Course Name: Turbulence Modelling

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Week - 5

Lecture - Lec28

28. Reynold's Averaged Navier-Stokes (RANS) models - I

Welcome you all to the class again. So last class we learned a lot about turbulent boundary layers and we particularly learned that, inside the what we call an inner turbulent boundary layer which is also called a constant stress layer. It is called constant stress because in the limit of infinite Reynolds number the stress inside will be constant and I am I also showed you the data I am just re-plotting it here. Basically this is for the same turbulent plane Couette flow. The straight line is the turbulent shear stress and the dotted line red one is the viscous shear stress and the total stress as you see is not constant.

It can be improved with some increased number of sampling or so. This is already sampled a lot but it would never go to a perfect constant line, okay. So you can see here. has some variation nearly I am not sure whether you are able to see there is a dotted blue line here it is not constant goes to constants only in the limit of Re_{δ} tending to infinity ok.

So we learned that about this constant stress layer and this concept will be used in the modeling community as well as I said experimentalists use it because you do not have to measure the near wall flow. All I can do is I can measure the turbulent shear stress here and then that gives me the viscous shear stress on the wall that is the idea. So I do not have to measure that part. experimentalists will make use of that and modeling community would say now I do not have to use mesh closer to the wall to capture the viscous shear stress. I can just use the turbulent shear stress data to know what is the skin friction.

But there will be a challenge here. We have seen from the data from the budgets of RMS. I showed you the production rate, dissipation rate, redistribution rate all this. So what is the nature of the dissipation rate? Is it positive negative? On a graph right in the budget it always had a loss and a gain. So the dissipation rate was occurring.

it was negative right epsilon is always positive and minus epsilon when we plotted it is on the it is a loss to the system right and where is the maximum epsilon was there did you notice the maximum epsilon was on the wall ok. If you recall the budget the epsilon goes maximum on the wall so viscous dissipation rate or the viscous effects are strong close to the wall just like viscous shear stress is maximum on the wall and turbulent shear stress is minimal or zero on the wall and therefore the viscous dissipation rate is also epsilon right it is an epsilon dependent quantity so it goes maximum on the wall so now if you say i don't have to measure close to the wall I just compute turbulent shear stress and then I use this information to obtain viscous shear stress fine. But how are you going to get dissipation rates if you do not measure close to the wall right. So, experimentalists have difficulty not getting the dissipation rate on the wall if you do not measure it ok. It is very difficult to measure close to the wall as I explained in the last class the difficulties and challenges there.

So, this idea is useful provided you are not interested in the dissipation rates. right. So, dissipation rate is important to you if you are looking into the budget you need measurement close to the wall. Same thing goes with modeling community also right. It is not just important that I compute the skin friction.

If I want to know the dissipation rate in the system then I again need to have a mesh very fine close to the wall to compute epsilons. So, one must go near the wall we cannot ignore it that is the point. So constant stress layer fine the idea is interesting and we can use it in the modeling, but you have to be careful that near wall is also an important place viscous effects are dominant there ok fine. So now we move to so in the last class or to summarize what we learned is we have an inner turbulent boundary layer which we also call it as constant stress layer. In constant stress layer further we have three primary layers.

There are other layers for this course we will not go in to details of that, but we have seen that there are primarily three sub layers here ok, three sub layers. which is you have a linear sub layer and from data we know that here your u^+ is y^+. So, that is your velocity so close to the wall is just nothing, but your wall normal coordinate ok. It just linearly as you go above the wall the velocity just goes linearly. linear distribution very close to it and then you have a buffer layer.

This buffer layer is very important because most of the turbulence generation rate occurs in this zone and this is the zone where we have seen that there is a equal computation between turbulent shear stress and viscous shear stress. They were both becoming 0.5 in the previous graph right. If you just go back to that graph and recollect you can see here it is reaching about 0.5 both are becoming equally dominant in this zone and then we have an inertial sub layer, inertial sub layer.

This inertial sub layer we did not discuss much about it and I said that this is also an overlap zone where your outer layer and inner layer equations are both valid here and inertial sub layer is also called a log law region. Basically if you want to represent your velocity as a logarithmic of y^+. that is the idea and in the data we see for some flows it behaves like this and there is of course somebody wants to represent it as a power law y^+. Linear here in the linear sub layer in the inertial sub layer you want to do it in a logarithmic fashion. okay so it looks like this if you want to write it down what a log law looks like u plus is 1 by this is kappa logarithmic of y^+plus a constant b ok. So, here there are two constants here it is this kappa and b.

