17. Pressure-strain-rate and redistribution of turbulence in flows - II

So now to understand why these two stresses are non-zero that is who is making sure that they exist. Let us look at let us take help from the continuity equation for the fluctuating motion. So, recall continuity equation. for fluctuation. So, I have the ∂ ui or I can simply write the equation as $\frac{\partial u_i'}{\partial xi}$ or we use j equal to 0.

So, if I expand I get $\frac{\partial u_i'}{\partial x1}$ equal to if I move the other two terms I get - $\frac{\partial u_2'}{\partial x2}$ - $\frac{\partial u_3'}{\partial x3}$. So, now if I, let us call this equation star. If I now substitute this equation into let us say the above equations, one of the equations. Let us say we substitute star in substituting equation star in Ø 11.

What do I get? 2 $\frac{p'}{\rho}$ of $\frac{\partial u_i'}{\partial x1}$ I am substituting this part which is - $\frac{\partial u_2'}{\partial x2}$ - $\frac{\partial u_3'}{\partial x3}$ average. So, this is equal to - 2 $\frac{p'}{\rho}$ $\frac{\partial u_2'}{\partial x2}$ - 2 $\frac{p'}{\rho}$ $\frac{\partial u_3'}{\partial x3}$. So, the equation itself is telling us now without doing any experiments or simulation that your Ø 1 1 your Ø 1 1 is equal to - Ø22 - Ø33. Hence the name redistribution. This Ø11 is the term that was redistributing.

You understand? So, this Ø11 was stealing the stress or the turbulence along the direction 1 and redistributing it to the direction 2 and 3. because p 11 existed right. So, here p 11 is non-zero there was production along the x 1 direction and it redistributed the Φ 11 was responsible for this. Therefore, the pressure strain rate term Ø ik is the redistribution term that is responsible for redistributing turbulence from direction 1, from x1 direction. to x2 comma x3 directions.

Therefore, the name has come redistribution, it is pressure strain rate. So, the pressure strain rate correlation is responsible for this particular feature of turbulence being redistributed. Turbulence is diffusive, dissipative also it does redistribution. So, now I show it to you that this is the case. So, your Ø 11 is equal to nothing but - Ø 22 - Ø 33.

Now I see the data, now equation is telling the proof, obviously the data should also match. So, let us go back to the data. to se$\overline{u_1'u_1'}$e how it looks like. So now this is something called turbulence budget. So now I have all the left hand side and the right hand side terms.

I can compute them since I have the DNS data. Let us assume that you have the data. So if you have all the terms present, you can actually look at the budget, the total. That means you can submit and see whether it goes to 0. It should go to 0 if you have statistical sampling is correct.

non-zero indicates that you have not sampled it right either you have not waited enough to achieve statistical stationarity or you have not taken enough number of samples. So, in this particular case this particular dash dot line indicates the residue that means when I add up all the terms on left hand side and right hand side it goes to 0, residue is 0. So, that is why I call the entire thing as this budget here. and its gain and loss that we are looking into just like any financial budget or so. So, then we see for the term by term.

So, the first graph here the figure a shows $\overline{u_1'u_1'}$. So, this is basically the budget of so this is the budget of $u_1'u_1'$ stress from the stress equation itself it has all the terms I am not going to discuss the rest of the terms the diffusion rate terms the important terms it we have of course plotted all the terms. So, this circle here or the line here this indicates your production rate. So, this is your p 11 which is of course non-zero. So, p 11 is this there is enough production.

Right and I have this is the sorry so this is to be the triangle of this shape this is the redistribution term and you can see the it is here so it is completely negative this line is here completely negative and the dissipation rate is square it is completely negative here so the dissipation rate is draining the the turbulence along this direction. So, ε so, this is ε 11. So, this triangle this particular one is your Φ 11. So, this is completely negative and look at the value it is about you can say approximately you can say - 0.1 approximately it is right it is - 0.

1 approximately you can say. The rest of the terms are this is viscous diffusion rate, this is turbulent diffusion rate right and all this other things the diffusion rate let us not discuss. So, now look at the other stress which is your this is the budget of your $u_2'u_2'$ stress. So now if I see here obviously the circle is absent here p22 is 0 right in this particular even if you calculate it, it will be 0 the equation is 0 you can also calculate from the data it will be 0 no production rate and we have already seen that distress exists. So, you see here which is on the gain side.

one is this viscous diffusion rate, but that is only very, very close to the wall. So, what this for a moment ignore what this is. So, assume that this entire part is basically your wall normal x, let us say this is just x3, wall normal coordinate here, wall normal direction. So, you are basically looking into this will be your wall. the bottom wall is here both the sides and I have plotted this in the logarithmic coordinates.

So, as I so very close to the wall if you ignore it you can see that mostly the positive term is this one which is Φ 22 the redistribution rate Φ 22 right. So, that is that is the one which has taken some turbulence from direction 1 and giving it to $u_2'u_2'$ me and if you look at the magnitude approximately it is so this was - 0.1 let us say approximately. So, this is about let us say plus now let us say approximately right. So, this is about here maybe it is 0.

