Course Name: Turbulence Modelling

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Week - 7

Lecture – Lec41

41. Introduciton to wall-functions -II

So, let us look at as I said this log law is what we are going to use. When we have a node P here and we are solving let us say the two equation eddy viscosity model. We are solving for the velocities right and we are solving for the turbulence kinetic energy and dissipation rate or ω. So, we need a boundary condition here at node P. So, we need u_p, $\overline{u}_p$ actually, the mean velocity at p node.

You need k at p node, ε at p node, ω at p node and so on, whatever it is. That is what is the wall function. So how to get velocity at p? That is the first thing. You are computing the momentum.

How to get velocity at p node? Log law, I just told you. It is a log law. So you are just going to use this empirical formula. to give boundary condition at node P in your CFD calculation. You are not letting the flow calculate, you are not letting your discretized equation calculate the velocity at P, you will simply use the log law to give the velocity at P.

But this of course, this geometry as I am saying here right, so this kind of a complicated geometry, this is not a flat plate boundary layer, not smooth at all. But many times wall functions are used So, you need to know about this right that is the idea. So, remember how wall functions are derived and when you are using it be aware of it what you are doing. There can be wall functions available even for rough considerations also, but this kind of a general roughness is very complicated to get. You can get some kind of like the rib roughness that I have shown.

something where it is regular roughness regular periodically occurring obstacle that kind of a roughness there may be a wall function derived for that. But something very generic industrial flows no and this is more popular what I am going to do. So, this is what is

used in industrial flow even though they will have complicated geometries ok. So, first thing is to get the velocity at the P node. So, what do we need here is that at node p at node p we need $\overline{u_p}$, you need k_p, you need ε_p, you need ω_p and so on whichever model you are using depending on that ok.

So, $\overline{u_p}$ I already gave a clue. Now, let us look at the other things how to obtain the other parameters. Now, recall the log law zone right, log law region. So, what happens so special in this zone apart from that you can get a logarithmic fit for your velocity is that we have seen when we computed the c_μ right we use log law also there right.

So, what did we obtain there? $\sqrt{c_\mu}$ this was nothing but $\frac{\overline{u'v'}}{k}$. We looked at this ratio in the log law zone and we said this is from experiments I get the value 0.3 and therefore c_μ becomes 0.09. So, this we have already done right.

So, let us call this equation 1. we have looked at what is called a wall friction velocity in a constant stress layer right. So, now recall that also all these concepts are used now recall constant stress layer this constant stress layer has everything there linear sub layer buffer layer as well as the inertial sub layer all three. So, your wall friction velocity or the wall shear stress becomes viscous stress in the linear sublayer and as you go to inertial sublayer it becomes turbulent shear stress and somewhere in between it is a combination of both that we have learned right. So, using constant stress layer what we can get is that we have u^{*2} is nothing but your $\frac{\tau_\omega}{\rho}$ the wall the wall shear stress right.

So, let us call this equation 2 here. Now, you must know that this $\frac{\tau_\omega}{\rho}$. So, this is nothing but your or the τ_ω itself is ν in the linear sublayer. linear zone and this $\frac{\tau_\omega}{\rho}$ becomes - $\overline{u'v'}$ in the log law zone that we have seen to first order what would happen. appear so that means the u^* the wall friction velocity you can compute this I told you both experimentalists like it because they do not want to they do not have to measure close to the wall they can compute only $\overline{u'v'}$ from that they can compute wall friction velocity and use it to compute skin friction coefficient and so on.

Now modeling community why leave them out right we also want to use this concept. So, now I have this formula the equation 1 $\overline{u'v'}$. Now, that is nothing but $\frac{\tau_\omega}{\rho}$ which is nothing but u^{*2}. So, I can substitute $\frac{u^{*2}}{k} = \sqrt{c_\mu}$. That is what I am going to do now.

This is clear what I have done. So, therefore, using the above equations what I get is the $\frac{\overline{u'v'}}{k} = \sqrt{c_\mu}$. So, now I am going to substitute $\frac{\overline{u'v'}}{k}$ as the $\frac{\tau_\omega}{\rho}$ by k = $\sqrt{c_\mu}$. it is nothing but $\frac{u^{*2}}{k} = \sqrt{c_\mu}$. So, why did I do all this? That is because now I can get a k or I can simply write here k at the p node I can compute now simply as u^{*2} which you can obtain from the logarithmic law over $\sqrt{c_\mu}$. So, k_p k at node p you need u^{*2} or u* and then c_μ value already know right c_μ is 0.

