Course Name: Turbulence Modelling

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Week - 6

Lecture – Lec31

31. Different approaches to solve turbulence closure problem - II

So, zero equation does not mean that there is no equation. So, what we mean by zero equation is there is a zero transport equation ok, we are using an algebraic expression here. Zero equation implies zero transport equation model, ok. So, this transport is omitted in the form. So, zero equation model means zero transport equation and then we are only considering, only algebraic expression is used.

There are many models under this category of zero equation. As I mentioned in the course outline in the beginning, I will not discuss every model that is present in the literature. I will only give you one particular model under each class. Let us say zero equation, I will discuss one particular model.

Go to one equation model, two equation model like that. So, I will give one example and I will tell you how this model is built. So, there are many popular models here one particular example is called, what is called a Prandtl's mixing length model, ok? An example is Prandtl's mixing length model. So, what they mean by a mixing length? I will discuss here. So, this idea comes from, if you recall, the turbulent boundary layer itself has many sub-layers.

So, you had, for example, if you recollect, this was the wall. So, you had a region called linear sub-layer where and then above you had an inertial sub-layer, right? So, in the linear sub-layer, you had u^+equal to y^+, and far away, you had an inertial sub-layer where u^+ is a function of y^+ again, but it was the function was basically logarithmic of y^+ in a log-law zone. So, somehow you are able to express your velocity profile based on a wall normal coordinate. A non-dimensional y-coordinate right. So, the idea here comes from that part that I am going to use something like a length, ok? We will call it mixing length, which is basically a wall-normal coordinate.

So, the modelling idea here is that the modelling idea comes from this turbulent boundary layer, turbulent boundary layers where u^+ is a function of y^+, where y^+ can be considered as a length, a mixing length. either a linear function of y^+or a logarithmic of y^+. So, using this log-law region or the linear sub-layer we can construct a model now. So, what we do is now we say simply eddy viscosity also I would like it to be a function of y^+that is why the name mixing length. But it has to be a meter square per second dimensionally.

We have to construct something right. So, this can be as I mentioned it can be your *l* square and of course and a time square, right? So, for the *l* square, I will like to call this name *l* mix here; *l* mix is your mixing length. So, the model has what is called $(\kappa y)^2$, and then a turbulent time scale is required, but here, the only available turbulent time scale is not really a turbulent time scale, but some time scale is basically your $\frac{\overline{du}}{dy}$. This is giving you a time scale, but note that here this is eddy viscosity here this is a turbulent term at least you want it to be a turbulent term you defined it as turbulent viscosity, but now this is becoming this is purely a geometrical length scale right geometry dependent term and this is flow dependent term. So, both are not turbulent here, unfortunately, right, but this is what you have access to.

So, one particular unknown nu t is replaced by two known things you have access to mean velocity and its gradient. So, I get $\frac{\overline{du}}{dy}$ as a time scale right. So, this is your time scale part, and this is your length scale part, κy. So, here, κ is the von Karman constant, which is equal to 0.41, and y is your wall-normal coordinate, a wall-normal coordinate just a distance from the wall ok like this.

y direction in a boundary layer and $\frac{\overline{du}}{dy}$ this as I said, this is your time scale. So, there are some advantages of this model, but mostly, this model is no longer being used. It is a too crude approach to model turbulence. The first one, you already has realized that you want a turbulent term, but now you have a geometric length scale and then a flow time scale, right? So, this is becoming non-turbulent, basically, the model. The model works simple, but it has more disadvantages than advantages here.

So, note so you have model is too very basic. The main advantage you can say is that we can say, advantages or one advantage I can think of, is that you can generate initial conditions in CFD calculations. So, when you initialize your velocity u v w and then you are solving for pressure and everything you need to give an initial condition in your CFD calculation what it has to be usually we do not put it as 0. We make a guess. So, then if you have a field ν_t right then you need to guess eddy viscosity right.

So, you can use this formula to guess to give an initial condition. So, you can use this to guess initial conditions. Model can be useful to give initial condition in CFD simulations.

That is an advantage. Disadvantage of course are many as I said the model is relying on flow and geometric scales ok.

Turbulence term is modelled based on a geometric and flow scale, not really a turbulent quantity at all, right, yes. And another disadvantage anybody can take a look at here? One of the biggest disadvantage here that is there for all eddy viscosity models, ok? So, whenever you construct a model, this should not be working only for one type of flow. As I told you right, the model should be fairly working for many types of flows, and also it should be giving reasonable accuracy, it should be easy to implement in your software, and it should be numerically stable, right? So, the schemes that you use should be such that it should work and the code should not crash. So, all these things there are many ways the model can be popular. So, now is this model applicable to other classes of flows? For example, let us take a jet flow.

Do you have y coordinate here? a wall normal coordinate right what about flow over a bluff body or let us take a plume or you take a mixing layer right. In generic class of flows where you have to search for a wall normal coordinate which does not exist here right and even if you give for a bluff body you can say ok there is a surface I will use y as a wall normal coordinate it will give erroneous values because there is no there is hardly any boundary layer downstream only upstream you have some ah boundary layer development downstream it is all flow separation and now turbulence coming in. So, this is not applicable to all kinds of flows here, right? So, works only for boundary layers. So, what about, you know, jets, wakes, mixing layers, plumes, all these things, that is what is y in those flows. Therefore, it is only specially made only for let us say a boundary layer flow, it does not even work for other flows.

So, you cannot even use it, I am not even talking about accuracy here. Therefore, all 0 equation models are become least popular, it would be nobody uses it. The only use you can say is that you can use it for generating initial conditions. So, some value is there other than 0, at least something is there to start with for your calculation. So, we will move to yeah before this we would like to see whether model behaves physically for example eddy viscosity this is a turbulent parameter.

