}{\partial y}$. So, it is a fluctuating quantity. So, it is a correlate average of a correlation functions of space and time. So $\overline{a}_1^2 y^2$ and so on. Similarly, I get so this of course, will have behavior of y^2.

So, $\overline{v'v'}$ this the smallest term here is b2 square right v'is $\overline{b}_2 y^2$ plus higher order terms. So, that means this becomes $\overline{b}_2^2 y^4$ plus higher order terms. So, this has a y^4 behavior the wall normal stress. and $\overline{w'w'}$ is giving me $\overline{c}_1^2 y^2$ plus higher order terms. So, it is giving me a y^2 behavior and then the shear stress u' v' if I take this.

I would get of course, 1 u' has a y behavior and v' has a y square behavior. So, this will give me a y cube behavior. So, this will give me $\overline{a_1 b_2} y^3$ plus higher order terms. So, this is giving me a y^3 behavior in the near wall region. So, we have the Reynolds stresses and we also need turbulence kinetic energy because that appears in the diffusion rate and so on.

So, the turbulence kinetic energy $k = \frac{1}{2}\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right)$. So, that means it will have essentially $k = \frac{1}{2}\left(\overline{a}_1^2 y^2 + \overline{b}_2^2 y^4 + c_1^2 y^2 + ...\right)$. So, this is giving me the y^2 behavior only. So, turbulence kinetic energy is giving the y^2 behavior to first order ok. So, what else we have? We have we also have the mean gradient right in the production rate of turbulence kinetic energy P_k, Reynolds stresses multiplied by dou u bar by dou y and so on.

So, we have that term also. So, $\frac{\partial \overline{u}}{\partial y}$. Before that we can say what is the $\frac{\partial u'}{\partial y}$ term. this occurs in your dissipation rate of turbulence kinetic energy in the exact one $\nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$. So, $\frac{\partial u'}{\partial y}$ will give me if I differentiate the u' I would get basically a_1 plus higher order terms. So, I have y_0 behavior $\frac{\partial u'}{\partial y}$.

Similarly, $\frac{\partial \overline{u}}{\partial y}$ will also give me the same one $\overline{a_1}$ plus so I would get y_0 behavior as well that is no y dependency basically it is telling me this these two components have no dependency on the y at all ok. and then I have ε which is the dissipation rate. This one is the exact one if you recall I will have $\nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}$. So, here to first order as I said $\frac{\partial u}{\partial y}$ terms we are looking into essentially this will be function of $\frac{\partial u'}{\partial y}$ $\frac{\partial u'}{\partial y}$ this will be sum of of course 9 different terms ok. So, we will have this but to first order if you take it up it essentially comes to u' $\frac{\partial u'}{\partial y}$ dependency the other are higher order terms.

So, this will give me again no y dependency for an ε also. And then what else I have? I have $\frac{\partial k}{\partial y}$ term, this comes in the diffusion rate $\frac{\partial k}{\partial xj}$. I am looking into only this y because I am going to take up a statistically stationary and homogeneous in the two direction that is a plane turbulent couette flow. If I take it then $\frac{\partial}{\partial x}$ $\frac{\partial}{\partial z}$ terms vanish. So, only this $\frac{\partial}{\partial y}$ terms become dominant.

So, I am looking into only that part. So, $\frac{\partial k}{\partial y}$. So, k has y^2 behavior. So, this will give me y behavior. y dependency for the $\frac{\partial k}{\partial y}$ term. And then apart from this we have the in the model term of course, the eddy viscosity is also there that I will see.

I also need what else I would need that triple velocity correlation. So, the p prime u i prime over bar the pressure diffusion rate is not modelled if you recall we said this is small in certain class of flows. So, we did not model and therefore, its counterpart does not exist in the model equation. So, we will not consider that. So, we are in the diffusion we are looking into only viscous diffusion and turbulent diffusion and its counterpart in the model.

So, viscous diffusion will have this $\frac{\partial k}{\partial y}$ $\nu \frac{\partial k}{\partial y}$ term will come and the other component is the turbulent diffusion rate. So, the turbulent diffusion rate is let us say the v' $u_i' u_i'$ let us say. This this will give me behavior of v' is y^2 behavior and this one each one will have $u_i' u_i'$' that is the sum of this. this particular thing is giving me to first order only turbulence kinetic energy behavior $u_i' u_i'$. So, this gives y^2 behavior this is giving me y^2 behavior.

So, this gives me essentially y^4 behavior this is like turbulence kinetic energy this part. So, we will use all these terms and substitute it in the equations to see whether the two terms have same y dependency as you approach the wall, if not which is the case then we will see what has to be done in the next class. Any questions on this so far we will take it.