09. So, this is we have figured it out at least how the turbulence kinetic energy k at p node we can introduce which is $\frac{u^{*2}}{\sqrt{c_\mu}}$. But how to get u* then? I said from your logarithmic log law region right. So, from log law we have $u^+ = \frac{1}{K} \ln ln\left(y^+\right) + B$. So, now I can rewrite this b, b is some constant 5.

2 or where b is 5.2 or 5.5 and so on some constant value. So, all I can do now is I can rewrite this as $\frac{1}{K} \ln ln\left(y^+\right)$, plus I would like to express this constant also as a function of von Karman constant and in a logarithmic way. So, I would write this as $\frac{1}{K} \ln ln\,(E)$, E turns out to be 9 here ok. So, I get the value here.

So, E will be 9. So, now I can write this as $\frac{1}{K} \ln ln\left(y^+\right) + E$ that is So, this is how you can compute your at the p node you can get this value, but I am interested in u* right. So, u* appears here naturally if I expand the y^+. What is y^+? It is $\frac{u^*}{\nu}$. So, $\frac{u^*}{\nu E}$. So, all I would need now is to obtain u^*.

So, but if you recall what is $\overline{u^+}$ this is nothing but $\frac{\bar{u}}{u^*}$, it is a non-dimensional quantity right. So, I need $\frac{\bar{u}}{P}$ value you can compute that as well as to get u^*now. So, this implies u^*you can get using $\bar{u}$ at the p node of course the $\frac{k}{\ln ln\left(\frac{u^* y_p E}{\nu}\right)}$. So everything is available for u here to compute u^*, von Karman constant is available right kappa 0.41, up you can compute from log law right, yp is just a distance is known, y_p is your distance why is the y_p is the distance of y from wall to the node p.

E value is known kinematic viscosity is known you can compute this. But is there a

challenge here to compute this? What is it? u *. u *yeah u *is depending on u *itself. So, how do you do this? You have to iterate right. So, you must iterate since u *is depending on itself or appearing on both sides, both LHS and RHS, you must iterate u *.

So, iteratively you must find in a CFD calculation. So, now K p is found. So, u *can be obtained using this that means first you obtain your u_p value from that you obtain u *after u * is available K_p is obtained. So, all these are boundary conditions that you have to implement K_p u_p and let us say ε_p. Now, how do I get ε_p? we introduce a scale for ε_p.

So, if you recall again recall the log law region again. So, what is so special happening in the log law zone is that we saw in canonical flows the p_k is nearly balancing the ε to first order. and we introduce a scale for this that is $\frac{u^3}{L}$. So, we can set this as u^{*3} by the mixing length idea K y_p. So, ε_p is now available to you.

So, therefore, ε_p nothing but $\frac{u^{*3}}{K\,yp}$. So, the boundary condition is available for u now or the so called wall function is available to you now k_p ε_p * everything is available. for ω also it is straightforward since you know the relationship between ω k and ε you can obtain this I have k_p and ε_p using these two I can obtain a relationship for ω_p also. If you are using let us say ω_p it is in a k ω model right in a k ω model so we know ω is $\frac{\varepsilon}{\beta^* k_p}$. So, I can simply say ω_p therefore should be $\frac{\varepsilon p}{\beta^* k_p} = \frac{u^{*3}}{k y_p \beta^*} \frac{1}{k_p}$, k_p is $\frac{u*2}{\sqrt{u}}$.

So, it becomes $\frac{\sqrt{C_\mu}}{u*2}$. So, this cancels out giving you $\frac{u^*\sqrt{c_\mu}}{k y_p \beta^*}$. But here β^* we know that is equal to c_μ equal to 0.

09. So, therefore, ω_p can be simply $\frac{u^*}{k y_p \beta^* \sqrt{c_\mu}}$. Sometimes this β^* is dropped. So, the formula appears slightly different. In some commercial codes I have seen that this β^* is dropped they would like to have ω_p simply as a ratio of $\frac{\varepsilon_p}{k_p}$. So, then it will be $\frac{u^*}{k y_p \sqrt{c_\mu}}$ itself.

Sometimes β^*r is included sometimes it is not. So, also this one sometimes it is also you can say ω_p is simply considered as $\frac{\varepsilon_p}{k_p}$ itself. So, which will give you $\frac{u^*\sqrt{c_\mu}}{k y_p}$ These are

like if the results are not good enough sometimes some constants are dropped or you would even see that in some commercial codes the in a k ε model it is not $\frac{c_\mu k^2}{\varepsilon}$ it will be $\frac{\sqrt{c_\mu} k^2}{\varepsilon}$. If they feel that gives a better result they would use that because you are going to use this model for many types of flows not just boundary layer. So, they have all these small modifications. okay