Course Name: Turbulence Modelling

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Week - 5

Lecture – Lec24

24. Inner and Outer TBL: Order of magnitude analysis - I

So, let us get started. So, in the last class I promised you that I will show you the data or revisit this data. So, we did the scale analysis for this turbulent boundary layers where we considered that for the turbulence velocity we are going to use the same scale right. Remember like for the for this component right. We said we are going to use a single velocity scale right and I told that is because the order of magnitude is same for all this changes in also for the shear component.

So, that is what we see here. So, you see, there is anisotropy even in the simplest flow, like a plane Couette flow, but the order of magnitude is the same. So, urms is about twice that of vrms and maybe 3 times that of wrms. It is not 10 times or 100 times more, same order of magnitude and therefore, we consider the same scale in the order of magnitude estimate.

So, you can choose any scale, but you must come up with an argument why you are doing that. And here I have chosen the same velocity scale for the turbulent quantities and I chose a different scale for the mean velocity and its gradient terms, changes in mean velocity and so on. So now we go back to the turbulent boundary layers that we were looking at. So we were looking into turbulent boundary layers. So, what did we learn in the last class that we can choose some scales and look into the U momentum equation the mean momentum equation and upon introducing the scales we learned that to first order what terms are balancing each other.

So, we learned that by order of magnitude estimates by order of magnitude estimates to first order. The other terms are not negligible, it is just that they are of second, third order importance. We are only looking into to first order which term is balancing on the left and the right hand side. So, to first order, we learned that we have a left-hand side term. The largest left-hand side term is of the same order of magnitude as the largest turbulent term.

To first order, this is what we achieved after doing the scale analysis by introducing the scales that we have chosen. The largest turbulence term balancing the left hand side term. So, this is the largest turbulence term on the RHS and this was the LHS term largest LHS term. And with this of course, we learned that when this happens we learned that essentially the boundary layer growth depends on the turbulence intensity that is $u^2/$

$U_s \delta U_s$. the boundary layer growth in a thin boundary layer grows like turbulence intensity ok.

So, this is fine, but there were two three consequences here. Somebody asked me why we did not consider the pressure term ok. So, there are one reason I can tell you now. So, the pressure term we did not include. If I am going to make the pressure term, the pressure gradient term along the x direction dp dx term if that balances against this largest left-hand side term, then what will be the turbulent terms? They are first-order terms or second-order terms or higher order they are higher-order, right? So, in a turbulent flow, how can turbulence terms be of second-order importance, right? In a turbulent flow, turbulence dominates, so the turbulent term has to be big.

So, we do not want pressure term balancing it, and that is what we would see. That does not mean that the pressure term is not important. It has its own importance, and we will soon see that one. So, the first position the point that I want to make is that the so you can see that we have considered the largest turbulence term to balance LHS term, the largest LHS term and not the pressure gradient term. That is because if pressure gradient balances if or I can simply write if this particular term balances this to first order then the turbulence term is of second order importance that we do not want right.

So, turbulence term dominates in a turbulent flow. this is not a valid argument. Since turbulence dominates in a turbulent flow, maybe pressure can balance the other term in a barely turbulent flow or weak turbulent flow or something like that. But we are looking into high Reynolds number flows right, the Re_δ was large, $Re_L$was large. So, high Reynolds number implies turbulence is dominant and turbulence term must balance the other big term.

So, this was one thing. The other consequence I will tell you or when pressure term will become important. So, there is another one more thing that we have not considered. What is that? Did we miss anything here? while studying so far or we are just happy that we have achieved this. No doubt this is an interesting information, the boundary layer growth depends on turbulence intensity is a good information.

But are we happy with this or did we miss the big picture here? This is a big picture, very important. Fine, even high Reynolds number did we miss a big picture. are we missing out anything here like a like the elephant in the room we are not looking into that at all what is the topic of this chapter turbulent boundary layer turbulence we saved? What about the boundary layer to get the boundary layer what term is important We need to save the viscous term, right? Recall the argument Prandtl placed to save the viscous term right. He said the gradient becomes so large so that it offsets 1 by Re effect. But here, our viscous term, the scales that we chose, says that viscous terms, both $1/Re_\delta$ and $1/Re_{\mathrm{L}}$ both terms are small for high Reynolds numbers.

So, it is of second order importance. So, how do I now save this viscous term? Now, the question is to achieve. Let us call this star to achieve equation star. We neglected viscous terms, and we said it is second-order importance. So, how to save viscous term now? How to save at least one viscous term, the largest viscous term? How to save viscous terms to get a boundary layer in place.

Then what term is balancing that to first order? See, on the left-hand side, we already have a large term; this is the large term, this particular term. On the right-hand side, we are searching in the U-momentum equation, which will balance, and we found that the turbulence term can balance it now if you want the viscous term to balance using the same argument as Prandtl, then what will happen to the turbulence term that will become negligible. You can say only one term on the right-hand side if you want the viscous term to survive turbulence will vanish but turbulence is important in turbulent flows If the turbulent term survives, the viscous term vanishes, no boundary layer. So, there is something that we are missing here, and what we are missing or what is that important information that we have missed is there must be another layer inside the turbulent boundary layer, just like the boundary layer concept itself, right? Prandtl thought of there must be a layer close to the wall where these gradients are sharp all these things. Now, there must be maybe one layer or many layers inside a turbulent boundary layer right.

