19. Turbulence Kinetic Energy and it's dissipation rate - II

So, again, we will do the same exercise as before that do we have enough terms here to solve, enough equations, sorry, enough equations to solve for the unknowns? So, if I do the same exercise here, so if I simply say how many equations, how many unknowns I have, So I have one equation for k, ok? So, I have one equation for turbulence kinetic energy. I am only going to solve this particular equation. This is my turbulence kinetic energy equation k. And uj bar I am going to get from solving the mean momentum. Let us say I have access to it.

So, it is not going to be a problem. On the right hand side, I have unknowns. So, I have p prime ui prime. So, that is three unknowns.

So, I have three unknowns there. The viscous diffusion rate is not a problem because it is depending on the turbulence kinetic energy k itself. That is why modeling community like this equation. If you start with the other, as I said, you can derive this equation differently, this will introduce unknowns, the viscous diffusion rate. This, the way we have defined, derived, this is now useful because turbulence kinetic energy, the transport of it due to viscosity is depending on turbulence kinetic energy itself.

So, one can iteratively solve this in CFD techniques. So, this is not an unknown for us. This, now ui prime ui prime is sum. So, it loses the directional dependency. So, what is the order of this tensor? How many unknowns are there here? 3 unknowns.

It has become a vector. Now, $u_i' u_i'$, it is summed. Summed up means it becomes a scalar. So, only uj prime is the vector. So, this is a vector now.

So, we have this $u_i' \; u_i' \; u_j'$. This is 3. or j equal to 1, 2, 3, you get 3 unknowns here. But what about the production rate of turbulence kinetic energy? For that, we need to solve Reynolds stresses. To solve Reynolds stresses, it has more unknowns.

So, in any case, to solve turbulence kinetic energy, I need Reynolds stresses, which is an unknown for me. So, this is an unknown. So, Reynolds stress is an unknown here$u_i' u_j'$. That is, since it is a symmetric tensor 6 and then the dissipation rate $\frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j}$, it is a second order tensor, again 9 or 6 terms as you can think of. So, the dissipation rate is other unknown here, if I just remove this, make it a bigger equations unknowns.

I have the dissipation rate epsilon which is $\nu \overline{\frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j}}$. So, this is giving me additional 6 or 9 unknowns, the way you can think of your 6 or 9. So, again turbulence closure problem. I found a way to make the Reynolds stress equation simple by deriving an equation for turbulence kinetic energy, but it is still throwing me more unknowns. One equation, many unknowns, 21 unknowns.

So, hence there is a turbulence closure problem again. So, no matter what you do this turbulence closure keeps coming and therefore, we have to start modeling, make some approximations. So, before we do that we have this I think the number of equations are sufficient now, we do not need to derive more. If needed, I will. Of course, I will derive, but right now, I think we have enough equations to start considering modelling.

But before that, we have not understood certain terms, right? We have readily understood that why this pressure strain rate term is called pressure strain rate, this particular term. Right? that I showed to you that it is during redistribution. But the diffusion rate and the production rate term, and the dissipation rate term, these three terms, I have not told you why we are calling them as such. So, we will look into this term. So, the first term is dissipation rate.

Dissipation, or I can say epsilon here, right? ϵ is the dissipation rate of TKE. If you notice carefully, there is a negative sign there. If you go back, you see here it is minus of this particular term and what is epsilon here? What is the sign of epsilon? Positive, it is always positive here. This was not revealed in the Reynolds stress equation where we cannot tell, right? But here, it is always going to be positive. That means it is a sink term in the context of CFD.

It is a sink term. This is draining the energy from your system, ok? Hence, the name destruction rate or dissipation rate of turbulence kinetic energy and it is a negative rate of change. If you look at every term as the rate of change, like the first term is the unsteady rate of change of turbulence kinetic energy, then the last term, ϵ, is the negative rate of change of turbulence kinetic energy. So, epsilon, this term is essentially you have $\nu\overline{\frac{\partial u_i^{'}}{\partial x_j}\frac{\partial u_i^{'}}{\partial x_j}}$ and therefore, ϵ is always greater than or equal to 0, and the sign is negative in the TKE equation. that is you get minus epsilon indicating it is a negative rate of change, indicating that it is a negative rate of change of TKE. Therefore, the name destruction rate or dissipation rate.

dissipation or destruction rate. This was straightforward to understand why we call epsilon a dissipation rate term. Now, there is an another term which is called diffusion rate. The other term is diffusion rate of turbulence kinetic energy. So, the diffusion rate term, to understand this, we need to make some kind of an idealization to see, but I hope you will get what I am trying to convey here.

