Course Name: Turbulence Modelling

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Week - 6

Lecture – Lec30

30. Different approaches to solve turbulence closure problem - I

So in the last class I told you that there is also relevance for the pressure term. Some of you asked why we did not consider the pressure term in the scale analysis right. So when we looked at this inner coordinates or inner turbulent boundary layer, we did not consider the pressure term also in the outer layer, but I did mention that pressure will also become relevant. We discussed this three important sub layers within an inner layer that is we have this inertial sub layer here. where the turbulent terms are dominant and I we also saw that in the outer layer equation the left hand side inertial term balances the turbulence term to first order and in the buffer layer both the turbulence term and the viscous term balances to each other to first order and then I said when we go to this linear sub layer where only viscous effects are dominant turbulence is negligible then how can we leave this term by itself somebody must balance this right.

This is where in the linear sub layer this is where the pressure term is balancing the viscous term to first order. Let us say if you have a pressure driven channel flow ok. So, you are imposing a pressure gradient $-\frac{dp}{dx}$ in your pipe flow or a channel flow then this $\gamma\frac{\partial \bar{u}}{\partial y}$ is the the friction that is that it is opposing and balancing the driving pressure gradient they has to balance each other to first order. So, the pressure gradient becomes dominant in the linear sub layer ok, this is clear ok.

So, now we go back to our turbulence modeling. So, in the last class we looked at Boussinesq hypothesis right we started with what is called an eddy viscosity model. eddy viscosity models under the framework of the Ryan's models ok. Ryan's model is little bit bigger you can have many different class of models. So, we started to look at eddy viscosity models in that we learned what is a Boussinesq hypothesis.

So, we had six unknowns So, $-\overline{u_i' u_j'}$ we had 6 unknowns and this is of course, the exact terms. This is now being modeled using Boussinesq hypothesis as $2\nu_t s_{ij}$ mean strain gradient $-\frac{2}{3}k\delta_{ij}$. So, here of course, this is the modelled part. So, 6 unknowns is now replaced with 2 unknowns that is this and this. So, 2 unknowns now, 2 unknowns ν_t, k and this is your model equation.

So wherever you see this Reynolds stress stresses we are replacing this by this model form. So, we will see what are the consequences of modeling like this. So, we also discussed that the $-\frac{2}{3}k$ the use of this $-\frac{2}{3}k$ that is if the flow is completely homogeneous in all three directions to save Reynolds stresses we have this $-\frac{2}{3}k$ and I also discuss why $\frac{2}{3}$ and all these things. And k is used because you want this Reynolds stresses are essentially turbulent stresses. So, you want turbulent stresses to be modeled by a turbulent terms not the flow term right.

So, here you have base essentially three quantities here right. So, this particular quantity here. So, if you see this $\overline{S_{ij}}$ that is a mean strain gradient. So, it is a flow term. So, you are modeling turbulent stresses based on a flow term which is not really great, but that is ok.

But there are two other terms that you want at least to be turbulent or you have a choice now. You can decide how ν_t as well as k to be modeled ok. Because $\overline{S_{ij}}$ is already a flow dependent parameter it is not a turbulent parameter right. So now there is also kind of misconception in some literature that eddy viscosity models I have seen some students say eddy viscosity models or isotropic models that is actually not true. It only promotes isotropy.

So eddy viscosity models by itself is not isotropic ok. So, it is simply called EVM or eddy viscosity models by itself is not isotropic. model because you still have this strain rates S_{ij} which can help you to get anisotropic nature of the Reynolds stresses. But, but two terms that is your ν_t comma k these two terms promote promotes or encourages isotropy because these two are scalar quantities. But two terms are scalar and promotes isotropic behavior in the model.

S_{ij} will still help you. The reason I said that it is not isotropic, but it only promotes isotropic behavior. That is like for example, if I take i = 1, j = 1. So, this is your - $u_1'u_1'$ stress component. This is still now 2 ν_t.

You have S_{ij} will give you half of that. So, it is basically you get 2 $\nu_t \frac{\partial \overline{u}_1}{\partial x_1}$. This is what you get. So, and then for i = 2, j = 2, the other component you get - of course you get - $\frac{2}{3}$ k. So,

$- \overline{u_2' u_2'}$.

the exact one is replaced by the model which is model is 2 $\nu_t \frac{\partial \overline{u}_2}{\partial x_2}$ - $\frac{2}{3}$ k. So, you see these two terms here this term and this term in a given flow in a generic I am just talking in a generic terminology in a given flow the these two terms need not be the same. right though $\overline{u}_1$ by the gradient along the x direction or the gradient along the y direction of this two velocities need not be the same in general right. And therefore, $\overline{u_1' u_1'}$ and $\overline{u_2' u_2'}$ are not isotropic. If these two terms are same of course, then it is promoting isotropy.

