Course Name: Turbulence Modelling

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Week - 5

Lecture – Lec26

26. Inner layer equation, constant stress layer, and inner velocity scaling - I

Welcome you all again. Let us get started with the turbulent boundary layers that we are looking into, right? So, in the last class, we looked at that there is a need for an inner turbulent boundary layer because there was no way of saving the viscous term in the outer boundary layer, right? So, the outer layer other terms were dominant the turbulent terms and some inertial terms were dominant. So, we need to therefore, there was a need for an inner turbulent boundary layer, and then we introduced two new scales inner scales, that is η_w and u_w. We still do not know the nature of them, but we just said that this like this η_w should be so tiny, right? So, this η_w should be smaller than δ by an order of magnitude, which is at least an order of magnitude smaller than L ok. And therefore, the ratio of η_w by L was very small ok is very small and this value depends on the Reynolds number.

This η_w by value is very small, and the value depends on the value depends on your Re delta. So, L, η_w are all functions of x, and as you move along the x direction, the Re delta is changing, your boundary layer thickness is changing, and therefore, the ratio depends primarily on the Reynolds numbers that you are working with. So, with this η_w by L being small some of the terms in the U momentum equation were of second order importance. So, to first order what did we learn? So, the u momentum equation, we had this u equation to first order, what this equation reduced to? Essentially 0 to first order I am writing.

So, we have the first term, which is $\frac{\partial \overline{u' v'}}{\partial y}$, the turbulent term and one of the viscous terms, the viscous shear stresses, which is $\nu \frac{\partial^2 \overline{u}}{\partial y^2}$. The largest viscous term is balancing the largest viscous term balances the largest turbulence term, largest turbulence term to first order where in the inner layer right inner turbulent boundary layer. In this layer these two terms are dominant to first order and when we introduce the scales this led to essentially, we get our $u_w \eta_w / \nu$ was of the order of magnitude 1 right. And this is nothing but, this is nothing but Re_{η_w} is 1 inside the inner turbulent boundary layer. This is what we learnt yesterday.

So, let us call this equation one here. And this equation will be the equation, let us say the inner layer equation. So, we can call this inner layer equation here, inner layer equation. So, now we have an equation for the inner turbulent boundary layer. Of course, the outer one had the turbulent term, the same turbulent term, but we had on the left-hand side $\bar{u}\frac{\partial \bar{u}}{\partial x}$.

Those two are balancing each other in the outer turbulent boundary layer. So, now another thing we can see here, we can easily see here is that the η_w takes a form based on the viscosity here. So, now I have a length scale η_w which is the near wall length scale or an inner scale which depends on the viscosity right. So, I have η_w is nothing but ν/u_w of course, the nature of u_w is required. Because in a given flow, the kinematic viscosity is known.

If you know what is u_w you can find out the η_w the inner length scale that is useful to you. We will see what that happens to. So, for that to be determined we will take this equation 1 and then integrate this within the turbulent boundary layer. But within the layer we are not going all the way to the outer flow. within the layer.

So, consider equation 1 ok. So, I have 0. This is approximately equal to because there are other second order terms. And I already told you this equation 1 resulted because η_w by L is small. But if η_w by L becomes 0.

Then this equation 1 becomes exact, that means it will become 0, is equal to this. Now, I cannot integrate equation 1 because still there are second order terms. So, I would like to consider in the limit Re_δ tends to infinity what would happen then in the limit Re_δ tends to infinity η_w by L. will become 0. It is small now and it depends on Re_δ.

When the scale separation becomes so large, the limit Re_δ tends to infinity η_w by L becomes 0. So, we will consider this, consider equation 1. If Re_δ tends to infinity then η_w by L will be 0, not small. Then your equation 1 will become 0 equal to, now you see this is becoming equal to instead of approximate. So, the second order terms are all gone now.

In this theoretical consideration, Re_δ tending to infinity, only these two terms are surviving inside the inner turbulent boundary layer. So, 0 equal to I have if I now integrate this, I get I will integrate this from y equal to 0 from wall to some distance y, but this y has to be within your turbulent boundary layer. You should not go all the way where your free stream velocity is; you are approaching the free stream velocity. So, it should be y within the boundary layer. So, then, so this is only a theoretical consideration.

In reality, we never approach this equation what we are going to. We simply want to integrate it to see what happens. I will show you the data. In the data, of course, this η_w by L is not 0. That means all the other terms on the left-hand side, the two inertial terms, as well as the viscous and other turbulent terms; all exist in actual flows where we are working with finite Reynolds numbers.