So, I can write it here kappa and b here. So, you are essentially representing again your velocity in the inertial sublayer as y^+. So it is an empirical relation. So you can take the data and fit and see. So this knowledge has not come from theory.

Both u plus is y plus and there is a logarithmic zone. This is coming from the data. If you take many data from many turbulent boundary layers from pipe flow, channel flow boundary layers, wherever you have a smooth surface, a flat plate boundary layer, you take data and then you fit and then you see that you get an empirical relation and then they see that this constant now here or the kappa here, kappa is the von Kármán constant. which is usually if you see the value as 0.41, sometimes it is 0.

40 also, but it is 0.41 is commonly used and the other constant b this value you will see it as 5.2. Now logarithmic law or the this so called log law people say it is universal, but sometimes you do see this value changing to 5.5 also or 5.

3 and so on. That is to see that whether it is fitting to their data. Yeah so the data is like I said it is an empirical correlation and if you have happen to have a flow where your data fits for this it is fine otherwise it is not a problem. You do not have to worry so much about it and somebody asked a question a good question that whatever we have discussed this three layers or the other layers is it all going to be there all the time in all types of turbulent flows? right as I mentioned turbulent flows are different types of flows each flow is different and even in a turbulent boundary layer the The boundary conditions that we have applied is important. Now we have taken a smooth wall. If you take a rough wall and depending on the roughness height many of these layers do not exist they vanish.

So, usually in a rough boundary layer what happens is something called a roughness sub layer comes in replacing this inertial and buffer layers sorry replacing the linear and the buffer layer. So, depending on the height of the roughness its influence changes ok. And so I will not go into a rough turbulent boundary layer, but you just need to know that as long as your surface is smooth whether it is a pipe or a channel flow or a boundary layer this formula should work ok. So, this is this formula here call it equation 1 which is for same for a boundary layer or a pipe flow or a channel or a channel flow, different types of internal flows or semi-confined boundary layers ok. Is this clear? I think with this we can move to the next chapter RANS modeling chapter 4 right RANS models.

So now we are going back to the Reynolds average Navier Stokes equation the first governing equation that we derived. and we see what will happen to when we try to model here. But if you recollect I said there are primarily two class of turbulent models. One is called eddy resolved models that means you are trying to capture some of the turbulent phenomena and then model some of some of the turbulence. So, some turbulence is captured and some is modeled.

But when it comes to a RANS model this is a statistical approach. So, this is a statistical approach so the entire turbulence in your flow system is modeled here nothing is actually being captured so every turbulent eddy that you can think of is modeled in a statistical approach and here first class of models that we consider is eddy viscosity models eddy viscosity models. So, let us write the RANS equation to see what we learned from there. So, we have if you recall the RANS equation. the right hand side terms, pressure gradient term and then I will write the two stress terms together.

So, I have viscous part and the turbulent part. Let us call this equation 1 here. So now we know that this is the 6 extra unknowns. that needs to be modeled to close the equation otherwise the equation is not closed right. So, we proceed with this idea of this eddy viscosity here and the idea stems from essentially like very similar to the Newton you know the Newtonian idea that is the Newton's law of viscosity that it is a combination of two things.

First thing is that now I can write this as a sum of two different stresses a turbulent shear stress or a let us say turbulent stress and a viscous stress. And then we just saw that in flows in some of the turbulent flows this eddy viscosity model is not made only for a turbulent boundary layer it is being used for all types of turbulent flows. But if you just focus on a turbulent boundary layer we just learned that there is something called total stress and a constant stress right. So, that kind of an idea can be used here to model. So, this is your total stress right and in the according to this just like the analogous to Newtonian closure of your momentum equation they use the molecular viscosity or a kinematic viscosity component here.

So, the idea is here now if I rewrite this using what is called an eddy viscosity ok. So, the equation would be I have the left hand side a pressure term plus the $\frac{\partial}{\partial x_j}$ I would like to write this as $(\nu + \nu_t)\frac{\partial \overline{u_i}}{\partial x_j}$. Now, this is the exact equation this is exact equation no modeling is done, but here when we are moving to here this is already a modeled equation and here the idea of this total stress is being used to think it of like a lumping together and this particular is also called a nu effective viscosity. So, when you write codes or in many commercial codes they use this concept of an effective viscosity or a effective or effective eddy visco effective viscosity I would say. So here ν_t is what is called ν_t is the eddy or turbulent viscosity.

