Course Name: Turbulence Modelling

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Week - 4

Lecture – Lec23

23. Turbulent Boundary Layer: Order of Magnitude analysis - II

I am representing V_s in terms of the other three scales here velocity and the length scale. So, with this I can apply the scales to the U momentum equation. So, let us go and do this. So, we will take equation 2 now, equation 2 that is your $\overline{u}$ equation, U momentum equation. Let me just copy it.

So, I have the equation ready here. which is my u momentum equation. So, I would now start to introduce the scales. So, introducing the scales.

So, for u bar, I have u_s, this is $\frac{\Delta u_s}{L}$. and for the next one $\overline{v}$ r it is v_s. So, that is v_s $\frac{\Delta u_s}{\delta}$ and for the $\overline{p}$ we did not do anything. I intentionally did not do anything there. I will tell you why.

So I have not introduced a scale here. I am going to keep it as a question mark here. This does not mean that this term is not important. It is certainly an important term, but we would like to keep it as a question mark. I will reveal why I do not want to take this into the game now.

So let us look at the other scales here. So I have this new $\partial^2\overline{u}$ so change all changes I have the same scale here Δ u_s by ∂x this is ∂x ∂x by basically it is not the square term here right so it is the way it is written I have written like this it is ∂x ∂x ∂y ∂y and so on so this will be L^2 u Δ u_s by L^2 And this $\frac{\nu\Delta u_s}{\delta^2}$ simply introducing the scales. And for the turbulent quantity, this is u^2, a single turbulence velocity scale $\frac{u^2}{L}$ and this is $\frac{u^2}{\Delta}$. I told you U RMS is of the same order of mind as V_{rms} and therefore, both the normal stress and the shear stress the Reynolds stresses I am using v^2 that is the argument I already placed. So, now if I look into the LHS and the RHS here right.

So, this is the left hand side part ok and the right hand side part. So, which is the largest left hand side term here that will reveal if I going to substitute V_s here ok. So, if I substitute the V_s value which is $\frac{\Delta u_s \delta}{L} \frac{\Delta u_s}{\delta}$. So, essentially this cancelling out. So, if I compare between the two terms here, this is $u_s \frac{\Delta u_s}{L}$ right and this is $\frac{\Delta u_s \Delta u_s}{L}$ ok.

I rewrite this as like this. So, you see this is so, which is the smallest term on the left hand side. first term oh sorry the largest term i'm talking about the second term is the smallest one so which is the largest term obviously us is an order of magnitude this particular term is an order of magnitude larger than Δu_s right so this is the largest term on the left hand side this is the largest lhs term the first term. So, because of this I would like to know which term will balance this particular term. Is this clear? Why the first term is the largest left hand side term? Because u_s is an order of magnitude larger than Δu_s.

So, let us divide throughout by the largest LHS term. So, divide throughout by largest LHS term to see which term balances this in the entire equation. So, if I do this, here I will get as this becomes 1 ok. So, this if I divide, so I get $\frac{\Delta u_s \frac{\Delta u_s}{L}}{u_s \frac{\Delta u_s}{L}}$. So, obviously, this is going out giving me only $\frac{\Delta u_s}{u_s}$term ok.

This I have retained as it is this pressure I have not done anything with that I have not introduced a scale there. So, now, we have the other term which is $\nu \frac{\Delta u_s}{L}$, $\frac{1}{L}$ divided by have $u_s \frac{\Delta u_s}{L}$. So, this and this cancels out giving me this and then I have the next one which is mu $\Delta \frac{\nu \frac{\Delta u_s}{\delta}}{\frac{\Delta u_s}{L}}$. So, here only the Δu_s term goes away leaving out this. And then I have the two Reynolds stress terms which is I have $\frac{\frac{u^2}{L}}{\frac{u_s \Delta u_s}{L}}$.

So, the L term goes away. I have $\frac{\frac{u^2}{\delta}}{\frac{u_s \Delta u_s}{L}}$. Here no term goes away here, every term is retained. So, now if I rearrange, what do I get? If I rearrange here, I can write this as, so this is essentially 1 as before. I have here $\frac{\Delta u_s}{u_s}$.

This is a question mark. The second term on the left hand side we already know that this cannot balance the first term because it is already an order of magnitude smaller. So, this term cannot balance anything on this. So, the sum term on the right hand side must only balance this particular term. So, sum term on the right hand side must be of the order of magnitude 1 to balance this.

So, which one is that we have to choose we are optioned between four terms. First we will see which is the largest viscous term, which is the largest turbulence term. So, now we have the two viscous terms here. So, which is, if I rewrite this, I will get essentially this is $\frac{\nu}{u_s L}$. And what does this look like? Reynolds number Right So, I have, so I can write this as essentially $\frac{1}{L}$.

So, this is nothing but $\frac{1}{L}$. And this particular term, similarly I have $\frac{\nu}{u_s \delta}$ and I have the other term which is $\frac{L}{\delta}$ here ok. So, if I write this I would get $\frac{1}{Re_\delta}\left(\frac{L}{\delta}\right)$ ok. So, now, which is the largest viscous term here. The second one see the RE L.