See, this is only when in the limit ok in the limit Re_δ tending to infinity So, I have this equation now 0 equal to this, and then if I integrate dou by dou y of minus u prime v prime plus y equal to 0 to y, I have nu dou square u bar by dou y square.

$$0 = \int_{y=0}^{y} \frac{\partial}{\partial y} -\overline{u'v'} + \int_{y=0}^{y} \nu \frac{\partial^2 \bar{u}}{\partial y^2}$$

So, here y should be within the integration is within the boundary layer. So, here y within the turbulent boundary layer that is because if you have, for example if I recall this sketch that I had have a wall and then a boundary layer like this. So, this is the interface of your what we call a turbulent non turbulent interface ok. I have not introduced this term so far.

So, this we call a turbulent and a non turbulent interface. So, this zone is turbulent, and this zone is non-turbulent, and then there is an infinite, you know, thin interface here between the two zones. So, in the non-turbulent zone, that is where you have your free stream zone. Inside, you have a turbulent boundary layer, right? So, there is a discontinuity at this position. As I told you, turbulent flows are characterized by three-dimensional vorticity fluctuation.

So, after this interface, it suddenly vanishes. The three-dimensional vorticity fluctuation vanishes above this interface and below it survives. So, there is a sudden discontinuity here. And therefore, we do not integrate all the way. This is your y direction, right? So, we therefore y within the turbulent boundary layer is what I wanted to say.

Integration is not all the way where y is approaching the free stream within and anyway, I am interested only in the inner layer here. So, I do not want to go all the way up here, right? So, my inner layer is somewhere here. The inner layer equations are also valid in the inner layer right. They are obviously not valid outside because the inertial term becomes important, and the viscous term becomes 0 above ok. So, you should know that this is only a theoretical consideration here that is in the limit Re delta tending to infinity this happens ok.

So, if I integrate, I get now this is a definite integral. So, no constants of integration appear. y equal to 0 which is the wall here, right. This is y equal to 0, the wall to some distance. So, I get easily 0 equal to $-u' \; v'$ average at some distance y within the inner turbulent boundary layer, ok. And then I get this minus of the same $-u' \; v'$ is evaluated at y equal to 0.

$$0 = -\overline{u'v'} - \left(-\overline{u'v'}\right)\Big|_{y=0}$$

This is function of y the first term second one evaluated at 0 and then I have plus this particular term which is nu dou u bar by dou y at some distance y from the wall minus of nu dou u bar by dou y at y equal to 0.

$$0 = -\overline{u'v'} - \left(-\overline{u'v'}\right)\Big|_{y=0} + \nu \frac{\partial \bar{u}}{\partial y} - \nu \frac{\partial \bar{u}}{\partial y}\Big|_{y=0}$$

All terms survive here or any terms to be removed because we are applying boundary condition here for. Yes, this particular term at y equals 0; due to no-slip and kinematic boundary conditions, your shear stress turbulent shear stress must vanish. So, this term must vanish due to your no slip boundary condition fine.

So, now this means I have, and what is the Is there a name for any of these terms that you recognize from your fluid mechanics this particular term last term when you evaluate this gradient at the wall? What is this? Wall shear stress. Wall shear stress, right? You already know that this particular term is nothing but τ_w/ρ here your wall shear stress right. This wall shear stress ok. So, therefore, I have now a formula for wall shear stress that is τ_w/ρ.

$$\frac{\tau_w}{\rho} = -\overline{u'v'} + \nu \frac{\partial \bar{u}}{\partial y}$$

This particular equation of course, is valid in the inner layer right and it is also valid in the limit. So, if I now, this equation is valid only in the inner turbulent boundary layer and also or and in the limit Re_δ tending to infinity. So, for a finite Reynolds number, this will not be equal to, it will be approximate. So, wall shear stress is approximately equal to this, ok. And this is what we call a total stress here, total stress. Because this is a combination of your turbulent shear stress, turbulent shear stress and this is your viscous shear stress.

So, the combined total stress is what we are looking into here ok. So, here integration from some wall you can take a note here integrating integrating here from wall to some y distance. So, there is one more thing that we have to consider here. We have got an equation for total stress. But when we actually plot this data, we see that something peculiar happening here for this particular condition.

I am going to show you a graph, then you will see what this looks like. So, there is yes. So, now I am showing you data from this same turbulent plane Couette flow. So, I am plotting this $-\overline{u' \ v'}$. The prime is dropped here, ok? So, basically this is the u v means without the prime I have plotted.

