Course Name: Turbulence Modelling

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Week - 6

Lecture – Lec35

35. Standard k-ε model and the model constants - II

So the second one I would like to look at is $c_{\varepsilon 2}$. So now where does this $c_{\varepsilon 2}$ come in the equation? Does it come in K model or Epsilon model equation? Epsilon model equation, right? So obviously you already got the clue. You need to pick up a flow and a zone so that other terms become negligible and this particular term becomes dominant. So where does this $c_{\varepsilon 2}$ arrive occur within the epsilon model equation dissipation rate correct. So we need to consider a flow where dissipation rate term is dominant in the flow and other terms are negligible.

So what kind of flow is that? So there is something called grid decaying turbulence flow. There is a flow a turbulent flow which is completely decaying and I did mention in one of the classes that if you stop supplying energy to a turbulent flow it will decay. So, you need to consider a flow where production rate is 0 and the turbulence is only decaying and this occurs in let us say if you have gone and visited a wind tunnel right if you turn off the blower. there is a grid generating turbulence in every wind tunnel ok.

We have wind tunnels you can go and visit in our own lab. So, this there will be a grid essentially a mesh and then an all coming flow passes through that generating turbulence downstream. So, if you turn off the blower turbulence will decay. So, we consider such a flow ok. So, here we consider grid decaying grid decaying turbulence ok.

So, in that of course, we look at far far downstream of the grid in the far downstream zone . This if you have a mesh as I mentioned some grid is there and then the flow you have an oncoming flow passing through this generating turbulence and then it is eventually dying. So, I am going in that far far downstream zone of a grid generated turbulence. So, if I take such a region far downstream of a grid generating or grid decaying turbulence. So, at that particular location far far downstream your velocity will be nearly constant right.

Your stream wise velocity will eventually attain the free stream velocity whatever the background flow is and other two velocity components are 0 there. So, in that zone what happens is essentially your $\overline{u}_i$ is constant ok. So, the other two will be 0 the $\overline{u}_3 \overline{u}_2 \overline{u}_1$ is essentially achieved some u_∞ or free stream whatever value downstream ok. So, constant which implying that should have come to that is your let us say is basically u_∞ the $\overline{v}$ $\overline{w}$ are 0 that is what I mean by it has come to a constant at the far downstream location far far downstream of a grid. So, when this occurs when your mean velocity is constant what will happen to its gradient? $\frac{\partial \overline{u}_i}{\partial x_j}$ the gradient term what will happen to this? Your mean velocity is constant, so the gradient is 0.

When gradient is 0 what happens to your production rates? Your production rates if you look at the k model and epsilon model equation is essentially relying on a mean strain rate. So, we related the Reynolds stresses to strain using Boussinesq and that is omnipresent in the k epsilon model. Everywhere we go mean strain rate is present. So, when this is 0 $\frac{\partial \overline{u}_i}{\partial x_j} = 0$ or the strain rate is 0 this gives basically your P_k the model I am only looking at the model equation not the exact. So, the P_k will be 0 $P\varepsilon$ will obviously be 0 because $P\varepsilon$ is a function of P_k only.

So, P_k goes to 0, $P\varepsilon$ goes to 0 and we also let us say neglect the diffusion terms here, very small diffusion terms. Diffusion terms are also small, so it is omitted where in this particular zone far far downstream zone of a grid decaying turbulence not in a zone where turbulence is present generation right. So, we are for $c_{\varepsilon 2}$ to get it we are taking that kind of a flow. But for C_μ we took a turbulent boundary layer logarithmic zone where P_k and epsilon are balancing to each other here P_k is 0 completely. So, you have taken such a flow diffusion terms are gone.

So, if this is there then what will happen to the k model equation the k model equation essentially reduces to on the left hand side it is statistically stationary here right down downstream here this $\frac{\partial}{\partial t}$ will be 0 statistically stationary far far downstream ok. So, the left hand side $\frac{\partial k}{\partial t}$ term is 0 there are only 3 advective terms ok. So, your $\overline{u}$ or u_∞ is present $\overline{v}$ $\overline{w}$ is 0 there far far downstream of a decaying turbulence. So, one term survives here that is your $\overline{u}$ or u_∞ I call it u_∞ d ε sorry d_k sorry I am looking into $\frac{dk}{dx}$. So, d by $\frac{dy}{dz}$ terms or not present here because the other two velocities are essentially 0 when it has come to a free stream condition.