So, the solution to this is we need a boundary layer within the turbulent boundary layer. So, then what is that layer that we have already seen where this happened? This equation star happened. So, there is a region inside your turbulent boundary layer where the largest left-hand side term $\bar{u}\frac{\partial \bar{u}}{\partial x}$ is balancing the largest turbulence term. So, there is a region actually inside the boundary, but we must go to another region very close to the wall where this is not happening where we want to say viscous term also and the turbulence term ok. So, what we found so far in the scale analysis that we have done here is called this particular information that it has come this is called the outer turbulent boundary layer an outer layer or the turbulent part.

So, this is valid here. I will write it here: this is the outer turbulent boundary layer, so that means I do a schematic diagram. So, let us say we already chose a scale L of x and then we had a boundary layer growth. So, we said this is our delta of x turbulent boundary layer thickness. Now, all I am saying is there must be a region where viscous effects also should dominate ok.

So, we will call this outer boundary layer the outer turbulent boundary layer. I am going to use TBL, and there must be an inner turbulent boundary layer, and what is outside the boundary layer is what is called an outer flow ok. Many times it is not even turbulent or viscous this outer flow. This is your outer flow, not any layer there. So, this is usually non-turbulent; this outer flow is usually non-turbulent, very high in this one.

But we must now search for this so called inner turbulent boundary layer where we must save both the turbulence term and viscous term to first order. Another important note here is that the scale that we have chosen is not yielding the viscous term to be dominant, that means something is not right with the scales that we have chosen, and what is that scale not the l it is the vertical scale the δ. So, the δ was not small enough to tell us that there is actually an inner layer. So, the boundary layer thickness, the boundary layer thickness δ is not small enough to satisfy no slip for us. For that, we need to search for another scale so that at least one viscous term survives in our order of magnitude estimates.

So, any questions on this so far? Except that, I mean you can take this note also, as Arshdeep pointed out, why are we not using Prandtl's idea here? That is because the turbulence term becomes negligible. So, like Prandtl we can say largest viscous term must balance the largest LHS term, but that results in turbulence term to be negligible or second order important. That we do not want, turbulence has to be of first order importance. Turbulence is of first order importance in turbulent flows. So, then we search for this new scale; this is clear so far what we are doing except for the pressure gradient.

I will come to that everything else is clear why we are not using pressure gradient that I will come back. So, there must be an inner turbulent boundary layer, and for that, we choose a new scale, and we redo the whole analysis. So, we will call this inner turbulent boundary layer, inner turbulent boundary layer. So, we have the same momentum equation u momentum equation right, u momentum equation mean momentum which $\bar{u}\frac{\partial \bar{u}}{\partial x}$ plus I am taking the same equation dou y equal to minus 1 by rho the pressure gradient along the x direction. And then I have the turbulent term, the two turbulent terms which is u prime u prime minus u prime v prime, the turbulent shear stress term.

And the two viscous terms, which is nu dou square u bar by dou x dou x plus nu dou square u bar by dou y dou y.

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} + \nu\frac{\partial^2 \bar{u}}{\partial x \partial x} + \nu\frac{\partial^2 \bar{u}}{\partial y \partial y}$$

So, we just start with taking new scales here instead of what we used. So, for that we will define what is called an inner length scale and an inner velocity scale first. So, let η_w be the inner length scale. Let inner length scale be I call it η_w much smaller than delta.

So, we are searching for distance from the wall very close to it, delta is much much larger. This is let us say an order of magnitude at least smaller than delta, a thin zone very close to the wall. Let η_w be much smaller than delta and η_w we call it inner length scale. we do not know how small it is. So, how small is η_w? We put a question mark, we will revisit how small it has to be.

At least now we know that there is η_w has to be much less than δ and we also define an inner velocity scale. And then I have we call this u_w. Sometimes in literature you also call this u_*. We are coming to close to some of you asked what is that u_τ or u_*. When I showed the rms data, urms, vrms, wrms was divided by something.

I said it is a friction velocity. So, we are now coming closer to getting that answer. So, let inner velocity scale be uw or u star here. Again, how small is u_w? We retain a question mark. We will revisit what how small this has to be. At least now we have this particular scale that we are going to use, η_w and u_w.

Now, we have to make for the length scales that we are going to write here. So, I have this for dou x, I am going to retain l because no change in the longitudinal direction, only in the lateral direction we chose a different scale right? So, this I am going to use η_w not δ. So, I am choosing a near-wall scale here, w implies a wall near the wall. So, the w here, w implies wall here.

So, the length scale is sorted. So, then we have the velocity scales. So, here if you look into the equation, so I have u bar, I have changes in velocity also. So, in the first outer turbulent boundary layer, we said the u bar is an order of magnitude different than the changes in u dou u bar dou square u bar and so on. But as I approach very close to the wall, what will happen to the velocity gradients dou u bar by dou y? Let us say they become very large. So, close to the wall the changes in u or u bar is so large that it will be having same order of magnitude as u bar.

in the thin zone where eta w is dominant, right? So, dou u bar and dou square u bar all will have same order of magnitude close to the wall ok. So, here I am going to use this particular velocity scale inner velocity scale. An inner velocity scale I am going to use for both the mean momentum as well as its changes. What about the turbulent terms? Now, we have $\partial \overline{u' u'}$ as well as the shear stress $\overline{u' v'}$. What do I do here? So, I am going to use also the same scale.

I am going to choose the same scale u_w. So, why is that? So, for velocity or momentum I am using inner velocity scale, for turbulence velocity is also I am using inner velocity scale, both I am going to use the same that is because I am going to show you the data otherwise this argument is not good.