So, now what will happen to turbulence close to the wall even in a boundary layer. It becomes 0 right the Reynolds stresses. If you are modelling based on eddy viscosity, eddy viscosity has to go to 0. It is fully viscous in the linear sub-layer. It is a viscous dominant flow.

So, on the wall what happens at y equal to 0. So, you need to see, for example, at y equal to 0, what happens to this ν_t value according to the model ν_t exactly has to be 0, right?

So, I can write it here So, at y equal to 0 ν_t has to be 0 right, but according to this model what will this be y goes to 0 right. So, y going to 0 will make this that is κy is equal to 0. So, this is completely fine, right? And if you take a boundary layer, what will happen when you go to free stream for eddy viscosity? What should that be? Again, eddy viscosity has to be. There is no free stream, and there is no turbulence, right, unless you have some value.

Let us say there is no free stream turbulence you are going to free stream conditions u infinity then there is no turbulence. So, eddy viscosity has to be 0. Is it giving this at very large y values? See, this is ν_t is 0; nu t is essentially 0 here because your κy is 0, right? That is, κy is 0. So, at y very large distance at y very very large far away from the wall then what will how will ν_t be 0. Exactly, it is still 0 that is because your gradient goes to 0.

So, model is intact. Now they have thought about this that model falls back to physical values on the wall far away from the wall. The only problem is it works for a boundary layer. So, now we go to jump. We do not discuss one equation. Now, we will come back to that.

I will go to the equation model. It is much easier to construct one equation if you have already constructed a two equation. As the name says there are two equations that means there are there is actually two transport equations here. as against the 0 equation right 2 equation mean 2 transport. So, when we go here to do transport now first thing as I said we would like to address eddy viscosity.

In a Boussinesq we already use Boussinesq to close the RANS equation, but ν_t and k is still an open-ended question. So, we started attacking eddy viscosity now. So, we would like to see what ν_t will be in a 2 equation model. and ν_t , as I said, is a turbulent quantity. So, we want it to be based on some turbulence quantities, ok? So, we will take up, for example, the u.

I would like to model this as the square root of k, which is the square root of your turbulence kinetic energy, the right turbulence kinetic energy square root of k. So now a turbulence term is included. This is a turbulence term. So this is good.

the velocity scale, it is addressed. Unlike previously, we are using a flow time scale and geometric length scale. At least now, I am thinking of using a turbulent quantity. This has two purposes: one, I am modelling eddy viscosity based on a turbulence quantity; another thing, in Boussinesq, there are two unknowns, ν_t and k. If my ν_t is based on k, then that is also a great idea, right? So, in the Boussinesq hypothesis, right ν_t and k are unknowns.

Therefore, by modeling ν_t based on based on k is a good idea.

So, one of the scales is set. Now we would like to introduce another scale, a time scale here. I will put up a turbulent time scale which is simply the ratio of k over ε. If you recall, this is a meter square per second square, and epsilon is a meter square per second cube. ε is the dissipation rate of turbulence kinetic energy. It is the rate at which the k is changing.

So, it will have the same unit as $\frac{\partial k}{\partial t}$, is same as ε, right? So, it is a meter square per second cube. So, this is giving me a turbulence time scale, right? Again, this is a turbulence term, ok? Turbulent time scale here. Turbulent time scale. But of course, it is not associated with a particular eddy because k and ε is statistical it is representing all the turbulence that is inside the system. So, we do not know it is only on dimensional grounds we can say that the unit comes to a second and therefore, is a turbulent time scale and u is of course, or square root of k is the turbulent velocity scale.

So now, if I use these two, I can construct what is ν_t. So, I can say now, ν_t, I would like this to be equal to I want this to be a function of k and ε. So, this forms the basis of what is called $k\varepsilon$ models in the under the class of eddy viscosity models a two-equation problem. So now ν_t is equal to a model constant comes in.

I will tell you what the C_μ becomes later. Let us take $C_\mu\ k\varepsilon$, but we need dimensions to be exact, correct. ν_t has to be a meter square per second here, right? So, we want that to be the same as on the other side as well. So, how do I get this when I want to make dimensions to be correct on both sides? Is it meter square per second? I want here also, I want it to be meter square per second. How do I do this? Sorry, dimensional analysis, right? So, using dimensional analysis, ok? So, I have a meter square per second on the left-hand side, and on the right-hand side, I have a meter square per second square a meter square by second cube to b. So, I get two equations 2 equal to 2 a plus 2 b that is for the m.

$$\nu_t = C_\mu k^a \varepsilon^b$$

2 = 2a+3b and -1 = -2a-3b

and for s, I get minus 1 equal to minus 2a minus 3b. So, two simultaneous equations, so if I get rid of that I get easily b is equal to minus 1. Substituting it back, I get an equal to 2, right? Write it as a equal to 2. So, therefore, what does this yield the model now will be ν_t is equal to $C_\mu\ k$ rise to 2 k square over ε. This is the expression used in the so-called k-ε eddy viscosity models.

ν_t is computed using $C_\mu k^2/\varepsilon$. But of course, we still do not know, or at least we have not touched upon what it is ε, how it is modelled, how k is modelled, and how C_μ is modelled; at least we have got an expression here, right? So, what is C_μ? How to compute k, how to compute epsilon, these three I need. So in the Boussinesq ν_t is solved at least. Even in the Boussinesq I need k, turbulence kinetic energy. So if I am going to find out a way to compute turbulence kinetic energy k that will solve the purpose for both ν_t and k. In addition, of course, I need to figure it out what is C_μ and ε ok.