So, it is $-\overline{u' \ v'}$ is your turbulent shear stress, and the other one is $\nu \frac{dU}{dy}$, Uis the mean velocity here. So, which one is which you do not know? I have not put the legend here, but I know what it is, or you can also guess. So, viscous term, was it dominant as you go away from the wall? This is y, y coordinate is your wall. So, this is your, so y equal to 0 is the wall, absolute wall and you are going away from the wall when you are going in the x axis. So, away from the wall, what terms were important? Turbulent term, and then there was one inertial term.

So, obviously, the viscous term was negligible. So, which curve is following that? So, this dash line you see the viscous term is of negligible importance here. So, this dash line is your viscous shear stress and that becomes maximum on the wall. What about the turbulent term? The turbulent term is dominant. obviously this term is not balanced by this that $\bar{u}\frac{\partial \bar{u}}{\partial x}$ was balancing this straight line which is the turbulent shear stress minus u prime v prime over bar.

And as it approaches the wall, obviously on the wall due to the boundary condition, it is 0, and somewhere in between, it is balancing each other ok. And when you add these two, it is not exactly 1, closer to 1. So, if I am looking into the total stress, that is why the equation told in the inner layer I have total stress. So, if I am going to add these two, this plus this, it would more or less look like this dotted line here. So, this dotted line which is values coming 1, what is it? Nearly constant, right? In the limit of Re_{δ} tending to infinity, it will be constant because all other terms drop, but for a finite Reynolds number it is nearly constant.

So, there is something amazing here. Now, the total stress is constant. So, this layer is also called constant stress layer. So, now you may not realize the importance of this. The experimentalists are so happy looking into this graph. The modelling community is also thrilled by looking into this graph.

I will tell you why experimentalists are happy. Now, I want to know the wall shear stress. Wall shear stress is important for your skin friction coefficient, also right for the drag. Now, it is very difficult for anybody experimentalists here. In this class, who has done some measurements like this velocity measurements close to the wall? No, obviously you are turbulence modeling students, not experimentals.

But ask your friends who have done experiments, it is not possible to measure the velocities so close to the wall to get the du dy accurately. It is extremely complicated. The experimental methods are not easy to get du dy close the wall at few mic you know micrometers distance from it you need to get those gradients that is very difficult challenging or next to impossible you can say. But now they say I do not have to do this. I will measure u prime v prime correlation away from the wall, ok? And that is nothing but my turbulent, and that is nothing but my wall shear stress.

τ_w/ρ is the sum of these two, but when you are going away from the wall, τ_w/ρ is nothing but $-\overline{u' \ v'}$ because the another term is practically 0. So, τ_w/ρ the wall shear stress becomes turbulent shear stress away from the wall. So, I measure $u'v'$ and look at its correlation. It is easy to measure away from the wall. Because this ah they usually use this what is called hot wire anemometry or laser PIV equipment.

So, when you put lasers very close to the wall you get reflections. So, you cannot get data close to the wall ok and then if you want to bring this hot wire anemometry, it is having a finite it is an intrusive technique. So, very close to the wall it gets damaged. So, basically, very close to the wall, they cannot get data. So, this data is obviously coming from my direct numerical simulation where we can have a mesh very close to the wall and get it.

What is the value? So, experimentalists are happy there is a constant stress layer. I measure turbulent shear stress here at a y very, very far, and I get wall shear stress. They are happy. Now the modeling community is also happy. So, I do not need to put lot of mesh very close to the wall.

For example, let us say I am not really interested in near-wall behaviour of turbulence. For example, let us say in a turbulent combustion people do not really worry about what is happening very close to the wall because walls quench the flame. So, if somebody is modelling turbulent combustion, then they say I am happy. I already know what should be the wall shear stress because anyway, I am going to compute the turbulent shear stress.

And I have some idea about what should be. So, modeling community is also happy. So, this has a very profound influence this graph or the idea of this total shear stress or the total stress. This is clear? So, we will go back to our graph or the slides here. So, this particular total stress or this particular equation is also now this essentially implies this total stress is constant in the limit tends to infinity ok. And the total stress or the τ_w/ρ approximately constant at finite Re_δ and therefore, this particular layer, the inner layer.

Therefore, the inner turbulent boundary layer is also called constant stress layer ok. So, it is almost constant when we work with actual data here. So, we have learned that turbulent boundary layer has at least two layers an outer and an inner layer. Inner layer also has a name called constant stress layer which is sum of turbulent shear stress and viscous shear stress ok.

So, this is what we learned. And so, before I go into show you that there are other layers within a turbulent boundary layer, let us go and look at what is this η_w and u_w that we have not finalized, right? So, that question we have left it open. So, let us go and see what it is.