Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 7

Lecture – Lec36

36. Standard k-ε model, RNG k-ε model, and Prandlt's one equation model - I

So, if you see here, I was a bit lazy and I did not expand the tensors properly. So, this is $\frac{\partial \overline{u_j}}{\partial x_i} \frac{\partial \overline{u_i}}{\partial x_j}$. So, I did not expand all the term, at least the three terms I should have expanded. Here I have set i equal to 1, 2, 3, the sum of 3, j remain the same, I should have done that. so if I do this basically this is only the 1 is expanded, but it is a sum of 3 terms summation of summation in i.

So, it is 1 plus 2 plus 3. So, I get the same $\nu_t \frac{\partial \overline{u_j}}{\partial x_2} \frac{\partial \overline{u_2}}{\partial x_j}$ and then plus I will have $\nu_t \frac{\partial \overline{u_j}}{\partial x_3} + \frac{\partial \overline{u_3}}{\partial x_3}$. The end result is the same, it is these two terms still vanish the entire part because here the

$\overline{u}_2$ is 0 or your v velocity is 0, this is 0 here and same here u_3 is 0, this. So, I did not expand that term.

here due to homogeneity this entire term goes away and here of course this is 0. So, all the terms all the 9 terms goes to 0 this entire part. So, here Nirupam asked a very good question that why like I gave the value here experimental data M comes out to be 1.25 plus or minus 0.06 here 0.

0. not much space 0.06. So, if you actually set this in you are not getting 1.2, you are getting like 1.8 ish something.

So, that is true the m value is correct. This $c_{\varepsilon 2}$ is still used as 1.2, 1.92 in the standard k ε model. And the argument they place is they have also accommodated the free shear flows.

So, now this is we are looking into you know decaying turbulence. So, they are also looking into to accommodate for free turbulent shear flow. So, this is used in standard k ε model. So, if you set this value you will get what is the value we got the ratio do you remember Nirupam? this if you substitute the $C_{\varepsilon 1}$ comes in a range 1.8 plus or minus some value 0.

14 plus or minus 0, 0.4 plus or minus 0.4. is what you get if you substitute you do not get this. But in standard k ε model they recommend 1.

92 value saying that they have also considered the behavior for free shear flows free turbulent shear flows or free shear turbulent flows. So, this 1.92 is used in the standard k ε model, this constants can slightly vary as you change a different k ε model. So, what I am doing right now is for standard k ε ok, how these 5 constants are appearing right ok. So, we will go ahead with we have other 3 more constants to look at σ_k, σ_ε and $c_{\varepsilon 1}$, we have looked into sorry.

is C_ε oh yeah correct $C_{\varepsilon 2}$, $C_{\varepsilon 2}$yes. So, $C_{\varepsilon 1}$is still open and σ_k and σ_ε two more constants to look at. So, we have three more constants σ_k, σ_ε and $C_{\varepsilon 1}$. So, σ_k and σ_ε is obtained by what used to be called computer optimization and today of course you can use some other technique. So, using computer optimization σ_k is found to be 1.

and σ_ε is found to be 1.3. Again this is in standard k ε model. All the values that I am giving out is on for the standard k ε model. So, now $c_{\varepsilon 1}$ is the one that we have to look at.

So, to get $c_{\varepsilon 1}$. I will go back to the logarithmic region of the flow again ok. So, to get c_μ we looked into log law region and then for $C_{\varepsilon 2}$ we looked into a decaying turbulence. Now, $c_{\varepsilon 1}$ appears in the production rate of ε right. So, we will go back to the zone of the so called log law region.

So, for $c_{\varepsilon 1}$ consider consider the log law region again that is the inertial sub layer, log law region of a smooth turbulent boundary layer. Again same considerations as before that is statistically stationary homogeneous along the two directions. homogenous along, homogenous means statistical homogenous always, statistical, statistically homogenous along your x1, x3 direction that is your in the x, y, z coordinate it is x and z. Statistically homogenous along x1 or x3 direction and the flow is fully developed. fully developed flow which gives to your $\overline{u}_2 = \overline{u}_3 = 0$, that is $\overline{v}$, $\overline{w}$, same consideration as before.

So, now we need $c_{\varepsilon 1}$ and for that we will consider both the, especially we consider the ε equation. To get the $c_{\varepsilon 2}$, we took ε equation, we took a flow where some terms are negligible, we drop them and then obtain $c_{\varepsilon 2}$. So, we go back to ε, modeled ε equation.