06 or so. so that means the rest of this would have gone to the other component right it has stolen this from one direction it has given this and it is not giving it equal this redistribution is also not like giving everything equal that is there is anisotropy in the redistribution also here right and then again this is your ε 22 which is draining so ε 22 - ε 22 indicates that

dissipation rate is there There is no production, but there is dissipation. It is a square term as we saw in the equation. Now, if I look into the third one, which is the $u_3'u_3'$ stress. So, again this is the x3 direction.

and this is the wall. So, here again you see the there is no production obviously p33 is 0, but this stress exists we saw from the data and the dissipation rate is again here which is your ε 33 there is dissipation rate. and you can see here this particular triangle here the right facing triangle this one this is your redistribution Φ 33. It has gone negative here to balance out the other term which is the viscous diffusion rate. So, the very close to the wall the redistribution rate is let us not go into deep detail of each particular problem. But just to clarify because you may see that why is it negative here because this has to balance out because the residue has to be 0 at the end.

So, which term is balancing at every wall normal direction also you can understand. As you go close to the wall away from the wall what terms are balancing each other to first order. so here very close to the wall or at this location I can say that okay the pressure strain rate is balancing the viscous diffusion rate they are the bigger terms and the rest of them are small players but as I go away from it dissipation rate is larger and the biggest positive term is the redistribution rate V33 and therefore redistribution rate is the term responsible here resulting in a positive x2 direction and x3 direction stresses. What happens to note here? So, if i equal to 1 for example and j equal to 2 that is it will give you your $\overline{u_1'u_2'}$ k equal to correct sorry. k equal to $\overline{u_1'u_2'}$ if I expand it I get $\frac{\partial \overline{u_1'u_2'}}{\partial t}$ equal to this is the your left hand side.

On the right hand side I have this d12 the viscous diffusion rate plus turbulence diffusion rate plus pressure diffusion rate the combination of all the three plus the pressure strain rate the Φ12 term which is $\frac{p'}{\rho}\frac{\partial \overline{u_1'}}{\partial x2} + \frac{\partial \overline{u_2'}}{\partial x3}$, sorry ∂ x1 average. And then this is your Φ 12 term and then I have the production rate terms which is - $u_1'u_2'\frac{\partial \overline{u1}}{\partial x1}$ + I get the divergence comes summation over j $u_2'u_2'\frac{\partial \overline{u1}}{\partial x2}$ + u_2' u 2 u 1 u 2 $u_2'u_3'$ $\frac{\partial \overline{u1}}{\partial x3}$ and then I have extra terms because it is two of this. three more terms come which is $u_1'u_1'\frac{\partial u2}{\partial x1}$ + $u_1'u_2'\frac{\partial \overline{u2}}{\partial x2}$ + $u_1'u_3'\frac{\partial \overline{u2}}{\partial x3}$. It is a very long equation here.

This entire part is your P12. Of course, some terms will go away. The $\frac{\partial}{\partial x_1}$ in this particular case due to homogeneity and $\frac{\partial}{\partial x_2}$, x3, x3 is the third direction. So, we may have probably changed the directions here. So, x3 was our homogeneous direction or x2 was the homogeneous direction. x1 is homogeneous, the out of plane component was, out of plane is 2.

So, here I have taken the opposite one here. So the data I have shown is for the, so this was x3 and this was x2. Please correct that. So I have taken this is $\frac{\partial}{\partial x_2}$then, this becomes x2, this is 3 and this is 2, $\frac{\partial}{\partial x_2}$here.

So, I have Φ 12 terms. So, here I have the other one which is homogeneous which is x dou by if I take x3, x3 was wall normal. Maybe in the data we have to change not in this one. Anyway one of the terms will go away whichever you take as homogeneous one here the u2 should go away, u2 velocity itself will go away giving rise to if I take x3 as homogeneous then this term will go away if I take x2 depending on the coordinate system one of them will go away and only so P12 basically survives only one term remains. If I take x3 as the homogeneous direction then this would go away $\frac{\partial}{\partial x_3}$ and this term would survive. So, you cannot learn much from this 1, 2 direction apart from that we are expanding it and we would get this dissipation rate also - 2 u of $\frac{\partial u1'}{\partial x1}\frac{\partial u2'}{\partial x1}$.

So, here I get much more complicated terms average plus $\frac{\partial u1'}{\partial x1}\frac{\partial \overline{u_2'}}{\partial x2} + \frac{\partial u1'}{\partial x3}\frac{\partial \overline{u_2'}}{\partial x3}$. This is your ε 12. so you can obviously expand it but very difficult to learn because here earlier the ε 1 1 ε 2 2 ε 3 3 they were square terms so we definitely knew that it is a sinc term it is removing the turbulence from the system but here this is a correlation u_1’ and u_2’ so we do not know its nature whether it is positive or negative what it is we do not know in these terms so it is difficult to understand from the the correlation or the shear stresses u_1’ u_2’, u_2’, $\overline{u_3'}$ and those