So, this is the only term surviving on the left hand side of your k model equation. So, you are essentially looking into k decaying along x as you walk along the flow direction k is decaying continuously and on the right hand side as I said diffusion terms are omitted considering that they are small production is 0 ok right. So, that means what time is surviving - ε sorry. So, only - ε survives the entire equation comes to this form the k model equation for this particular flow at that particular location. similarly epsilon model equation.

Here also $\frac{\partial}{\partial t}$ term is gone. So, on the left hand side again your $u_\infty \frac{d\varepsilon}{dx}$ term survives ok. and on the right hand side the diffusion terms are small even for the epsilon model equation Pε is 0. So, only the dissipation rate of epsilon term survives which is that is where the $c_{\varepsilon 2}$ is there. So, I would like to find out only $c_{\varepsilon 2}$ not its friend $c_{\varepsilon 1}$ ok.

that is why we killed the friend term $c_{\varepsilon 1}$ is killed because Pε is 0 because P_k is 0 there all right. So we have only - $c_{\varepsilon 2} \frac{\varepsilon^2}{k}$. If you go back and see the equation this is what it is the destruction rate of the dissipation rate of turbulence kinetic energy. So these two are my equations now 1 and 2. So, now I am going to assume how the k is decaying along x.

I would like to have the turbulence kinetic energy decay as 1 over x to some power as I walk along x it is decaying. So, I consider or I can say let k turbulence kinetic energy decay as k $=c\, x^{-m}$, m is unknown I do not know what m is here, I do not know basically it is 1 over x 1 over x raise to some value k is decaying as I walk along the x. So, if I now substitute here and use your high school mathematics calculus. So, if I substitute in equation 1 what will happen? I get $u_\infty \frac{dk}{dx}$ differentiating this equation here this particular equation So, it is $\frac{d}{dx}$ of $c\, x^{-m}$ = - ε. So, I get basically C - m or - C m u_∞ , right? I get x^{-m-1} = - ε .

So, this gives me basically the formula for ε now. Epsilon is now the way it is decaying, right. ε is now decaying as C m $u_\infty x^{-m-1}$. So, now I can substitute epsilon in equation 2 and differentiate that. So, I get $u_\infty \frac{d\varepsilon}{dx}$, so $\frac{d}{dx}$ of $cm\, u_\infty\, x^{-m}$ - okay =- $c_{\varepsilon 2}$ ε square.

So I have c^2 m^2 u_∞^2 okay $\frac{\left(x^{-m-1}\right)^2}{k}$, k is also known here which is $c\, x^{-m}$ okay. So if I go ahead and differentiate the left hand part I get c m u_∞^2 and then I get - m - 1, correct. This is m -, yes, x^{-m-1} - 1 = the other side which is - $c_{\varepsilon 2}$, 1 c will go away. I get c m^2 u_∞^2 also goes away here, right. This and sorry, I will write it here, u_∞^2, u_∞^2.

This $\frac{(x^{-1})^2}{x^{-m}}$. this goes away and then the C also goes away here. So, if I look now, it should essentially this formula should be m of m + 1, wait, yes, -, okay. So, this is essentially - of - m of m + 1 x^{-m-2} =- $c_{\varepsilon 2}$ m^2 of course and then I have $\left(x^{-m-1}\right)^2$. and then x^m here, ok.

So if I do this, I look into this particular form now, everywhere it is becoming this x^{-m} -, ok this becomes - 2m - 2 and then there is m. So this essentially becomes x^{-m-2} only, this particular part, right. So this is only, so this is $x^{-2m-2+m}$. So, this becomes - m - 2.

So, this cancels out. so i get essentially $c_{\varepsilon 2}$now is = just the ratio of m (m + 1) / m, okay, m and m also I can cancel, this also I can cancel out. So, I get essentially (m + 1) / m at the end. Now I can go and measure a grid decaying turbulent flow far far downstream I can go and see what is this m right? How is k decaying as 1 over x^m? So I can get m from the experimental data. What is the decay rate there? And from that I can get c_ε. So from experimental data we know that this experimental sorry.

experimental data in a grid decaying turbulence of course, gives me m =1.25 plus or - 0.06. So, if I substitute it here I get $c_{\varepsilon 2}$ value comes out to be 1.

92. So, this is your another model constant we have used standard k ε somewhere you would have seen this value C_μ is 0.09, C ε is 1.92 ok. This is how we have achieved two So, we will see in the next class how the other three constants come out to be.