So, the strain rates will help you out in generic flows to get anisotropic behavior of the Reynolds stresses but this this two terms this turbulence kinetic energy here right. So this k as well as the ν_t these two are scalar and they are promoting isotropic prediction inside the flow. So they are working hard to get some isotropic behavior which is not desirable but the nature of the model is like this ok. So this you have to remember. So, by default it is not isotropic, but there is some isotropic effect in the model when you use any eddy viscosity model ok.

So, now still we have not found the final solution because in the RANS equation we had 6 unknowns, 6 unknowns is replaced by now 2 unknowns eddy viscosity and turbulence kinetic energy k I still need access to this. So, we will start attacking one term at a time we will take up the eddy viscosity what it is ok. So, the eddy viscosity eddy viscosity. Now, already I told you that this has nothing to do with either any eddy or you can say it is a some turbulent term it has nothing to do with nothing to do with any eddy or viscosity ok. So, it is better to call it some turbulence term, but anyway the name has become popular.

So, we call it eddy viscosity models or turbulence viscosity. And you see that now to get this we go purely on dimensional terms. So, dimensionally this has to be meter square per second ok for equation to be consistent. it is appearing next to the turbulent stresses are appearing together with viscous stresses. So, ν and ν_t can be written together strain rate outside and therefore, meter square per second is the dimension for the eddy viscosity.

So, now that means, I can come up with many arguments and options. For example, I can say now I want my eddy viscosity to be for example, let us say depending on a velocity scale. a turbulent velocity scale and a turbulent length scale or I can say the eddy viscosity now purely on dimensional grounds depends on a length scale and a time scale ok. So, where this u, l and t are turbulent velocity, length and time scales respectively all right. So, this is a turbulent scale at the end of the day.

We want ν_t to be depending on some turbulent scale and as I mentioned turbulent flows are characterized by many scales there are many eddies right. So, which scale you want to consider here is a question right. So, note turbulent flows turbulent flows are multi scale multi scale phenomena right. are many scales to be used right, eddies of many eddies of many sizes. They all have their own turnover times or time scales associated with it, they are all moving at their own velocity and they are of different length.

So, now the question is which which U, L and or T to consider for modeling. What do you think? What can we choose here? The new T we need to model and I am mentioning that its turbulent flows are multi-scale. many many eddies are there of different length time and velocity. And therefore, I mentioned eddy viscosity models or also statistical models we go purely at statistically statistical approach without considering any random component here. The moment we think of eddies some random random nature of the flow has to be considered for modeling and this is a purely statistical approach ah.

So, therefore, it is like a lumped approach we are not really going to quantify and model any particular eddy here. right. So, this question is very difficult to address in eddy viscosity models which particular eddy are we modeling here right. We are trying to replace the entire Reynolds stresses in your flow here the 6 particular terms with this everywhere in the flow and everywhere in the flow you will have many different eddies. and we are not trying to at look at what is the energy associated with a bigger eddy and intermediate eddy and all these things.

We are just lumping everything together and we are saying the 6 Reynolds stresses are simply depending on an eddy viscosity, a turbulence kinetic energy and so on right. So, that is why this model is very different, it is very basic approach and obviously ah you know there will be errors associated with this ok very statistical approach. So now we will purely proceed based on dimensional arguments. So the answer of course is not favorable it is a bit disappointing that no particular scale is being considered here right.

No particular or group of eddies. are considered here right. So, it is a purely statistical approach. The largest eddy is of course, being resolved you are not modeling it that is the flow. Now, the let us say in this particular room what what could be the largest eddy size as big as this room right. So, that is you are going to capture your flow is going to capture that, but anything else you are not being capturing in this right.

Everything else is being modeled except the largest eddy that is sitting every other eddy that is there associated with turbulence is being modeled in eddy viscosity models. So, we purely proceed. So, the model modeling will proceed on dimensional grounds. that that is that is eddy viscosity has the same units as kinematic viscosity $\frac{m^2}{s}$ right. So, we need to find some velocity length or time scale and then we have different models for that.

So, we will proceed with looking into a zero equation model. So, to get eddy viscosity now we will proceed with there are many options the first and the easiest option is using what is called a zero equation model.