So, to understand the diffusion rate or it is also there is a divergence term. So, there is a divergence is essentially you can say this as a divergence term here. So, the diffusion rate of turbulence kinetic energy, this particular term looks like you have dou by dou xj of I have minus 1 by rho p prime ui prime delta ij minus half ui prime ui prime uj prime plus I have this nu dou k by dou xj, right? is it correct, nu dou k by dou xj, let me go back and see, yeah, nu dou k by dou xj, correct.

$$\frac{\partial}{\partial x_j}\left\{-\frac{1}{\rho}\overline{p'u_i'}\,\delta_{ij} - \frac{1}{2}\overline{u_i'u_i'u_j'} + \nu\frac{\partial k}{\partial x_j}\right\}$$

So, the pressure diffusion rate, turbulent diffusion rate, viscous diffusion rate of turbulence kinetic energy, the transport of k due to pressure, viscosity and turbulence itself, three terms. So now, to understand this, let us consider that turbulence is already existing in a box, ok? So, now you consider turbulence in a bounded box.

So, that means let us say I have turbulent structures, let us say I have some turbulence and I am going to consider this in a particular box here. So, I have a box, a bounded box. Let's say there are turbulent eddies inside. Turbulence is already present and I am considering a closed box around it. So, why I am doing this is now, if I have this box of some volume, let us say V, ok? and or V0, let us say and the surface S0.

Now I can integrate, I can make a volume integral to see what does it do this particular term. So, to do that I simply say consider a volume integral here, volume integral. Of course, I have dv. I am considering a volume integral of this entire diffusion rate term. Any particular theorem that we can use here? Gauss divergence theorem.

$$\iiint_{V_0}\frac{\partial}{\partial x_j}\left\{-\frac{1}{\rho}\overline{p'u_i'}\,\delta_{ij} - \frac{1}{2}\overline{u_i'u_i'u_j'} + \nu\frac{\partial k}{\partial x_j}\right\}dv$$

So, using the Gauss divergence theorem here, that is why I mentioned this diffusion rate are divergence because you have a dou by dou xj or nabla dot in vector format. So, using the Gauss theorem or divergence theorem, so you would get you can convert the volume integral into a surface integral. That is, if I have a volume integral, let us say dou j dv, I can make it into a surface integral with the nj unit vector nj ds. This is your Gauss divergence theorem.

$$\int_V \partial_j\{\ \}\,dv = \int_S n_j\{\ \}\,ds$$

So, if I apply this here, what do I get? I get essentially the surface integral is 0 and then I have the nj of minus 1 by rho p prime ui prime delta ij minus of ui prime, ui prime, uj prime, the entire bracket term as it is mu dou k by dou xj ds.

$$\iint_{S_0} n_j \left\{ -\frac{1}{\rho} \overline{p' u_i'} \delta_{ij} - \frac{1}{2} \overline{u_i' u_i' u_j'} + \nu \frac{\partial k}{\partial x_j} \right\} dS$$

So, now I have considered this box. Box, in a sense, I have a fixed box that means walls are there, and a bounded box that means six walls are there here. walls or solid surfaces. I am just assuming that turbulence is already existing and there is a box here. Now, if I have a surface integral, I can evaluate the surface integral by applying the boundary condition.

Now, what is the boundary condition for? It is essentially containing even the k here, this particular k, k is nothing but your $\frac{1}{2} u_i' \ u_i'$. So, essentially, what I have is u_i' velocity, fluctuating velocity. What does fluctuating velocity, what happens to that on the wall? For using your no slip and kinematic condition, right? It becomes 0. 1 and 3 become 0 due to your no slip, and 2 becomes 0 due to kinematic condition. The wall normal u_2' is kinematic.

So, it cannot go inside, it is a kinematic condition and u_1', u_3' are wall parallel. So, they stick on the wall. So, you have a no slip condition. So, by applying the boundary conditions on the wall surfaces, it is your no-slip, kinematic boundary conditions. If I do this, I get u_i' is going to be 0.

Since u_i' is 0, the entire term is zero. So, what does this mean? That means that the net integral is 0. So, the rate of change of this particular term, the contribution of this particular diffusion rate to the net rate of change of k is 0. That means if turbulence is already produced there inside, it is merely transporting it from one direction to the other direction. It is only diffusing, it is transporting due to viscosity, due to turbulence and due to the pressure field.

It is not generating, not destructing, not redistributing. The net contribution is not there. So, it is only transporting within the box. That is the way you can understand this, and hence, we have given the name diffusion rate or transport, ok? Therefore, the net integral, the integral here, is 0, which implies the net contribution to the rate of change of turbulence kinetic energy.

by this transport term is 0. It is neither creating nor destructing turbulence kinetic energy. But only transporting from one point to the other position, only diffusion, but not transport only transporting ok. It is neither creating nor destructing it, but only transporting that is diffusing due to viscosity, due to pressure, due to turbulence itself. this particular term is now, is it ok to understand? So, the dissipation rate, we understood why it is dissipation rate, diffusion rate, we understood why it is diffusion rate. And then the only particular term that is left out in the equation is the production rate.

For that, I need to derive one more equation. So, you have to bear with me that one more equation, if I derive it, that equation itself will tell you that it is a production rate. So, we will stop here.