So, we said L is at least an order of magnitude larger than δ . right. So, 1 by Re Lmust be then smaller than $\frac{1}{Re_\delta}$. Let us say RE L is 10 raise to 6 1 million then $\frac{1}{Re_\delta}$ or the Re_δ will be at least 10 power of minus 10 power of 5 right at least. So, this will be a larger term because L by δ is at least 10, the ratio is at least 10, it can be 100 or more.

So, this will be the largest viscous term here. But still if RE L is very large, this term should be negligible. So, we will see the other two terms before I come to this. We have this turbulence term which is $\frac{u^2}{u_s \Delta u_s}$ and then you have same $\frac{u^2}{u_s \Delta u_s}\frac{L}{\delta}$ okay so i have the the term in between these two which is larger obviously the last term. So, this last term is nothing but this one if you see it is $\frac{\partial}{\partial y}$ in a boundary layer the wall normal gradient of the your Reynolds shear stress $\frac{\partial}{\partial y}$ part is becoming larger than the $\frac{\partial}{\partial x}$ term.

So, this is the largest turbulent term here because L is at least an order of magnitude larger than δ . So, this is the largest turbulence term because L is at least an order of magnitude larger than δ. If δ is 10, L is 100. minimum that is the consideration we have. So, the largest turbulence term is identified.

So, now there is a competition between the largest turbulence term and the other two viscous terms. Obviously, $\frac{1}{L}$cannot compete here for large Reynolds numbers ok. If I have Reynolds numbers being very large, so for let us say RE L is very large. high Reynolds numbers, this term becomes negligible for high Reynolds numbers. What about this term? This is Re_{δ}.

This particular term, if ReL is very large, then this term also can be small. But if Re_{δ} is not that that means, if you have low Reynolds numbers then you can retain this term. But for very high Reynolds numbers it is still small. For example, if I take Re L to be 10 raise to 6 as I said 1 million Reynolds number and considering that L and δ is one order of magnitude different then Re_{δ} will be 10 raise to 5. okay and $\frac{L}{\delta}$ is 10 so this will give me 1 by 10 raise to 4 which is still small okay so this will be still small for still small for large Re_{δ} So that means, there is no other computation here now, the largest turbulence term must be balancing the left hand side term here.

Of course, unless we want the pressure term to participate. So, now we have the largest turbulence term. So, therefore, I can say therefore, I can say here it is also negligible. Therefore, the largest turbulence term must be balancing the LHS or the largest largest LHS term. So, this is we are looking at first order.

This does not mean that other terms are all negligible and dropped. They are of second order, third order importance. One can also find which is the second order important term and that is being balanced by the other term. To first order we are looking into what is the what are the two terms that are balancing each other right. So, this is to first order we are looking to first order.

So, what does now we have learnt is that this particular thing implies that 1 is of the same order of magnitude as $\frac{u^2}{u_s \Delta u_s}\frac{L}{\delta}$ which is nothing $\frac{\delta}{L}$ is of the same order of magnitude as $\frac{u^2}{u_s \Delta u_s}$. So, to first order we have an expression here in the equation $\frac{\delta}{L}$ is of the same order of magnitude as $\frac{u^2}{u_s \Delta u_s}$. Of course, we have not let the pressure participate here, I will tell you why. But in this, is there something to learn from this equation? $\frac{\delta}{L}$ is of the same order of magnitude as $\frac{u^2}{u_s \Delta u_s}$. What is $\frac{\delta}{L}$? The growth, growth of boundary layer, the boundary layer growth.

What is it depending on now? Sorry? velocity turbulence velocity sometime something like a non-dimensionalized turbulence velocity this is this we call it turbulence intensity right. So, we have let us say u the small u is let us say 1 meter per second and u_s Δ u_s is we are using it to non-dimensionalize it. or let us say in a flow you have 10 meters per second easy as your the inflow velocity and the turbulence velocity let us say is 1 meters per second then you can say the small u by capital U is 0.1 so you can say 10% turbulence intensity you know a rough value how much turbulence somebody can ask you how much turbulence you have you can say okay 10% turbulence So, this is essentially what this means is I have this δ is the boundary layer growth, boundary layer growth is of the same order of magnitude as turbulence intensity.

a non-dimensional turbulence velocity. So, as turbulence in intensity increases your boundary layer is growing. So, something we learned that what is the turbulent boundary layer is depending on you may know about a laminar boundary layer growth Δ what does that depend on. But in a turbulent boundary layer maybe we at least know without doing any experiment or simulation that to first order the boundary layer grows based on turbulence intensity. Higher the turbulence intensity, higher the turbulent boundary layer growth. Of course, with the considerations that we had, we took a flow where there is no boundary layer separation that means our some of the terms may pop in if you have those things like we have considered a thin boundary layer and all these things this is what we have got and we have an another consideration here the pressure term that we have not taken into consideration.

So, I will continue in the next class why this pressure term is not being considered.ok