You should remember that this ν_t neither represents viscosity, neither represents any eddy ok. It is just the name has come. You can definitely call it turbulent term because the this nu is representing the molecular viscosity effects. This nu t is actually being used actually to represent the reynolds stresses or the turbulent stresses and therefore, I would agree that that is why the name t is here ν_t. So, it is a turbulent component yes, but physically it is not anything to do with viscous effects or it does not represent or capture any of the eddies.

You are completely taking a nonlinear effect here. This has u'u' v'v' w'w'. So this nonlinear nature of turbulent behavior is completely replaced by your strain rate and a scalar quantity. So this is also your the moment you made it a scalar obviously, you lose the directional dependence. The anisotropic nature of turbulence is actually taking a loss here by modeling it as a scalar eddy viscosity ok .

But the idea is as I said very similar to your analogous to the Newtonian closure. So, this modeling idea is modeling idea is analogous to Newtonian closure that is relating the stress to strain strain rates. And therefore, you have .$-\overline{u_i' u_j'}$ the Reynolds stresses that you relate it to $\nu_t \frac{\partial \overline{u_i}}{\partial x_j}$, the strain rate. But then this is not the general expression that you have. ν_t is said okay it is a we want dimensionally it should have same value, it should be meter square per second because there must be dimensional consistency between ν and ν_t

when it is modeled like this. So, it will have the same dimensions as a viscosity, but its effect is not like a viscosity here. So, but this is not the general expression what we are doing it here. So, general expression is actually coming from what we call a Boussinesq approximation. Boussinesq approximation which gives you a general expression for this Reynolds stresses. Generic closure for the Reynolds stress term which is minus-$\overline{u_i' u_j'}$this is now being closed using a model.

So, this is now set equal to 2 ν_t Sij over bar that is the mean strain rate minus $\frac{2}{3}$ k δ_{ij} ok. So, I take this. So this is your Boussinesq approximation. Now I have 6 unknowns here modeled as 2 unknowns ν_t, k 2 unknowns have come here. So the equation is still not closed for you to work with, but at least the 6 unknowns which represents the anisotropic nature is moved to 2 scalar quantities ok.

This is a tensor here. So this had anisotropic nature. This is move now this has become scalar. k is the turbulence kinetic energy here ok. So k is turbulence kinetic energy. Now, if I just started this course with chapter 4, obviously you would not know what these things are.

You would not know what is a reynolds stress, you would not know what is turbulence kinetic energy, right. Now, the theory chapter 2 will help you reconnect with all these terms. But of course, all those equations are not closed. So, we go and close these equations with some arguments that we are going to make.

right. So yes, so this is the expression that we have. Now the question comes is of course if I now little bit expand this equation, let us call this equation 2 here. So equation 2 is going to be substituted in equation 1 to close equation 1 right. So, you have this viscous stress part and then this turbulent stress is replaced by the strain rate.

So, that goes into that part. So, I have now if I expand it I get $-\overline{u_i' u_j'}$ is equal to I get essentially $2\nu_t \partial \overline{u}_i$, I am expanding the mean strain rate tensor $\partial x_j + \frac{\partial \overline{u}_i}{\partial x_j} - \frac{2}{3} k \delta_{ij}$. Obviously, this goes away. when you substitute this reynolds stresses inside your equation here in equation 1 ok. So, and then apply the incompressibility another term will also vanish. So, this is $\frac{\partial \overline{u}_i}{\partial x_j}$ survives, but there is also $\frac{\partial \overline{u}_j}{\partial x_i}$.

So, $\frac{\partial}{\partial x_j}$ divergence of $\frac{\partial \overline{u}_j}{\partial x_i}$ that term will vanish and then it will recover to this. So, this is the final equation that is coming in here that will drop out. But the generic expression is here. So, this is your equation. Now, the question is stress is being related to the strain rate.

As I said this is your stress and this is the strain. But then what is this? Why is this term coming into picture? So the question we are asking is why this $-\frac{2}{3} k \delta_{ij}$ has come? Why

$\frac{2}{3}k$ and why k? You can model this by any other term. Why are we introducing this particular idea here? So question we have to ask is why $\frac{2}{3}k$ and why k itself? So we will see why this has been introduced. So this is not arbitrary, there is some thought process in this.