So, modeled ε equation, this if I rewrite this, I would get $\frac{\partial \varepsilon}{\partial t} + \overline{u}_j \frac{\partial \varepsilon}{\partial x_j}$ the left hand side and then of course, the right hand part you would get your diffusion rate. So, you have $\frac{\partial}{\partial x_j}\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial \alpha_j}\right)$.

and the production rate of ε which is $c_{\varepsilon 1} \frac{\varepsilon}{k} P_k$ - $c_{\varepsilon 2} \frac{\varepsilon 2}{k}$ or you can write $\frac{\varepsilon\varepsilon}{k}$ here. E by k is the timescale that we introduced. So, now if I consider all this all the statistically stationary, statistically homogeneous, fully developed flow all these considerations I would get this term will go away. This upon expanding all the three will go away $\frac{\partial}{\partial x_1}$ $\frac{\partial}{\partial x_3}$ will go away and in other one it is $\overline{u_2}$ will go away $\overline{u_2}$. So, this term also goes away due to all these considerations.

So, I would get 0 equal to here $\frac{\partial}{\partial x_1}$ and $\frac{\partial}{\partial x_3}$ term goes away, again due to statistically homogeneous. So, that is this part $\frac{\partial}{\partial x_1} \frac{\partial}{\partial x_3}$ of this average terms though they go away. I get only $\frac{\partial}{\partial x_2}$ term which is $\frac{\partial}{\partial y}\left(\nu \frac{\partial \varepsilon}{\partial y^+}\right)$ I am just splitting the, splitting this viscous and turbulent contribution to the transport. And then I get $\frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial y}\right)$. So, this $\frac{\partial}{\partial x}$ or the $\partial y \partial z$ terms, they go over here.

So, the viscous term and the turbulent diffusion rate plus I have this $c_{\varepsilon 1} \frac{\varepsilon}{k} P_k$ - $c_{\varepsilon 2} \frac{\varepsilon 2}{k}$. Now, in a log law region and when we look into the transport terms which term is dominant here to first order and which one will be negligible. We have this viscous, this is viscous diffusion, this is turbulent diffusion rate and the pressure we dropped in the k model and therefore, it was dropped also in the ε model turbulent diffusion rate. So, which is dominant? in the log law region, the turbulent diffusion rate, the viscous is negligible. So, this is negligible in the log law region, viscous effects are not dominant.

viscous effects. So, therefore, the equation is let us call this equation 1 which has three terms, one diffusion term and then the production term and the destruction term. So, now I would like to introduce scales here for the velocity and the length scale. I would like to introduce a scale to this equation. So, we have So, the idea is that the modeling idea is to introduce a turbulent scale in equation 1. So, for the velocity scale, what do we use for a turbulent velocity scale? I would like to use u star that is the friction velocity, wall friction velocity.

And I would like to introduce a turbulent length scale. A turbulent length scale I use the mixing length the kappa y. So, I have where kappa is your one common constant. So, I mean essentially introducing turbulent scales here u star and kappa y and that I am going to introduce for the terms in equation 1 which is essentially ε I need to introduce the scales to ε here and then ν_t t of course and then k and p_k. These are the four terms that are there and I need to introduce these scales inside these four terms.

So, first thing I will do is use the logarithmic region for the k model equation. So, recall k model equation in the log law region. What do you get in the log law region? To first order we saw that P_k is balancing ε all other terms are negligible. So, P_k equal to ε to first order. So, P_k equal to ε and what is ε dimensionally, $\frac{m^2}{s^3}$.

So, this is dimensional arguments now, $\frac{m^2}{s^3}$. So, placing dimensional arguments, using dimensional arguments. I get both for P_k and ε, I can use $\frac{u^3}{L}$ which is $\frac{u^{*3}}{Ky}$. So, wherever I see P_k and ε, I am going to use this.

Let us call this equation 2. I am simply using some dimensional arguments here and introducing these two scales. Now, P_k and ε are done, eddy viscosity and turbulence kinetic energy I need to introduce. So, eddy viscosity we already have the formula. So, eddy viscosity mu t is $c_\mu \frac{k2}{\varepsilon}$ that is c_μ I need for the k^2. So, from this I can obtain well before this I can say again dimensional arguments also for ν_t, I can say here mu t is also a velocity and a length scale $\frac{m^2}{s}$.

So, ν_t I can simply introduce. u*Ky. Let us call this equation 3 here. So, all I need is turbulence kinetic energy. For that I will use the relationship between eddy viscosity and k and ε.

$c_\mu \frac{k2}{\varepsilon}$. So, I can use it to obtain the scales for k. So, nu t is equal to $c_\mu \frac{k2}{\varepsilon}$. So, I get k^2 therefore, ν_t which is u*Ky and then ε which is $\frac{u^{*3}}{Ky}$. and then divide by c_μ. So, some terms goes away Ky Ky goes away giving rise to essentially $\frac{u^{*4}}{c_\mu}$.

So, with this I get the expression k I get this as c_μ raise to minus half u * 2 . So, this is equation 4. So, now I can introduce equation 2, 3 and 4 in one. these scales or the equivalent of these scales for these four terms ε p k, ν_t and k inside 1 to see what is the value for $C_{\varepsilon 1}$.